Beyond NP Revolution

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Boolean Satisfiability

Boolean Satisfiability (SAT); Given a Boolean expression, using "and" ($\wedge$) "or", ($\vee$) and "not" ($\neg$), *is there a satisfying solution* (an assignment of 0's and 1's to the variables that makes the expression equal 1)?
Example:

$$(\neg x_1 \vee x_2 \vee x_3) \wedge (\neg x_2 \vee \neg x_3 \vee x_4) \wedge (x_3 \vee x_1 \vee x_4)$$

Solution: $x_1 = 0$, $x_2 = 0$, $x_3 = 1$, $x_4 = 1$

History:

- William Stanley Jevons, 1835-1882: "I have given much attention, therefore, to lessening both the manual and mental labour of the process, and I shall describe several devices which may be adopted for saving trouble and risk of mistake."
- Ernst Schröder, 1841-1902: "Getting a handle on the consequences of any premises, or at least the fastest method for obtaining these consequences, seems to me to be one of the noblest, if not the ultimate goal of mathematics and logic."

Complexity of Boolean Reasoning

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- Clay Institute, 2000: $1M Award!

Algorithmic Boolean Reasoning: Early History

- Davis and Putnam, 1958: "Computational Methods in The Propositional calculus", unpublished report to the NSA
- Davis and Putnam, JACM 1960: "A Computing procedure for quantification theory"
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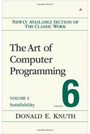
- Conflict-Driven Clause Learning (MSS96a;)
- Two decades of *Moore's Law for SAT solvers*

The Tale of Triumph of SAT Solvers

Modern SAT solvers are able to deal routinely with practical problems that involve millions of variables, although such problems were regarded as hopeless just a few years ago. (Donald Knuth, 2016)



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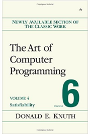

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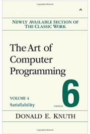

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Now that SAT is “easy”, it is time to look beyond satisfiability

The Disruption of NP Revolution

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Practitioners There are no powerful SAT solvers, so design problem-specified algorithms

Theoreticians Assume access to all-powerful SAT oracle.

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Beyond CNF Solvers Just handling CNF solving is not sufficient

- Need to handle CNF+XOR formulas;
- XORs can be solved by Gaussian elimination
- CryptoMiniSAT: Solver designed to perform CDCL and Gaussian Elimination in tandem (Soos 09; SM, AAAI19)

Constrained Counting and Sampling

- Given
  - Boolean variables $X_1, X_2, \cdots X_n$
  - Formula F over $X_1, X_2, \cdots X_n$

- $\mathsf{Sol}(F) = \{$ solutions of F $\}$

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- $W(F) = \frac{1}{3} + \frac{1}{3} + \frac{1}{6} = \frac{5}{6}$

Today's Menu

Testing of AI systems

Network Reliability

Hardware Validation

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Testing of AI systems

Network Reliability   Constrained Counting

Hardware Validation

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Network Reliability Constrained Counting Hashing Framework
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Network Reliability	Constrained Counting	Hashing Framework
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- Classical verification/testing setup for traditional systems
  - System captured as a model $M(\mathcal{I}, \mathcal{O})$ via logical constraints
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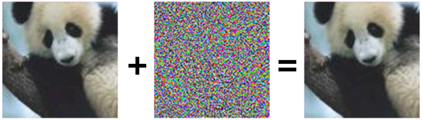

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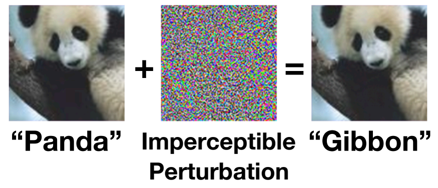

- Acceptable despite multiple executions with error: From satisfiability to counting

(BSSMS, 2019)

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Can we predict likelihood of a region facing blackout?

Reliability of Critical Infrastructure Networks

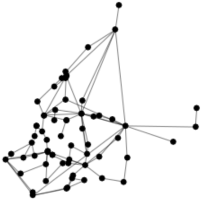

Figure: Plantersville, SC

- $G = (V, E)$; source node: s and terminal node t
- failure probability $g : E \to [0, 1]$
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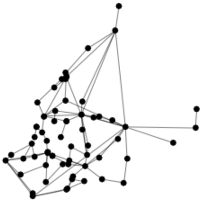

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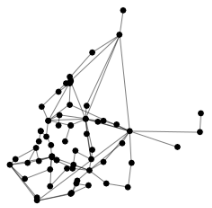

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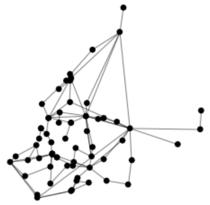

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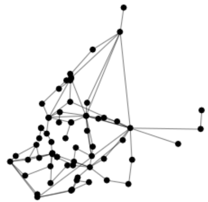

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(DMPV, AAAI 17, ICASP-13, RESS 2019)

Constrained Counting

Prior Work

Strong guarantees but poor scalability

- Exact counters (Birnbaum and Lozinskii 1999, Jr. and Schrag 1997, Sang et al. 2004, Thurley 2006)
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Weak guarantees but impressive scalability

- Bounding counters (Gomes et al. 2007,Kroc, Sabharwal, and Selman 2008, Gomes, Sabharwal, and Selman 2006, Kroc, Sabharwal, and Selman 2008)
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How to bridge this gap between theory and practice?

