Quantitative Automated Reasoning: The Quest for Scalability

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Computing: The Story of an Endless Quest for Scalability

Watson, 1940s: “I think there is a world market for about five computers.”

Gates & Allen, 1970s: “A computer on every desk and in every home”

2020: 22 billion IoT connected devices

Automated Reasoning

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PC1 (char[] SP, char[] UI) {
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Boolean Satisfiability (SAT): Given a Boolean formula, is there a solution, i.e., an assignment of 0's and 1's to the variables that makes the formula equal 1?

Example: $(X_1 \vee \neg X_2 \vee \neg X_3) \wedge (X_2 \vee \neg X_3)$ $\qquad X_1 = 1, X_2 = 1, X_3 = 1$

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[Circa 2012]: Now that SAT is "easy", it is time to look beyond satisfiability

Beyond SAT I: Quantification

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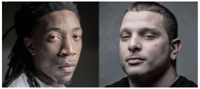
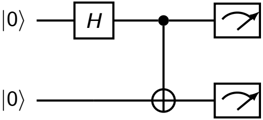
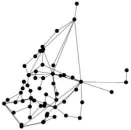
Information Leakage



Fairness



Quantum Simulation



Critical Infrastructure

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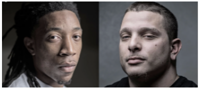

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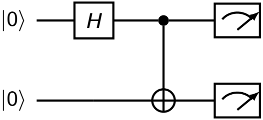

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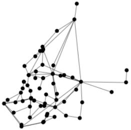
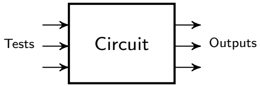


Critical Infrastructure

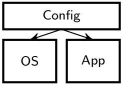
Quantification: How often does $\mathcal{M}$ satisfy $\mathcal{P}$?

Counting

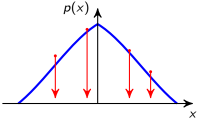
Beyond SAT II: Reasoning about Distributions



Constrained-Random Verification



Configuration Testing



Sampling

- Multiple application domains require *reasoning about distributions*
- Constraints define the structure of relevant input spaces
- **Challenge**: How to efficiently sample from and analyze these distributions?

Beyond SAT III: Automated Synthesis

$f(u,v) \geq u;$
$f(u,v) \geq v;$
$f(u,v) = u \vee$
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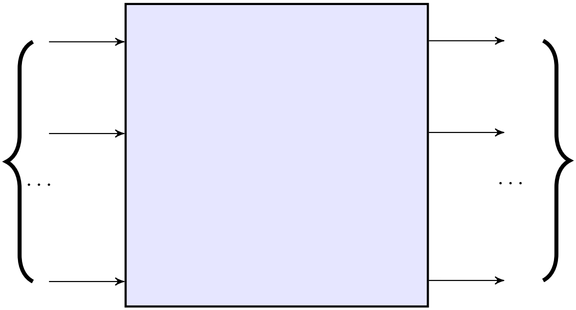
Specification: $\mathcal{P}(\mathcal{I}, \mathcal{O})$

age	25
capital-gain	4000
occupation	coach

Inputs
$\mathcal{I}$



Outputs
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Synthesis

Beyond SAT III: Automated Synthesis

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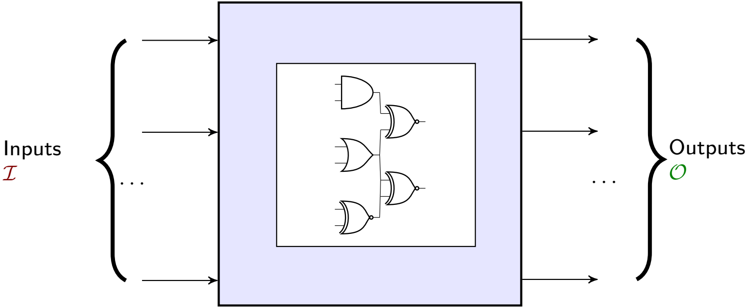

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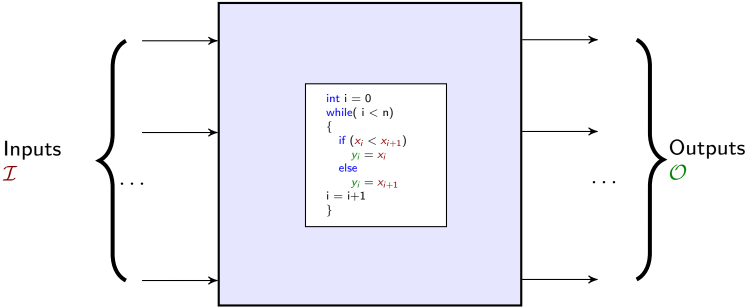
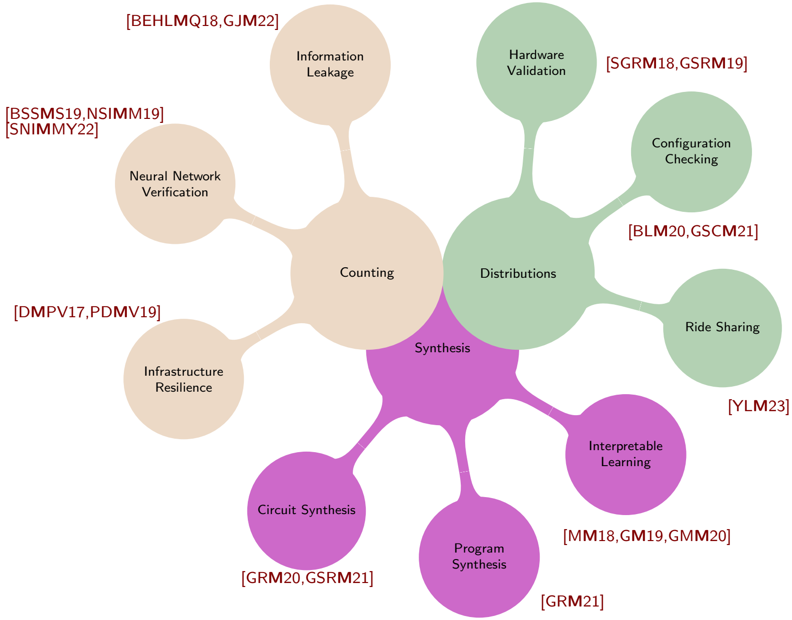


Synthesis

Research Overview



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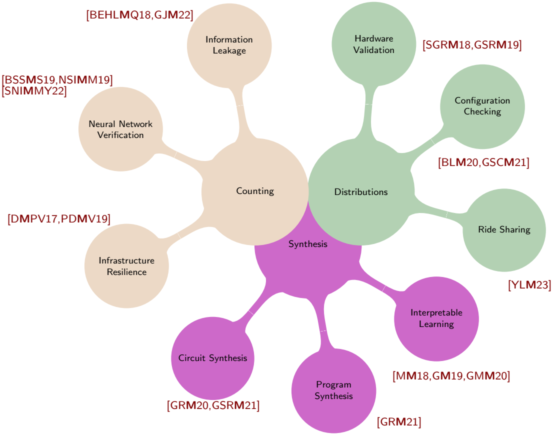

