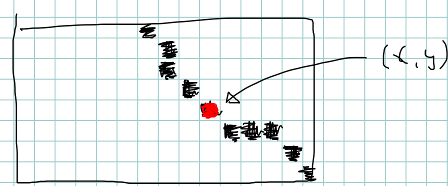
THE FILL FRONT IS A BINARY IMAGE OF PIXELS TO BE FILLED NEXT. IMAGINE THE FOLLOWING FILL FRONT IMAGE $I(x,y) \in [0,1]$ WHERE:

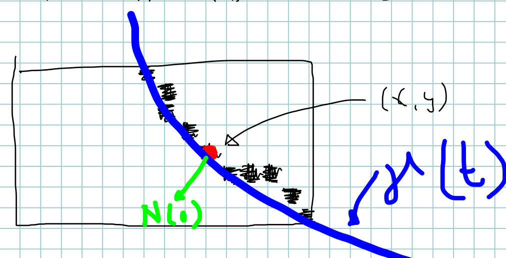
$I(x,y)=0$ WHEN □

$I(x,y)=$ WHEN ■

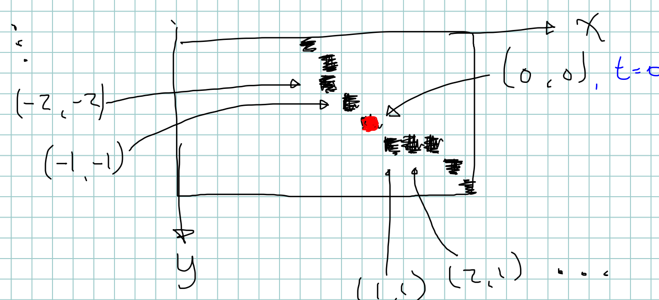


ASSUME WE ARE ESTIMATING THE NORMAL $N(t)$ AT PIXEL (x,y) MARKED IN RED

A SECOND DEGREE CURVE FIT TO THIS DATA WOULD LOOK LIKE:



FOR CONVENIENCE, ASSUME THE ORIGIN IS NOW AT (x,y), SO THAT:



THIS MEANS THAT THE X COORDINATES OF THE FRONT LINE ARE

$x=[-3,-2,-2,-1,0,1,2,3,4,5]$ AND FOR y THEY ARE:

$y=[-4,-3,-2,-1,0,1,1,1,2,3]$

IF WE USE A WINDOW OF WIDTH $w=3$ WE ARE LEFT WITH

$x=[-2,-2,-1,0,1,2,3]$

$y=[-3,-2,-1,0,1,1,1]$

COPIED HERE FOR CONVENIENCE:

$x = [-2, -2, -1, 0, 1, 2, 3]$

$y = [-3, -2, -1, 0, 1, 1, 1]$

WE NOW FIT 2^{nd} ORDER POLYNOMIALS TO EACH OF THESE TWO VECTORS TO OBTAIN THE APPROXIMATIONS TO THE COORDINATE FUNCTIONS:

$$\gamma(t) = [\underbrace{x(t), y(t)}_{\text{COORDINATE FUNCTIONS}}]$$

FOR INSTANCE, THE ONE FOR $x(t)$ HAS THE CONSTRAINTS:

$$\left.\begin{aligned} x(-3) &= -2 \\ x(-2) &= -2 \\ x(-1) &= -1 \\ x(0) &= 0 \\ &\vdots \\ x(3) &= 3 \end{aligned}\right\}$$

AND THESE CONSTRAINTS ARE USED TO ESTIMATE THE PARAMETERS OF A 2^{nd} DEGREE POLYNOMIAL:

$$x(t) = a_x t^2 + b_x t + c_x$$

THE LEAST SQUARES SOLUTION DEFINES THE SYSTEM:

$$\begin{array}{r} t=-3 \rightarrow \\ t=-2 \rightarrow \\ \vdots \\ t=3 \rightarrow \end{array} \begin{bmatrix} 1 & -3 & 9/2 \\ 1 & -2 & 2 \\ 1 & -1 & 1/2 \\ & \vdots & \\ 1 & 3 & 9/2 \end{bmatrix} \begin{bmatrix} x(0) \\ \frac{dx(0)}{dt} \\ \frac{d^2x(0)}{dt^2} \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \\ -1 \\ \vdots \\ 3 \end{bmatrix}$$

$x'(0)$

$x''(0)$

USE THE PSEUDO-INVERSE METHOD TO SOLVE FOR $[x(0), x'(0), x''(0)]$, REPEAT FOR $y(t)$ TO GET $y(0)$, $y'(0)$ AND $y''(0)$.

TO GET THE NORMAL TAKE THE FIRST DERIVATIVE $\left(\frac{dx}{dt}(0), \frac{dy(0)}{dt}\right)$ FUNCTIONS AND USE:

$$N(t) = \frac{1}{\left\|\left(\frac{dx}{dt}(t), \frac{dy}{dt}(t)\right)\right\|}\left(-\frac{dy}{dt}(t), \frac{dx}{dt}(t)\right) \text{ AT } t=0.$$

TO INCORPORATE THE GAUSSIAN WEIGHTS, YOU MUST SCALE THE ROWS OF THE EACH CONSTRAINT, SO IF THE ORIGINAL MATRIX IS

$$
\begin{array}{c}
t=-3 \rightarrow \\
t=-2 \rightarrow \\
\vdots \\
\\
t=3 \rightarrow
\end{array}
\begin{bmatrix} 1 & -3 & 9/2 \\ 1 & -2 & 2 \\ 1 & -1 & 1/2 \\ & \vdots & \\ 1 & 3 & 9/2 \end{bmatrix}
\begin{bmatrix} X(0) \\ \underbrace{\frac{dX(0)}{dt}}_{X'(0)} \\ \underbrace{\frac{d^2X(0)}{dt^2}}_{X''(0)} \end{bmatrix}
=
\begin{bmatrix} -2 \\ -2 \\ -1 \\ \vdots \\ 3 \end{bmatrix}
$$

THE FIRST ROW OF THE GAUSSIAN WEIGHTED MATRIX IS:

$$
\begin{bmatrix} \Omega(3)\cdot 1 & \Omega(3)\cdot(-3) & \Omega(3)\cdot(9/2) \\ & \vdots & \end{bmatrix}
\begin{bmatrix} X(0) \\ X'(0) \\ X''(0) \end{bmatrix}
=
\begin{bmatrix} \Omega(3)\cdot(-2) \\ \vdots \end{bmatrix}
$$

THEN YOU MAY USE $X'(0)$ AND $Y'(0)$ TO ESTIMATE THE UNIT NORMAL.