Designing Scalable Techniques for Dynamic Verification and Probabilistic Inference

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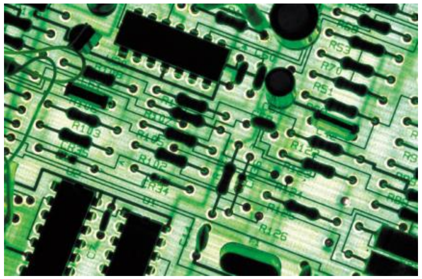
Joint work with Alexander Ivrii (IBM), Supratik Chakraborty (IITB), Daniel J. Fremont(UCB), Sharad Malik (Princeton), Sanjit A. Seshia (UCB), Moshe Y. Vardi (Rice)

How do we guarantee that systems work *correctly* ?



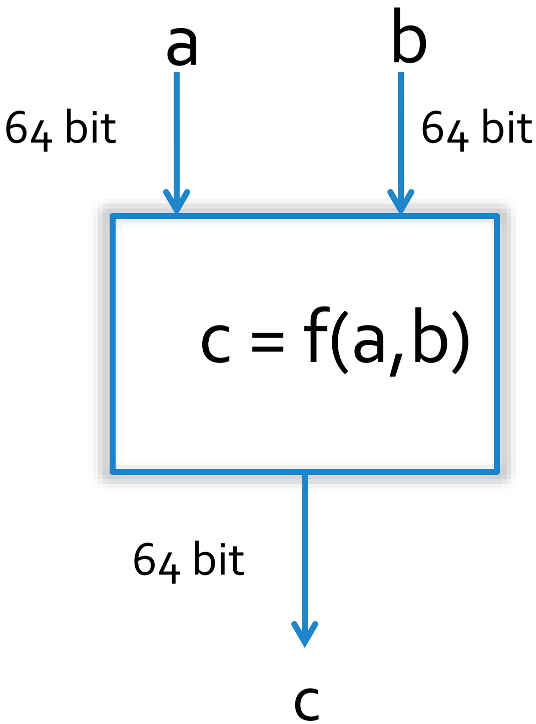
Functional Verification

- Formal verification
  - Challenges: formal requirements, scalability
  - 10-15% of verification effort (my estimate)
- Dynamic verification: ***dominant approach***

Dynamic Verification

- Dominant approach!
- Design is simulated with test vectors
- Test vectors represent different verification scenarios
- Results compared to intended results
- **Challenge**: Exceedingly large test space!

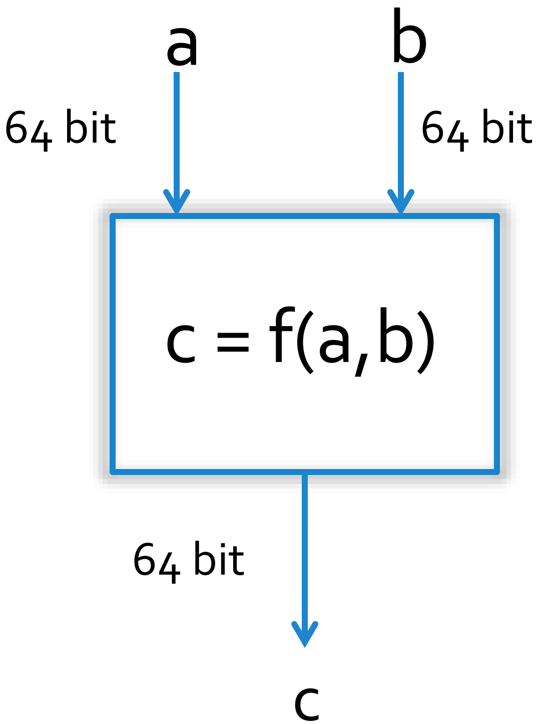
Motivating Example



How do we verify that circuit works ?

- Try for all values of a and b
  - 2^{128} possibilities
  - Sun will go nova before done!
  - Not scalable

Constrained-Random Simulation



Sources for Constraints

- Designers:
  1. $a +_{64} 11 *_{32} b = 12$
  2. $a <_{64} (b >> 4)$
- Past Experience:
  1. $40 <_{64} 34 + a <_{64} 5050$
  2. $120 <_{64} b <_{64} 230$
- Users:
  1. $232 *_{32} a + b != 1100$
  2. $1020 <_{64} (b /_{64} 2) +_{64} a <_{64} 2200$

- Test vectors: solutions of constraints

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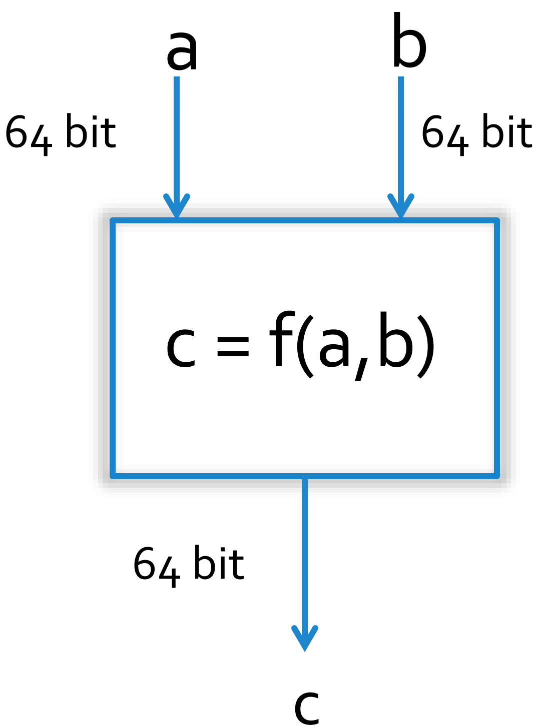

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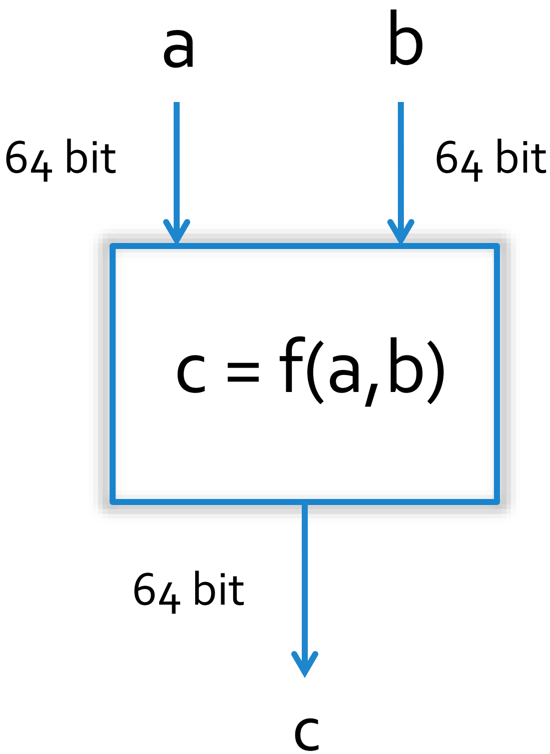
- Test vectors: solutions of constraints
  - Proposed by Lichtenstein, Malka, Aharon (IAAI 94)

Constraint satisfaction for random stimuli generation

Yehuda Naveh
IBM Haifa Research Lab

Constrained-Random Simulation



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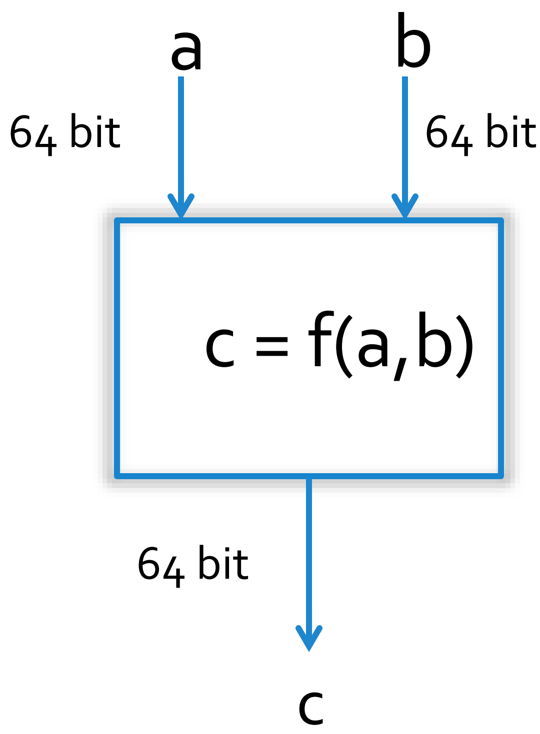
Problem: How can we *uniformly* sample the values of a and b satisfying the above constraints?

Problem Formulation



Set of Constraints

↓

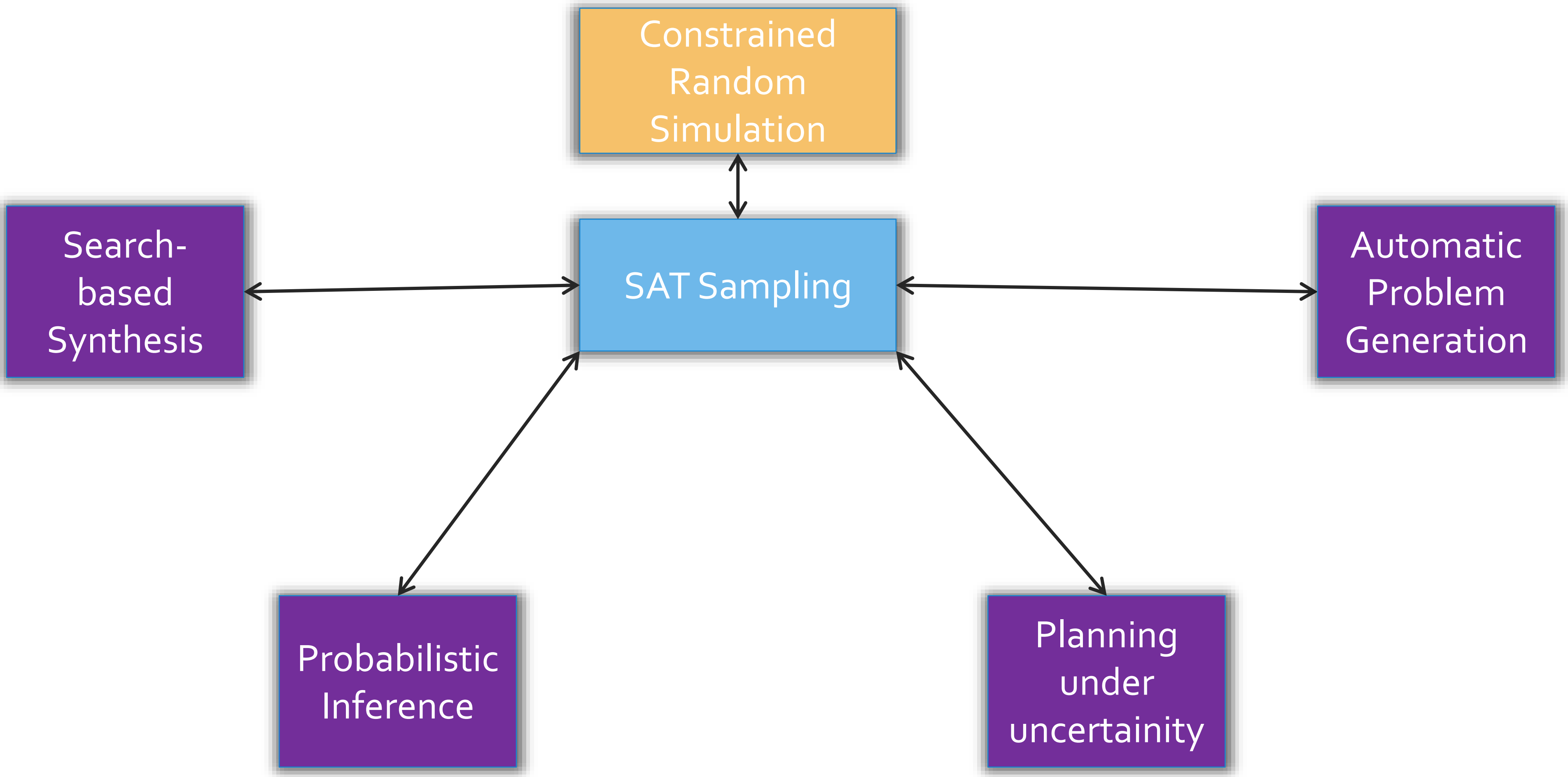
SAT Formula

↓

Given a SAT formula, sample solutions uniformly, while scaling to real world problems.

