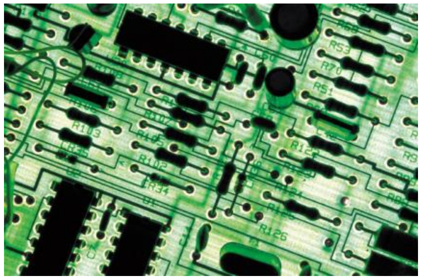
SAT Sampling and Counting: From Theory to Practice

Kuldeep Meel

Rice University

M.S. Thesis: Sampling Techniques for Boolean Satisfiability (2014)
Advisors: Moshe Vardi (Rice) and Supratik Chakraborty (IIT Bombay)

How do we guarantee that systems work <u>*correctly*</u> ?



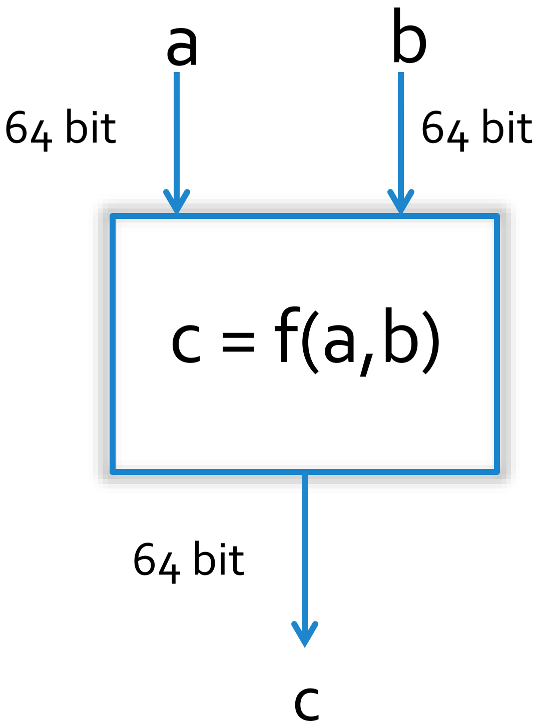
Functional Verification

- Formal verification
  - Challenges: formal requirements, scalability
  - ~10-15% of verification effort
- Dynamic verification: ***dominant approach***

Dynamic Verification

- Design is simulated with test vectors
- Test vectors represent different verification scenarios
- Results from simulation compared to intended results
- **Challenge**: Exceedingly large test space!

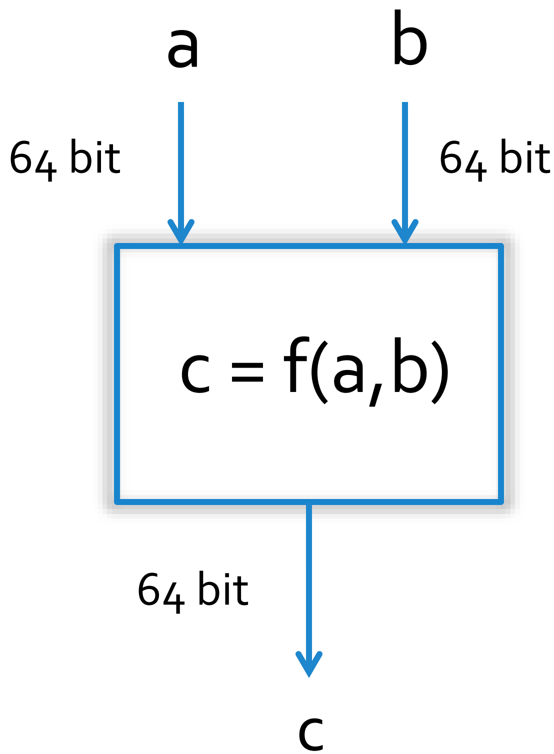
Motivating Example



How do we test the circuit works ?

- Try for all values of a and b
  - 2^{128} possibilities
  - Sun will go nova before done!
  - Not scalable

Constrained-Random Simulation

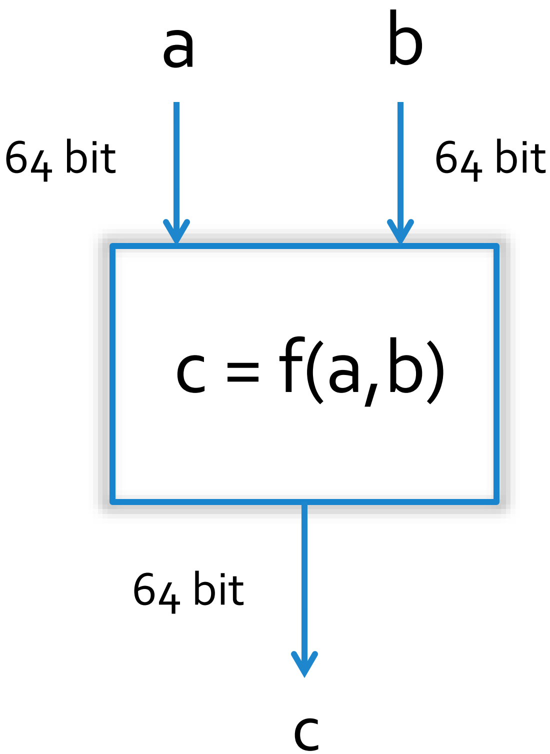


Sources for Constraints

- Designers:
  1. $a +_{64} 11 *_{32} b = 12$
  2. $a <_{64} (b >> 4)$
- Past Experience:
  1. $40 <_{64} 34 + a <_{64} 5050$
  2. $120 <_{64} b <_{64} 230$
- Users:
  1. $232 *_{32} a + b != 1100$
  2. $1020 <_{64} (b /_{64} 2) +_{64} a <_{64} 2200$

- Test vectors: solutions of constraints

Constrained-Random Simulation



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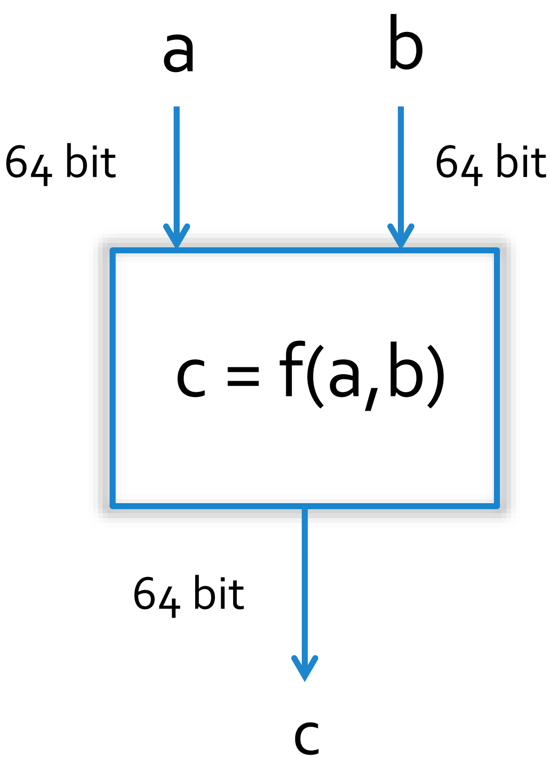
Problem: How can we *uniformly* sample the values of a and b satisfying the above constraints?

Problem Formulation



Set of Constraints

↓

SAT Formula

↓

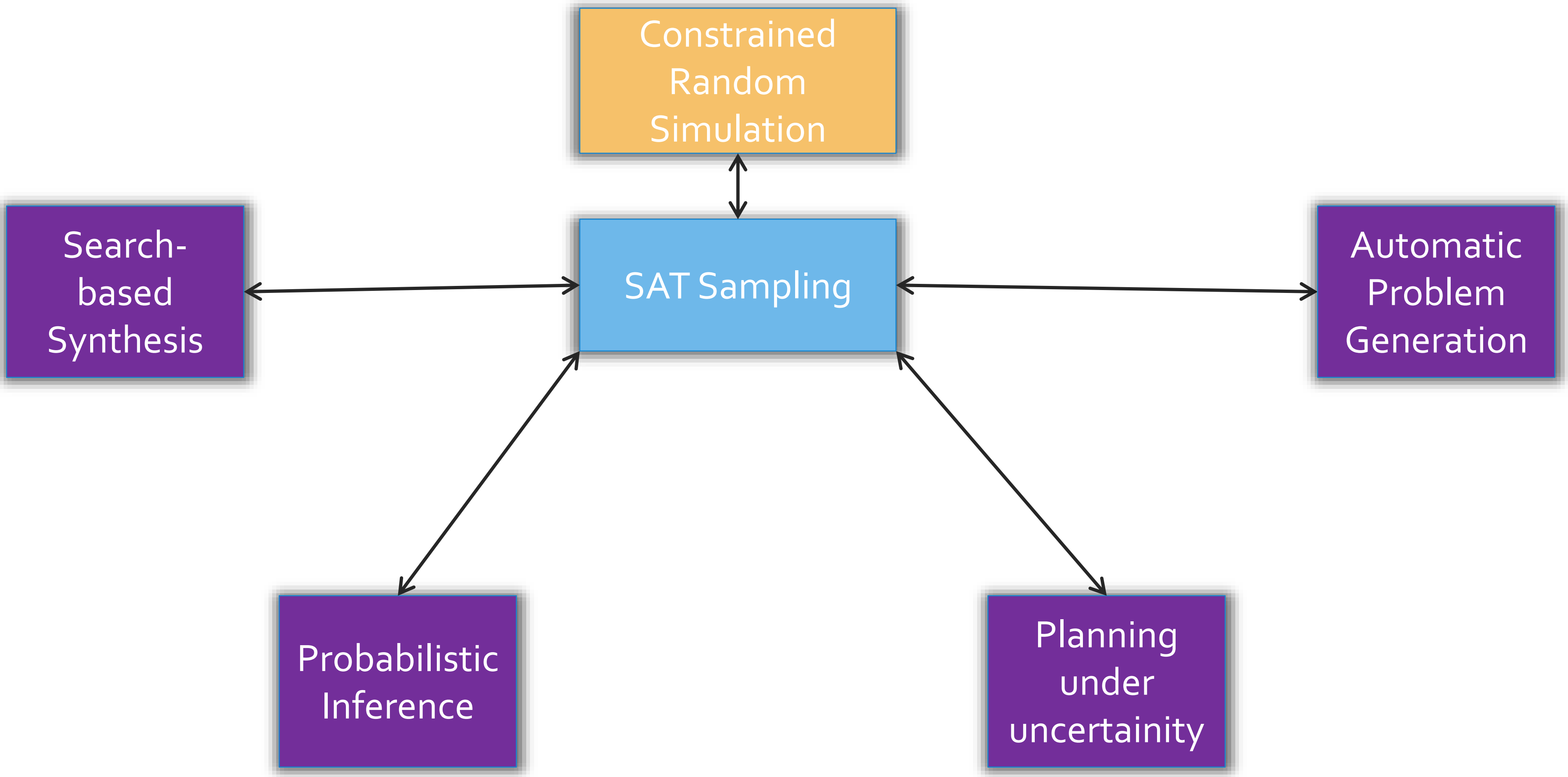
Sample satisfying assignments uniformly at random