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- ApproxCount$(F, W, \varepsilon, \delta)$: Compute C such that

$$\Pr[\frac{W(F)}{1+\varepsilon} \leq C \leq W(F)(1+\varepsilon)] \geq 1-\delta$$

From Weighted to Unweighted Counting

Boolean Formula F and weight function $W : \{0,1\}^n \rightarrow \mathbb{Q}^{\geq 0}$

Boolean Formula F'

$$W(F) = c(W) \times |\mathsf{Sol}(F')|$$

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How do we estimate $|\mathsf{Sol}(F')|$?

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How many people in Helsinki like coffee?

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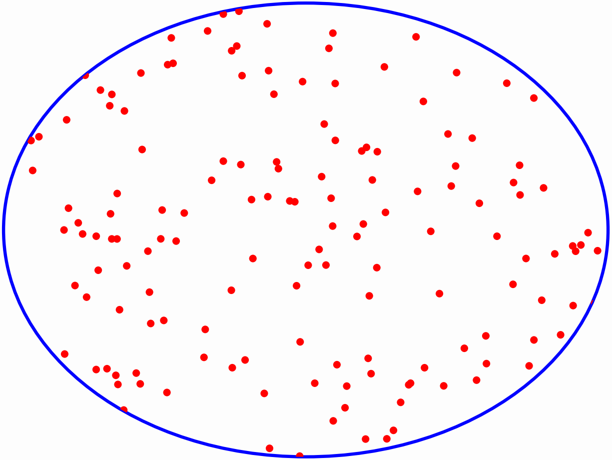
  - Potentially 2^n queries

Can we do with lesser # of SAT queries – $\mathcal{O}(n)$ or $\mathcal{O}(\log n)$?

As Simple as Counting Dots



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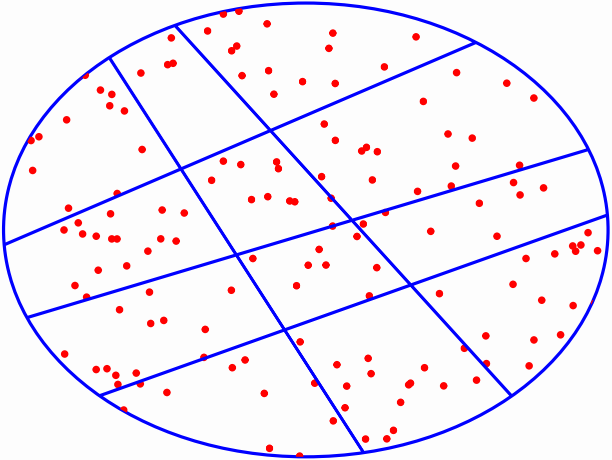
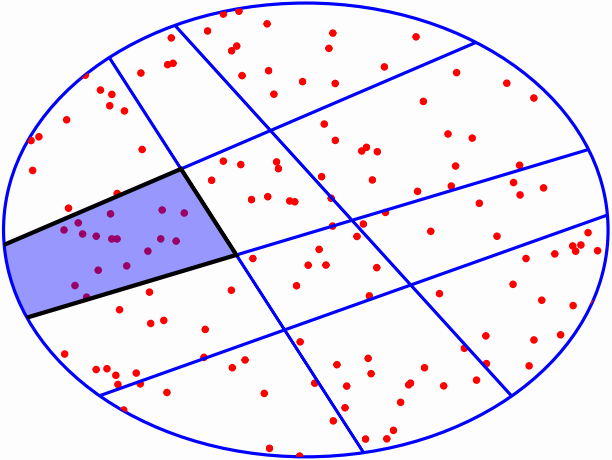


As Simple as Counting Dots

Pick a random cell



Estimate $=$ Number of solutions in a cell $\times$ Number of cells

Challenges

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Challenge 2 How many cells?