Scalable Uniform Generation of SAT-Witnesses

Diverse Applications



Search-Based Synthesis

- **Goal**: synthesize from under-constrained specifications (“sketch”)
- Large space of programs that satisfy correctness conditions
- Task: Find “optimal” program (wrt running time, memory, …)
- Method: ***Uniformly sample*** from the space of programs

Outline

- **Sampling Techniques for Dynamic Verification**
- Extension to approximate probabilistic inference
- Construction of Efficient Hashing functions
- Future Directions

Uniform Generation

Ref: “A Scalable Near-Uniform Generator” (CAV 2013)
“Balancing Scalability and Uniformity in SAT-Witness Generator” (DAC 2014)
“On Parallel Scalable Generation of SAT-Witnesses” (TACAS 2015)

Prior Work

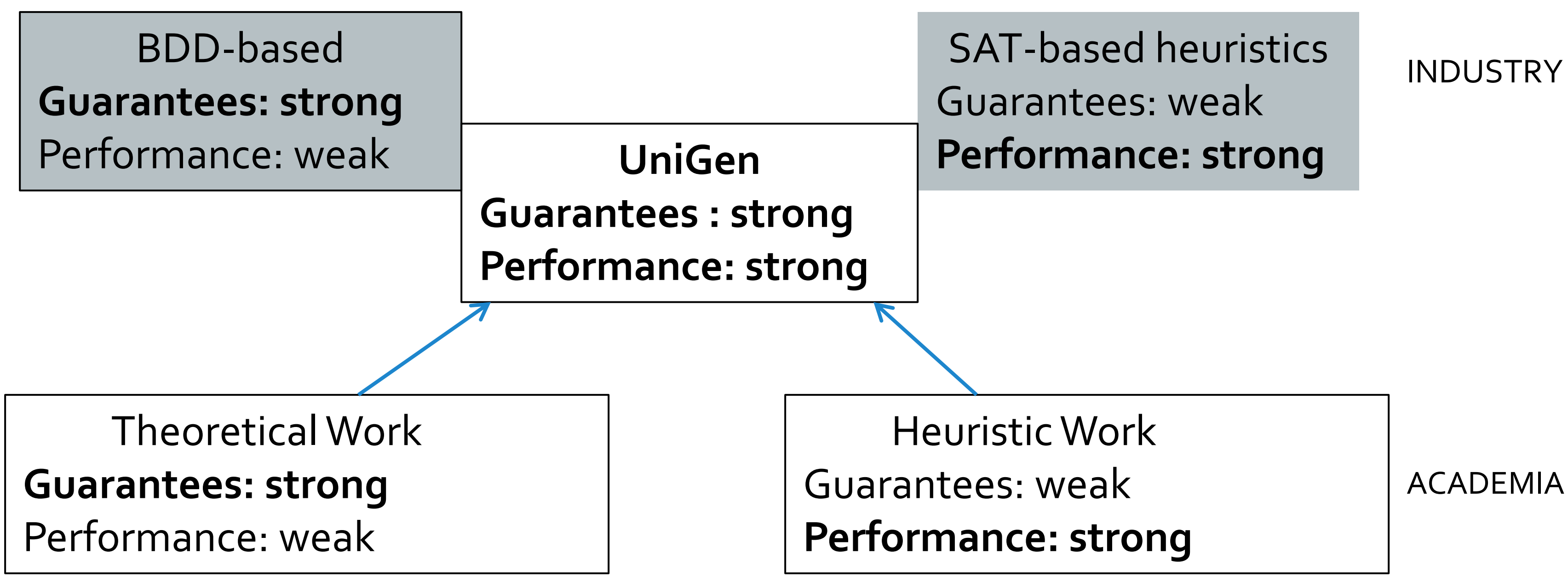
BDD-based		SAT-based heuristics	
Guarantees: strong		Guarantees: weak	INDUSTRY
Performance: weak		**Performance: strong**	

Theoretical Work	Heuristic Work	
Guarantees: strong	Guarantees: weak	ACADEMIA
Performance: weak	**Performance: strong**	

BGP Algorithm

XORSample'

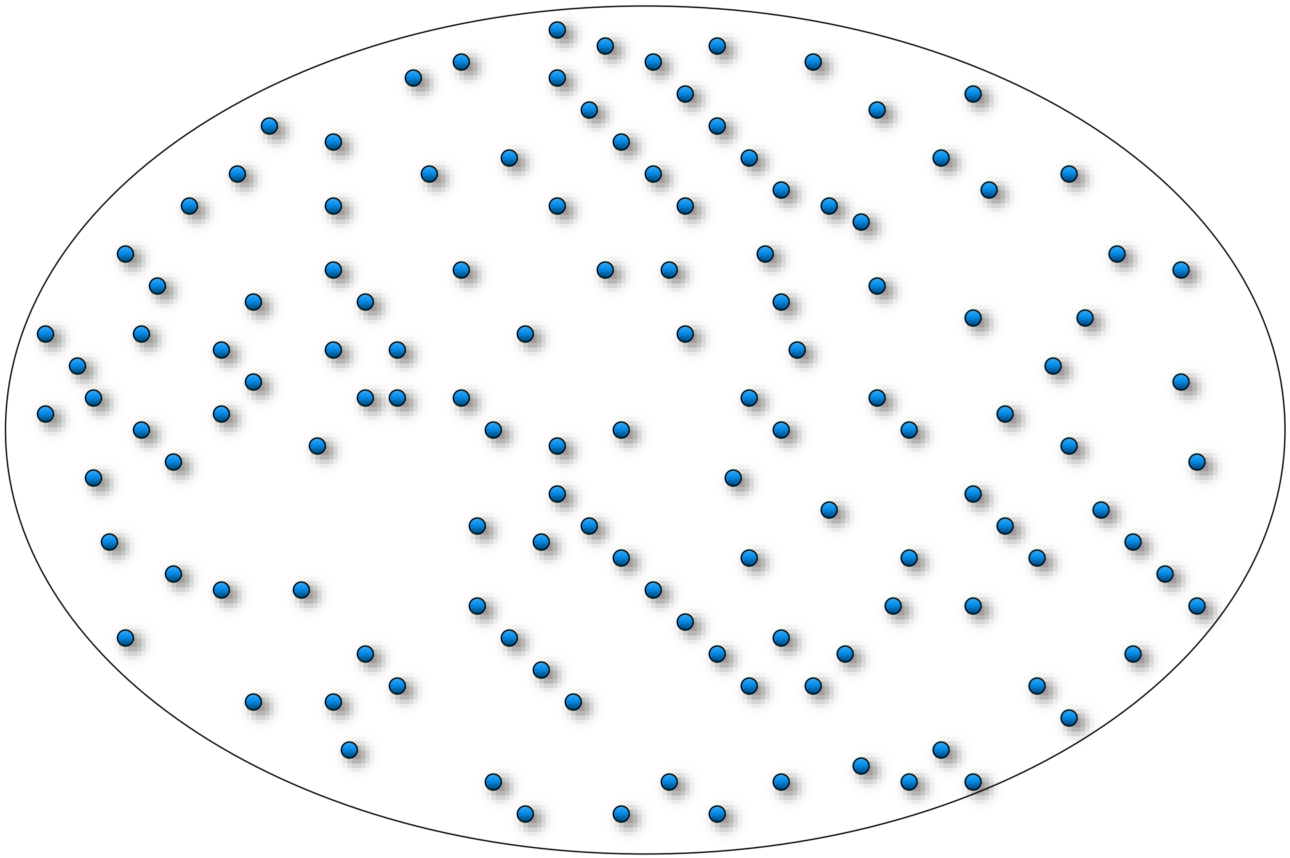
Our Contribution



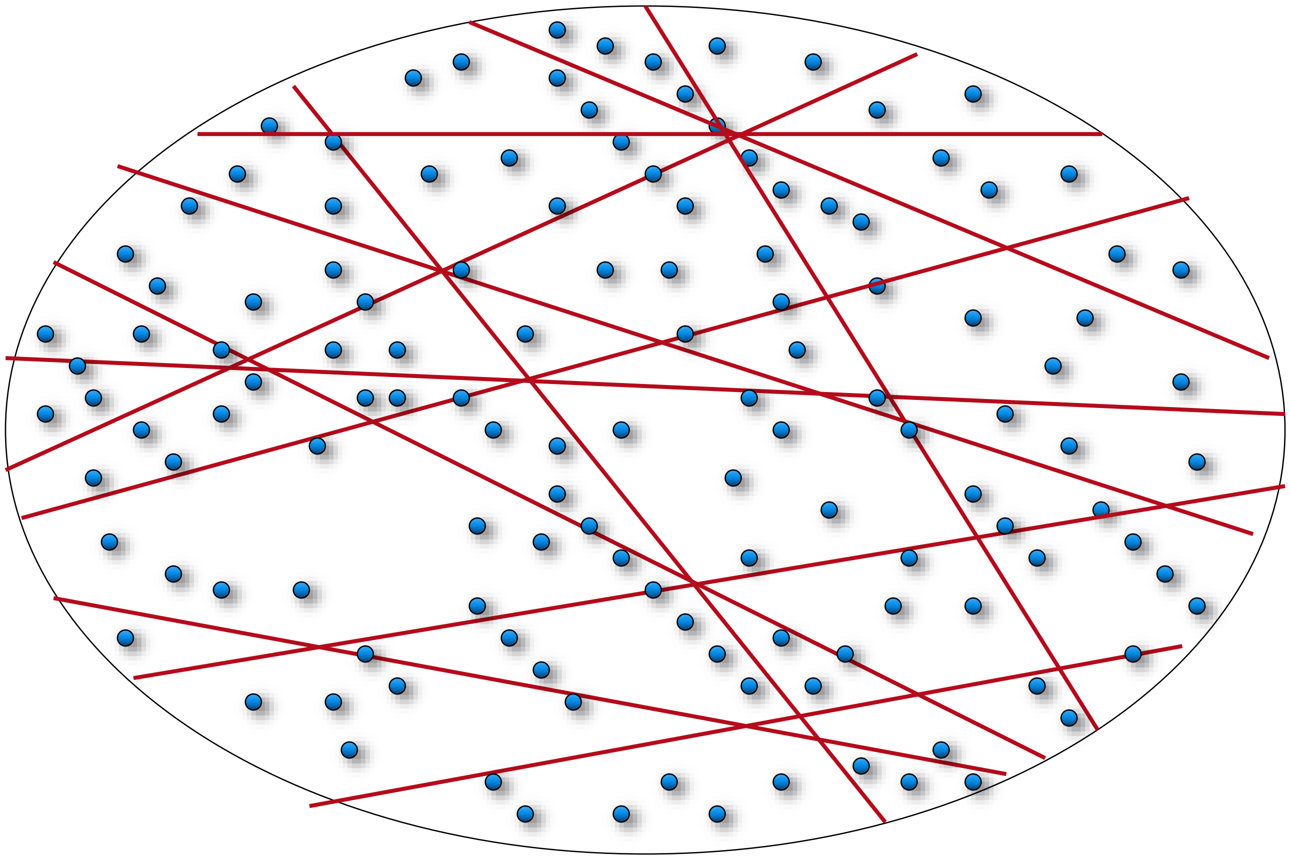
BGP Algorithm

XORSample'

Partitioning into equal “small” cells



Partitioning into equal “small” cells



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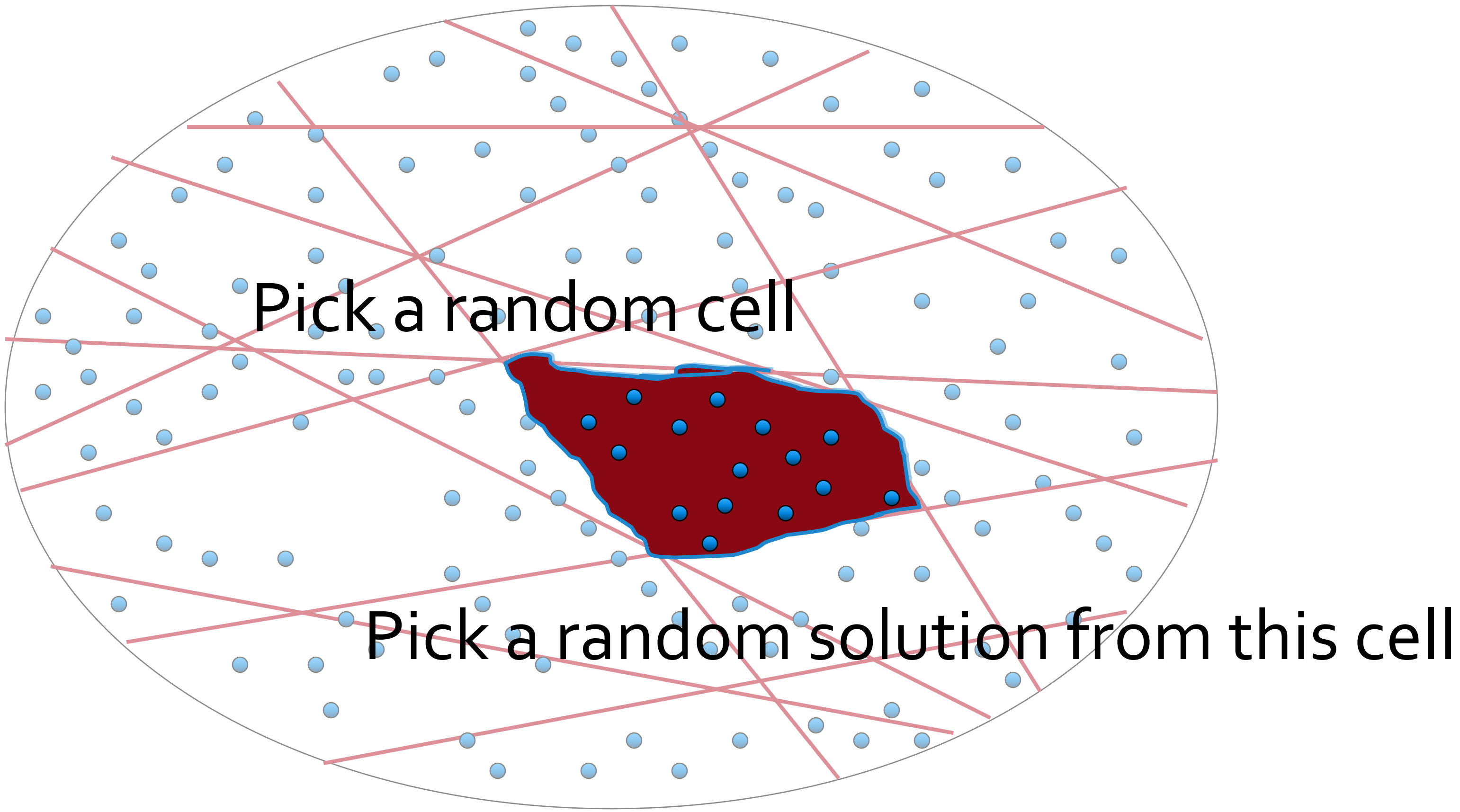


How to Partition?