SAT Sampling

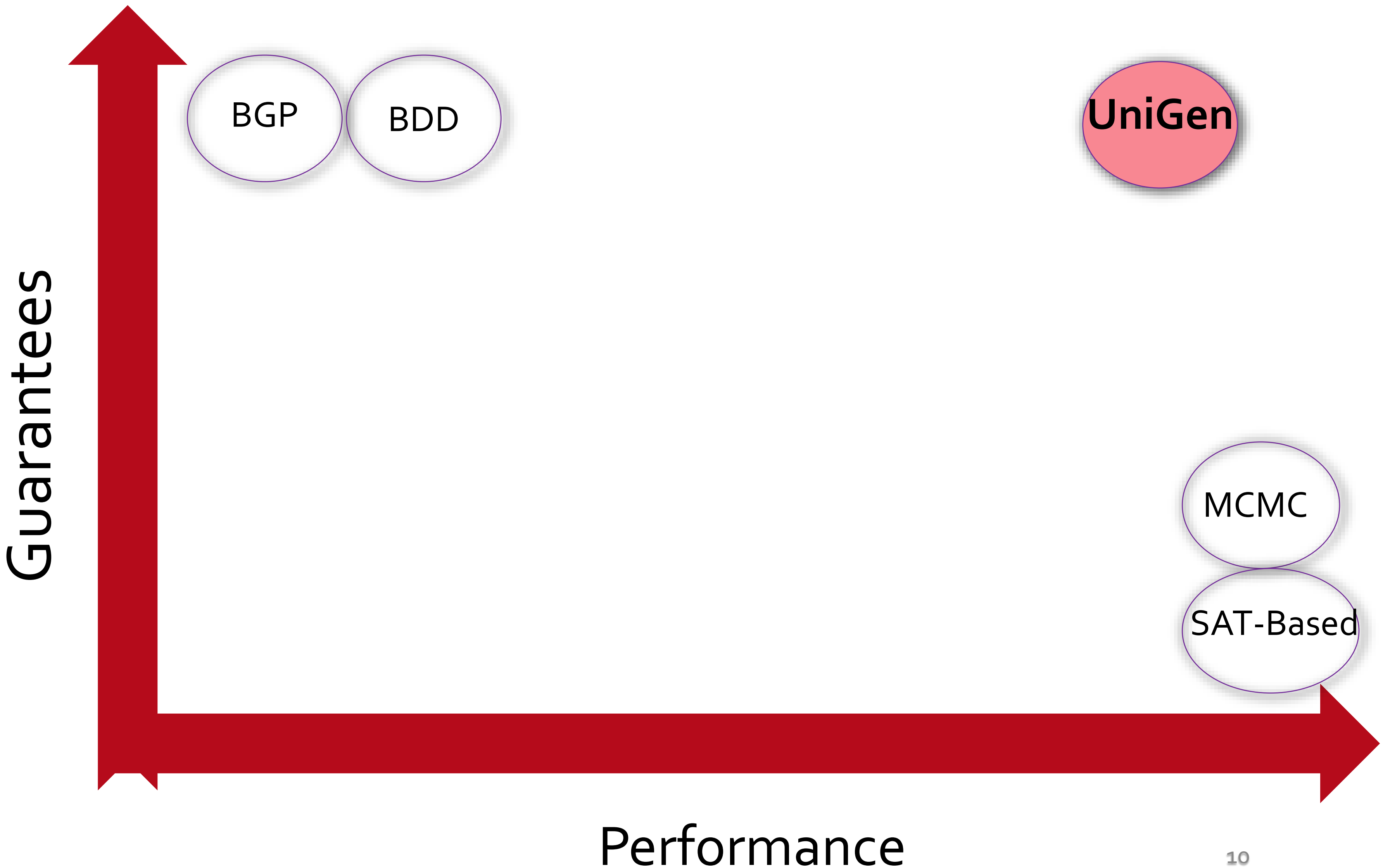
Roadmap

- **SAT Sampling**
- Model Counting
- Works inspired from core ideas
- Future Directions