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- Deterministic h unlikely to work
- Choose h randomly from a large family H of hash functions

  Universal Hashing (Carter and Wegman 1977)

2-Universal Hashing

- Let H be family of 2-universal hash functions mapping $\{0,1\}^n$ to $\{0,1\}^m$

$$\forall y_1, y_2 \in \{0,1\}^n, \alpha_1, \alpha_2 \in \{0,1\}^m, h \xleftarrow{R} H$$
$$\Pr[h(y_1) = \alpha_1] = \Pr[h(y_2) = \alpha_2] = \left(\frac{1}{2^m}\right)$$

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- The power of 2-universality
  - Z be the number of solutions in a randomly chosen cell
  - $\mathsf{E}[Z] = \frac{|\mathsf{Sol}(F)|}{2^m}$
  - $\sigma^2[Z] \leq \mathsf{E}[Z]$

2-Universal Hash Functions

- Variables: $X_1, X_2, \cdots X_n$
- To construct $h : \{0,1\}^n \rightarrow \{0,1\}^m$, choose m random XORs
- Pick every X_i with prob. $\frac{1}{2}$ and XOR them
  - $X_1 \oplus X_3 \oplus X_6 \cdots \oplus X_{n-2}$
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- To choose $\alpha \in \{0,1\}^m$, set every XOR equation to 0 or 1 randomly

$$X_1 \oplus X_3 \oplus X_6 \cdots \oplus X_{n-2} = 0 \qquad (Q_1)$$
$$X_2 \oplus X_5 \oplus X_6 \cdots \oplus X_{n-1} = 1 \qquad (Q_2)$$
$$\cdots \qquad (\cdots)$$
$$X_1 \oplus X_2 \oplus X_5 \cdots \oplus X_{n-2} = 1 \qquad (Q_m)$$

- Solutions in a cell: $F \wedge Q_1 \cdots \wedge Q_m$

2-Universal Hash Functions

- Variables: $X_1, X_2, \cdots X_n$
- To construct $h : \{0,1\}^n \rightarrow \{0,1\}^m$, choose m random XORs
- Pick every X_i with prob. $\frac{1}{2}$ and XOR them
  - $X_1 \oplus X_3 \oplus X_6 \cdots \oplus X_{n-2}$
  - Expected size of each XOR: $\frac{n}{2}$
- To choose $\alpha \in \{0,1\}^m$, set every XOR equation to 0 or 1 randomly

$$X_1 \oplus X_3 \oplus X_6 \cdots \oplus X_{n-2} = 0 \qquad (Q_1)$$
$$X_2 \oplus X_5 \oplus X_6 \cdots \oplus X_{n-1} = 1 \qquad (Q_2)$$
$$\cdots \qquad (\cdots)$$
$$X_1 \oplus X_2 \oplus X_5 \cdots \oplus X_{n-2} = 1 \qquad (Q_m)$$

- Solutions in a cell: $F \wedge Q_1 \cdots \wedge Q_m$
- Performance of state of the art SAT solvers degrade with increase in the size of XORs (SAT Solvers != SAT oracles)

Improved Universal Hash Functions

- Not all variables are required to specify solution space of F
  - $F := X_3 \iff (X_1 \vee X_2)$
  - X_1 and X_2 uniquely determines rest of the variables (i.e., X_3)
- Formally: if I is independent support, then $\forall \sigma_1, \sigma_2 \in \mathsf{Sol}(F)$, if σ_1 and σ_2 agree on I then $\sigma_1 = \sigma_2$
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Algorithmic procedure to determine I?

- FP^{NP} procedure via reduction to Minimal Unsatisfiable Subset
- Two orders of magnitude runtime improvement

(IMMV; CP15, Constraints16)

Challenges

Challenge 1 How to partition into roughly equal small cells of solutions without knowing the distribution of solutions?

- Independent Support-based 2-Universal Hash Functions

Challenge 2 How many cells?

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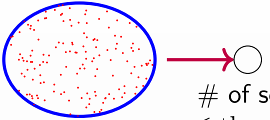
  - Check for every $m = 0, 1, \cdots n$ if the number of solutions $\leq \mathrm{thresh}$

ApproxMC(F, ε, δ)



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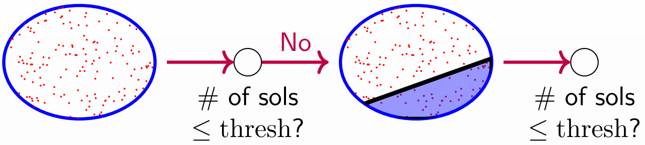

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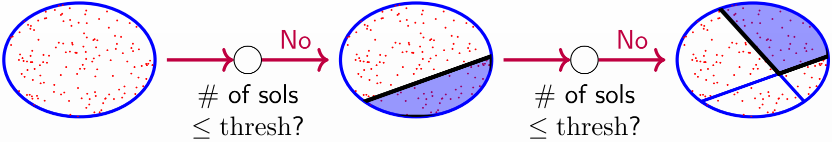

