

University of Toronto
Department of Computer Science

CSC420

Introduction to Image Understanding

Tutorial 6: radiometry and photometric stereo

Fall 2026

Presented by Javad Rajabi

Teaching Assistant

Instructor: David Lindell

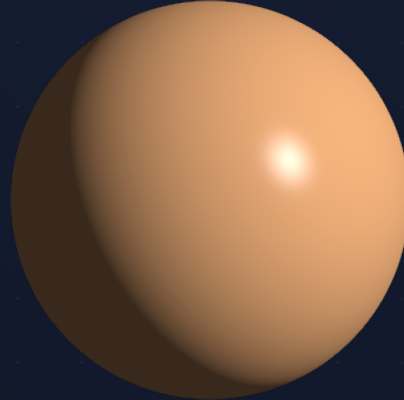


recovered surface normals

CSC420 Tutorial 6

From light to shape

Radiometry, reflectance and photometric stereo



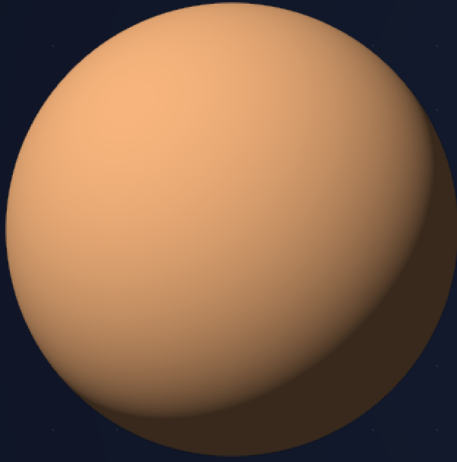
What we will cover

- 1 Why study light?
- 2 Solid angle
- 3 Flux, irradiance and radiance
- 4 Reflectance and the BRDF
- 5 Lambertian, glossy and mirror
- 6 Light sources
- 7 Colour and photometry
- 8 Vignetting and the moon
- 9 Shading and surface normals
- 10 Photometric stereo
- 11 From normals to depth
- 12 Practice problems
- 13 Recap

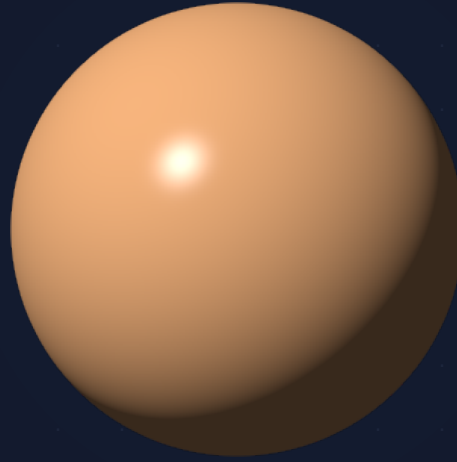
1 Why study light?

What appearance tells us

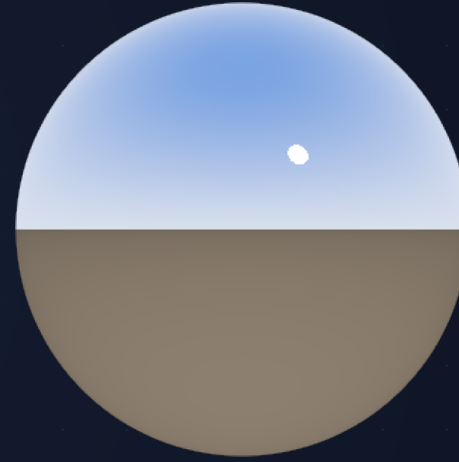
1 Why study light?