How to partition into roughly equal small cells of solutions without knowing the distribution of solutions?

Universal Hashing
[Carter-Wegman 1979] (IBM Research)

Universal Hashing

- Hash functions: mapping $\{0,1\}^n$ to $\{0,1\}^m$
  - (2^n elements to 2^m cells)
- Random inputs => All cells are *roughly* equal (in <u>expectation</u>)
- Universal family of hash functions:
  - Choose hash function randomly from family
  - For ***arbitrary*** distribution on inputs => All cells are ***roughly equal*** (in <u>expectation</u>)

Universal Hashing and Independence

- Hash functions from mapping $\{0,1\}^n$ to $\{0,1\}^m$
  - (2^n elements to 2^m cells)
- Universal hash functions:
  - Choose hash function randomly
  - For arbitrary distribution on inputs => All cells are *roughly* equal in expectation
  - But:
    - While each input is hashed **uniformly**
    - Different inputs ***might not*** be hashed **independently**

Strong Universality

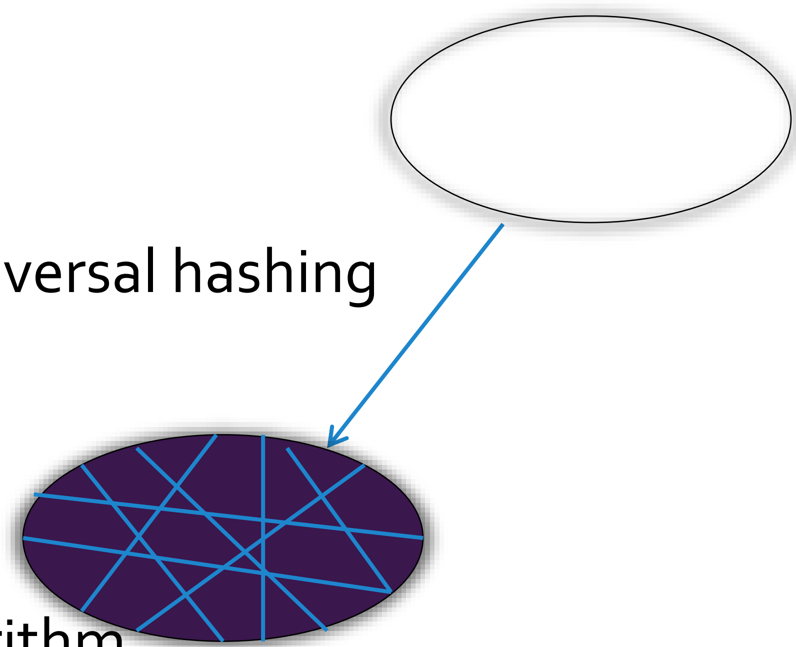
- H(n,m,r): Family of r-universal hash functions mapping $\{0,1\}^n$ to $\{0,1\}^m$ (2^n elements to 2^m cells)
  - r: degree of independence of hashed inputs

- Higher r => Stronger guarantee on ***range of size*** of cells

- r-wise universality => Polynomials of degree r-1

- Higher universality => Higher complexity

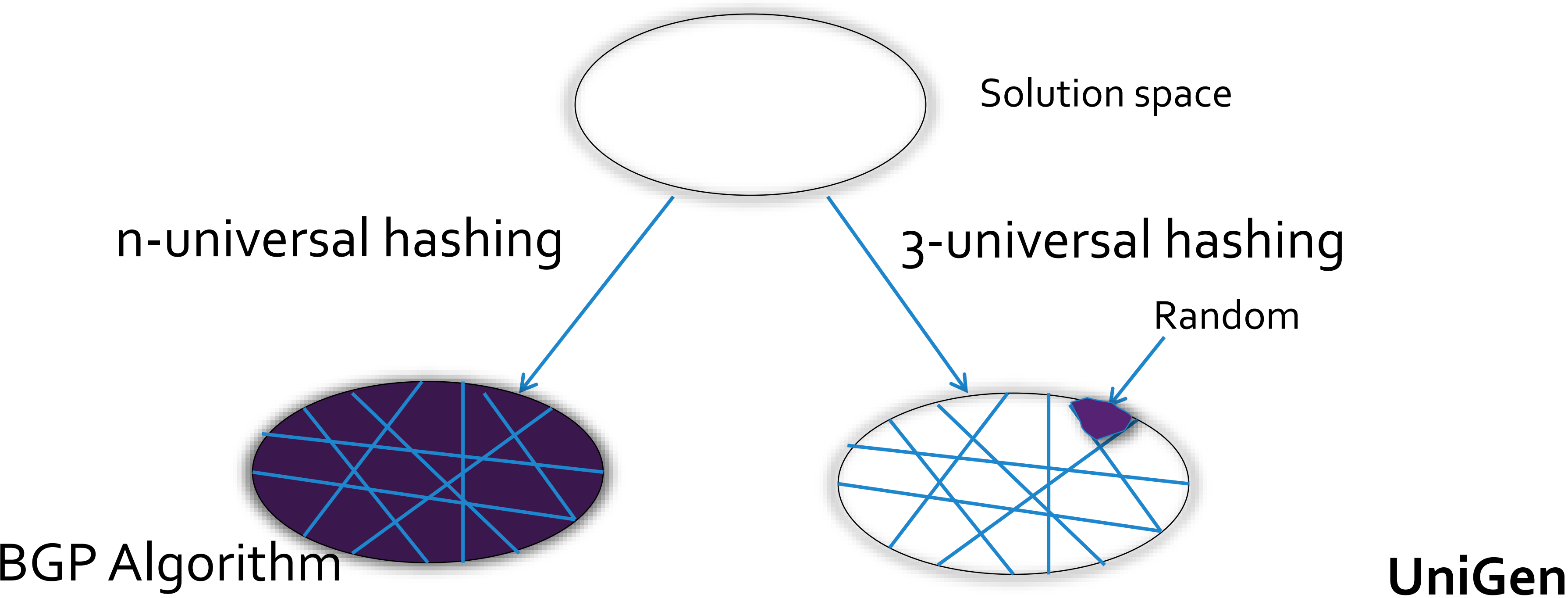
Hashing-based Approaches



All cells are small

Uniform Generation

Scaling to Thousands of Variables



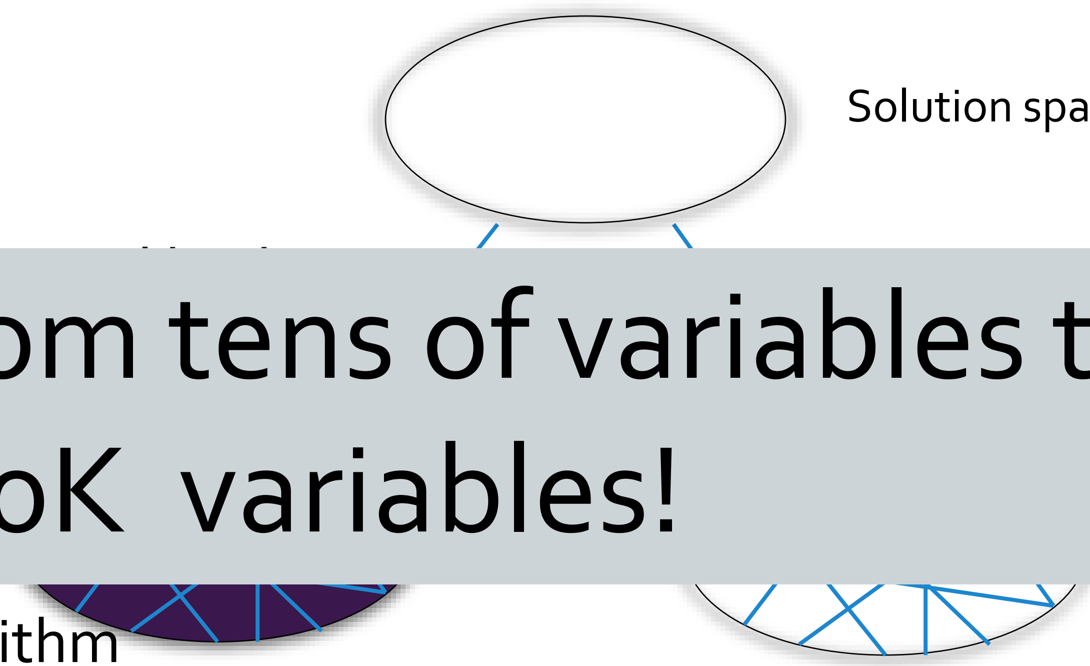
All cells are small

Only a randomly chosen cell needs to be "small"

Uniform Generation

Almost-Uniform Generation

Scaling to 100K Variables



From tens of variables to 100K variables!

All cells should be small

Only a randomly chosen cells needs to be "small"

Uniform Generation

Almost-Uniform Generation

Notions of Uniformity

- <u>Uniformity</u>

For every solution y of R_F

Pr [y is output] = $1/|R_F|$

- <u>Almost-Uniformity</u>

For every solution y of R_F

$1/(1+\varepsilon) \times 1/|R_F| <= Pr$ [y is output] $<= (1+\varepsilon)/|R_F|$

Partitioning

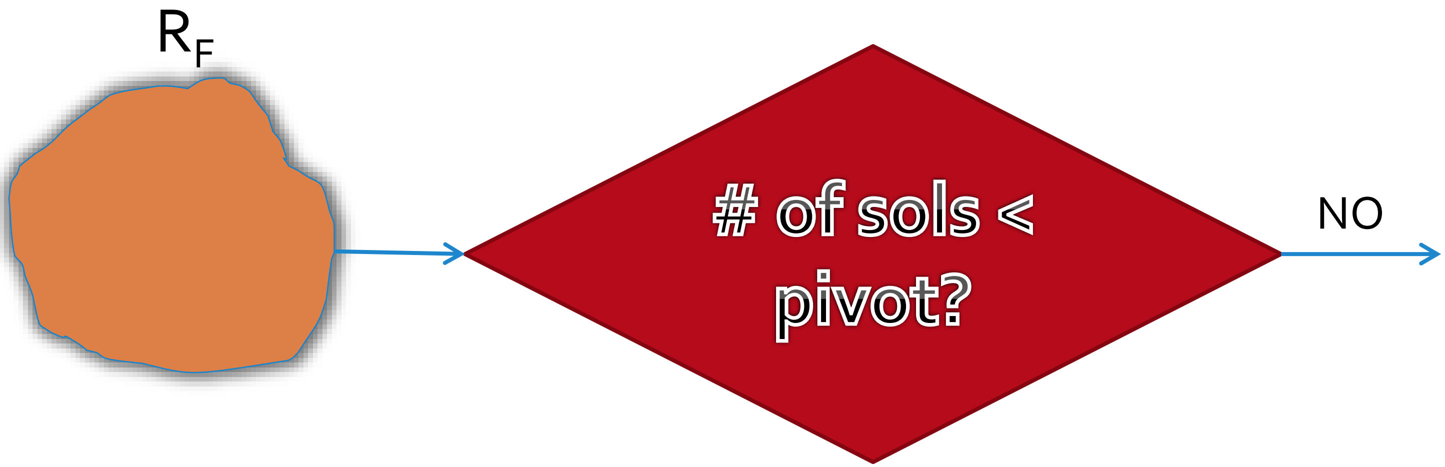
- How large should the cells be?
- How many cells?