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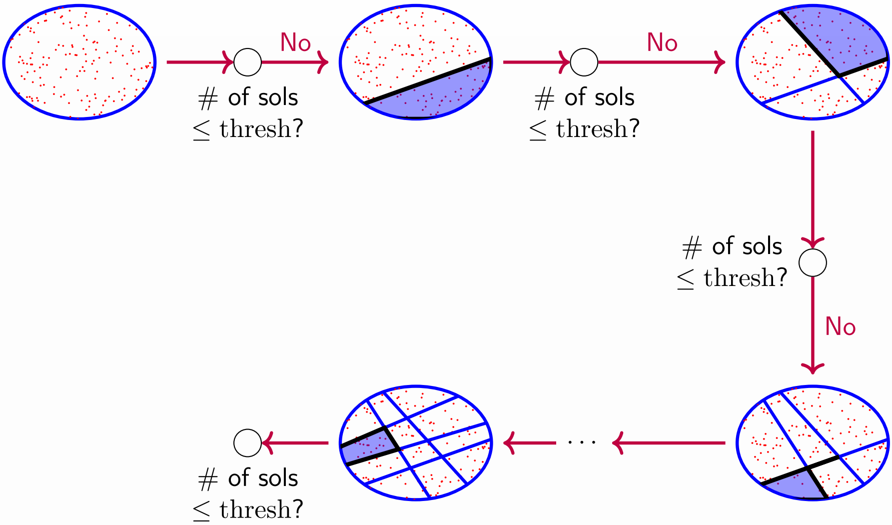

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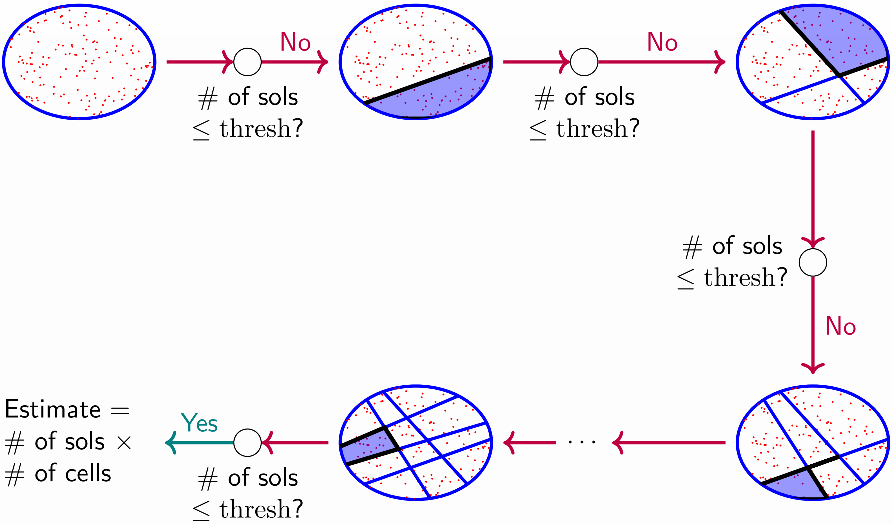

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- Stop at the first m where Query m returns YES and return estimate as $\#(F \wedge Q_1 \wedge Q_2 \cdots \wedge Q_m) \times 2^m$
- Observation: $\#(F \wedge Q_1 \cdots \wedge Q_i \wedge Q_{i+1}) \leq \#(F \wedge Q_1 \cdots \wedge Q_i)$
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  - Independence crucial to analysis (Stockmeyer 1983, $\cdots$)
  - Key Insight: The probability of making a bad choice of Q_i is very small for $i \ll m^*$

(CMV, IJCAI16)

Taming the Curse of Dependence

Let $2^{m^*} = \frac{|\mathsf{Sol}(F)|}{\mathrm{thresh}}$ ($m^* = \log(\frac{|\mathsf{Sol}(F)|}{\mathrm{thresh}})$)

Lemma (1)

ApproxMC (F, ε, δ) terminates with $m \in \{m^* - 1, m^*\}$ with probability $\geq$ 0.8

Lemma (2)

For $m \in \{m^* - 1, m^*\}$, estimate obtained from a randomly picked cell lies within a tolerance of ε of $|\mathsf{Sol}(F)|$ with probability $\geq$ 0.8

ApproxMC(F, ε, δ)

Theorem (Correctness)

$$\Pr\left[\frac{|\mathsf{Sol}(F)|}{1+\varepsilon} \le \mathit{ApproxMC}(F, \varepsilon, \delta) \le |\mathsf{Sol}(F)|(1+\varepsilon)\right] \ge 1 - \delta$$

Theorem (Complexity)

ApproxMC(F, ε, δ) *makes* $\mathcal{O}(\frac{\log n \log(\frac{1}{\delta})}{\varepsilon^2})$ *calls to SAT oracle.*

- *Prior work required* $\mathcal{O}(\frac{\boldsymbol{n} \log \boldsymbol{n} \log(\frac{1}{\delta})}{\varepsilon})$ *calls to SAT oracle (Stockmeyer 1983)*

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Theorem (FPRAS for DNF; (MSV, FSTTCS 17; CP 18, IJCAI-19))