Artificial Intelligence	AAAI:25×, IJCAI:13×, NeurIPS: 6×, KR:2×
Formal Methods/Constraints	CAV:11×, SAT:10×, CP:9×,TACAS: 3×, DAC: 3×, ICCAD: 2×, DATE:
Logic/Databases	PODS:7×, LICS:2×, LPAR:4×,
Software Engineering	ICSE:2×, FSE: 2×, CCS:1×

Research Program's Output

Recent Research Paper Recognition

- Best/Distinguished Paper Awards: CAV-23, CAV-24, and ICLP-24
- Best Paper Award Nomination: ICCAD-22 and DATE-23
- Research Highlights: CACM (2023), ACM SIGMOD (2022)
- Knuth's Article on Distinct Elements (and media coverage)

Job Placement Highlights

- Teodora Baluta (PhD) → Assistant Professor, Georgia Tech
- Priyanka Golia (PhD) → Assistant Professor, CISPA (and IIT Delhi, India)
  ACM India Doctoral Dissertation Award 2024
- Tim van Bremen (post-doc) → Assistant Professor, NTU Singapore
- Anna Latour (post-doc) → Assistant Professor, TU Delft
- Gunjan Kumar (post-doc) → Assistant Professor, IIT Kanpur
- Lawqueen Kanesh (post-doc) → Assistant Professor, IIT Jodhpur
- Yong Lai (post-doc) → Associate Professor, Jilin University

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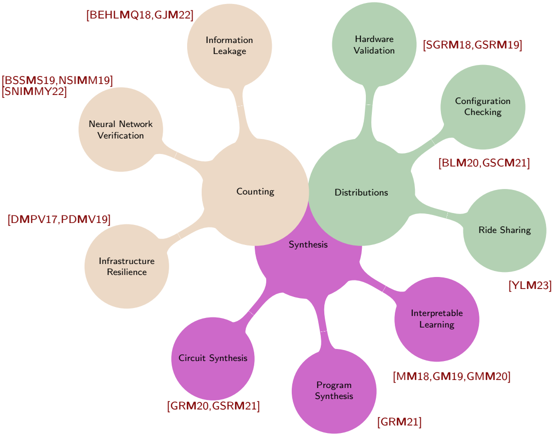

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Today's Focus: Counting

Counting

- Given: A Boolean formula F over $X_1, X_2, \cdots X_n$
- $\mathsf{Sol}(F) = \{$ solutions of F $\}$
- SAT: Determine if $\mathsf{Sol}(F)$ is non-empty
- Counting: Determine $|\mathsf{Sol}(F)|$

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- Generalization to arbitrary weights
  - Given weight function (implicitly represented) W: $\{0,1\}^n \mapsto [0,1]$
  - $W(F) = \Sigma_{y \in \mathrm{Sol}(F)} W(y)$
  - (Weighted) Counting: Determine $W(F)$

Today's talk: We focus on unweighted case, i.e., $|\mathrm{Sol}(F)|$

Today's Menu

Appetizer Applications

- Critical Infrastructure Resilience
- Quantitative Verification of AI Systems
- Quantum Simulation

Main Course ApproxMC: A Scalable Counting Framework

Dessert Future Outlook

Resilience of Critical Infrastructure Networks

[DMPV17,PDMV19, KM23]

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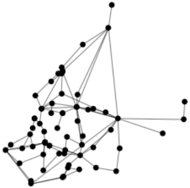

- $G = (V, E)$; set of source nodes S and terminal node t
- failure probability $g : E \to [0, 1]$
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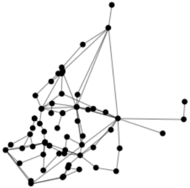

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Constrained Counting

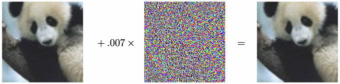
Key Idea: Encode disconnectedness using constraints

Impact: The first theoretically sound estimates of resilience in power transmission networks of ten medium sized cities in US

Quantitative Analysis of AI Systems

[BSSMS19,NSMIS19,NSMISV22,YTSM25]

Our Focus: Binarized Neural Networks



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$$\left| \{x : \mathcal{N}(x+\varepsilon) \neq \mathcal{N}(x)\} \right|$$

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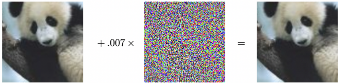

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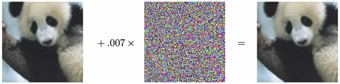

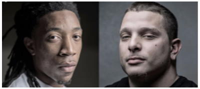

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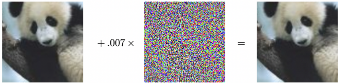

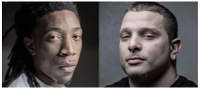

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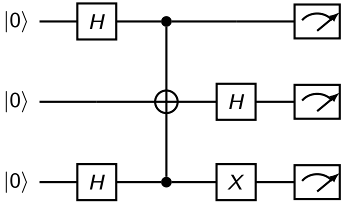
Impact: The first scalable technique for rigorous and certified quantification of robustness and fairness of Binarized Neural Networks

Quantum Circuit Simulation via Model Counting

[MBL24]



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- Measurement collapses to classical state with probability $|\alpha_i|^2$
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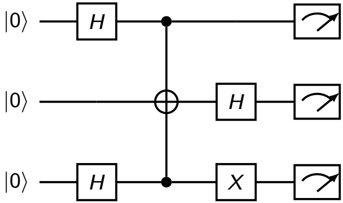

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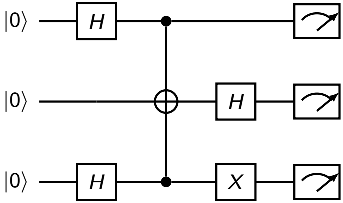

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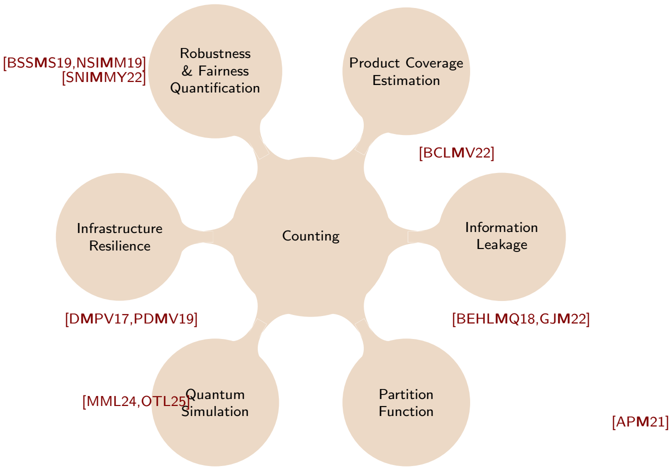
- $\Pr[\text{measuring outcome } x] = |W(F \wedge \text{outcome} = x)|^2$