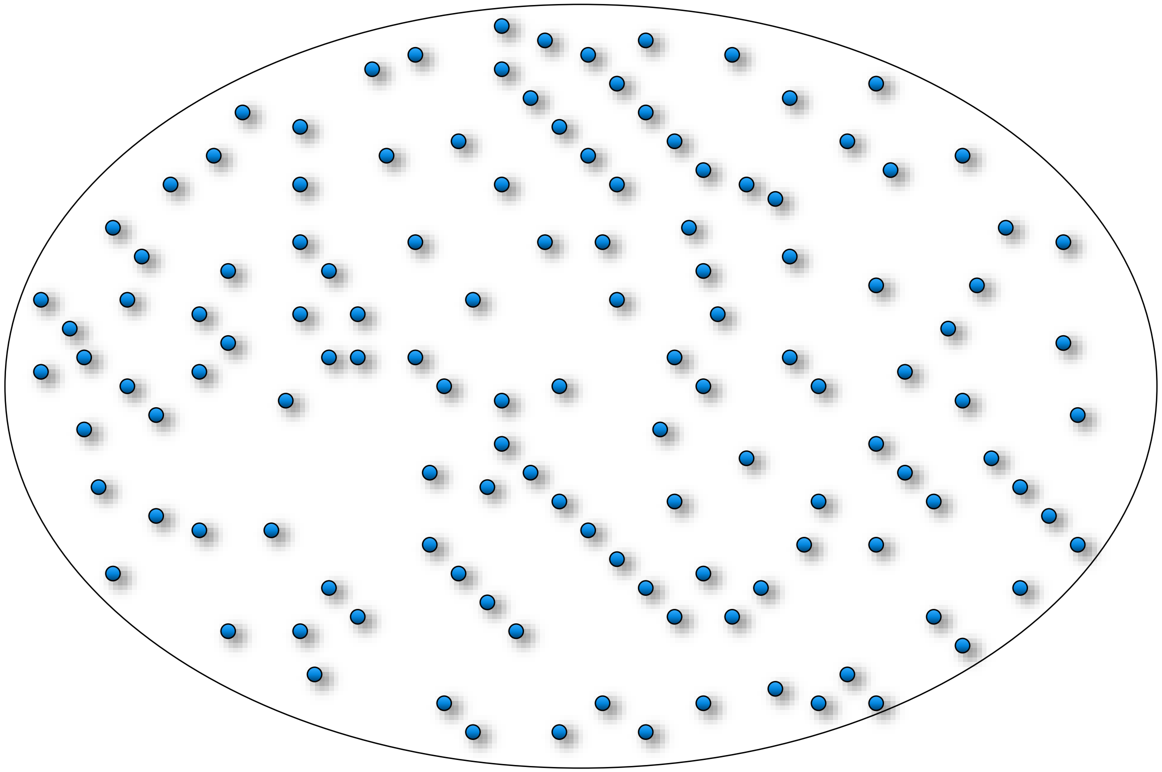
Diverse Applications



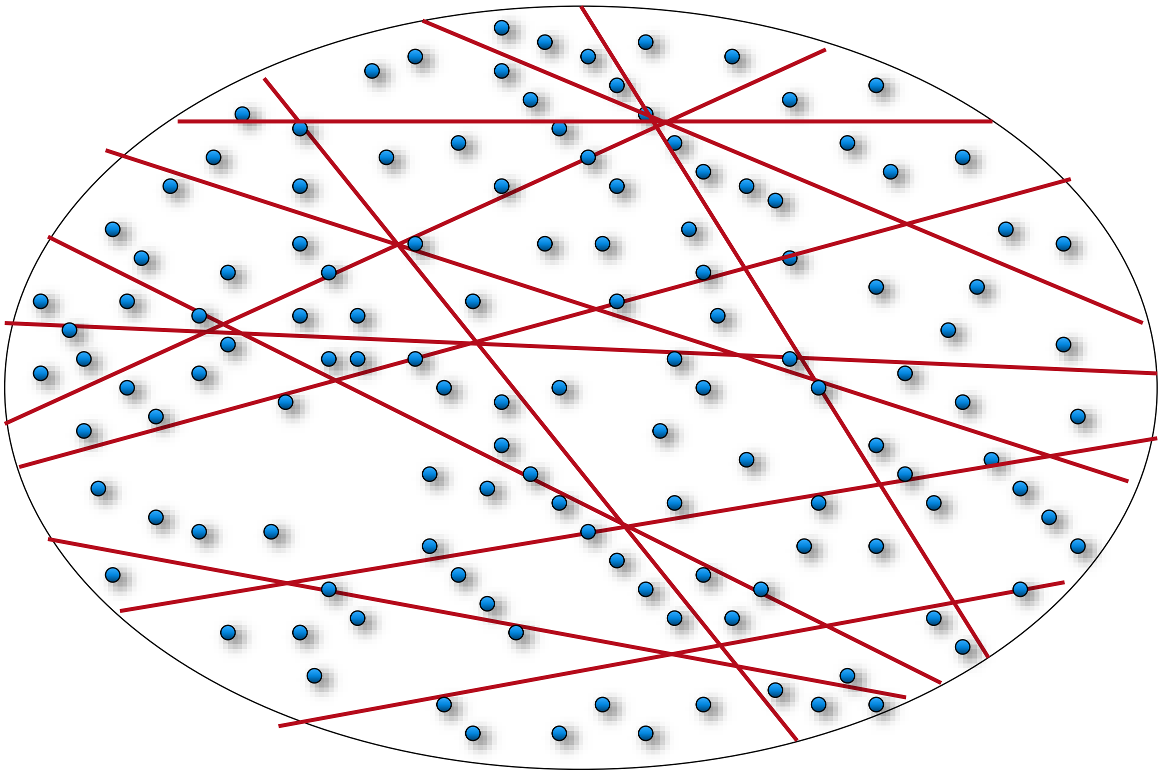
Prior Work



Core Idea

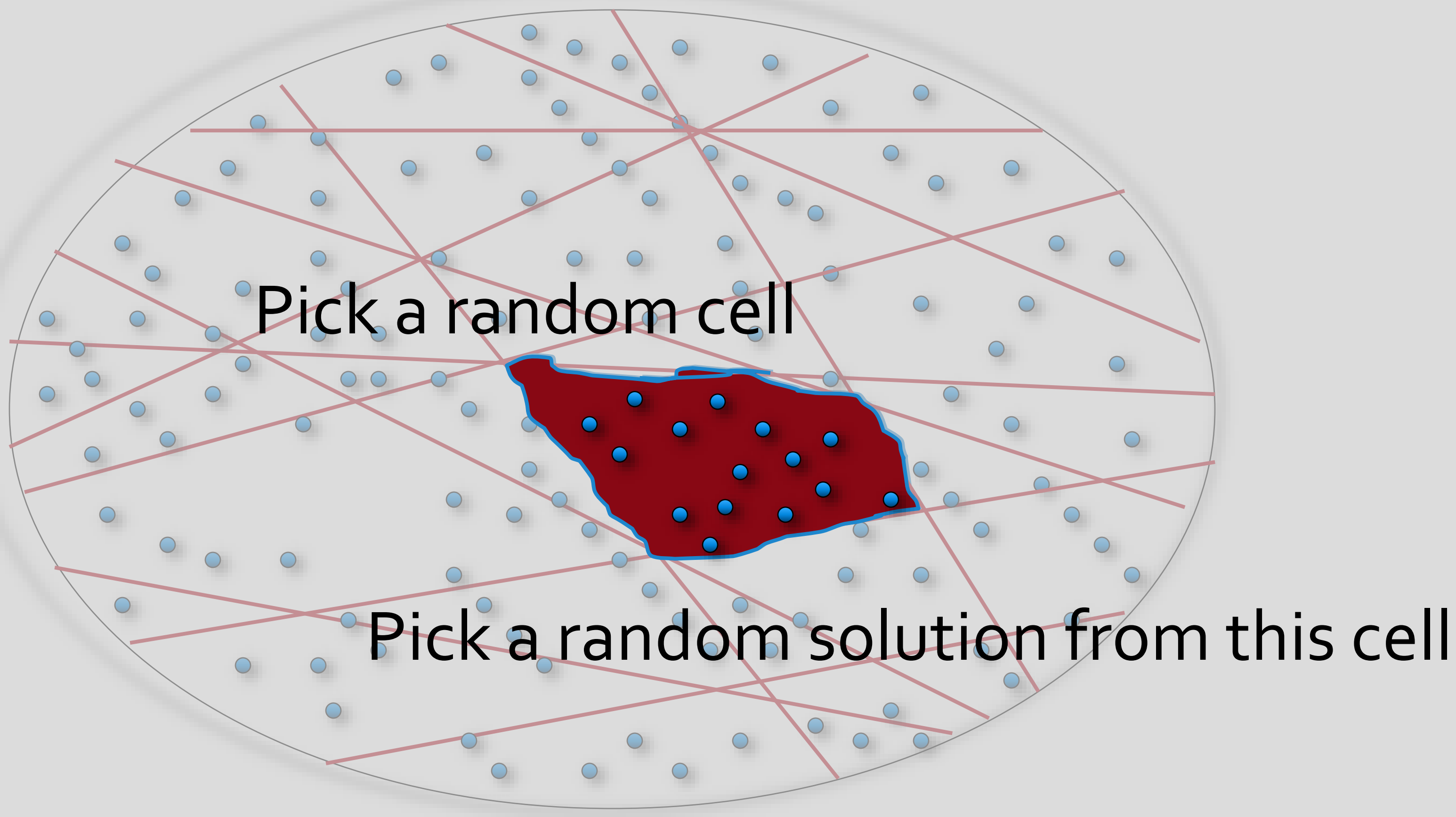


Partitioning into cells



Cells should be roughly equal in size and small enough to enumerate completely

Partitioning into cells



How to Partition?

How to partition into roughly equal small cells of solutions without knowing the distribution of solutions?

r-Universal Hashing
[Carter-Wegman 1979]

Universal Hashing

- Hash functions: mapping $\{0,1\}^n$ to $\{0,1\}^m$
  - (2^n elements to 2^m cells)
- Random inputs => All cells are *roughly* equal (in <u>expectation</u>)
- Universal family of hash functions:
  - Choose hash function randomly from family
  - For ***arbitrary*** distribution on inputs => All cells are ***roughly equal*** (in <u>expectation</u>)

Strong Universality

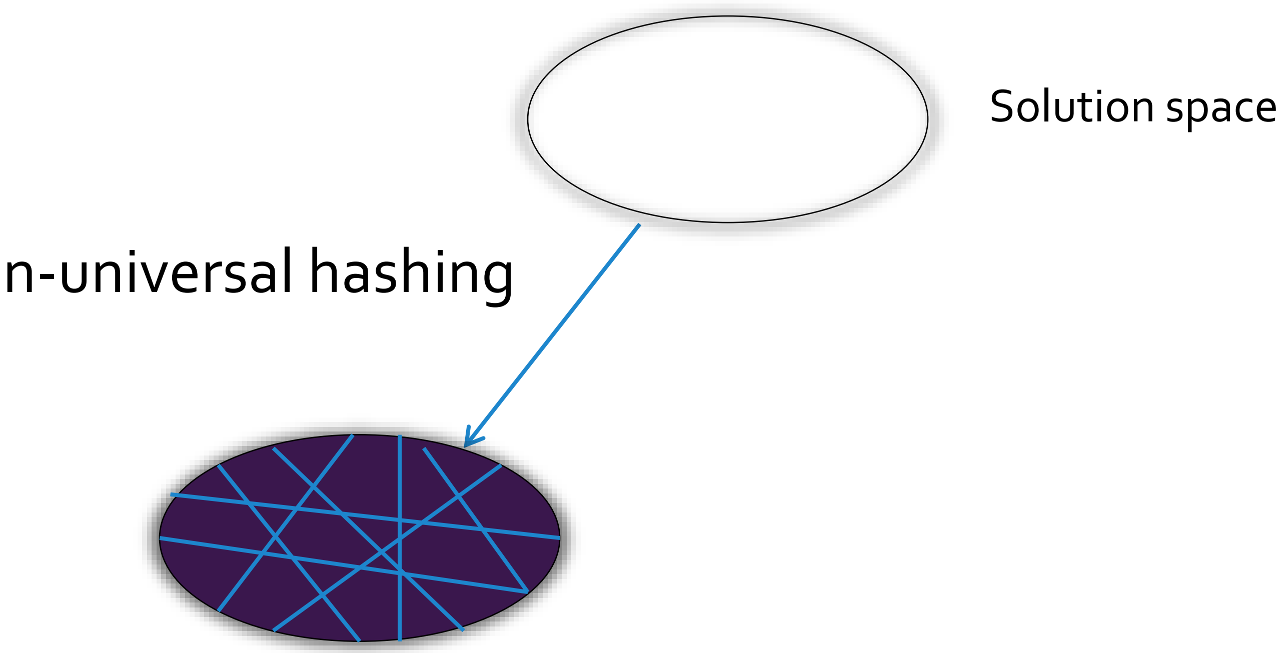
- H(n,m,r): Family of r-universal hash functions mapping $\{0,1\}^n$ to $\{0,1\}^m$ (2^n elements to 2^m cells)
  - r: degree of independence of hashed inputs

- Higher r => Stronger guarantee on ***range of size*** of cells

- r-wise universality => Polynomials of degree r-1

- Stronger universality => Higher complexity

Hashing-based Approaches

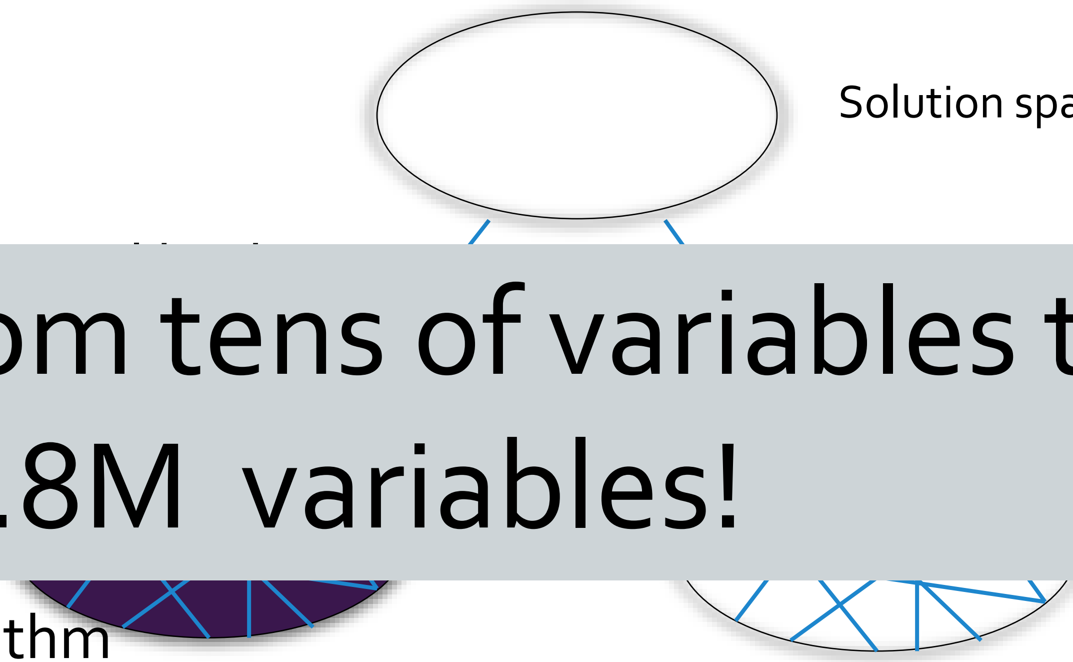


All cells required to be small

Uniform Generation

BGP Algorithm (Bellare et al, 2000)

Scaling to ~0.8M Variables



From tens of variables to ~0.8M variables!