If F is a DNF formula, then ApproxMC is FPRAS – fundamentally different from the only other known FPRAS for DNF (Karp, Luby 1983)

Reliability of Critical Infrastructure Networks

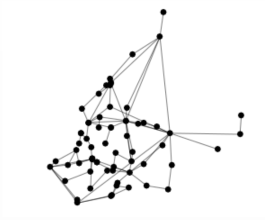
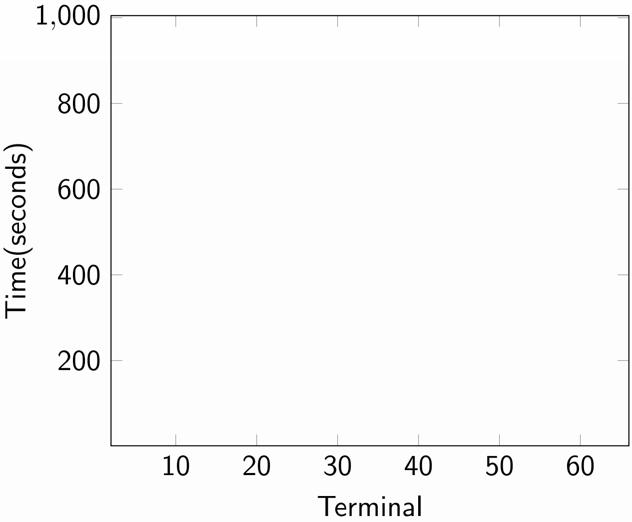


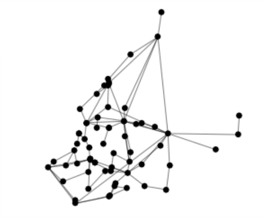
Figure: Plantersville, SC

- $G = (V, E)$; source node: s
- Compute Pr[t is disconnected]?

Timeout = 1000 seconds



(DMPV, AAAI17)



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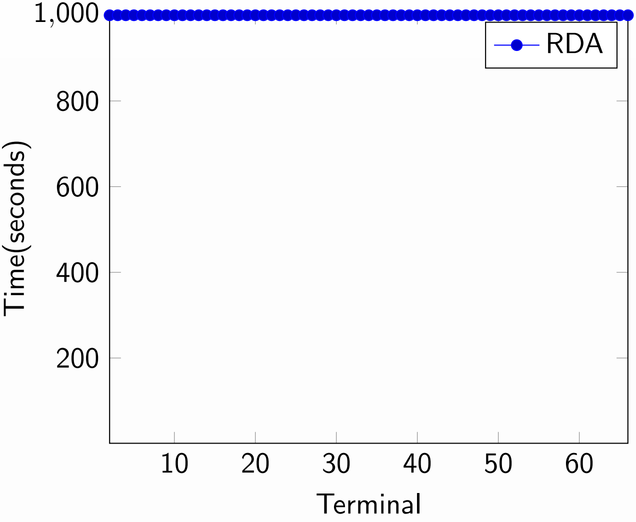

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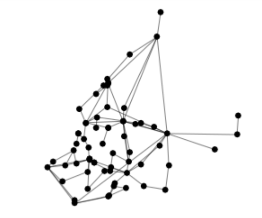

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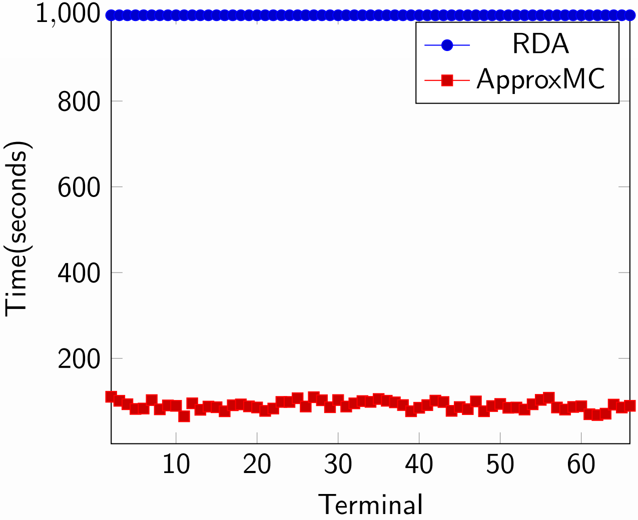