Impact: State of the art approach for large classes of circuits, enabling verification of quantum algorithms on classical hardware

Applications across Computer Science



Impact: Counting-based approach is now the state of the art for all these applications

So Fundamental Yet So Hard

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Stockmeyer, 1983: Probably Approximately Correct (PAC) aka (ε, δ)-guarantees

$$\Pr\left[\frac{|\mathrm{Sol}(F)|}{1+\varepsilon} \leq \mathrm{ApproxCount}(\mathrm{F}, \varepsilon, \delta) \leq (1+\varepsilon)|\mathrm{Sol}(F)|\right] \geq 1-\delta$$

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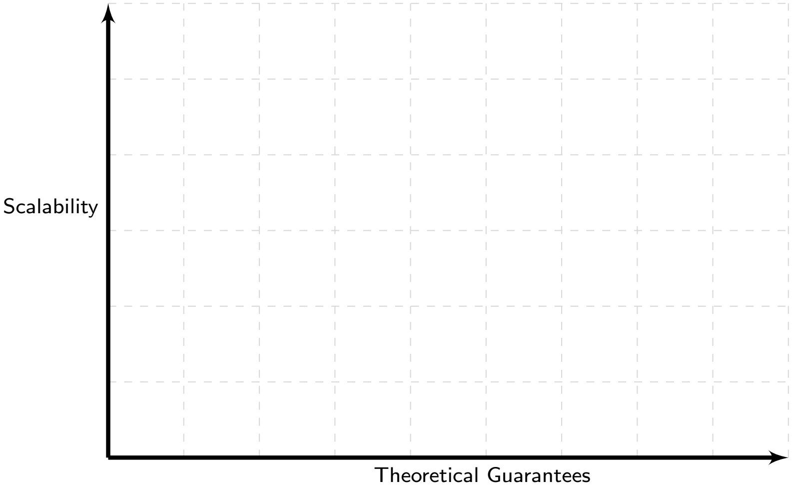
- Not practical

SAT Solver $\neq$ SAT Oracle

Performance of state of the art SAT solvers depends on the formulas

Snapshot from 2012



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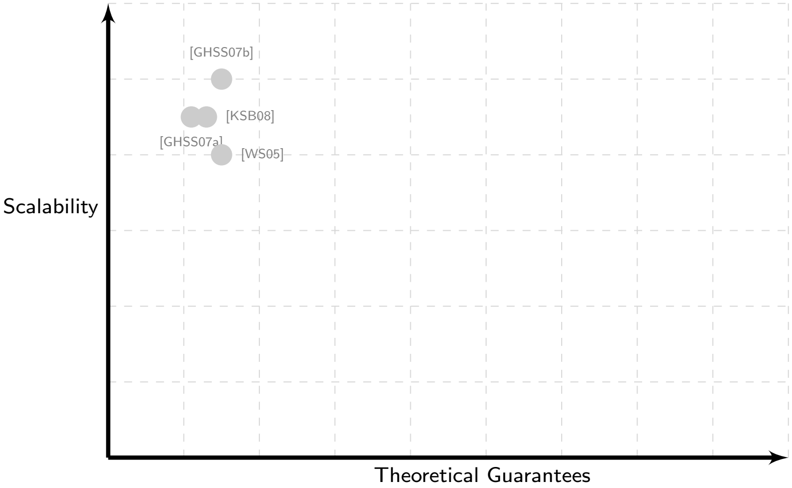

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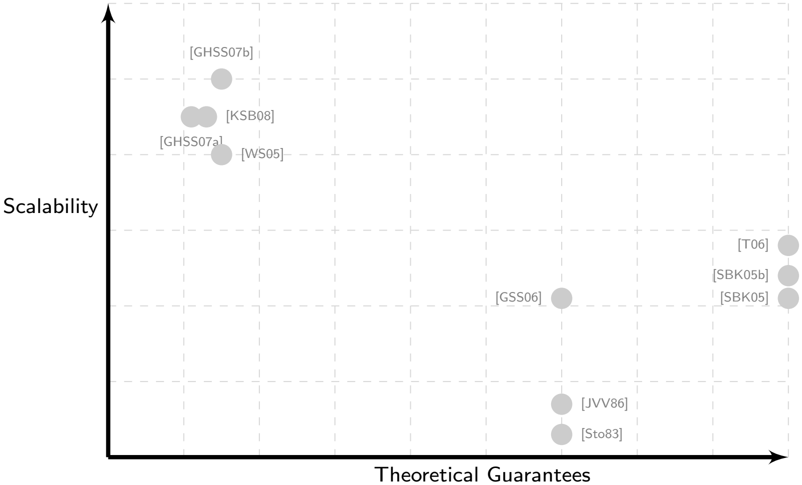
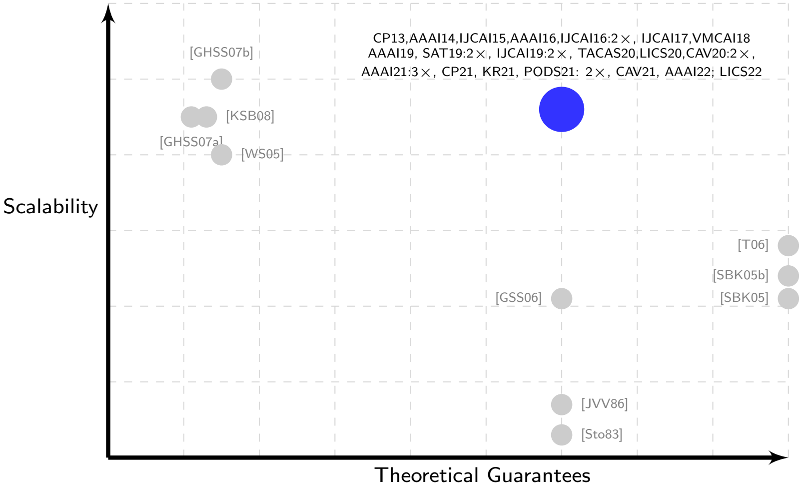


State of the art tool in 2012 could handle one out of 1076 robustness instances

Can we bridge the gap between theory and practice?



State of the art tool in 2012 could handle one out of 1076 robustness instances

Can we bridge the gap between theory and practice?

Counting in Singapore

How many people in Singapore like coffee?