All cells required to be small

Only a randomly chosen cell needs to be "small"

Uniform Generation

Almost-Uniform Generation

Underlying Hash Functions

- A cell can be represented as the conjunction of:
  - Input formula F
  - *m* random XOR constraints
- 2^m is the number of cells desired
- Use CryptoMiniSAT for CNF + XOR formulas

Strong Theoretical Guarantees

- Uniformity

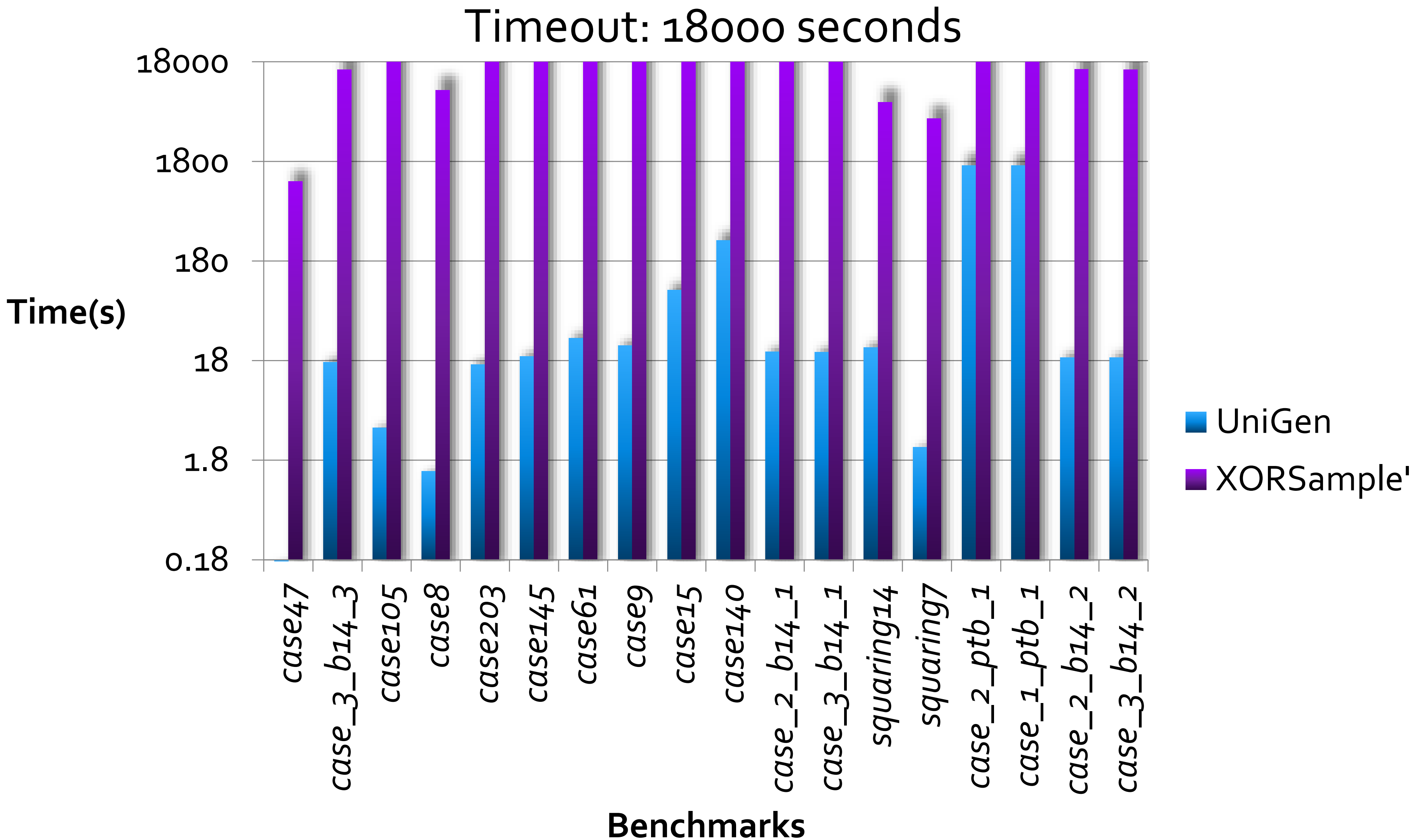
$$\Pr[\text{y is output}] = \frac{1}{|R_F|}$$

- Almost- Uniformity

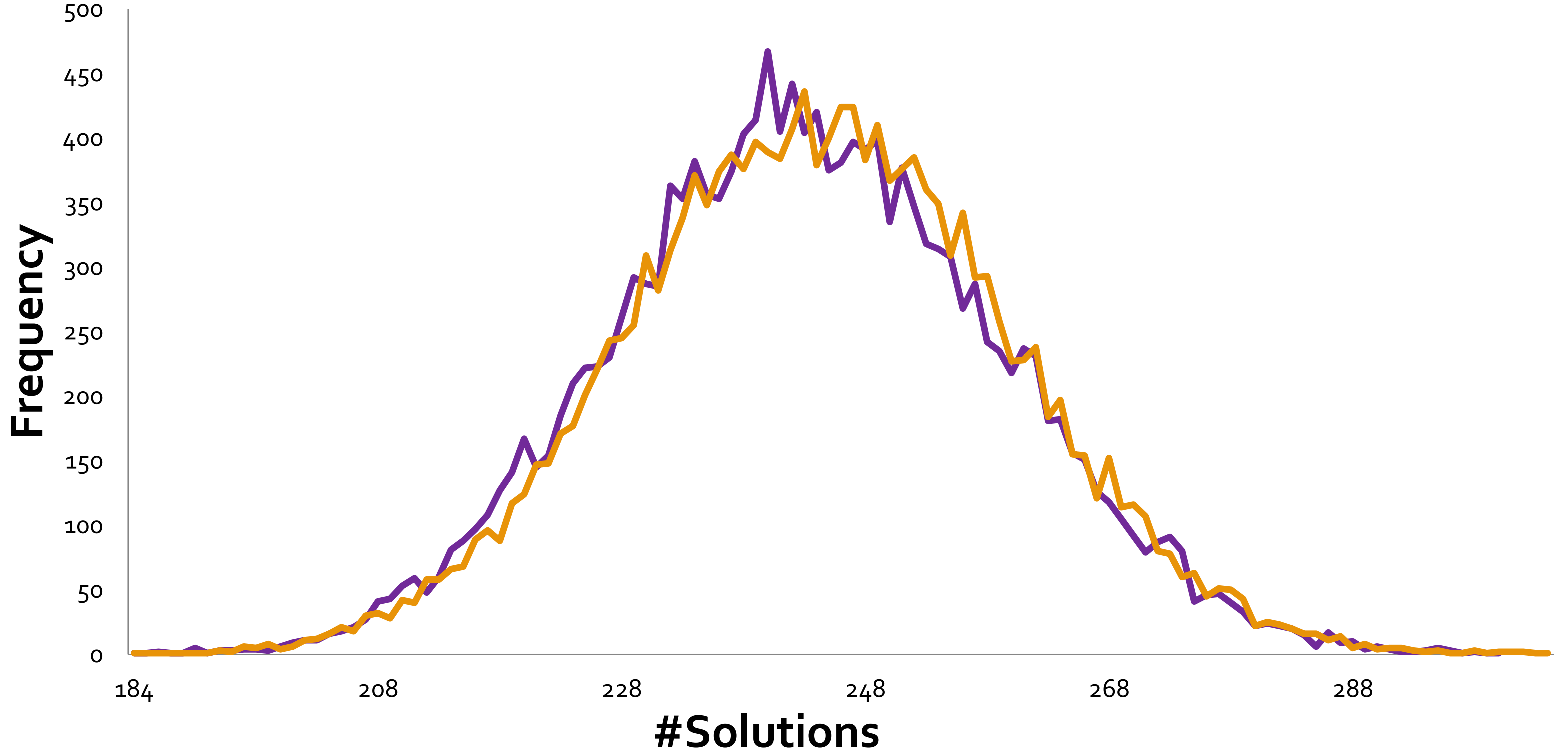
$$\forall y \in R_F, \frac{1}{(1+\varepsilon)|R_F|} \leq \Pr[\text{y is output}] \leq (1+\varepsilon)\frac{1}{|R_F|}$$

- UniGen succeeds with probability 0.52 (Previous best known: 0.125)