matte

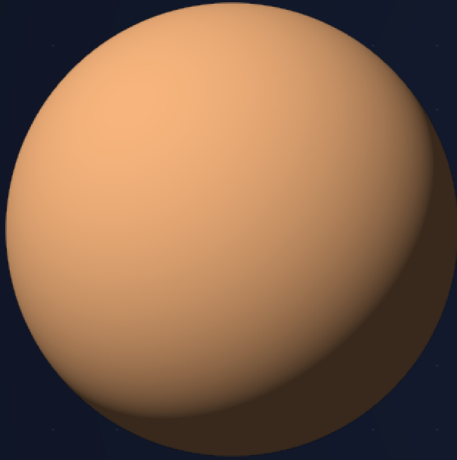


glossy



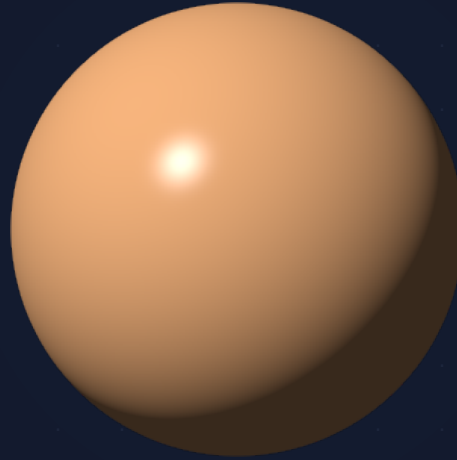
mirror

1 Why study light?



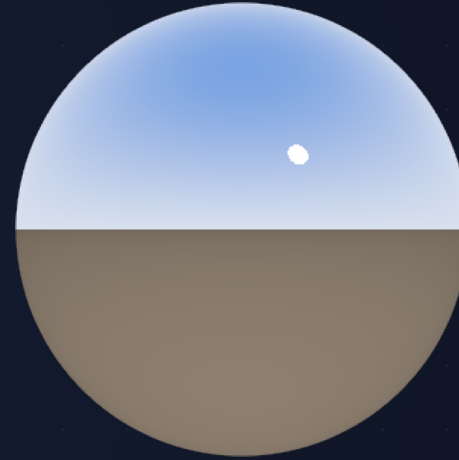
matte

shading → shape



glossy

highlight → material



mirror

reflection → surroundings

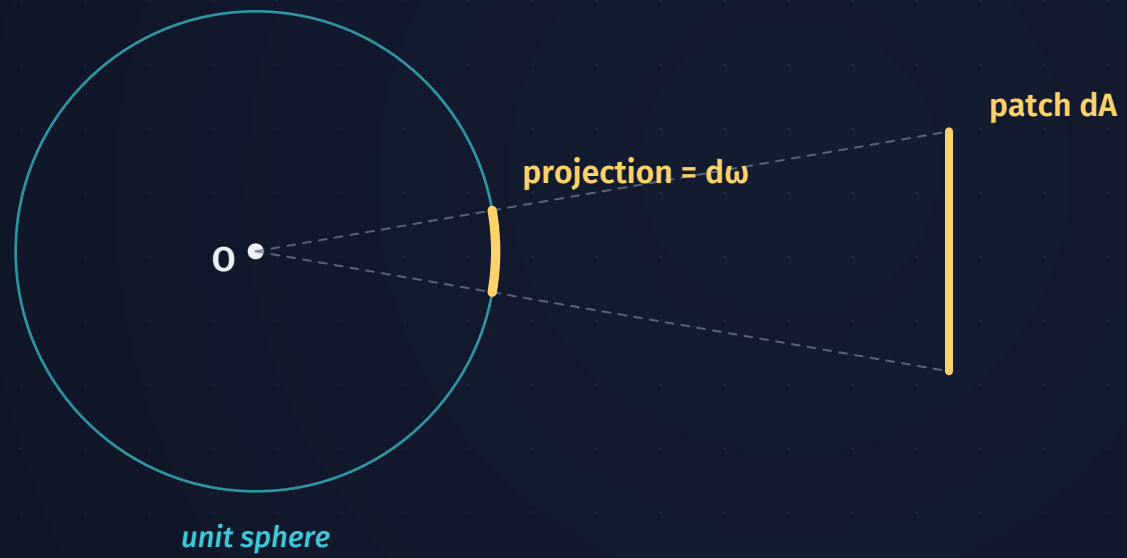
1 Why study light?