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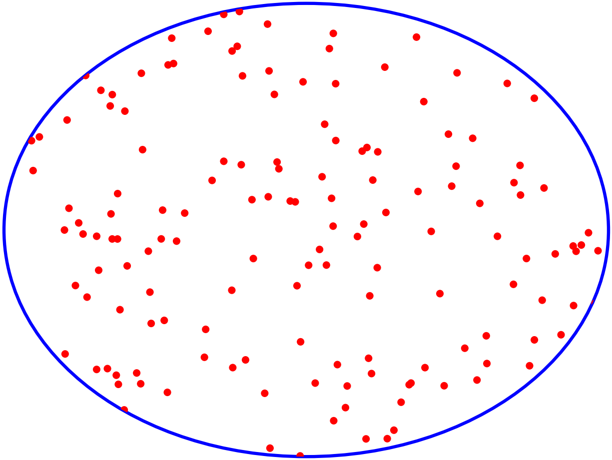
    - Potentially 2^n queries

Can we do with lesser # of SAT queries – $\mathcal{O}(n)$ or $\mathcal{O}(\log n)$?

As Simple as Counting Dots



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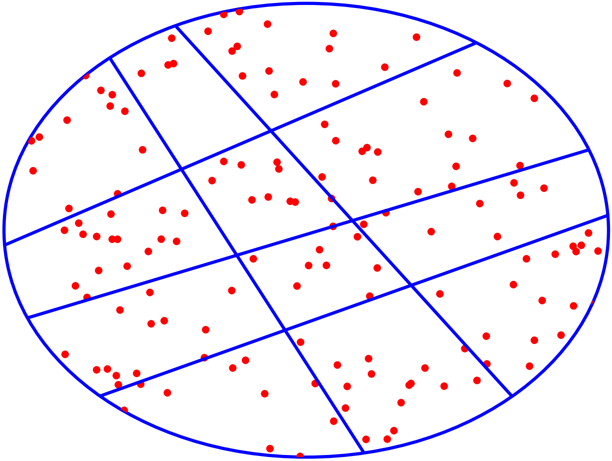
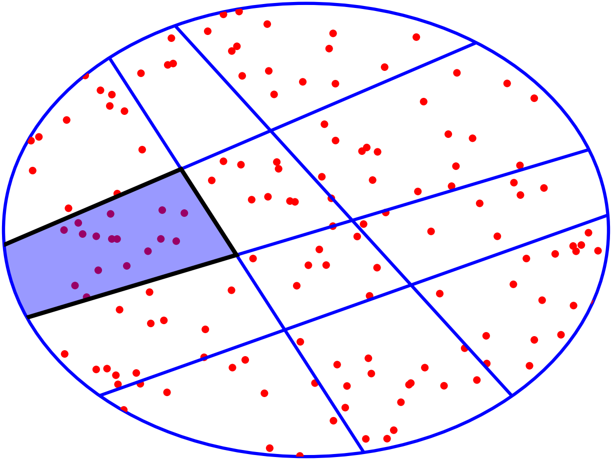


As Simple as Counting Dots

Pick a random cell



Estimate = Number of solutions in a cell $\times$ Number of cells

Challenges

Challenge 1 How to partition into roughly equal small cells of solutions without knowing the distribution of solutions?

Challenge 2 How many cells?

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- Designing function h : assignments $\rightarrow$ cells (hashing)
- Deterministic h unlikely to work
- Choose h randomly from a large family H of hash functions

$\boxed{\text{2-wise Independent Hashing}}$ [CW77]

2-wise Independent Hash Functions

- To construct $h : \{0,1\}^n \rightarrow \{0,1\}^m$, choose m random XORs
- Pick every X_i with prob. $\frac{1}{2}$ and XOR them
  - $X_1 \oplus X_3 \oplus X_6 \cdots \oplus X_{n-2}$

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- To choose $\alpha \in \{0,1\}^m$, set every XOR equation to 0 or 1 randomly

$$X_1 \oplus X_3 \oplus X_6 \cdots \oplus X_{n-2} = 0 \qquad (Q_1)$$
$$X_2 \oplus X_5 \oplus X_6 \cdots \oplus X_{n-1} = 1 \qquad (Q_2)$$
$$\cdots \qquad (\cdots)$$
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- Solutions in a cell: $F \wedge Q_1 \cdots \wedge Q_m$

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$\boxed{\text{Random XOR-based Hash Functions}}$ [CW77]

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Challenge 2: How many cells?

[CMV13,CMV16]

- A cell is small if it has $\approx \mathrm{thresh} = 5(1 + \frac{1}{\varepsilon})^2$ solutions
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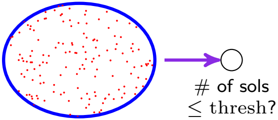

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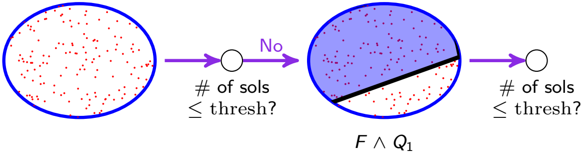

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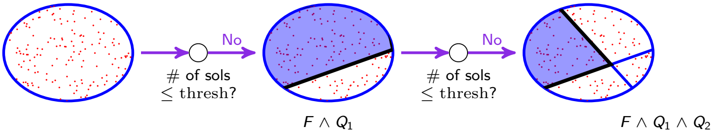

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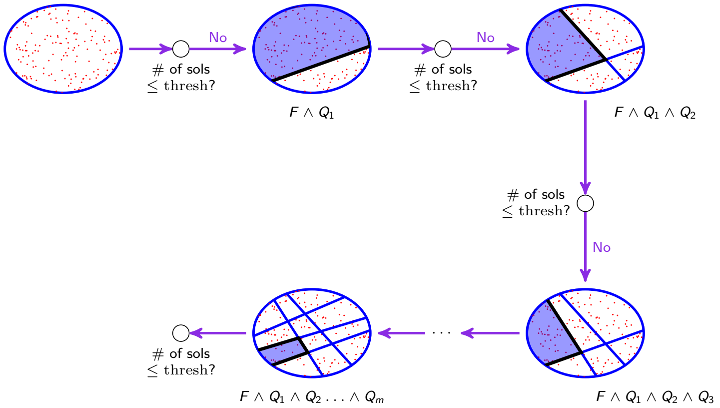

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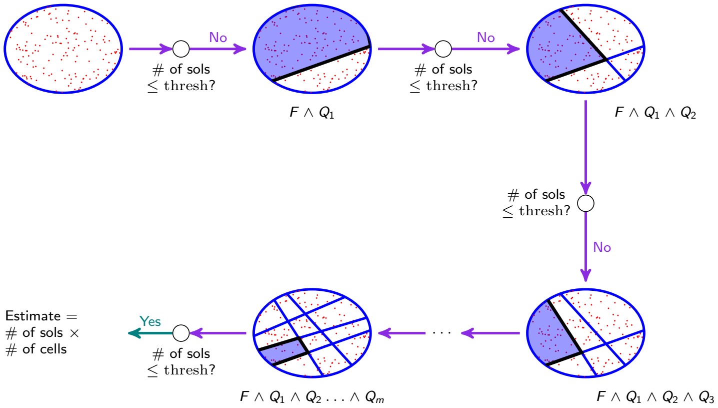