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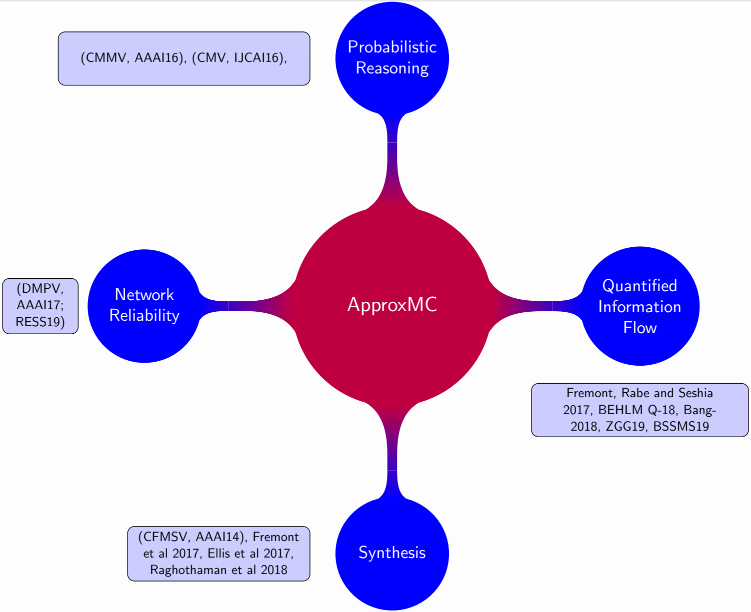
Beyond Network Reliability



Verification of AI systems

Network Reliability Constrained Counting

Verification of AI systems

Network Reliability

Constrained Counting

Hashing Framework

Verification of AI systems

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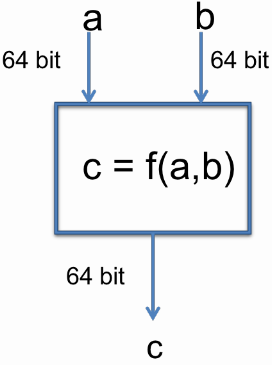
Hardware Validation

Hashing Framework



- Design is simulated with test vectors (values of *a* and *b*)
- Results from simulation compared to intended results

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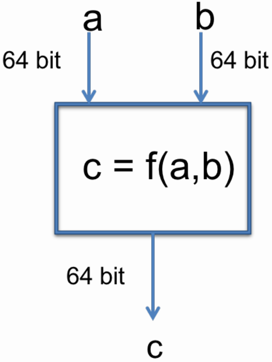

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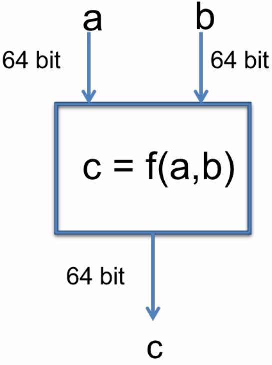

- Design is simulated with test vectors (values of *a* and *b*)
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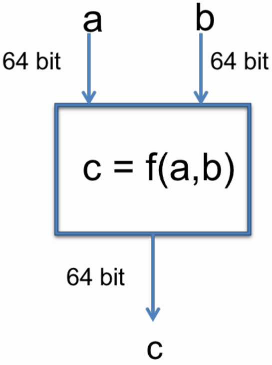
- Use constraints to represent *interesting* verification scenarios



Constraints

- Designers:
  - $a +_{64} 11 * 32b = 12$
  - $a <_{64} (b >> 4)$
- Past Experience:
  - $40 <_{64} 34 + a <_{64} 5050$
  - $120 <_{64} b <_{64} 230$
- Users:
  - $232 * 32a +_{64} b! = 1100$
  - $1020 <_{64} (b /_{64} 2) +_{64} a <_{64} 2200$

Test vectors: random solutions of constraints

Constrained Sampling

- Given:
  - Set of Constraints F over variables $X_1, X_2, \cdots X_n$
- Uniform Sampler

$$\forall y \in \mathsf{Sol}(F), \Pr[\text{y is output}] = \frac{1}{|\mathsf{Sol}(F)|}$$

- Almost-Uniform Sampler

$$\forall y \in \mathsf{Sol}(F), \frac{1}{(1+\varepsilon)|\mathsf{Sol}(F)|} \leq \Pr[\text{y is output}] \leq \frac{(1+\varepsilon)}{|\mathsf{Sol}(F)|}$$

Prior Work

Strong guarantees but poor scalability

- Polynomial calls to NP oracle (Bellare, Goldreich and Petrank,2000)
- BDD-based techniques (Yuan et al 1999, Yuan et al 2004, Kukula and Shiple 2000)
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Weak guarantees but impressive scalability

- Randomization in SAT solvers (Moskewicz 2001, Nadel 2011, Dutra Bachrach and Sen 2018)
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How to bridge this gap between theory and practice?