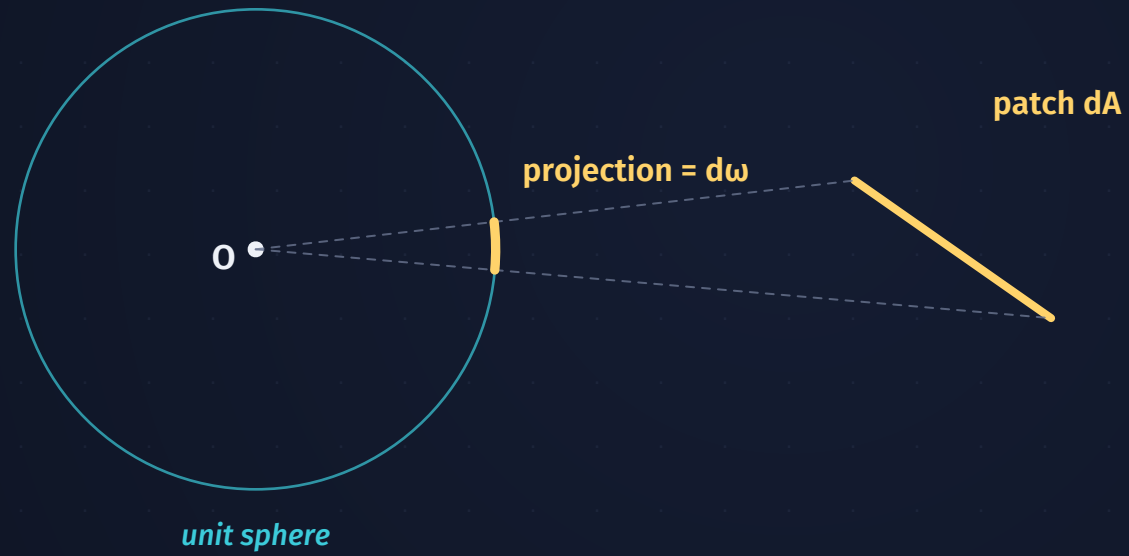
2 Solid angle

Measuring directions

2 Solid angle



2 Solid angle



$$d\omega = \frac{dA \cos \theta}{r^2} \quad [\text{sr}]$$

r grows $\rightarrow d\omega$ shrinks as $1/r^2$

tilt θ grows $\rightarrow d\omega$ shrinks as $\cos \theta$

2 Solid angle

$$d\omega = \frac{(0.02 \text{ m})^2}{(2 \text{ m})^2} = \frac{4 \times 10^{-4}}{4} = 10^{-4} \text{ sr}$$

$$\text{tilted } 60^\circ : 10^{-4} \cdot \cos 60^\circ = 5 \times 10^{-5} \text{ sr}$$

sphere : 4π sr hemisphere : 2π sr

$$\text{sun} : \pi (0.00465)^2 \approx 6.8 \times 10^{-5} \text{ sr}$$

the sun's angular radius is about 0.27°

3 Flux, irradiance and radiance

Units for light

3 Flux, irradiance and radiance

Radiant flux

Φ [W]