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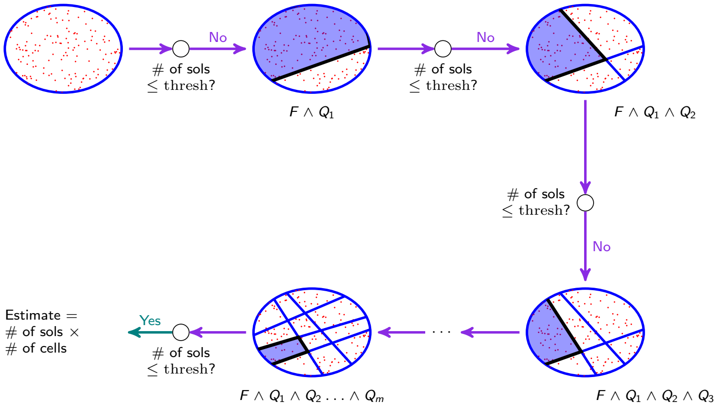

Theorem: $\Pr\left[\frac{|\mathsf{Sol}(F)|}{1+\varepsilon} \leq \mathsf{ApproxMC}(F, \varepsilon, \delta) \leq |\mathsf{Sol}(F)|(1+\varepsilon)\right] \geq 1 - \delta$

ApproxMC makes $O(\frac{1}{\varepsilon^2} \cdot \log \frac{1}{\delta} \cdot \log n)$ SAT queries.

ApproxMC: Early Years (2013-16)

Handle **reasonable** formulas: **reasonable** grids, **reasonable** programs

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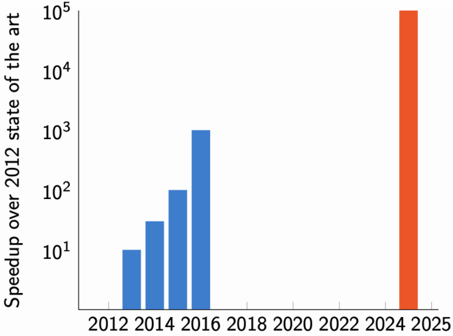
B. Cook: Virtuous cycle: application areas drives more investment in foundational tools, while improvements in the foundational tools drive further applications. Around and around.

The definition of "**reasonable**" changes after every iteration of the cycle

Closing Slide from Seminar at NUS in Feb 2017

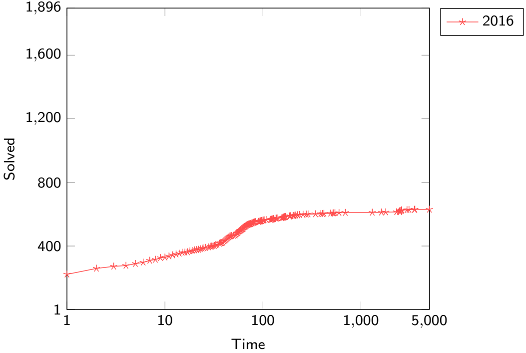
Mission 2025: Constrained Counting and Sampling Revolution



Requires combinations of ideas from theory, statistics and systems

2025 Target: 100× speedup over 2016

ApproxMC: In Pursuit of Scalability



1896 instances from diverse applications
All experiments on the same hardware

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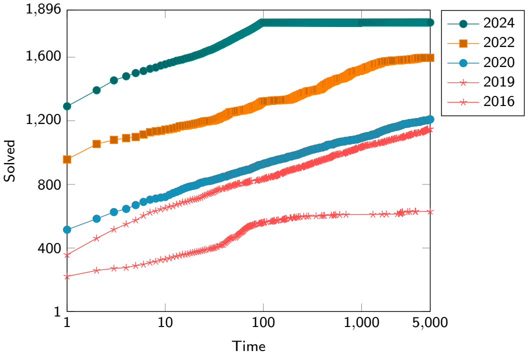

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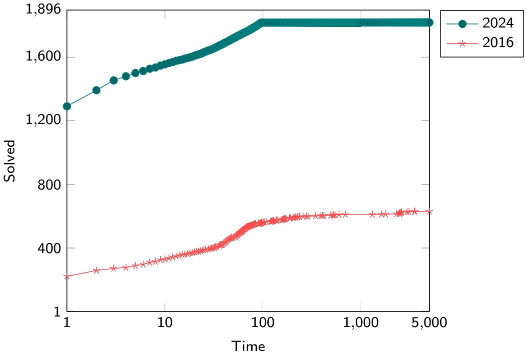

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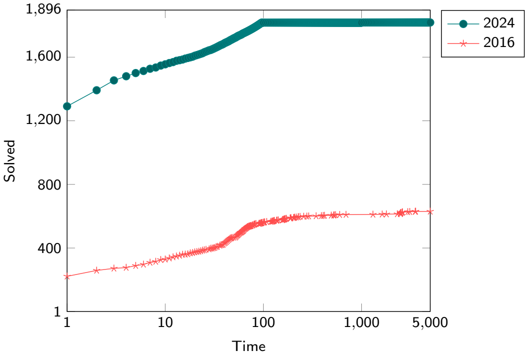

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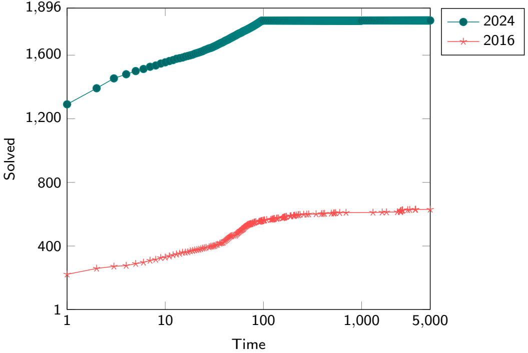

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Still provides (ε, δ)-guarantees

In Pursuit of Scalability

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Algorithmic Engineering	Indep Supp Constraints-16 ICCAD-22, DAC-24	Chain Formulas IJCAI-15 NeurIPS-20, SAT-25	Symmetry TACAS-20, AAAI-21	Prob. Caching IJCAI-19 AAAI-23, CAV-25
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Performance of state of the art SAT solvers depends on the formulas

SAT Solvers != SAT oracles

New Architecture for CNF-XOR Formulas

Modern SAT Solvers: Conflict-Driven Clause Learning (CDCL) paradigm

- Guess an assignment to subset of variables, if conflict, remember the reason
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level	dec		prop
0	x_1		
1	x_3	$\rightarrow$	x_5
2	x_4	$\rightarrow$	$x_2, \neg x_5$

CDCL

$$\left.\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{array}\right.$$

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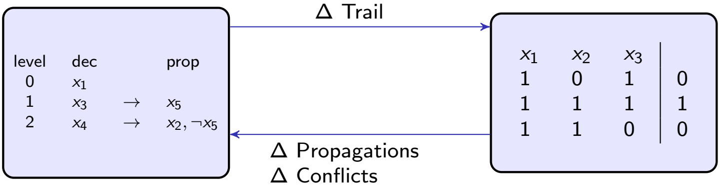