Size of cell

- Too large => Hard to enumerate
- Too small => Variance can be very high

$$\mathrm{pivot} = 5(1 + 1/\varepsilon)^2$$

UniGen



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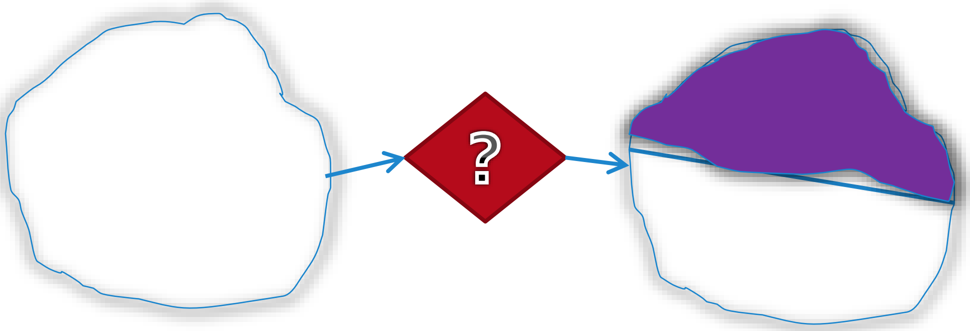

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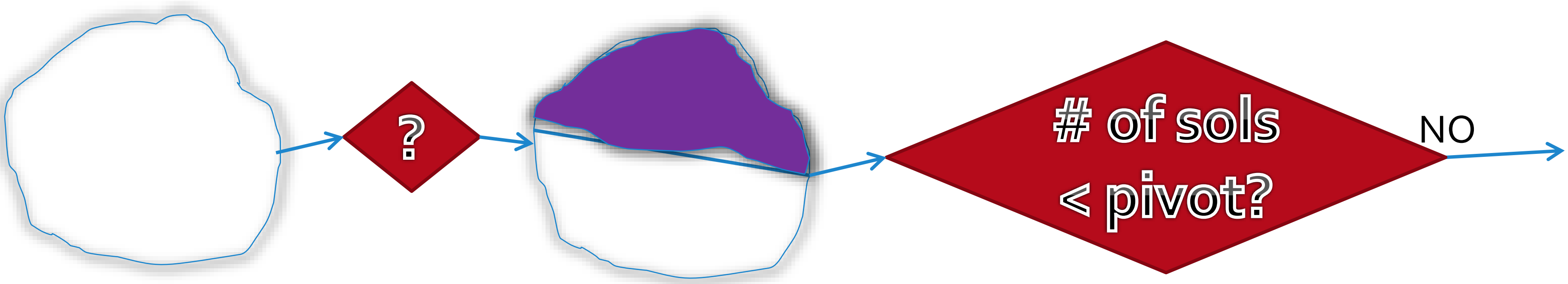

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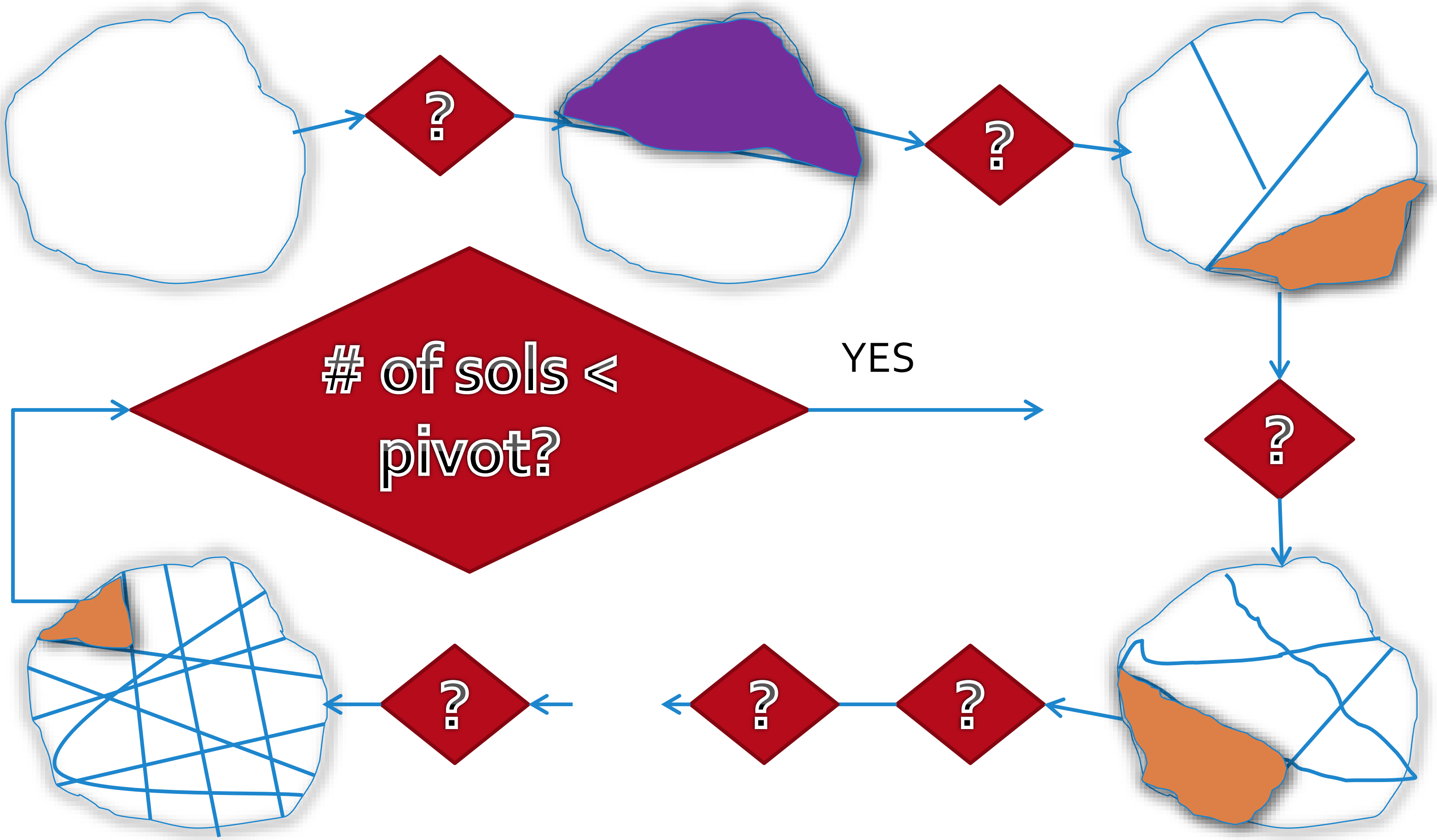

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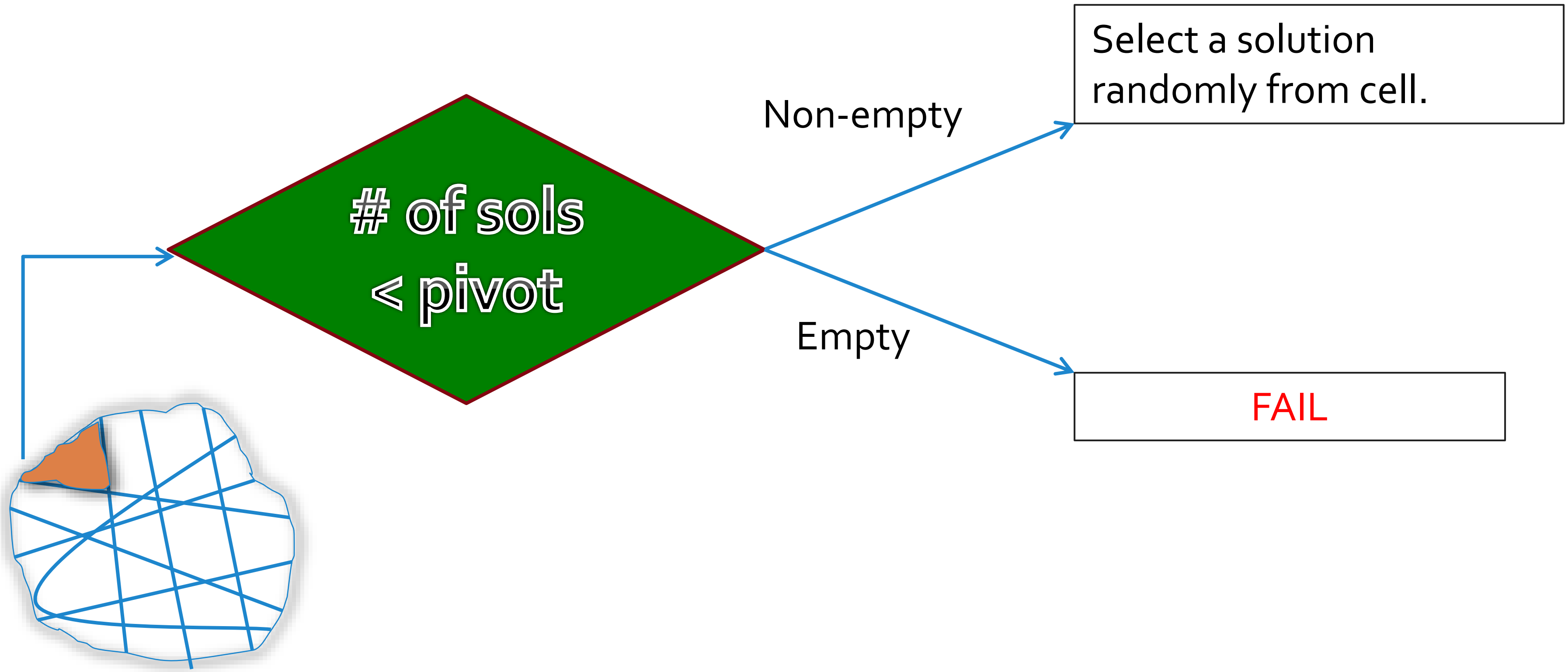


# of sols
< pivot
Non-empty
Select a solution randomly from cell.
Empty
FAIL

Strong Theoretical Guarantees

- Almost-Uniformity

For every solution y of R_F

$1/(6.84+\varepsilon) \times 1/|R_F| <= \Pr$ [y is output] $<= (6.84+\varepsilon)/|R_F|$