2-3 Orders of Magnitude Faster

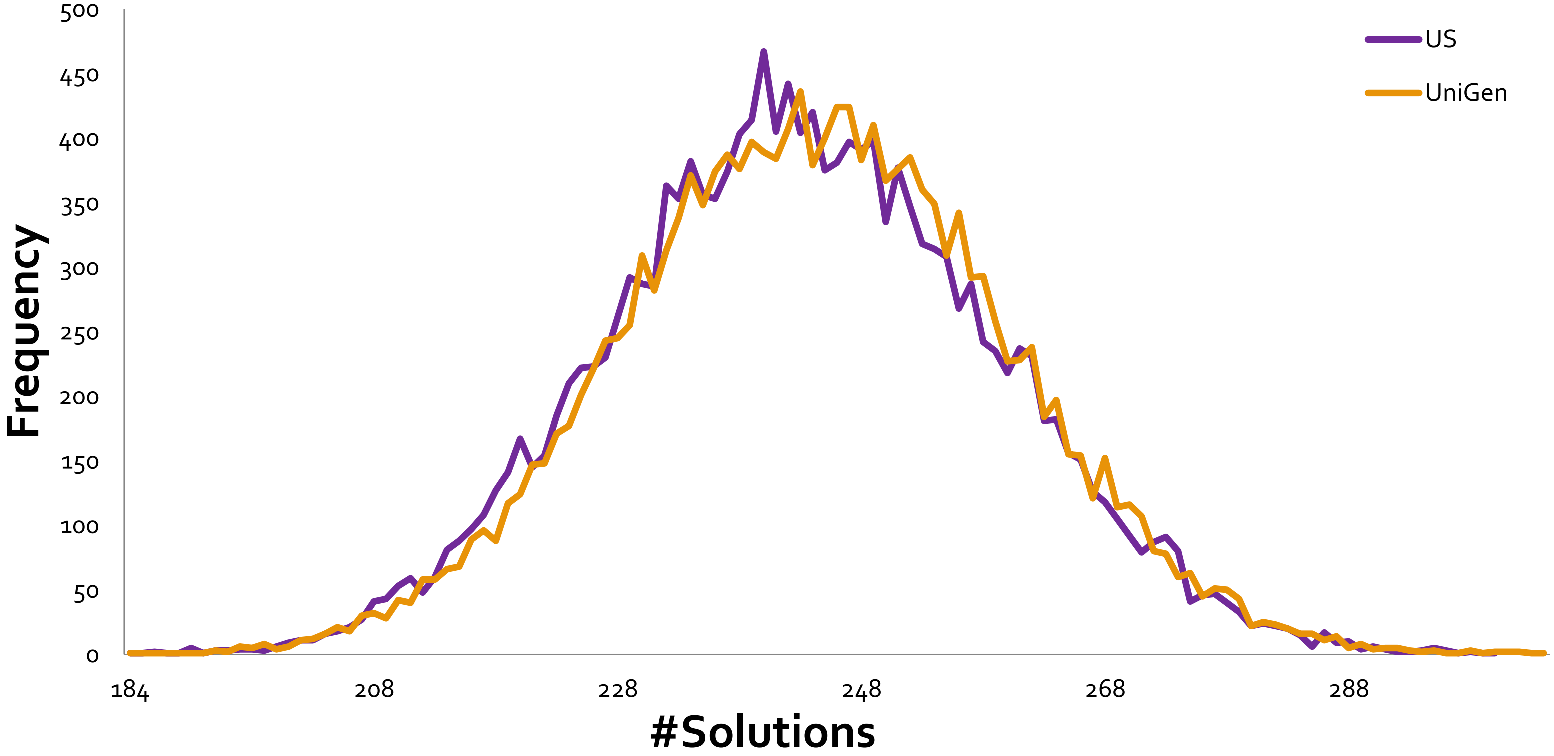


Results: Uniformity



- Benchmark: case110.cnf; #var: 287; #clauses: 1263
- Total Runs**: 4×10^6;** Total Solutions **: 16384**

Results: Uniformity



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Roadmap

- SAT Sampling
- **Model Counting**
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What is Model Counting?

- Given a SAT formula F
- R_F: Set of all solutions of F
- Problem (#SAT): Estimate the number of solutions of F (#F) i.e., what is the cardinality of R_F?
- E.g., F = (a v b)
- R_F = {(0,1), (1,0), (1,1)}
- The number of solutions (#F) = 3

#P: The class of counting problems for decision problems in NP!

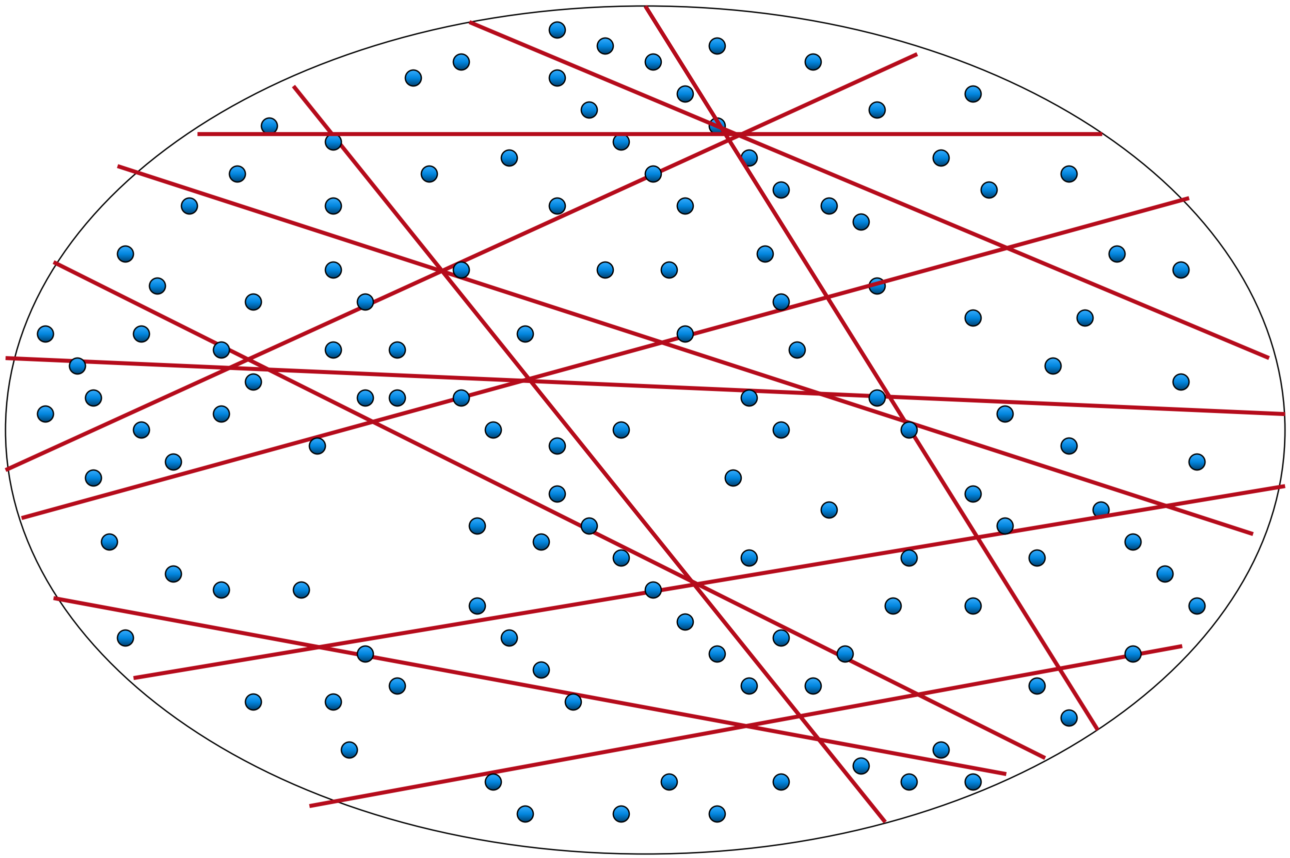
Practical Applications

Wide range of applications!

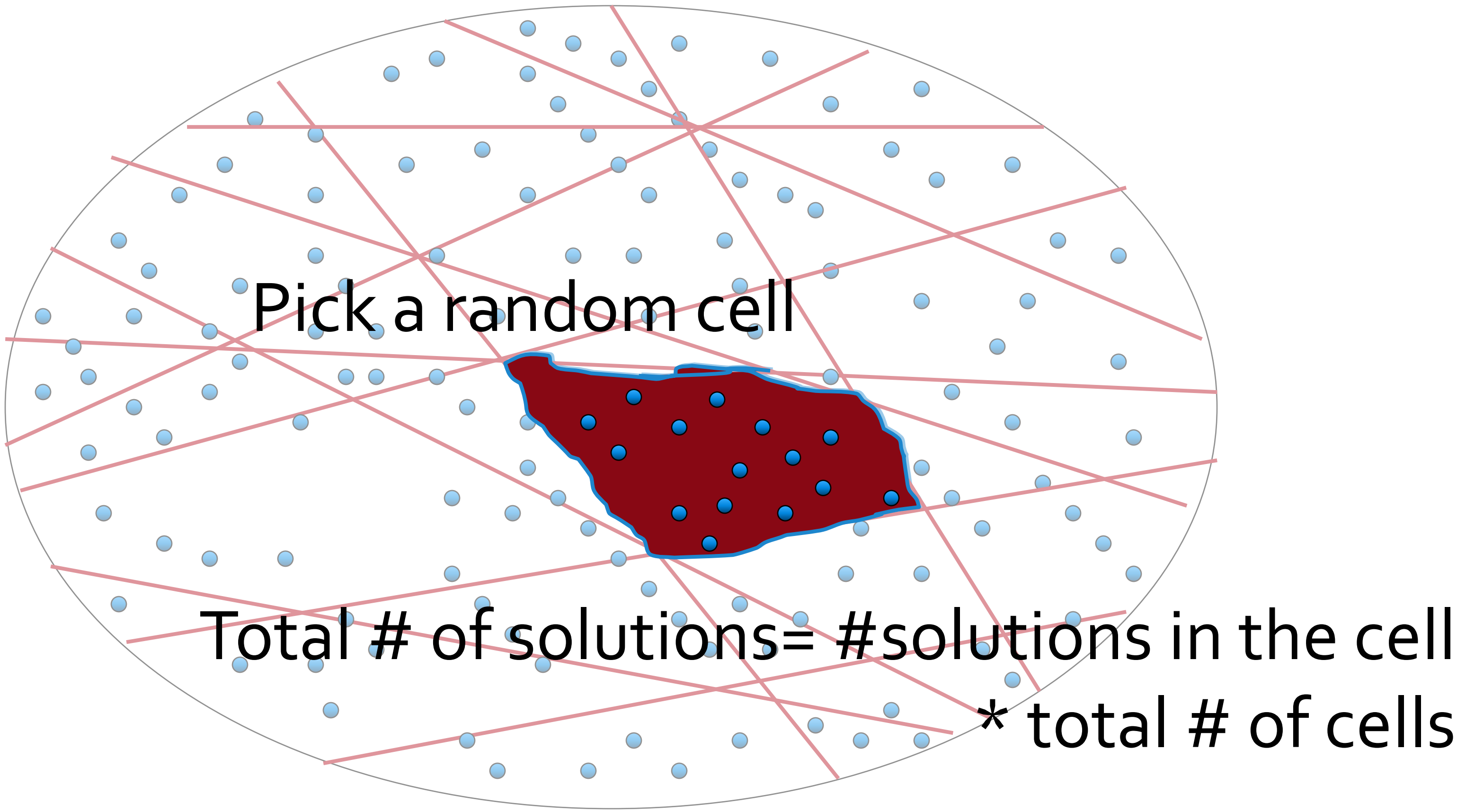
- Estimating coverage achieved
- Probabilistic reasoning/Bayesian inference
- Planning with uncertainty
- Multi-agent/ adversarial reasoning