Incremental CDCL

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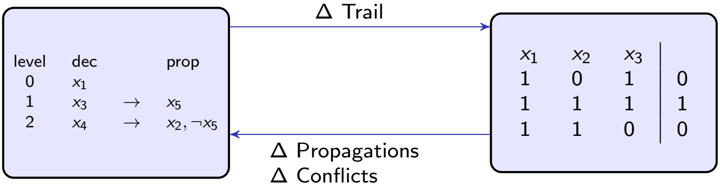

Incremental CDCL — Incremental Gauss-Jordan Elimination

Engineering an efficient CDCL-GJE solver [SM19; SGM20]

- Data-structures for efficient propagation and conflict analysis
- Supervised machine learning-guided heuristics

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Not All Variables Matter Equally

- Not all variables are required to specify solution space of F
  - $F := X_3 \iff (X_1 \vee X_2)$
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- Approach I: $\log n$ hard calls to SAT solver via reduction to GMUS [IMMV15]
  Best Student Paper, CP15
- Approach II: n easy calls to SAT solver via Padoa's theorem [SM22; SM24]

Approach II + ApproxMC is up to 100× faster than Approach I + ApproxMC

SAT Solvers $\neq$ SAT Oracles

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Algorithmic Engineering Pick $X_i \in \mathcal{I}$

Challenge 3 Do we have to really pick every variable X_i with **prob** $\mathbf{\frac{1}{2}}$?

- If we pick with prob $p < \frac{1}{2}$, then no guarantees of 2-wise independence
- Z_m : Number of solutions in a randomly chosen cell
- 2-wise independence $\implies \frac{\mathsf{Var}[Z_m]}{\mathsf{E}[Z_m]} \leq 1 \implies$ Concentration bounds

Beyond 2-wise Independence

[MA20]

Open problem (2013-19): Sparse ($p < \frac{1}{2}$) XORs that work in theory and practice

Beyond 2-wise Independence

[MA20]

Open problem (2013-19): Sparse ($p < \frac{1}{2}$) XORs that work in theory and practice

Theorem (Log-Sparse XORs suffice)

If we pick m-th XOR with $p_m = \frac{\log m}{m}$, *we have* $\frac{\mathsf{Var}[Z_m]}{\mathsf{E}[Z_m]} \leq 1.1$

Improvement of p from $\frac{m/2}{m}$ *to* $\frac{\log m}{m}$

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$$\frac{\mathrm{Var}[Z_m]}{\mathrm{E}[Z_m]} \leq 1 + |\mathrm{Sol}(F)|^{-1} \cdot \sum_{\sigma_1 \in \mathrm{Sol}(F)} \sum_{\substack{\sigma_2 \in \mathrm{Sol}(F) \\ w = d(\sigma_1, \sigma_2)}} \overbrace{r(w, p_m)}^{\approx \text{ collision probability}}$$

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[EGSS14,ZCSE16,AD17,ATD18]

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$$\text{Rewrite} \sum_{\sigma_1 \in \mathrm{Sol}(F)} \sum_{\substack{\sigma_2 \in \mathrm{Sol}(F) \\ w = d(\sigma_1,\sigma_2)}} r(w, p_m) = \sum_{w=0}^{n} C_F(w) r(w, p_m)$$

$$C_F(w) = \left| \{\sigma_1, \sigma_2 \in \mathrm{Sol}(F) \mid d(\sigma_1, \sigma_2) = w\} \right|$$

Isopmerimetric Inequalities: Possible to bound $C_F(w)$ if bound on $|\mathrm{Sol}(F)|$ is known

Barrier: But $|\mathrm{Sol}(F)|$ can be arbitrarily large

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Barrier: But $|\mathrm{Sol}(F)|$ can be arbitrarily large

Key Idea: In the context of Z_m, It suffices to assume $|\mathrm{Sol}(F)| < 2^{m+u}$ for small u.

Challenges in Pursuit of Scalability

How to partition into roughly equal small cells of solutions without knowing the distribution of solutions?

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Theoretical Advances Pick m-th XOR with $p_m = \frac{\log m}{m}$

In the Pursuit of Scalability

Theoretical Advances	Sparse hashing SAT-20 LICS-20	Duality CP-19	DNF CP-18,IJCAI-19 PODS-21,22	Phase Transition IJCAI-16,17,19 CP-20
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Software Development	CNF-XOR AAAI-19 CAV-20, CAV-24	Pseudo-Boolean CP-21 AAAI-24, AAAI-25 SAT-25	MaxSAT-XOR KR-21	Hardware Accelerator SAT-21

Reliability of Critical Infrastructure Networks

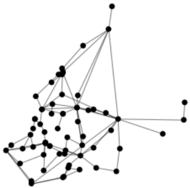
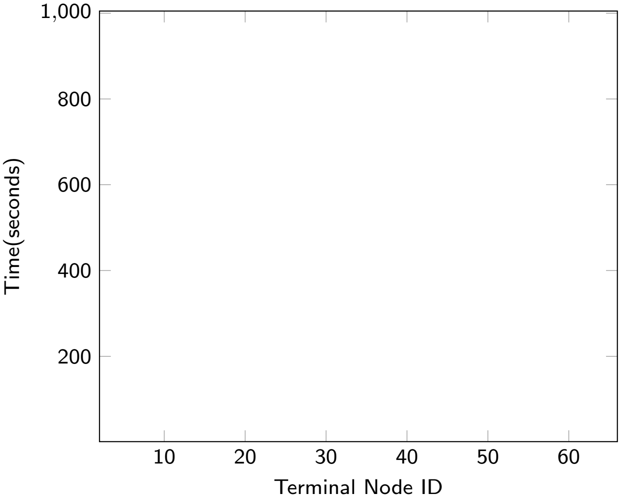
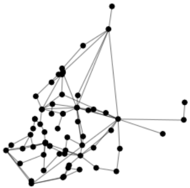


Figure: Plantersville, SC



Timeout = 1000 seconds

Reliability of Critical Infrastructure Networks



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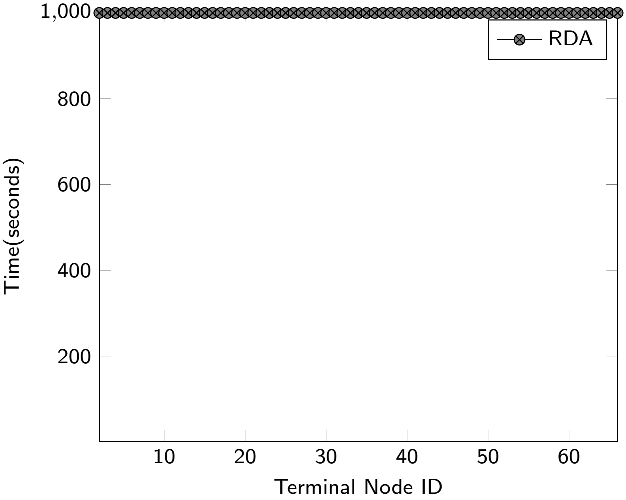