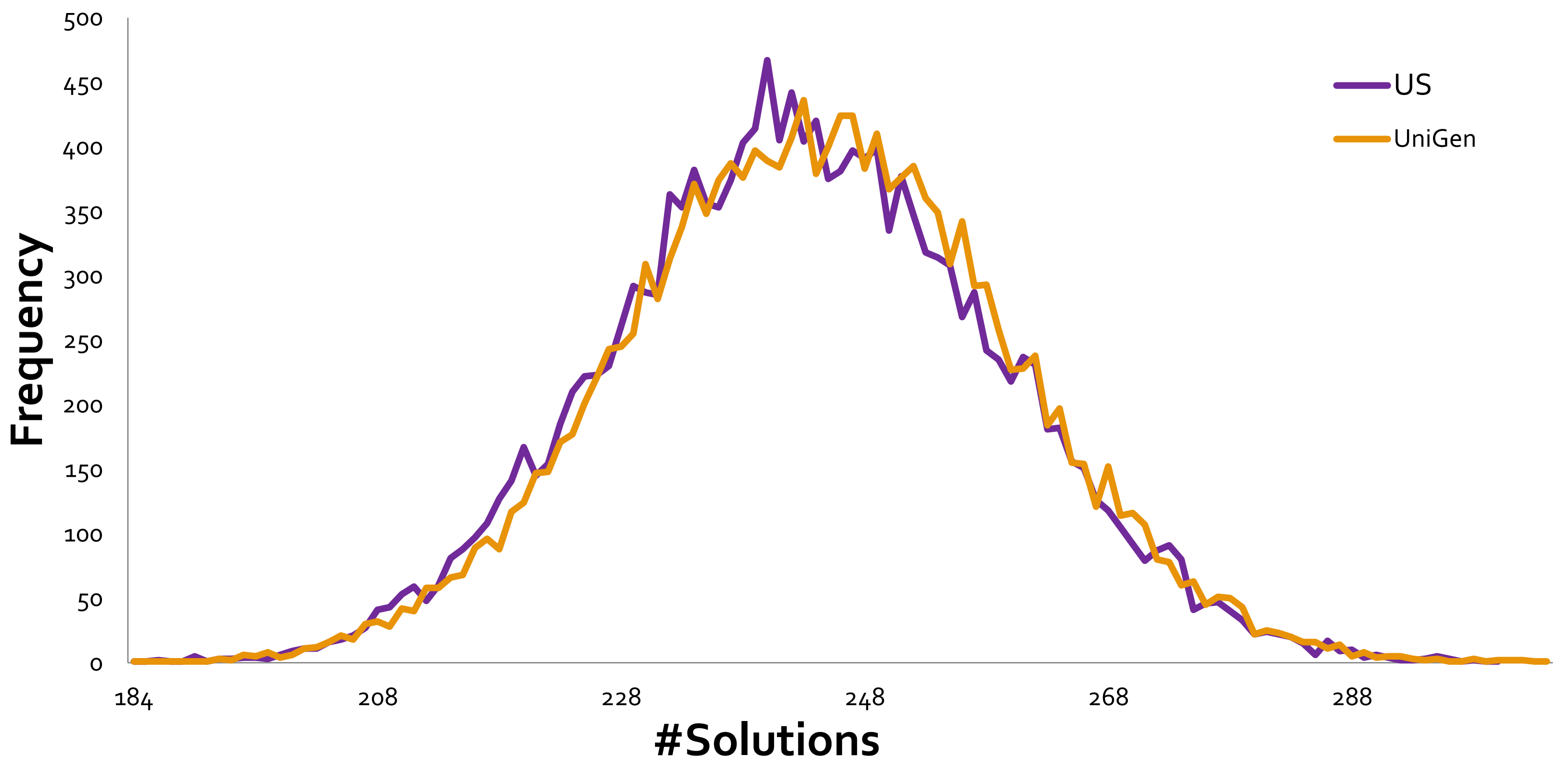
- Success Probability

UniGen succeeds with probability at least 0.52

  - In practice, succ. Probability ~ 0.99

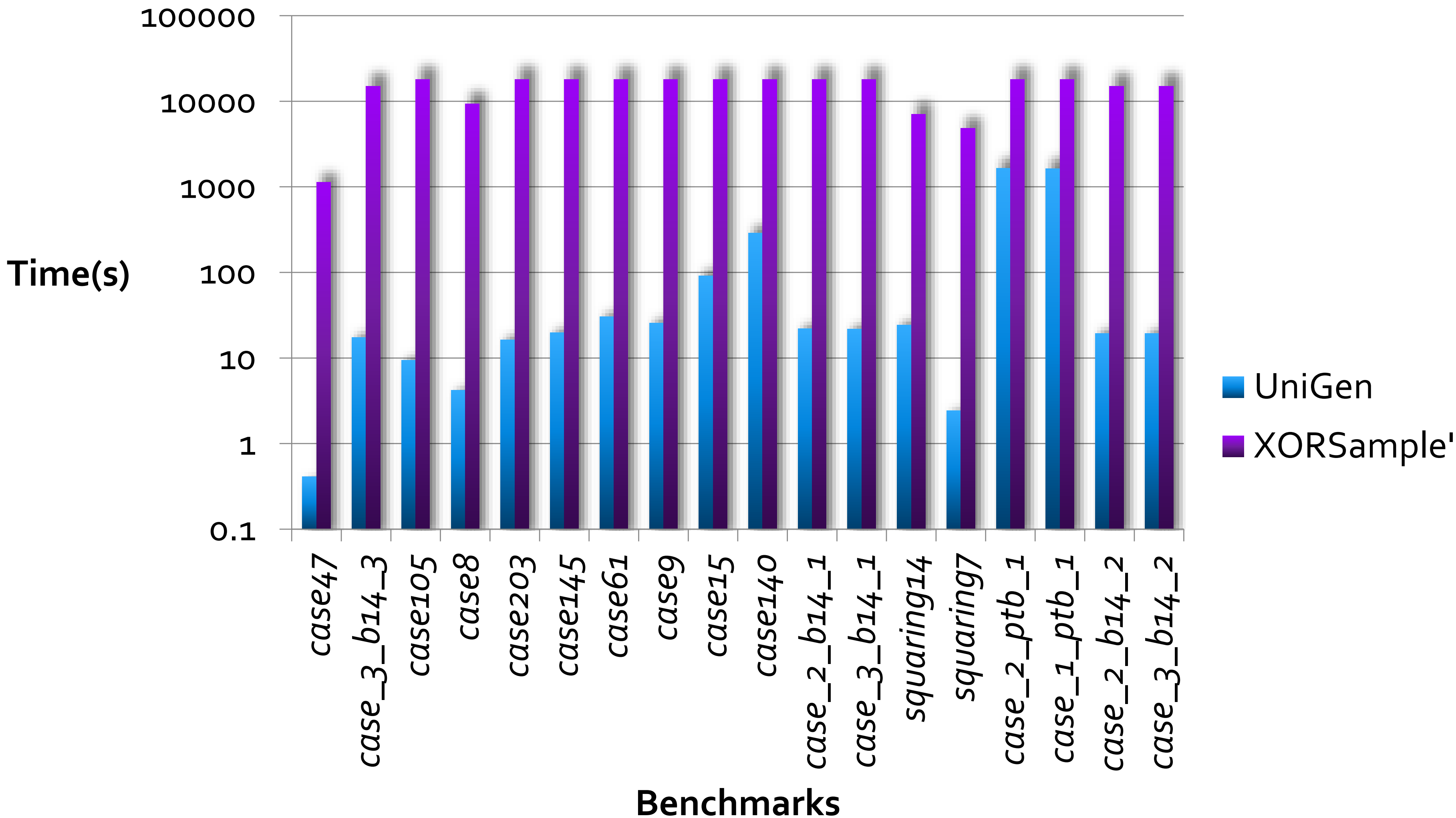
- Polynomial number of calls to SAT Solver

Results: Uniformity



- Benchmark: case110.cnf; #var: 287; #clauses: 1263
- Total Runs**: 4×10^6;** Total Solutions **: 16384**

2-3 Orders of Magnitude Faster



Outline

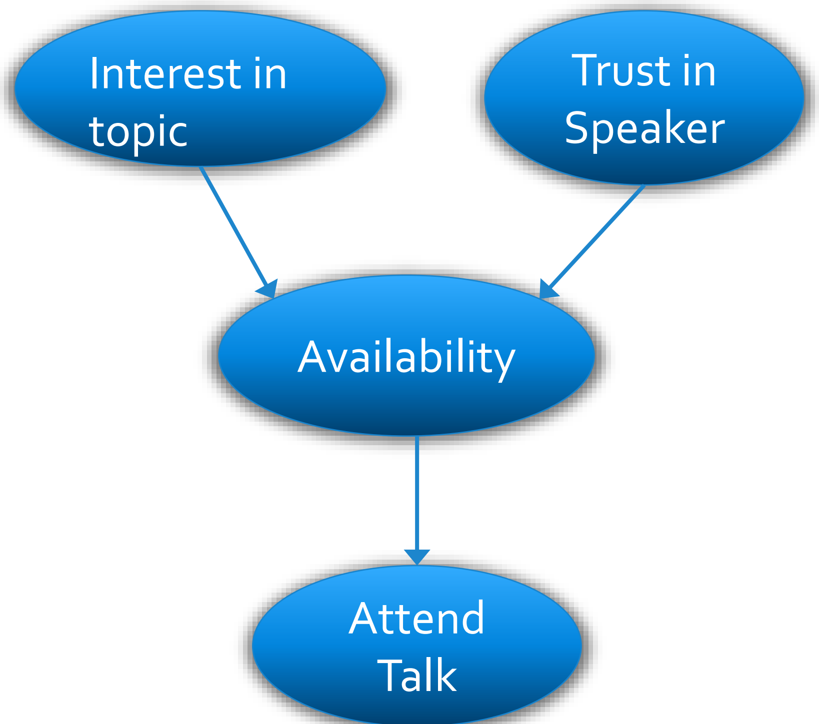
- Sampling Techniques for Dynamic Verification
- **Extension to approximate probabilistic inference**
- Construction of Efficient Hashing functions
- Future Directions

Extension to Approximate Probabilistic Inference

Ref: “A Scalable Approximate Model Counter” (CP 2013)

Probabilistic Inference

How do we infer useful information from the data filled with uncertainty?



Modeling Attendance for Today's Talk

+

Pr(Attending Talk |Interest in topic = True)

Roth, 1996

Model Counting

Model Counting

- Model Counting: Given a Boolean Formula F, count the number of models of F .

$$F = (a \lor b)$$

$$R_F := \{(a=0, b=1), (a=1, b=0), (a=1, b=1)\}$$

$$|R_F| = 3$$

- #P-complete
  - #P: Class of counting problem whose decision problems lie in NP

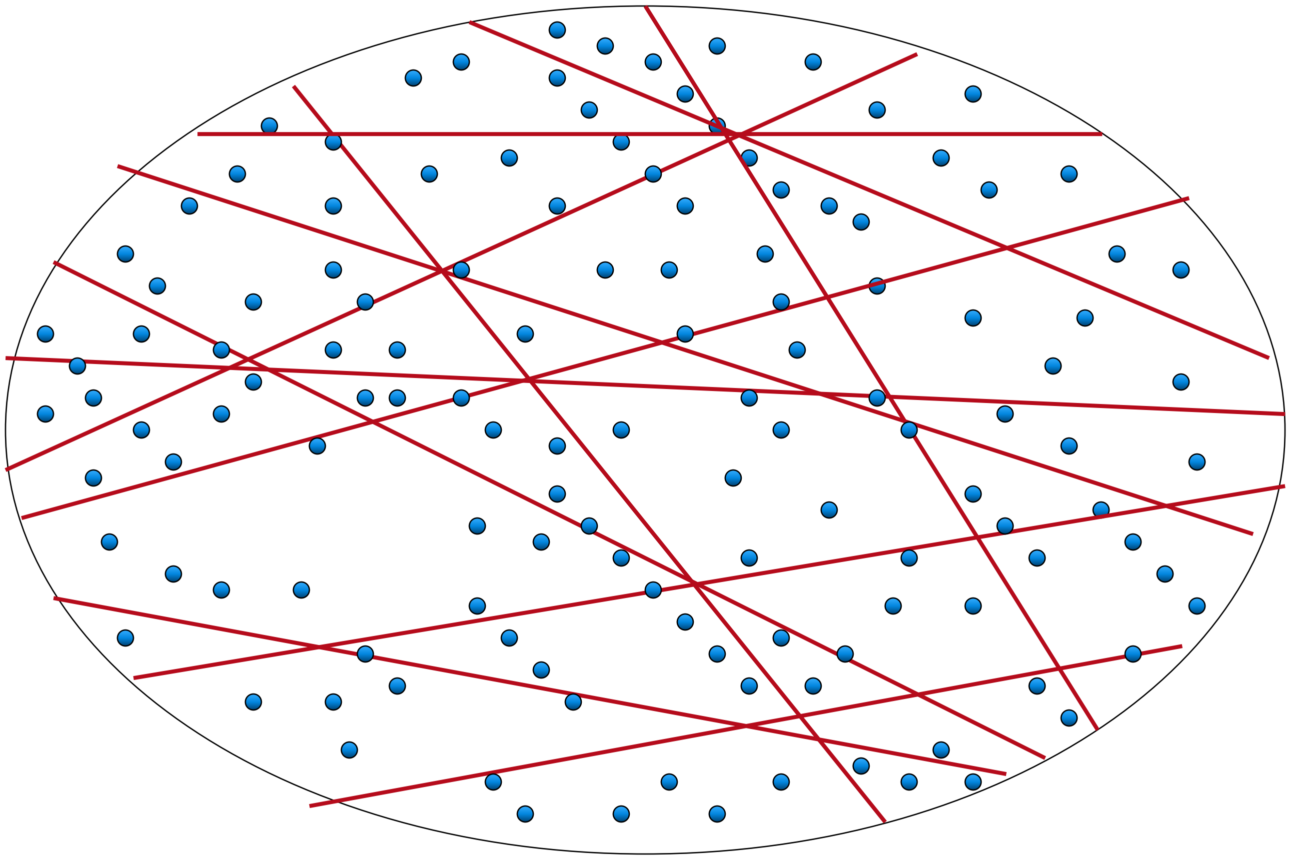
Practical Applications

Wide range of applications!

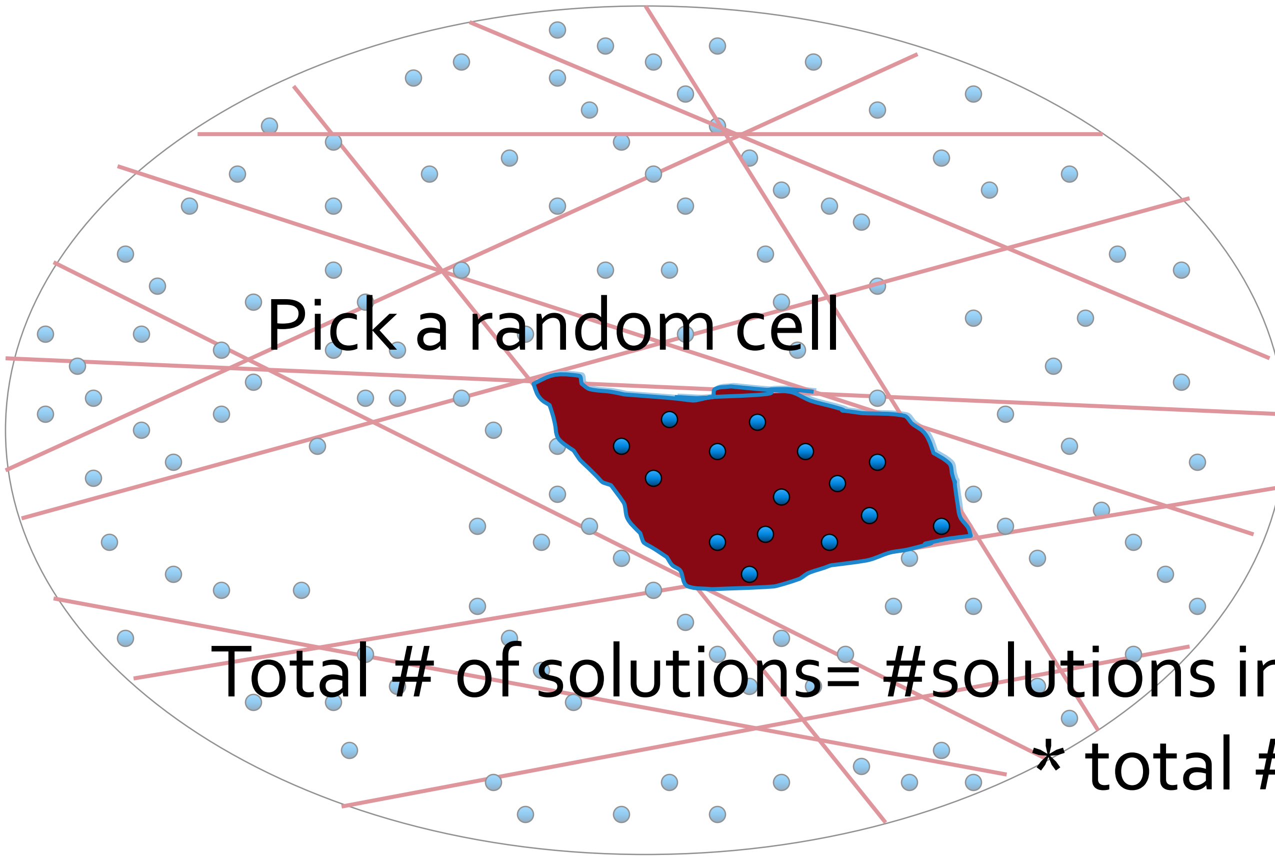
- Estimating coverage achieved
- Probabilistic reasoning/Bayesian inference
- Planning with uncertainty
- Multi-agent/ adversarial reasoning

[Roth 96, Sang 04, Bacchus 04, Domshlak 07]