power through a surface

3 Flux, irradiance and radiance

Radiant flux

$$\Phi \quad [\text{W}]$$

power through a surface

Irradiance

$$E = \frac{d\Phi}{dA} \quad [\text{W}/\text{m}^2]$$

per unit area

3 Flux, irradiance and radiance

Radiant flux

$$\Phi \quad [\text{W}]$$

power through a surface

Irradiance

$$E = \frac{d\Phi}{dA} \quad [\text{W}/\text{m}^2]$$

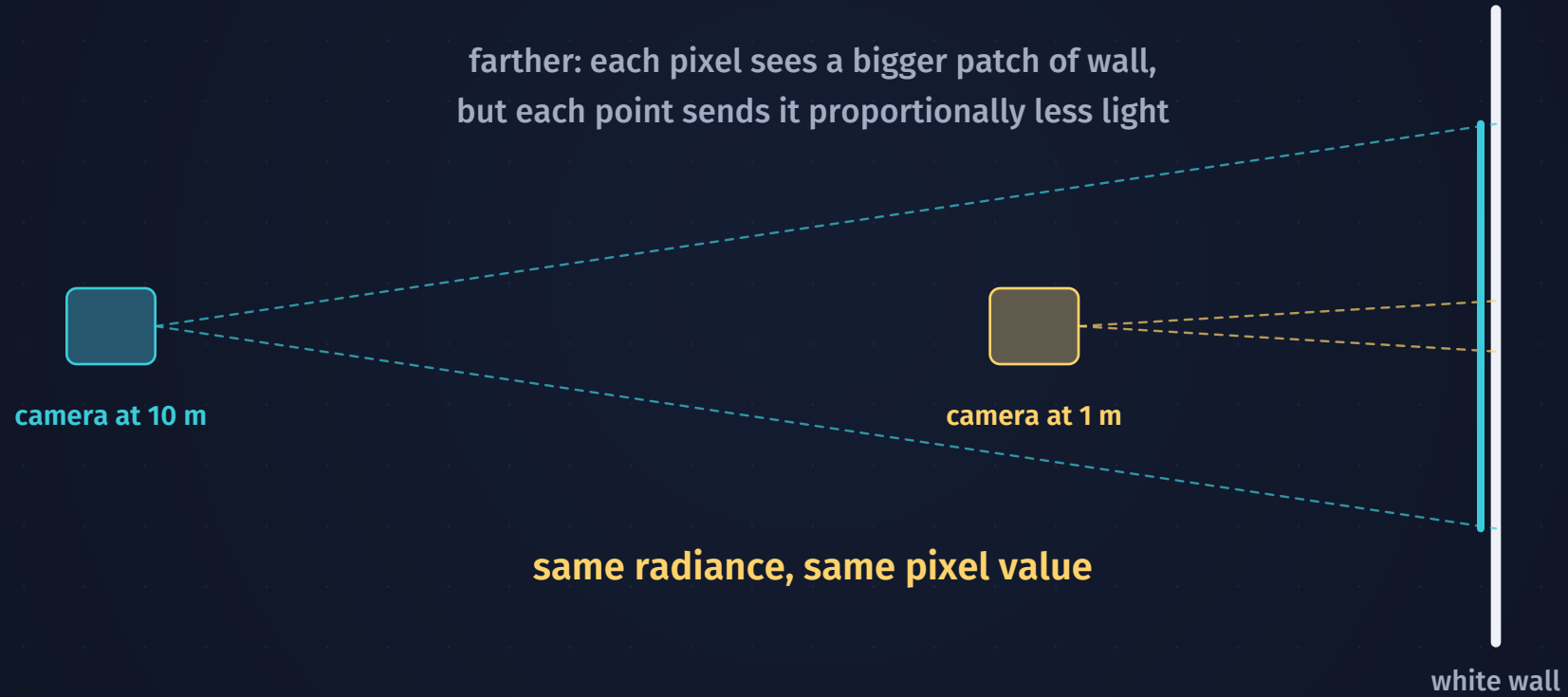
per unit area

Radiance

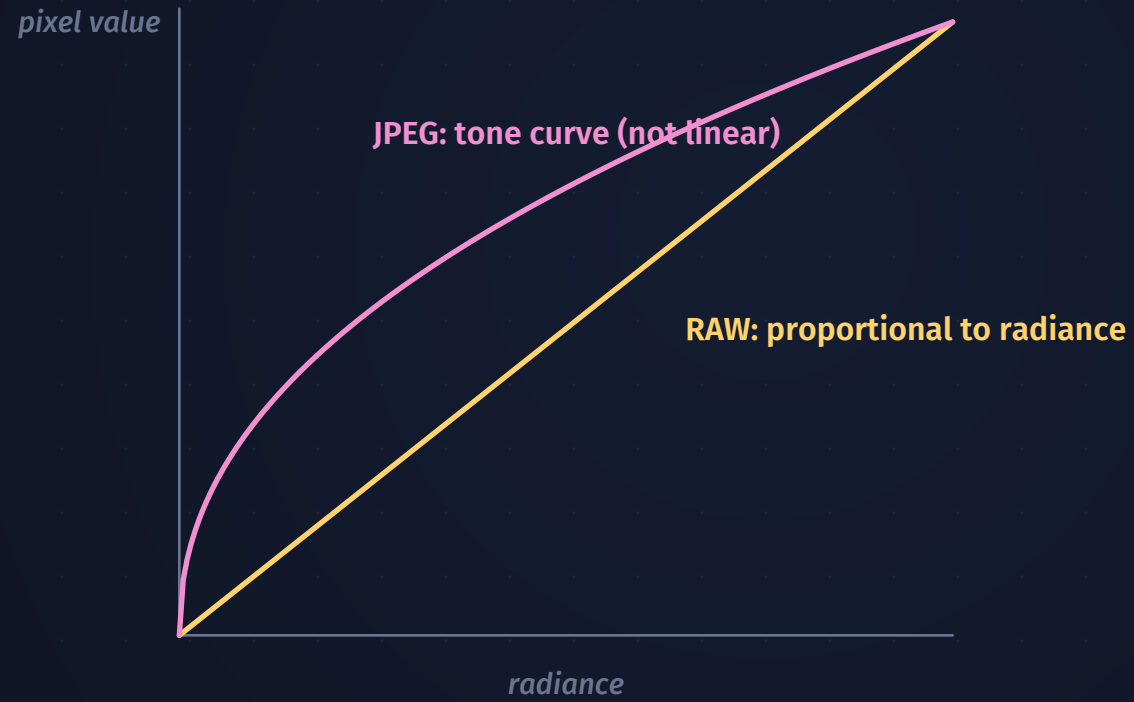
$$L = \frac{d^2\Phi}{dA \cos \theta d\omega} \quad [\text{W}/(\text{m}^2 \text{sr})]$$