[Roth 96, Sang 04, Bacchus 04, Domshlak 07]

Counting through Partitioning



Counting through Partitioning



Strong Theoretical Results

ApproxMC (CNF: F, tolerance: ε, confidence: δ)

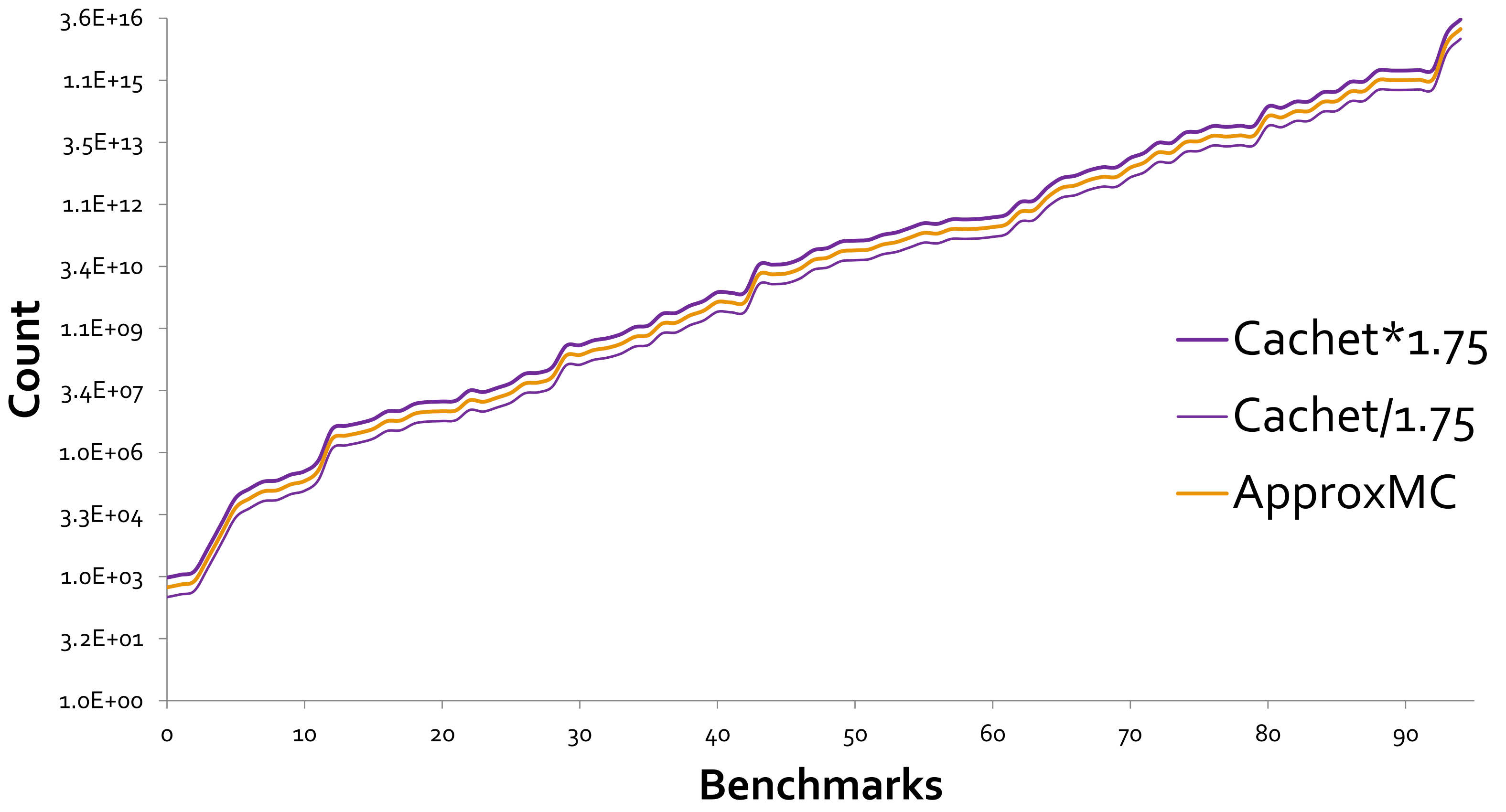
Suppose ApproxMC(F, ε, δ) returns C. Then,

$$Pr[\frac{|R_F|}{(1+\varepsilon)} \leq C \leq (1+\varepsilon)|R_F|] \geq \delta$$

ApproxMC runs in time polynomial in log $(1-\delta)^{-1}$, |F|, ε^{-1} relative to SAT oracle

The First Scalable Approximate Model Counter

Mean Error: Only 4% (ε: 0.75)



Mean error: 4% – much smaller than the theoretical guarantee of 75%

Roadmap

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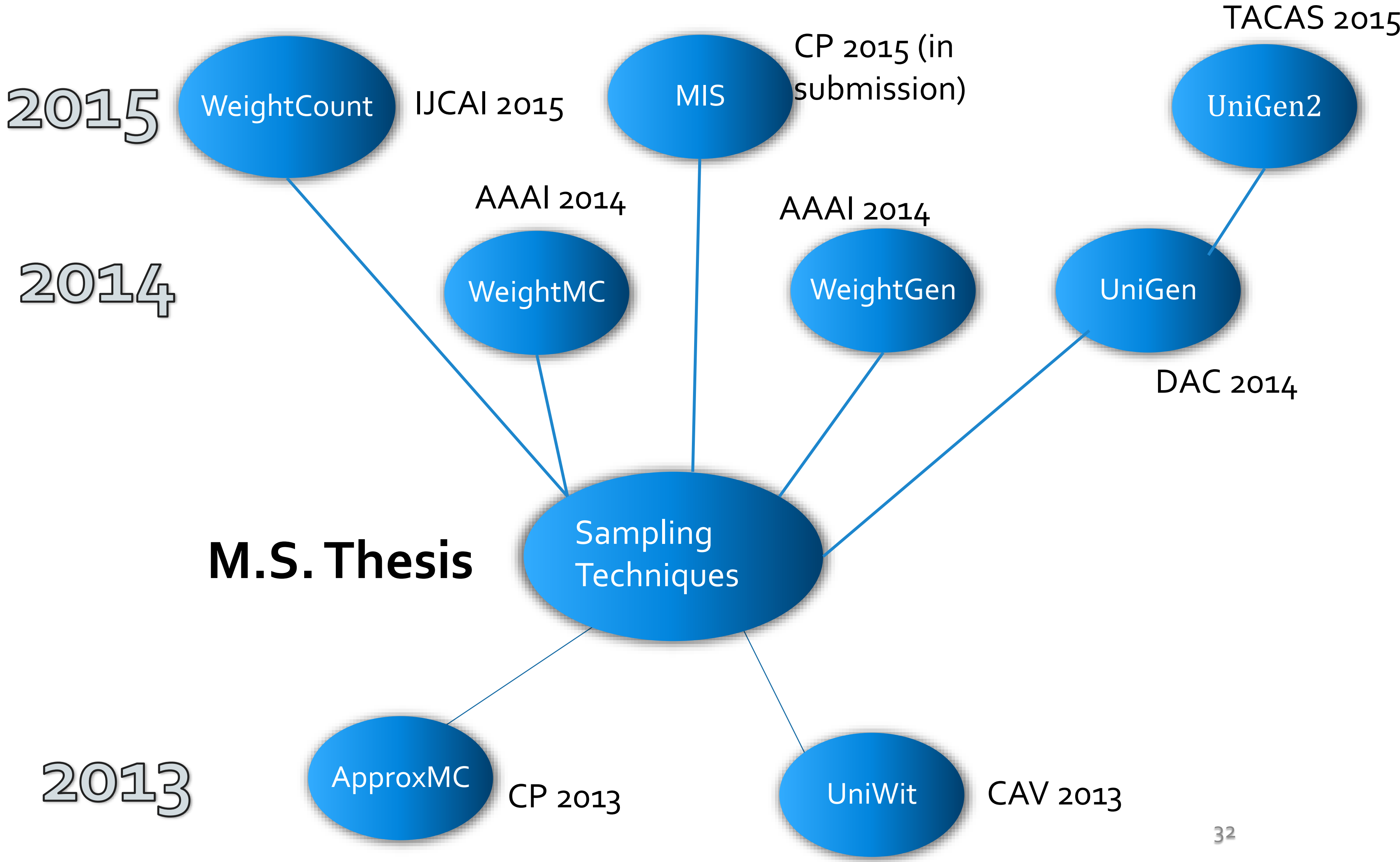
Extensions



2015
WeightCount
IJCAI 2015
MIS
CP 2015 (in submission)
TACAS 2015
UniGen2
2014
AAAI 2014
WeightMC
AAAI 2014
WeightGen
UniGen
DAC 2014
M.S. Thesis
Sampling Techniques
2013
ApproxMC
CP 2013
UniWit
CAV 2013

Applications and follow up

- Quantified Information flow (Fremont et al, 2014)
- Hashing-based integration (Ermon et al, 2014)
- Control Improvisation(Fremont et al, 2014)
- Probabilistic programming(Chistikov et al, 2015)

Roadmap

- SAT Sampling
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- **Future Directions**

Extension to More Expressive Domains (SMT, CSP, ASP)

- Efficient 3-universal/2-universal hashing schemes

- Solvers to handle F + Hash efficiently
  - CryptoMiniSAT has fueled progress for SAT domain
  - Similar solvers for other domains?

Deeper understanding of hashing

- Improved works on sampling require 3-universal hash functions while 2-universal is sufficient for counting
- Sampling and counting are inter-reducible via Jerrum, Valiant & Vazirani (1986)

Key Takeaways

- Sampling and counting are fundamental problems with wide variety of applications
- Prior methods failed to scale or offered very weak theoretical guarantees
- UniGen: The first scalable generator with theoretical gaurantees of almost-uniformity
- ApproxMC: The first scalable approximate model counter
- Extensions of underlying techniques in different contexts
- Visit: www.cs.rice.edu/~kgm2/ for papers/tools/source code!