Counting through Partitioning



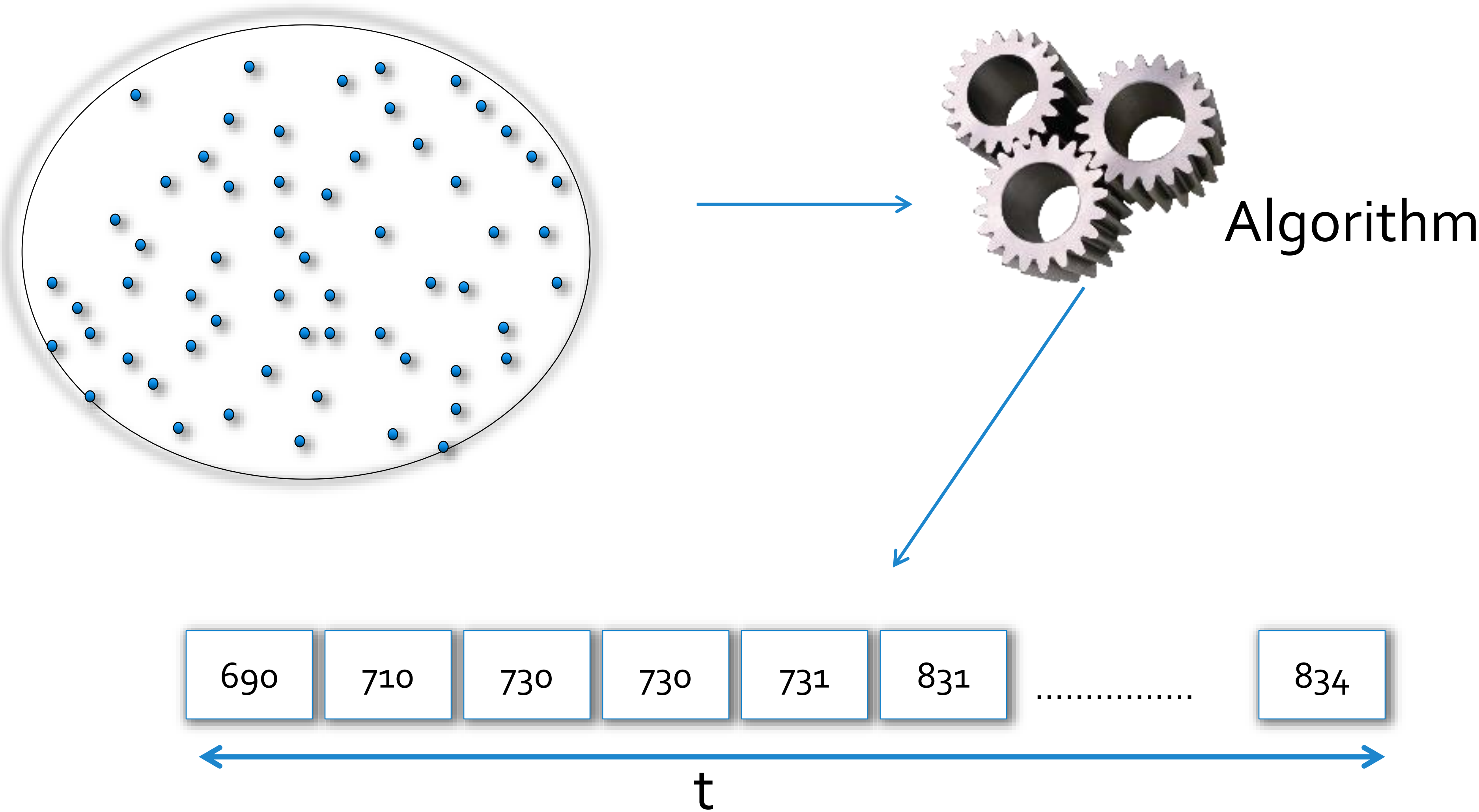
Counting through Partitioning



Pick a random cell

Total # of solutions= #solutions in the cell
* total # of cells

ApproxMC in Action



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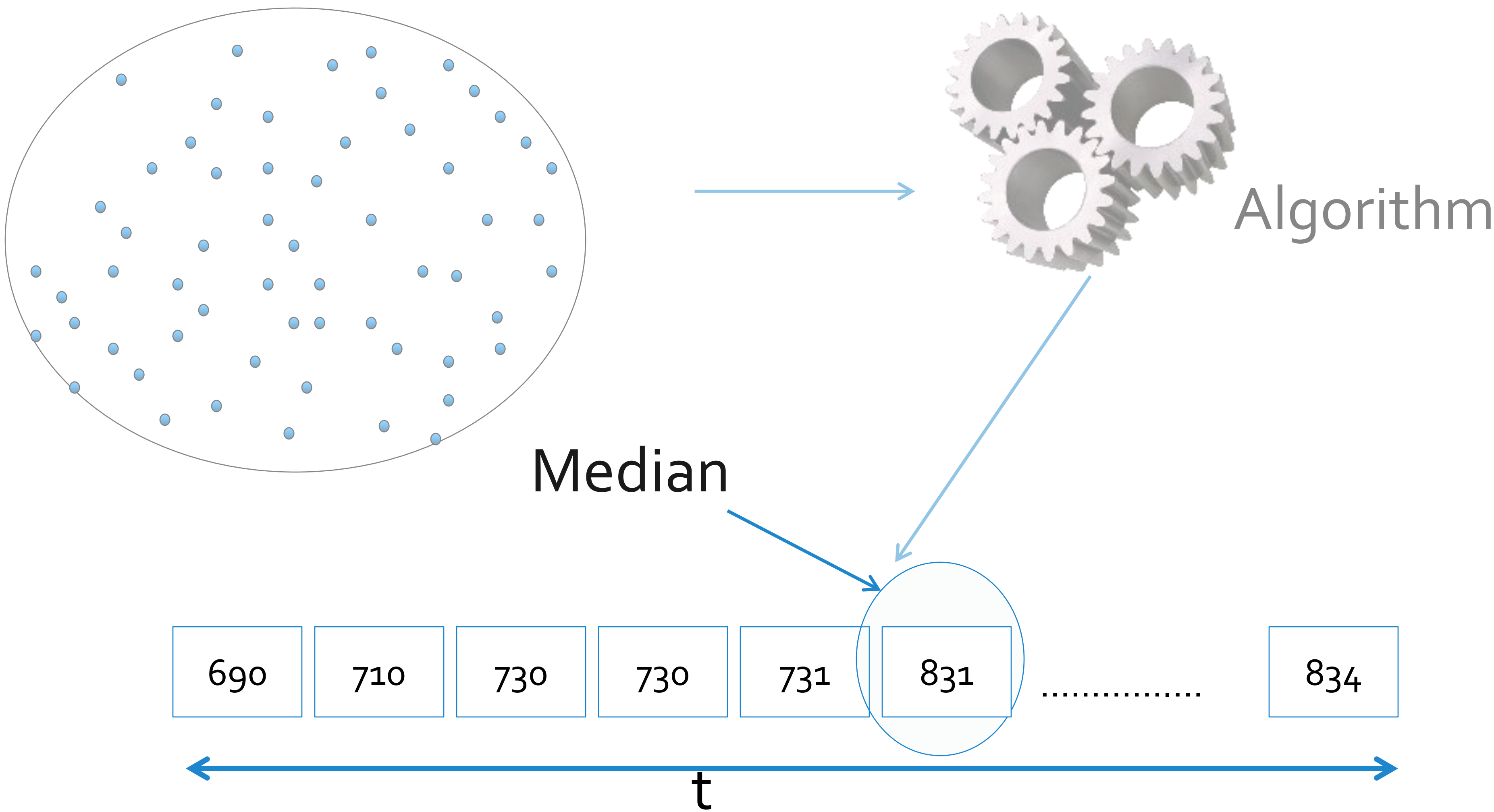


Strong Theoretical Results

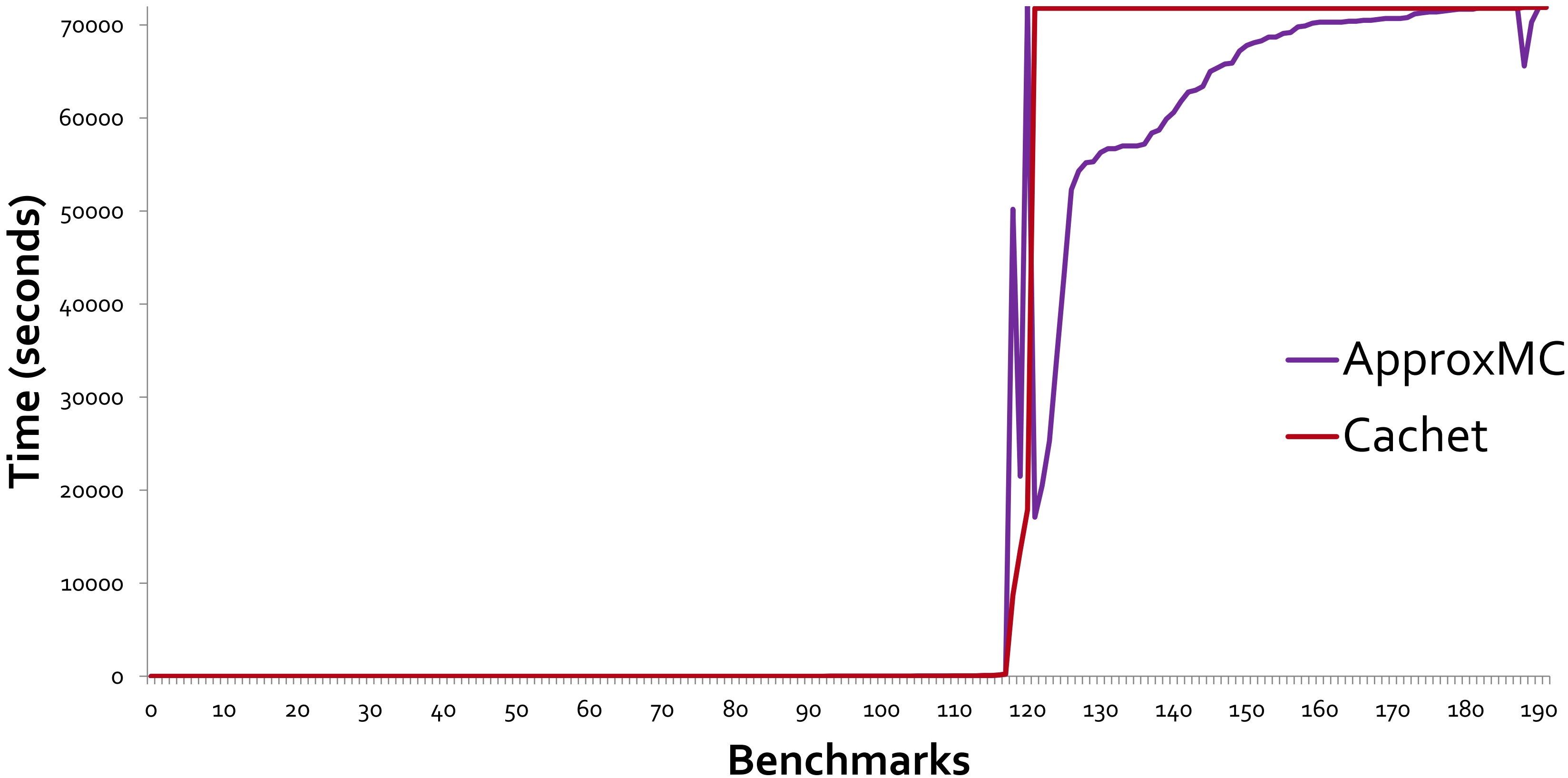
ApproxMC(F, tolerance: ε, confidence parameter: δ)

Suppose ApproxMC(F, ε, δ) returns C. Then

$$\Pr\left[\frac{|R_F|}{1+\varepsilon} \leq C \leq |R_F|(1+\varepsilon)\right] \geq 1-\delta$$

ApproxMC runs in time polynomial in $F, \varepsilon^{-1}, \log(1-\delta)$ relative to SAT oracle