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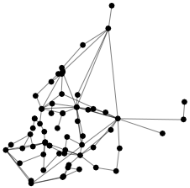

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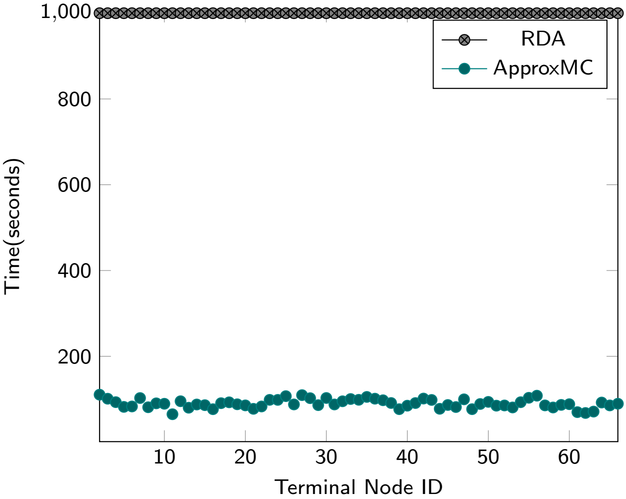
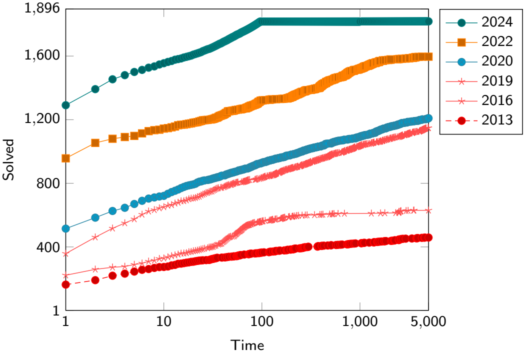


Timeout = 1000 seconds

Impact: The first theoretically sound estimates of resilience in power transmission networks of ten medium sized cities in US

ApproxMC: Progress over the years



1896 benchmarks from diverse applications

Time taken (seconds) for an instance

2016: 3552.16 2019: 32.83 2020: 19.59 2022: 0.15 2024: 0.02

A speedup of 100,000× over 2016

Another Iteration of Virtuous Cycle

B. Cook, 2022: Virtuous cycle: ...application areas drives more investment in foundational tools, while improvements in the foundational tools drive further applications. Around and around.

SharpTNI: Counting and Sampling Parsimonious Transmission Networks under a Weak Bottleneck

Palash Sashittal[1] and Mohammed El-Kebir[2*]

Check before You Change: Preventing Correlated Failures in Service Updates

Ennan Zhai[†], Ang Chen[‡], Ruzica Piskac[○], Mahesh Balakrishnan[§,*]
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Automating the Development of Chosen Ciphertext Attacks

Gabrielle Beck, Maximilian Zinkus, and Matthew Green,
Johns Hopkins University

Static Evaluation of Noninterference using Approximate Model Counting

Ziqiao Zhou, Zhiyun Qian, Michael K. Reiter, Yinqian Zhang

A Study of the Learnability of Relational Properties

Model Counting Meets Machine Learning (MCML)

Muhammad Usman, University of Texas at Austin, USA, muhammadusman@utexas.edu
Wenxi Wang, University of Texas at Austin, USA, wenxiw@utexas.edu
Marko Vasic, University of Texas at Austin, USA, vasic@utexas.edu
Kaiyuan Wang[*], Haris Vikalo, Sarfraz Khurshid

Quantifying Software Reliability via Model-Counting

Samuel Teuber[(✉)] and Alexander Weigl

IN SEARCH FOR A SAT-FRIENDLY BINARIZED NEURAL NETWORK ARCHITECTURE

Nina Narodytska, **Hongce Zhang[*]**

Quantifying the Efficacy of Logic Locking Methods

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Model Counting Competition

MC-2024

Part of SAT 2024

- Tracks: **Model Counting**, **Weighted Model Counting**, **Weighted Model Counting (negative weights)**, **Projected Model Counting**, **Projected Weighted Model Counting**
(description, StarExec Community, format, winners, report [tba], slides, solvers, competition instances, submitted benchmarks

MC-2023

Part of SAT 2023

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MC-2022

Part of FLoC Olympic Games

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(description, StarExec Community, format, winners, report, slides, solvers, competition instances, submitted benchmarks)

MC-2021

- Tracks: **Model Counting**, **Weighted Model Counting**, and **Projected Model Counting**
(description, StarExec Community, format, winners, report, slides, solvers, competition instances, submitted benchmarks)

MC-2020

The results of each Model Counting Competition are documented in a report.

- Tracks: **Model Counting**, **Weighted Model Counting**, and **Projected Model Counting**
(description, format, winners, report, slides, solvers, competition instances, submitted benchmarks)

Workshops

- 2024 Workshop on Counting, Sampling, and Synthesis (in person event)
(program)
- 2023 Workshop on Counting and Sampling (in person event)
(program)
- 2022 Workshop on Counting and Sampling (in person event)
(program, abstracts, slides)
- Workshop 2021
(program, abstracts, slides , recordings)
- 2020 Model Counting Workshop
(program, abstracts, slides , recordings)

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Generalizability

Union of Sets ApproxMC is Fully Polynomial Randomized Approximation Scheme (FPRAS) – fundamentally different from the Monte-Carlo based FPRAS

- IJCAI-19 Sister Conferences Best Paper Award Track [**M**SV19]
- Praise by Donald Knuth and coverage in popular media (Quanta Magazine) [CV**M**22]

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Streaming Counting over a stream: Distinct Elements
Example: How many unique customers visit website?
Fundamental problem in Databases

- CACM 2023 Research Highlights [PVB**M**21]
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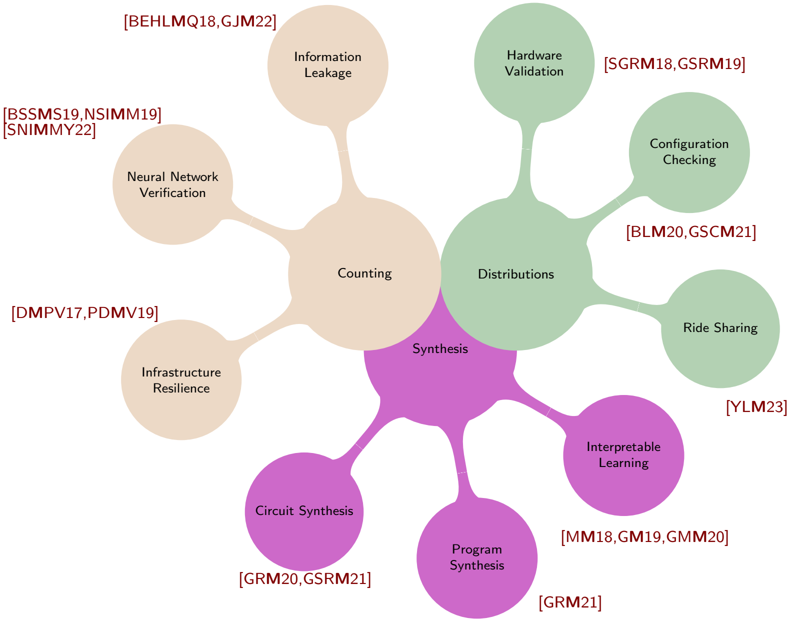
Unsatisfiable Subsets Count minimal subsets of clauses that are unsatisfiable.
Diagnosis metric for systems