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- Approximate counting and almost-uniform sampling are inter-reducible (Jerrum, Valiant and Vazirani, 1986)

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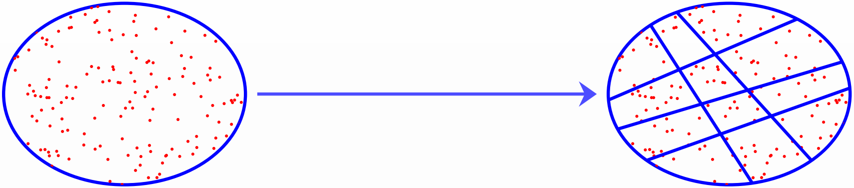
- Approximate counting and almost-uniform sampling are inter-reducible (Jerrum, Valiant and Vazirani, 1986)
- Is the reduction efficient?
  - Almost-uniform sampler (JVV) require linear number of approximate counting calls

Key Ideas



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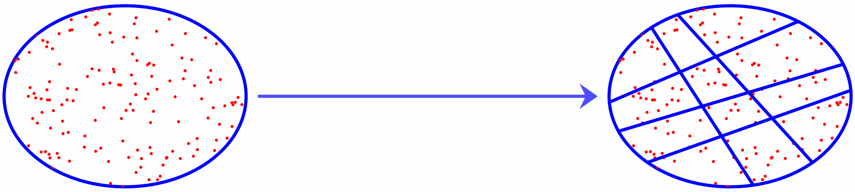

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  - Not just a practical hack required non-trivial proof

(CMV; DAC14),

(CFMSV; AAAI14, TACAS15),

(SGRM; LPAR18,TACAS19)

Theoretical Guarantees

Theorem (Almost-Uniformity)

$\forall y \in \mathsf{Sol}(F)$, $\frac{1}{(1+\varepsilon)|\mathsf{Sol}(F)|} \leq \Pr[y \textit{ is output}] \leq \frac{1+\varepsilon}{|\mathsf{Sol}(F)|}$; $\varepsilon > 1.71$

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Random XORs are 3-universal

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SAT Solver	1
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Experiments over 200+ benchmarks

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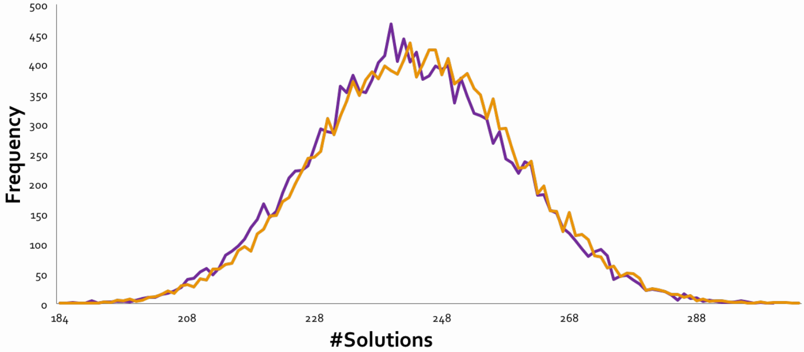
Experiments over 200+ benchmarks

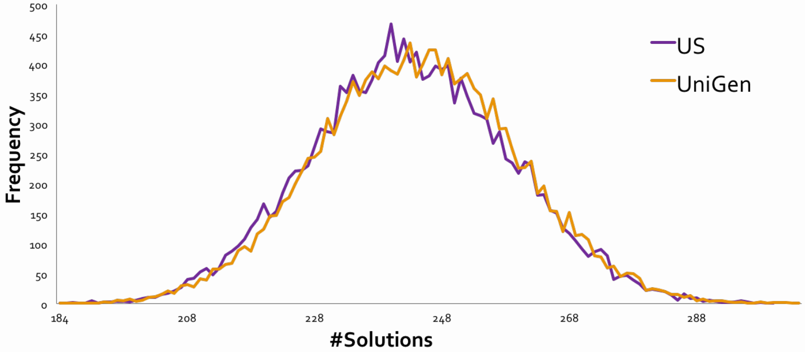
Closer to technical transfer

Quiz Time: Uniformity



- Benchmark: case110.cnf; #var: 287; #clauses: 1263
- Total Runs: 4×10^6; Total Solutions : 16384