Results: Performance Comparison



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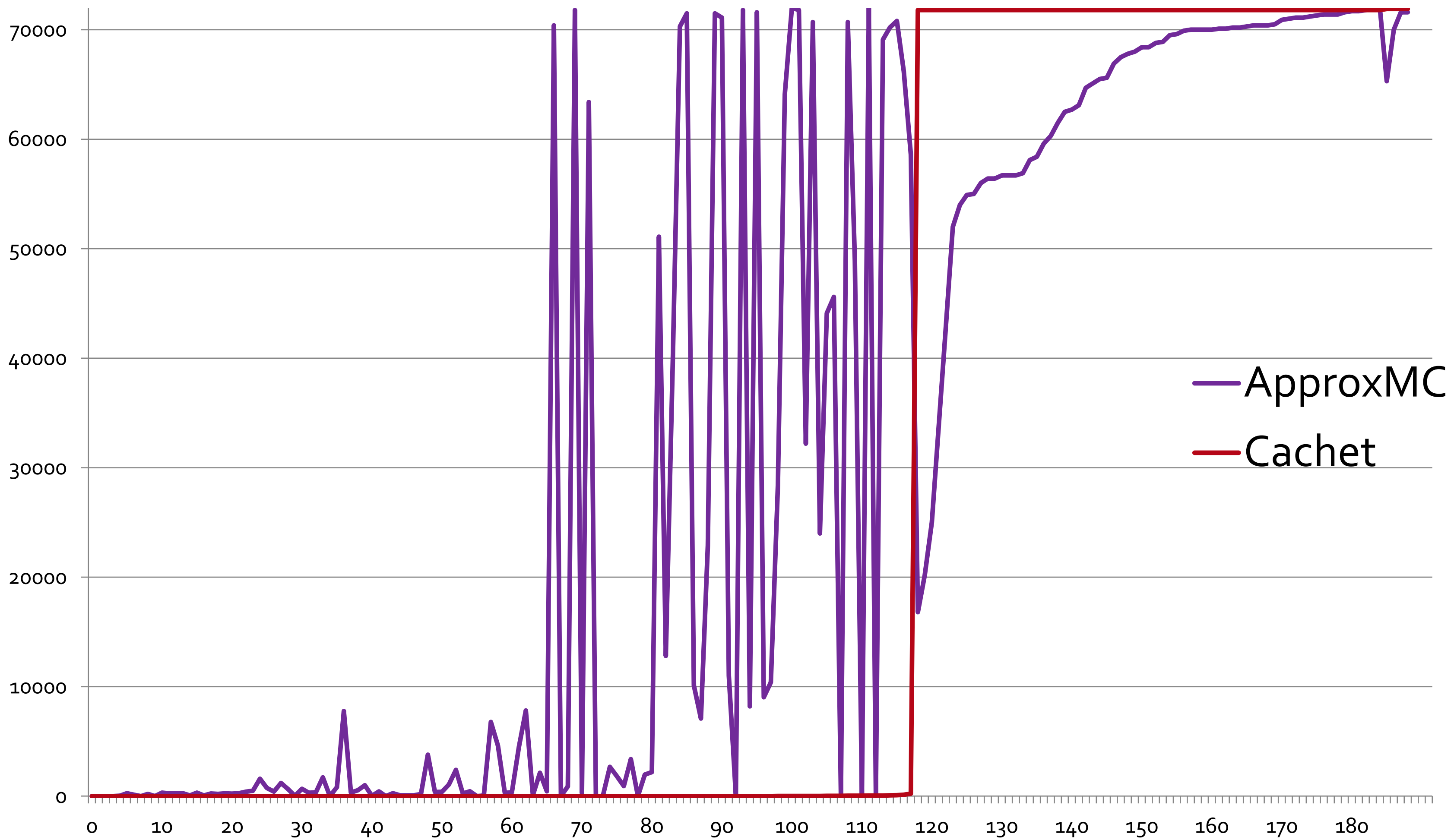
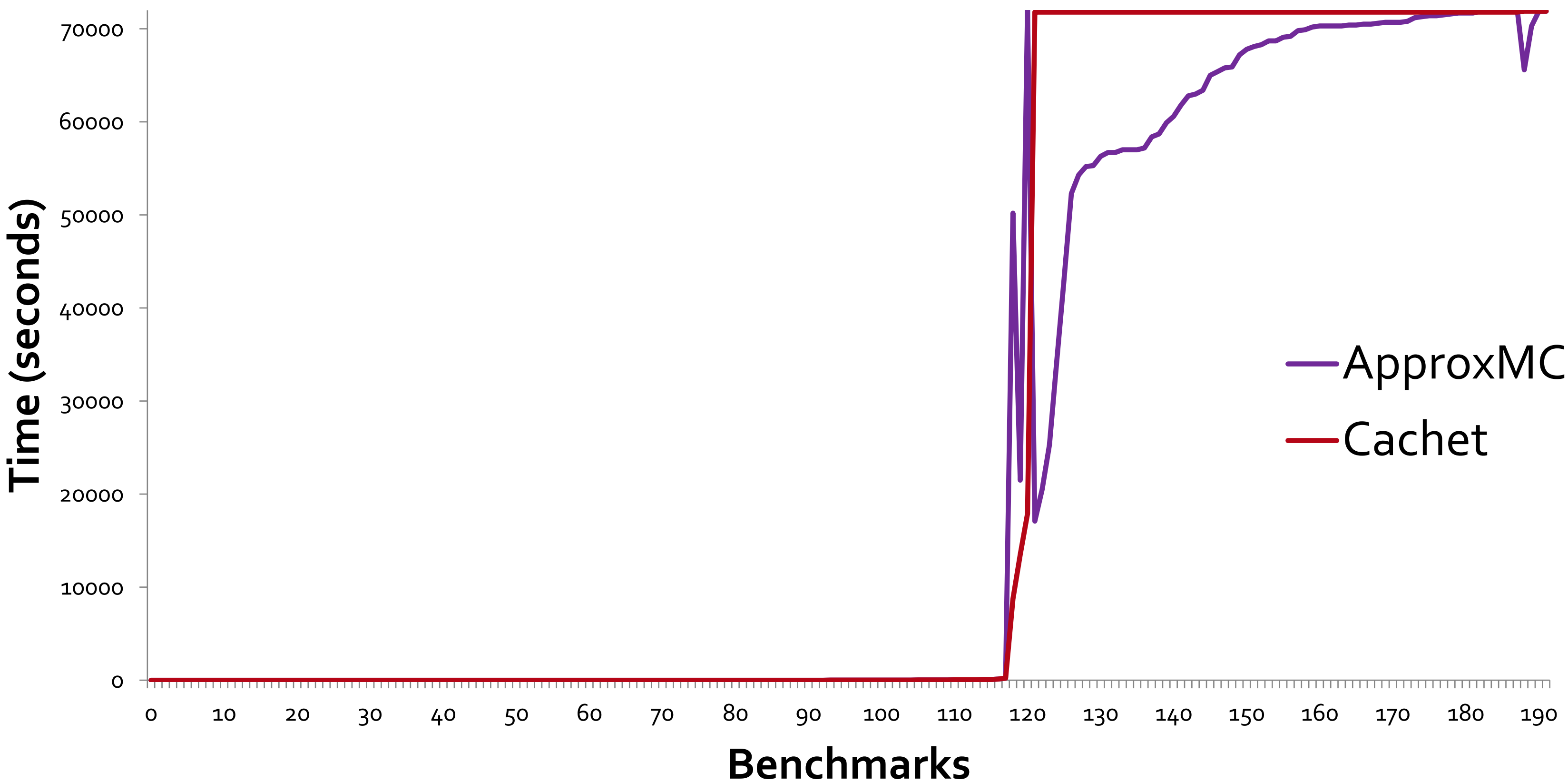
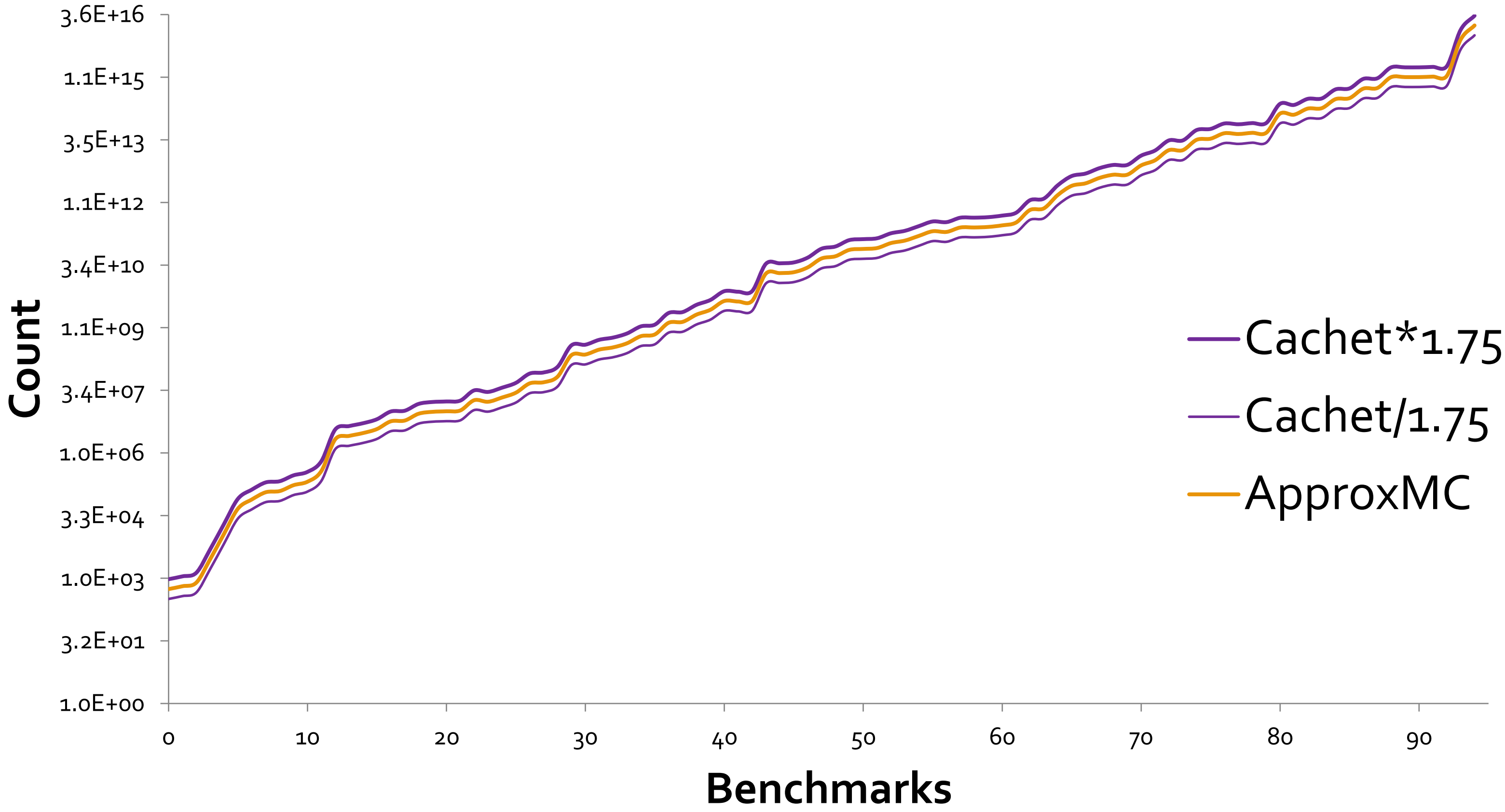


Can Solve a Large Class of Problems



Large class of problems that lie beyond the exact counters but can be computed by ApproxMC

Mean Error: Only 4% (allowed: 75%)



Mean error: 4% – much smaller than the theoretical guarantee of 75%

Approximate Weighted Counting

Ref: "Distribution-Aware Sampling and Weighted Model Counting for SAT" (In Proc. of AAAI 2014)

Weighted Counting

Given

- CNF Formula F
- Weight Function W over assignments

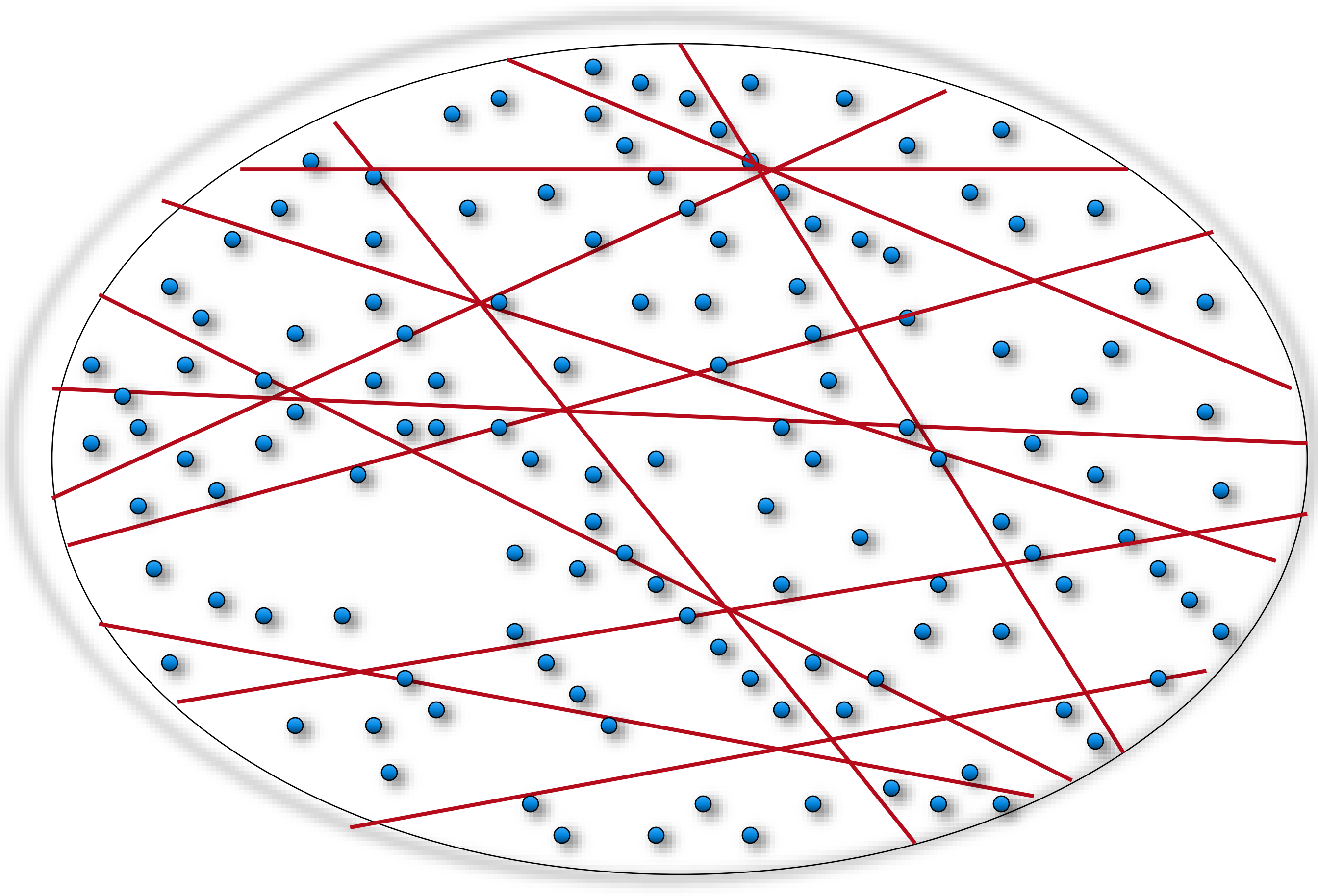
Problem

- What is the sum of weights of ***satisfying*** assignments?