per area, per solid angle: one ray

3 Flux, irradiance and radiance



3 Flux, irradiance and radiance



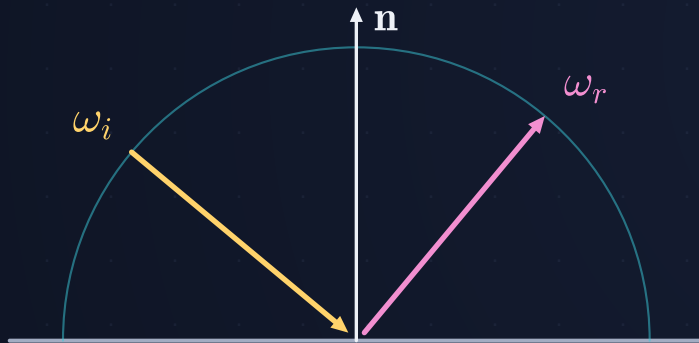
$$\text{RAW} \propto L$$

linearize JPEGs before physics

4 Reflectance and the BRDF

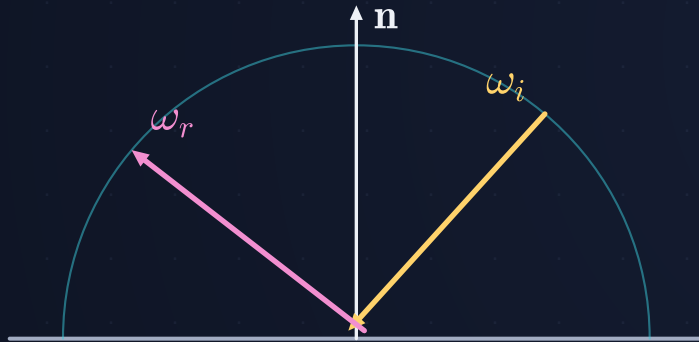
How surfaces redirect light

4 Reflectance and the BRDF



$$f(\omega_i, \omega_r) = \frac{dL_r(\omega_r)}{dE_i(\omega_i)} \quad [\text{sr}^{-1}]$$

4 Reflectance and the BRDF



$$f(\omega_i, \omega_r) = \frac{dL_r(\omega_r)}{dE_i(\omega_i)} \quad [\text{sr}^{-1}]$$

$$\int_{\Omega} f(\omega_i, \omega_r) \cos \theta_r d\omega_r \leq 1$$

energy conservation

$$f(\omega_i, \omega_r) = f(\omega_r, \omega_i)$$

Helmholtz reciprocity

$$f(\theta_i, \theta_r, \phi_i - \phi_r)$$

isotropy: 4D $\rightarrow$ 3D

$$L_r(\omega_r) = \int_{\Omega} L_i(\omega_i) f(\omega_i, \omega_r) \cos \theta_i d\omega_i$$