- ICLP-24 Best Paper Award [K**M**24]

Counting, Sampling, and Synthesis



In Pursuit of Scalability

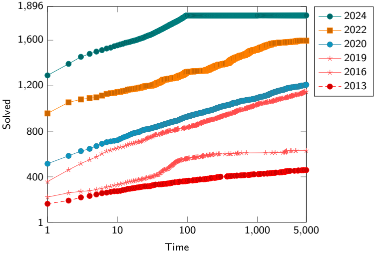
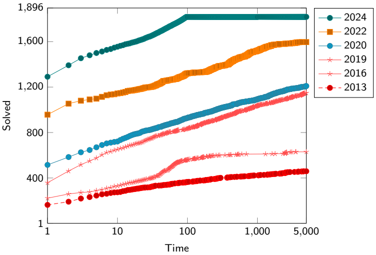


Figure: Counting over the years

In Pursuit of Scalability



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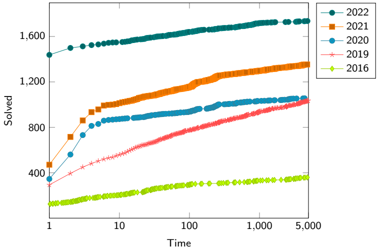
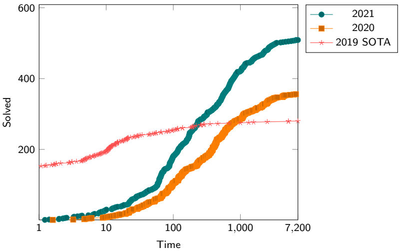


Sampling over the years



Synthesis over the years

ICCAD-21 & DATE-23 Best Paper Award Nomination

Where do we go from here?

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The Quest for Scalability is Endless

B. Cook, 2022: Virtuous cycle: ...application areas drives more investment in foundational tools, while improvements in the foundational tools drive further applications. **Around and around**.

Today's Quantitative Automated Reasoning tools

$$\approx$$

Qualitative Reasoning tools (SAT/SMT) in early 2000s

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Qualitative Reasoning tools (SAT/SMT) in early 2000s

The Time is Now

- LLMs enable rapid system creation $\rightarrow$ Critical need for verification
- The achilees heel of formal methods adoption: formalization of complex systems
- LLMs excel at enabling rapid formalization of complex systems

This is the perfect time to do automated reasoning!

Industrial Practice: 100$\times$ Speedup

The Pursuit of Scalablity

Mission 2030: 100$\times$ Speedup for Quantitative Reasoning Tasks

Challenge Problems (for Counting)

Civil Engineering Rigorous resilience estimation for power grid of Los Angeles

AI Safety Quantized neural network with 1M neurons

Quantum Simulation Quantum Simultion for 1000 qubits.

Technical Directions (for Counting)

Theoretical Advances Native reasoning over expressive theories (*Beyond SMT*)

Algorithmic Engineering Machine Learning-guided heuristic design

Software Development Hardware-accelerator aware tools

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Certification: Approximate count is "correct" or the distribution generated is correct

- Applications to verification of probabilistic programming
- Building on recent advances in distribution testing
- Preliminary Work: AAAI-19, NeurIPS-20, NeurIPS-21, CP-22, NeurIPS-22, ICML-24, ICLR-25

It Takes a Village

Research Group

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Bhavishya
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Bishwamittra Ghosh
Priyanka Golia
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Gunjan Kumar
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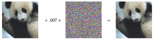
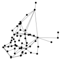
Kaiyuan Wang(Google,US)
Ziwei Xu(NUS, SG)

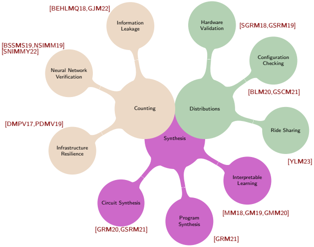
Funding Agencies: National Research Foundation, Ministry of Education, Defense Service Organization, NSERC
Industrial Support: Grab Taxiholdings, Amazon, Microsoft Research

Quantitative Automated Reasoning

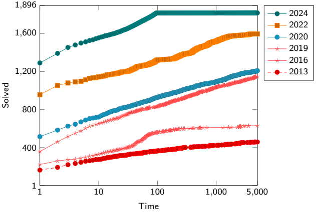
```
PC2 (char[] SP, char[] UI) {
        match = true;
        for (int i=0; i<UI.length(); i++) {
                if (SP[i] != UI[i]) match=false;
                else match = match;
        }
        if match return Yes;
        else return No;
}
```







Theoretical Advances	Sparse hashing SAT-20 LICS-20	Duality CP-19	DNF CP-18,IJCAI-19 PODS-21,22	Phase Transition IJCAI-16,17,19 CP-20
Algorithmic Engineering	Indep Supp Constraints-16 ICCAD-22, DAC-24	Chain Formulas IJCAI-15 NeurIPS-20, SAT-25	Symmetry TACAS-20, AAAI-21	Prob. Caching IJCAI-19 AAAI-23, CAV-25
Software Development	CNF-XOR AAAI-19 CAV-20, CAV-24	Pseudo-Boolean CP-21 AAAI-24, AAAI-25 SAT-25	MaxSAT-XOR KR-21	Hardware Accelerator SAT-21



These slides are available at https://www.cs.toronto.edu/~meel/talks.html

Detailed Future Directions

Applications: Infrastructure Resilience, Information Leakage, Prob. Databases, Configuration Testing, Partition Function, BNN Verification

Theoretical Advances

Formula-based Sparse-XORs DNF, Minimal Solutions, Chain formula
Revisiting FPRAS Permanent, Automata, Linear Extensions
Parameterized Complexity Addition of XORs
Streaming Delphic Sets
Synthesis A theory of learning from relations
Entropy Reduction in the number of queries

Algorithmic Engineering

Incremental Incremental Counting Queries
Bit-vectors Partitioning; Independent Support
Heuristic ML-guided heuristic synthesis
Distributed Streaming techniques
SMT Synthesis SMT Formula Learning
Beyond Qualititative Synthesis Optimal Functions, Approximate Synthesis

Software Development

Tighter Integration Multiple Queries
Hybrid Constraints Callbacks
XOR Handling PB-XOR, BNN-XOR, MaxSAT-XOR, ASP-XOR
Accelerators GPU
Knowledge Compilation SMT, Portfolio

Certification

Distribution Probabilistic Programming Equivalence
Counting Certificate for Approximation