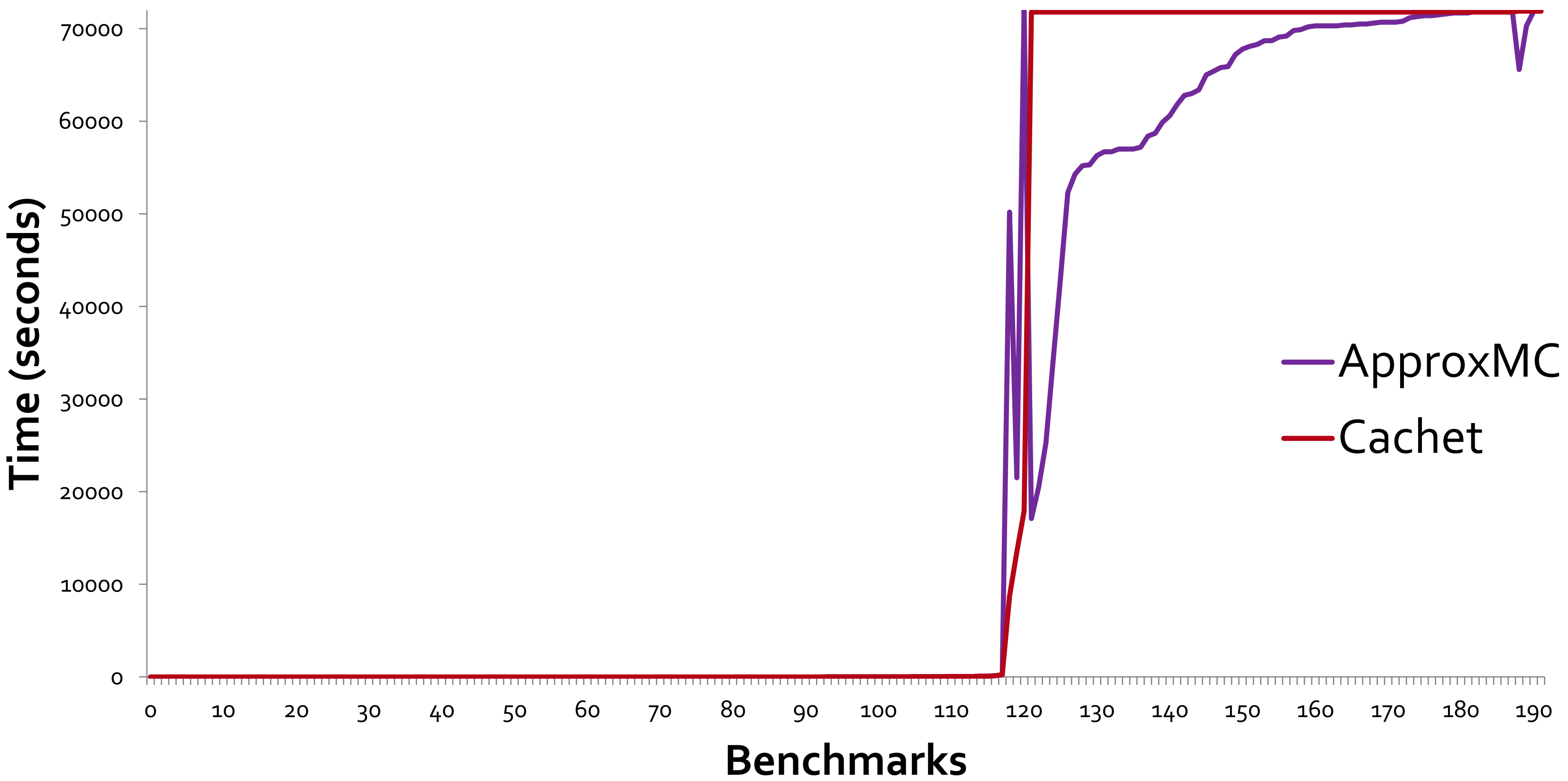
Acknowledgements

- <u>Advisors</u>
  - Moshe Vardi (Rice)
- <u>Collaborators</u>
  - Daniel Fremont (UCB)
  - Dror Fried (Rice)
  - Alexander Ivrii (IBM, Haifa)
  - Sharad Malik (Princeton)
  - Sanjit Seshia (UCB)

  - Supratik Chakraborty (IITB)

Backup Slides

Can Solve a Large Class of Problems



Large class of problems that lie beyond the exact counters but can be computed by ApproxMC

Exploring CNF+XOR

- Very little understanding as of now
- Eager/Lazy approach for XORs?
- How to reduce size of XORs further?

Weighted Counting

Ref: "Distribution-Aware Sampling and Weighted Model Counting for SAT" (In Proc. of AAAI 2014)

Weighted Counting

<u>Given</u>

- CNF Formula F
- Weight Function W over assignments

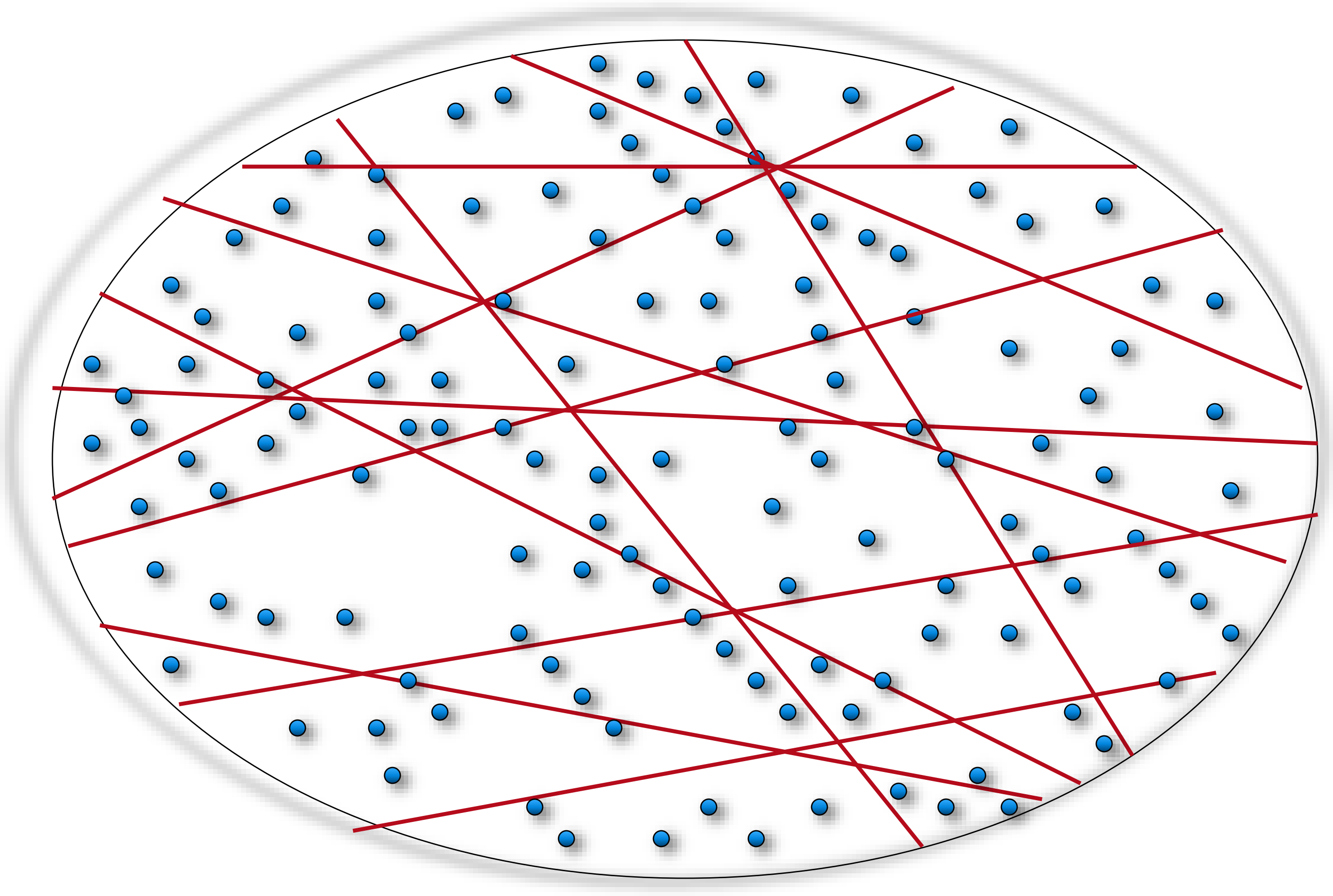
<u>Problem</u>

- What is the sum of weights of ***satisfying*** assignments?

<u>Example</u>

- $F = (a \vee b)$
- $W([0,1]) = W([1,0]) = 1/3 \quad W([1,1]) = W([0,0]) = 1/6$
- $W(F) = 1/3 + 1/3 + 1/6 = 5/6$