all incoming directions $\times$ BRDF $\times$ cosine

with $d\omega = \sin \theta d\theta d\phi$ to compute it in spherical angles

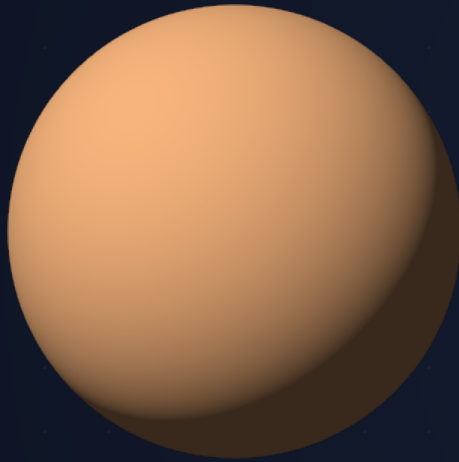
4 Reflectance and the BRDF



5 Lambertian, glossy and mirror

Three classic BRDFs

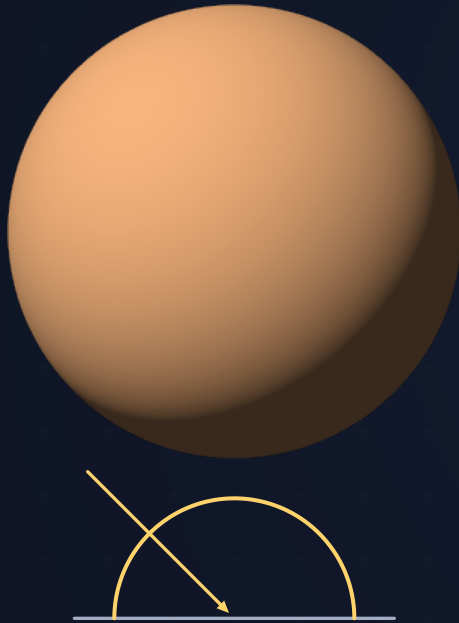
5 Lambertian, glossy and mirror



$$f = \frac{\rho}{\pi}$$

ρ : albedo, the fraction of light reflected

5 Lambertian, glossy and mirror



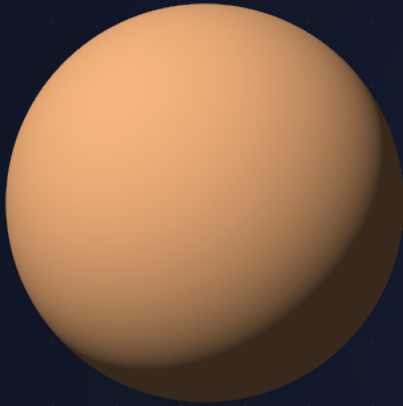
$$f = \frac{\rho}{\pi}$$

ρ : albedo, the fraction of light reflected

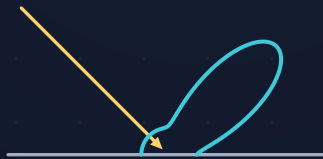
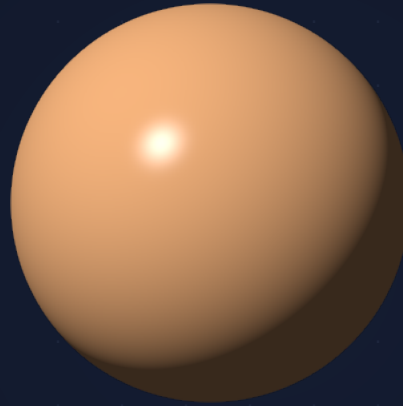
$$\int_{\Omega} \cos \theta \, d\omega = \int_0^{2\pi} \int_0^{\pi/2} \cos \theta \sin \theta \, d\theta \, d\phi = \pi$$

$$\int_{\Omega} \frac{\rho}{\pi} \cos \theta \, d\omega = \rho \leq 1$$

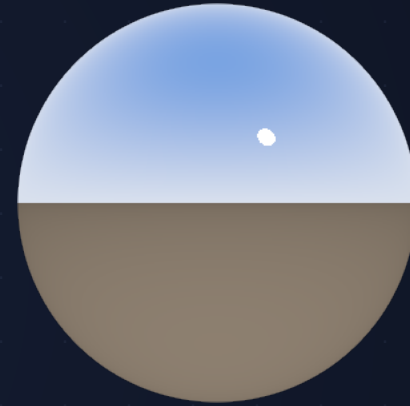
5 Lambertian, glossy and mirror



Lambertian

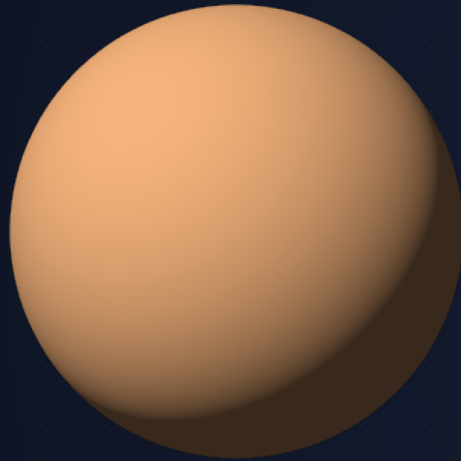


glossy



mirror

5 Lambertian, glossy and mirror



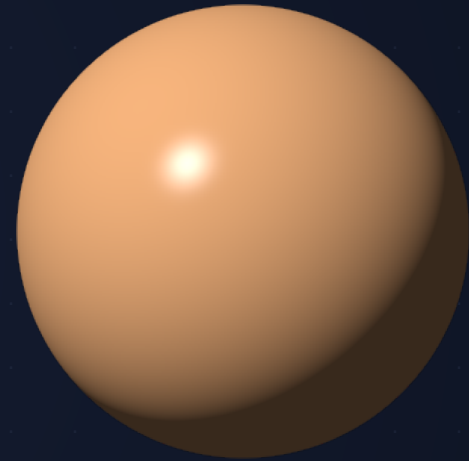
diffuse (coloured)

+



specular (white)