Example

- $F = (a \vee b)$
- $W([0,1]) = W([1,0]) = 1/3 \quad W([1,1]) = W([0,0]) = 1/6$
- $W(F) = 1/3 + 1/3 + 1/6 = 5/6$

Partition into (weighted) equal "small" cells



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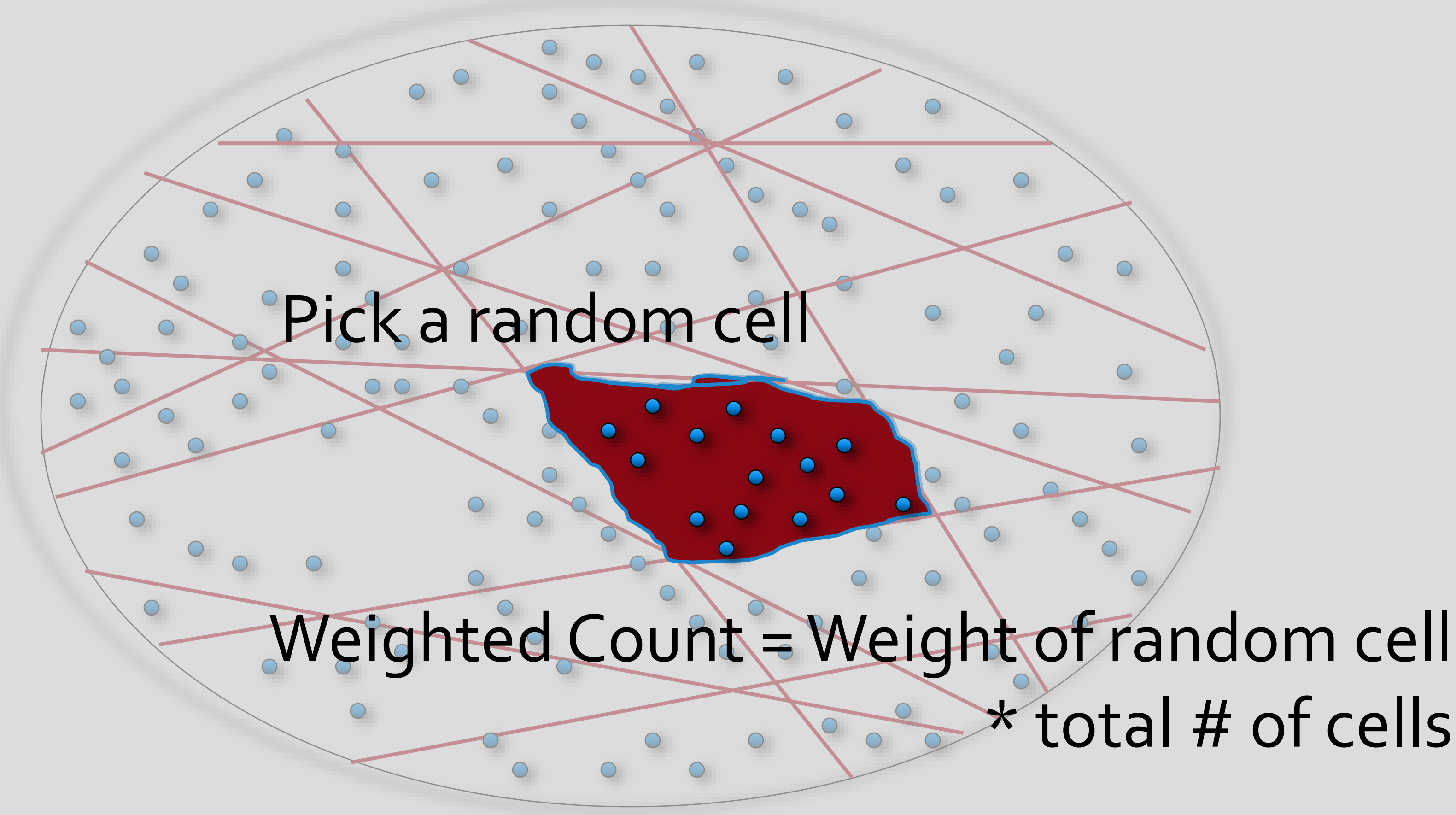


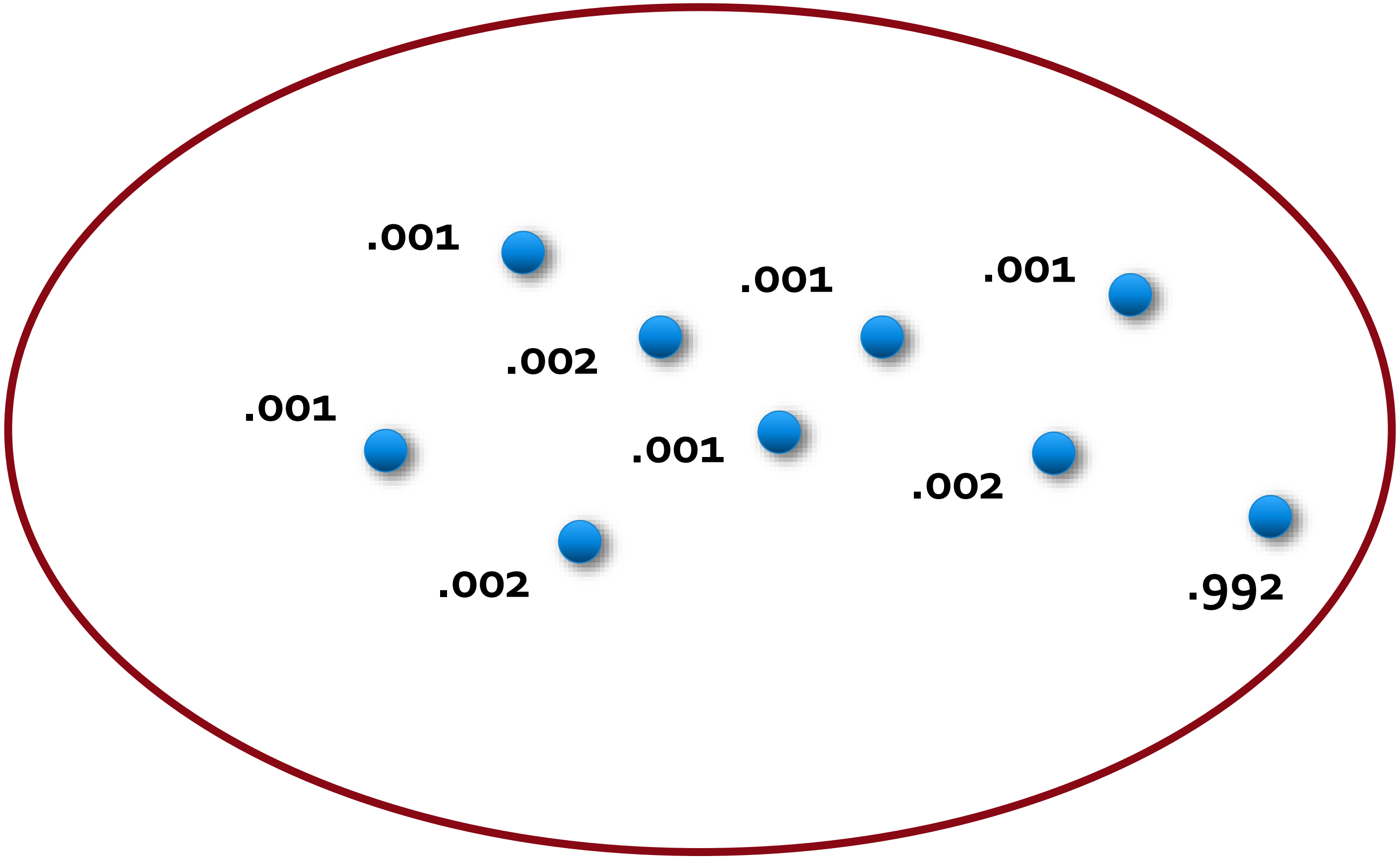
Can you always achieve partitioning?

What if one solution dominates the entire solution space

Tilt = w_{max}/w_{min}

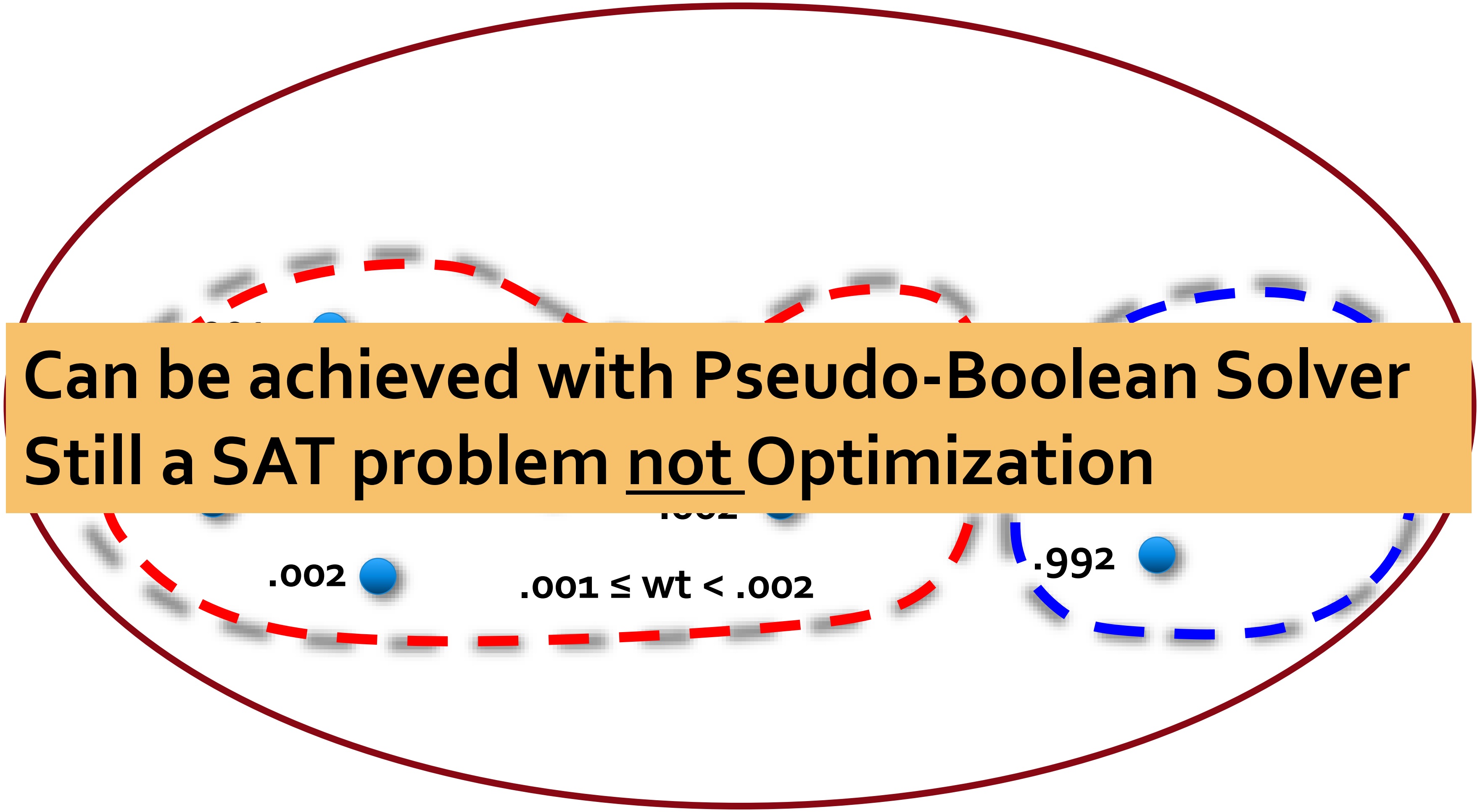
Small tilt $\longrightarrow$ All solutions contribute

How to handle large tilt?



Tilt = 992

Handling Large Tilt



Can be achieved with Pseudo-Boolean Solver
Still a SAT problem <u>not</u> Optimization

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- Sampling Techniques for Dynamic Verification
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- **Construction of Efficient Hashing functions**
- Future Directions

Construction of Efficient Hash Functions

Ref: "On Computing Minimal Independent Support and Its Applications to Sampling and Counting"
(In Proc. of CP 2015 and Invited to "Constraints" Journal)

Best Student Paper Award

XOR-Based Hashing

- 3-universal hashing
- Partition 2^n space into 2^m cells
- Variables: $X_1, X_2, X_3, \ldots, X_n$
- Pick every variable with prob. ½ ,XOR them and equate to 0/1 with prob. ½
- $X_1+X_3+X_6+\ldots X_{n-1} = 0$ (splits solution space)
- m XOR equations -> 2^m cells
- The cell: $F \cup$ XOR (CNF+XOR)

XOR-Based Hashing

- **CryptoMiniSAT**: Efficient for CNF+XOR
- Avg Length : n/2
- Smaller the XORs, better the performance

How to shorten XOR clauses?

Independent Support

- Set of variables such that assignments to these uniquely determine assignments to rest of variables for formula to be true
- c ⟷ (a V b) ; Independent Support (I): {a, b}
- If s_1 and s_2 agree on I then $s_1 = s_2$
- Hash only on the independent variables

Computing Minimal Independent Support

- Reduction to the problem of computing MUS (Minimal Unsatisfiable Subset)

- Minimal Independent supports are 1/100 – 1/1000 of the size of X

- Provides 1-2 orders of magnitude

Future Directions

Extension to More Expressive Domains (SMT, CSP)

- Efficient 3-independent hashing schemes
  - Extending bit-wise XOR to SMT provides guarantees but no advantage of SMT progress

- Solvers to handle F + Hash efficiently
  - CryptoMiniSAT has fueled progress for SAT domain
  - Similar solvers for other domains?

Handling Distributions

- Design of Pseudo-Boolean solvers to handle tilt
- Classification of problems according to tilt
- Online estimation of tilt
- Other techniques for high-tilt distributions

Questions?

Papers and tools: http://www.kuldeepmeel.com