Statistically Indistinguishable



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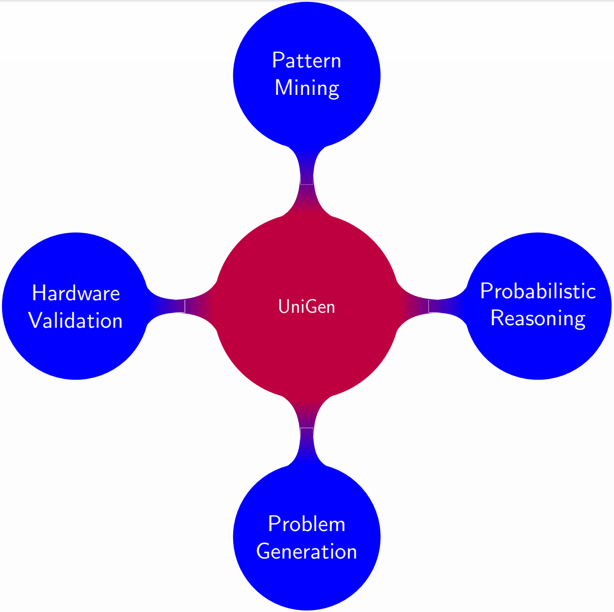
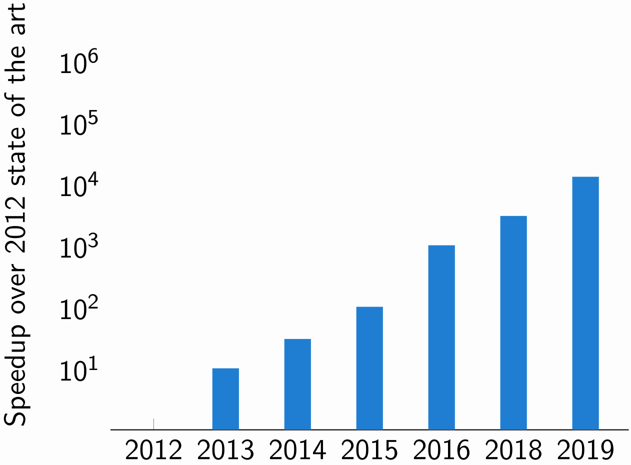
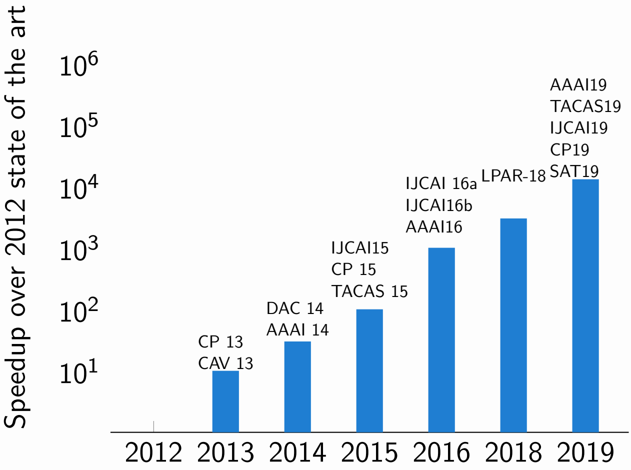
Usages of Open Source Tool: UniGen




Speedup over 2012 state of the art
10^6
10^5
10^4
10^3
10^2
10^1
2012
2013
2014
2015
2016
2018
2019

Mission 2025: Constrained Counting and Sampling Revolution



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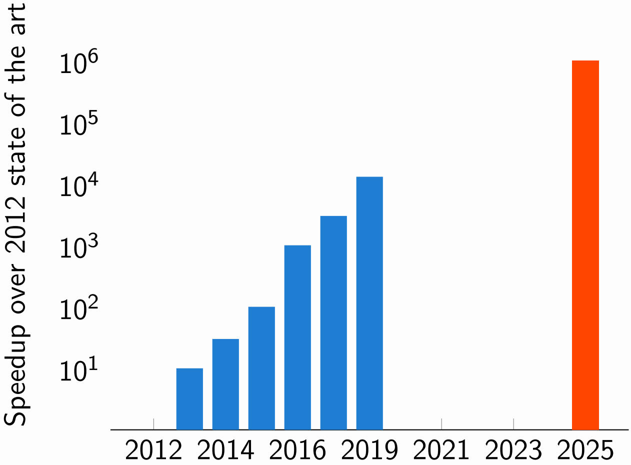

Requires combinations of ideas from theory, statistics and systems

Mission 2025: Constrained Counting and Sampling Revolution

Challenge Problems

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Civil Engineering Reliability for Los Angeles Transmission Grid

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  We can only see a short distance ahead but we can see plenty there that needs to be done (Turing, 1950)

The Amazing Collaborators

S. Akshay (IITB, India), Teodora Baluta (NUS, SG), Fabrizio Biondi (Avast, CZ), Supratik Chakraborty (IITB, India), Alexis de Colnet (NUS, SG), Remi Delannoy (NUS, SG), Jeffrey Dudek (Rice,US), Leonardo Duenas-Osorio (Rice,US), Mike Enescu (Inria, France) Daniel Fremont (UCB, US), Dror Fried (Open U., Israel), Rahul Gupta (IITK, India), Annelie Heuser (Inria, France), Alexander Ivrii (IBM, Israel), Alexey Ignatiev (IST, Portugal), Axel Legay (UCL, Belgium), Sharad Malik (Princeton, US), Joao Marques Silva (IST, Portugal), Rakesh Mistry (IITB, India), Nina Narodytska ((VMWare, US), Roger Paredes (Rice,US), Yash Pote (NUS, SG), Jean Quilbeuf(Inria, France), Subhajit Roy (IITK, India), Mate Soos (NUS, SG), Prateek Saxena (NUS, SG), Sanjit Seshia (UCB, US), Shubham Sharma (IITK, India), Aditya Shrotri(Rice,US), Moshe Vardi (Rice,US)

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