=



total

diffuse varies with wavelength; specular varies with polarization

5 Lambertian, glossy and mirror

Lambertian

1 parameter: albedo

Phong / Blinn

lobe width and strength

Ward

anisotropic gloss

Measured

gonioreflectometer, 4D table

6 Light sources

Models of illumination

$$L(x, y, z, \theta, \phi, t, \lambda)$$

position

direction

time

wavelength

6 Light sources



mirrored ball: a light probe

radiance depends only on direction

6 Light sources



mirrored ball: a light probe

radiance depends only on direction

low-pass filter



Lambertian sphere in this environment

smooth shading: 9 numbers are enough

6 Light sources

plenoptic function

a function on 5D and more

far-field environment

a function on 2D

low-frequency far-field

9 numbers

directional light

3 numbers: direction and strength

point light

4 numbers: position and strength

6 Light sources



light 1 m above the floor



light 2 m above: centre ¼ as bright

$$E = \frac{I \cos \theta}{d^2}$$

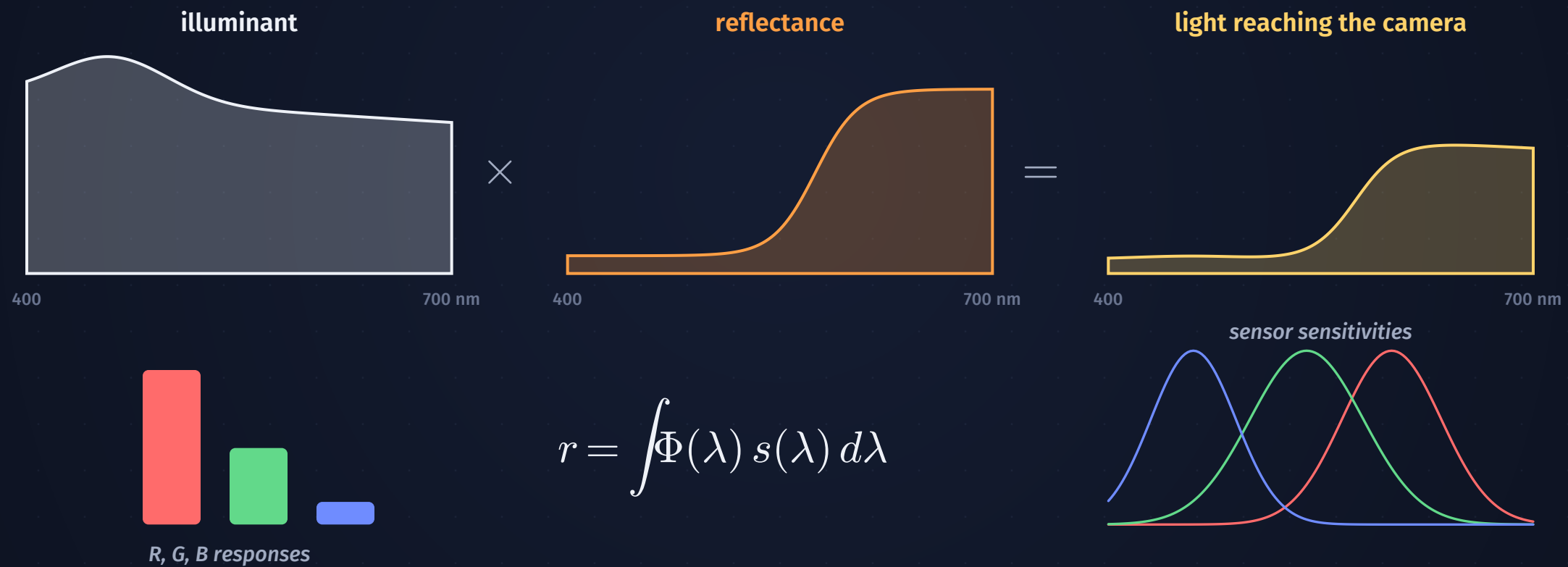
7 Colour and photometry

Wavelength and human vision

7 Colour and photometry



7 Colour and photometry



7 Colour and photometry

quantity	radiometric	photometric
flux	watt (W)	lumen (lm)
intensity	W/sr	candela (cd = lm/sr)
irradiance	W/m ²	lux (lm/m ²)
radiance	W/(m ² ·sr)	nit (cd/m ²)