Partition into (weighted) equal "small" cells



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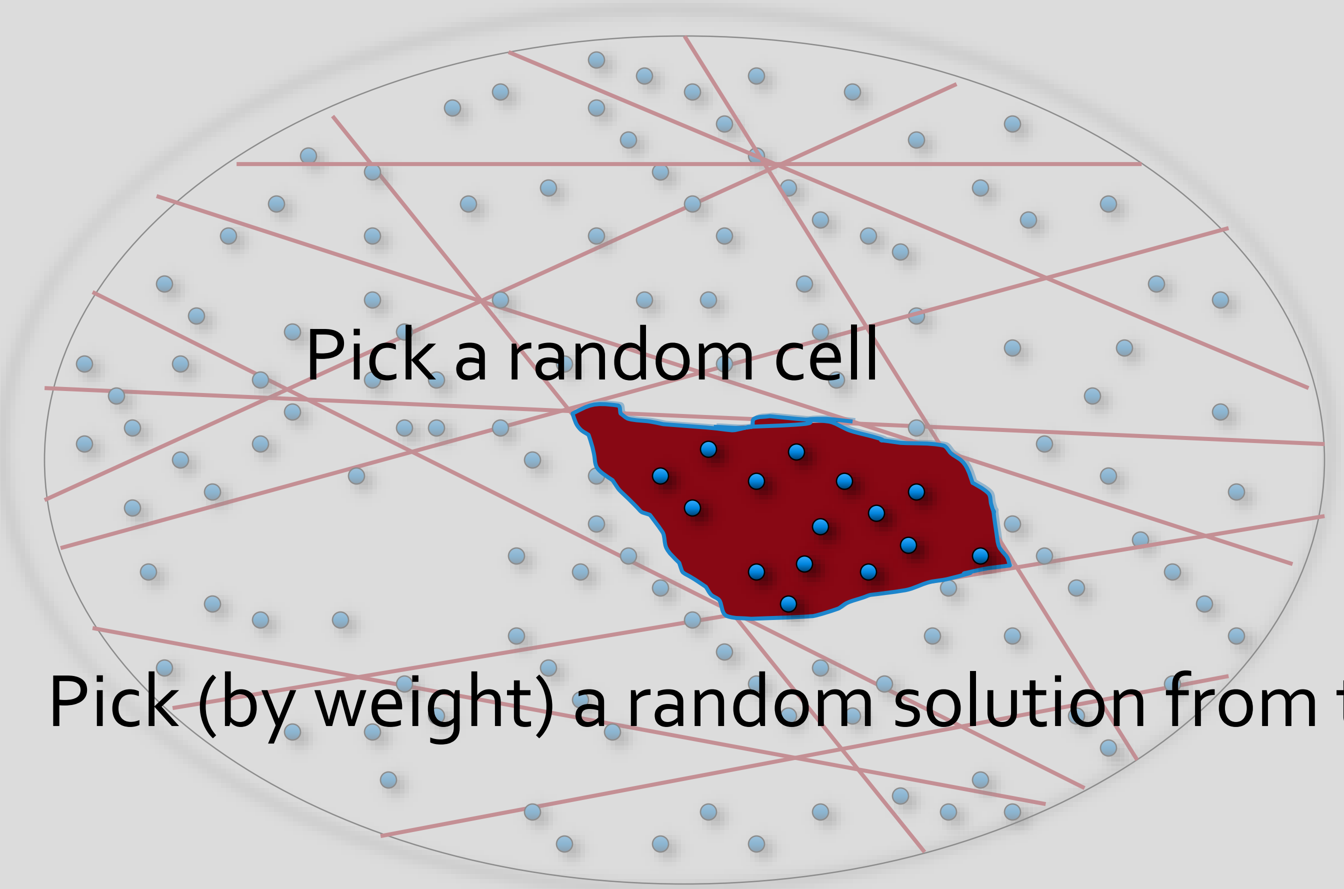


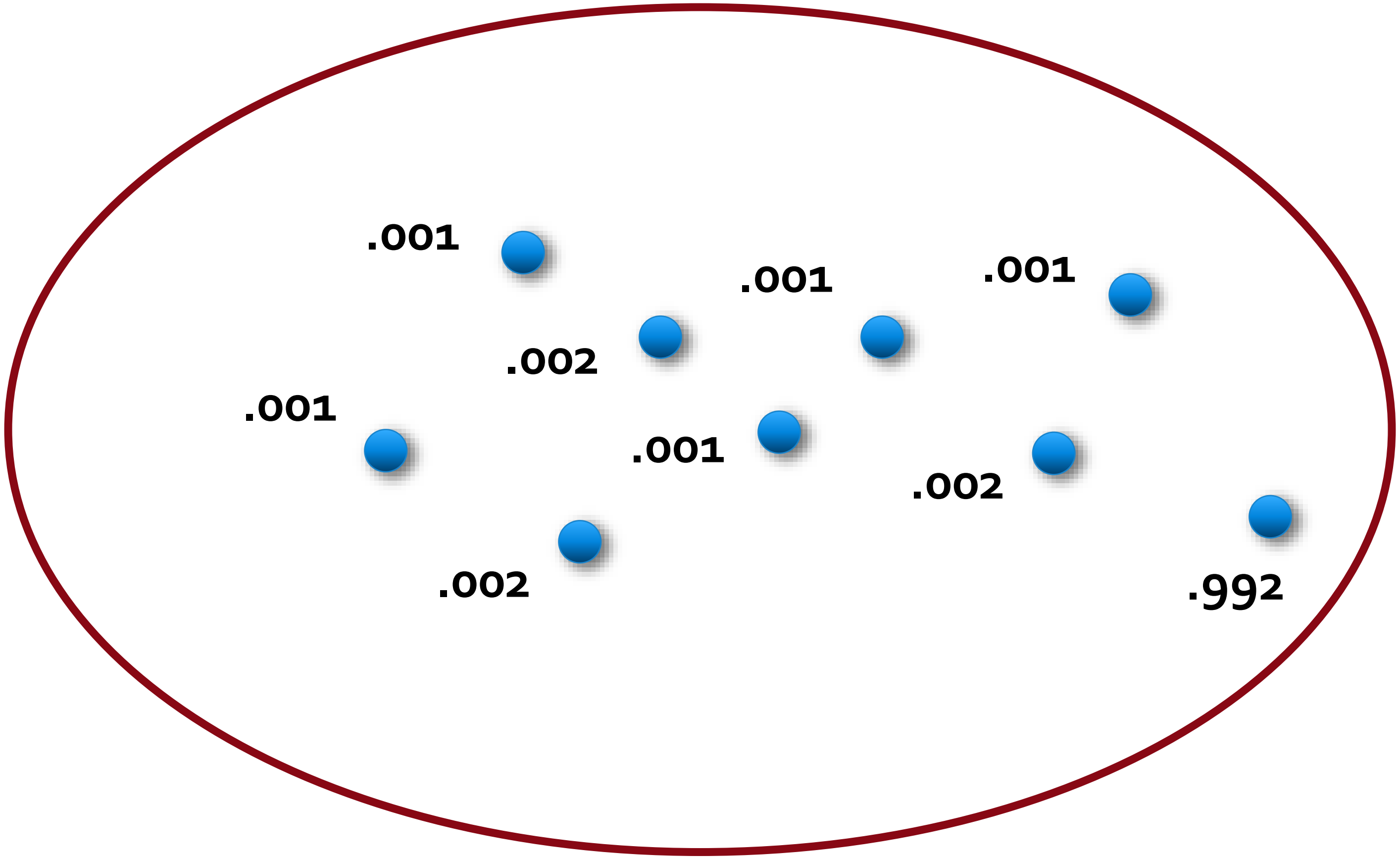
Can you always achieve partitioning?

What if one solution dominates the entire solution space

Tilt = w_{max}/w_{min}

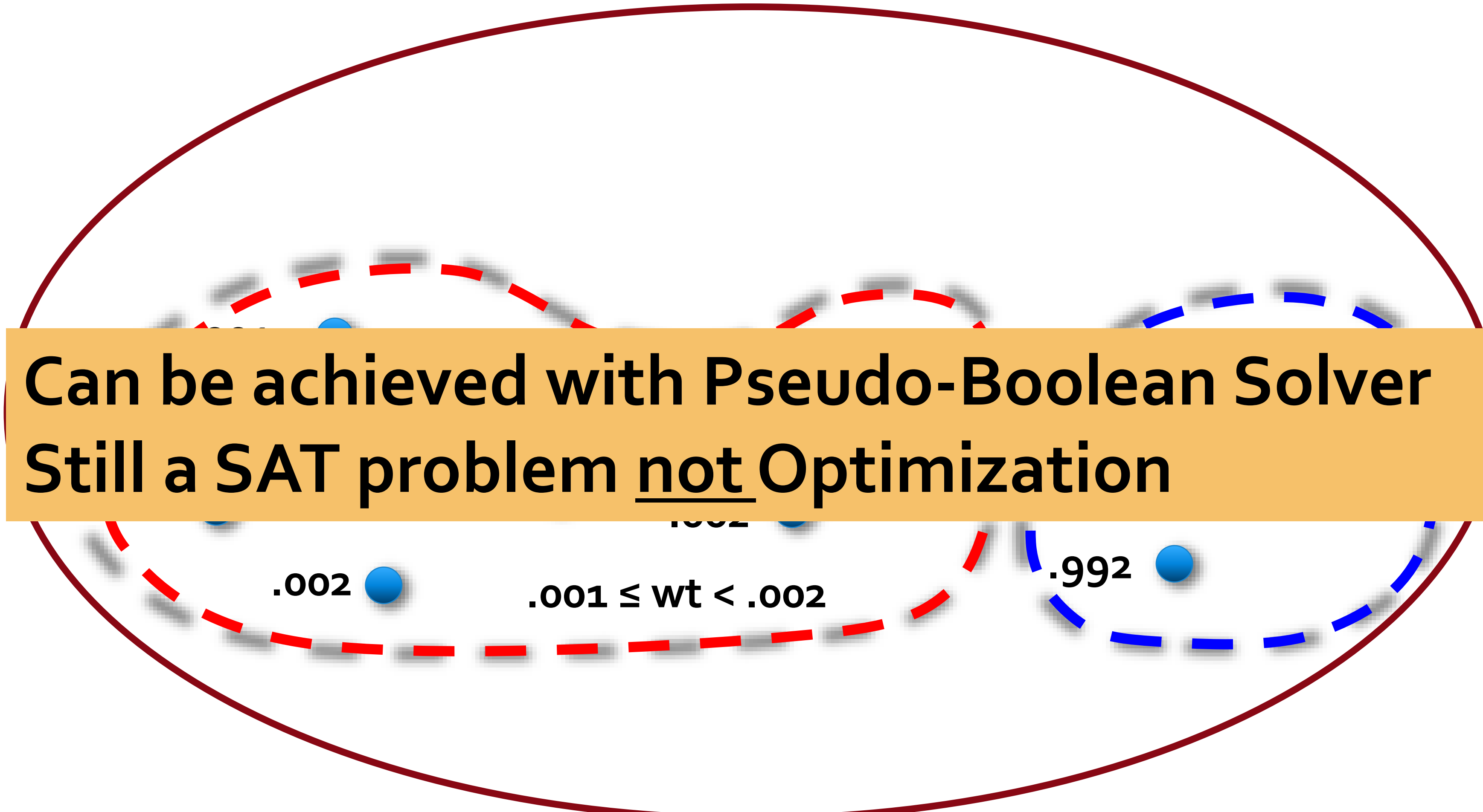
Small tilt $\longrightarrow$ All solutions contribute

How to handle large tilt?



Tilt = 992

Handling Large Tilt



Can be achieved with Pseudo-Boolean Solver
Still a SAT problem <u>not</u> Optimization