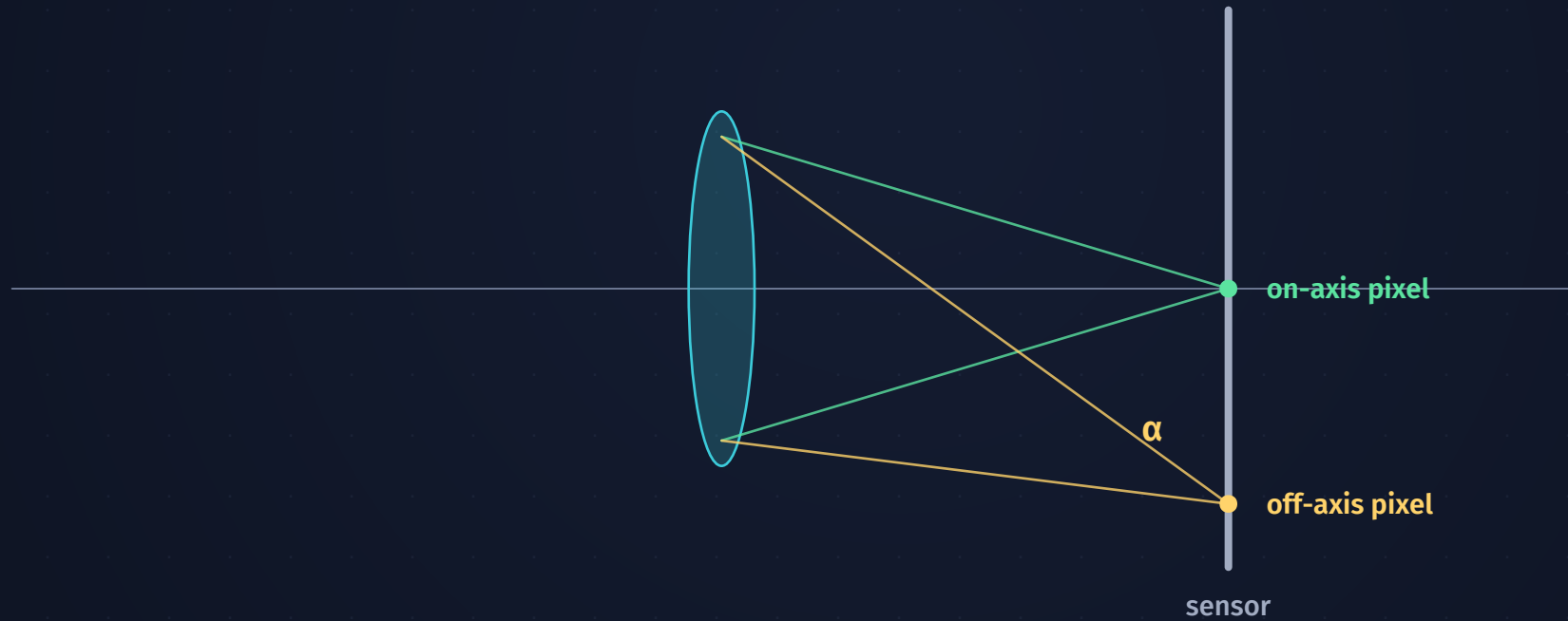
1 candela: 1/683 W/sr of green light at 555 nm

8 Vignetting and the moon

Two quizzes from the lecture

8 Vignetting and the moon

$$E = L \frac{\pi}{4} \left(\frac{D}{f} \right)^2 \cos^4 \alpha$$

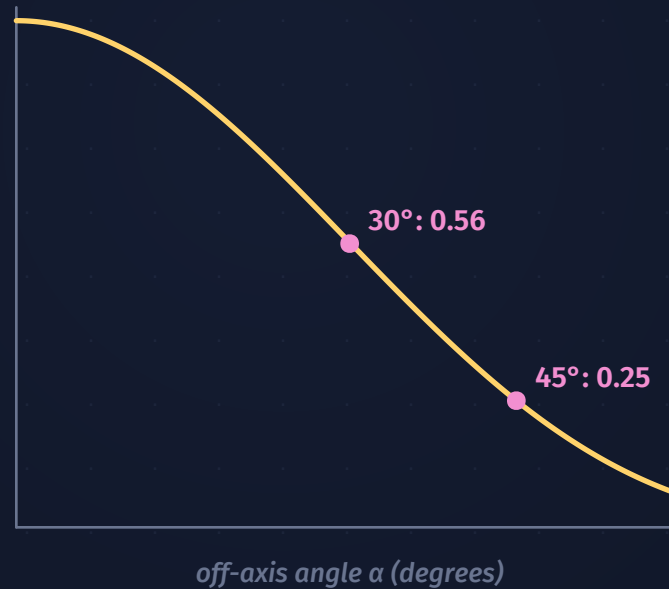


8 Vignetting and the moon



$\cos^4$ vignetting applied

$$E = L \frac{\pi}{4} \left(\frac{D}{f} \right)^2 \cos^4 \alpha$$



mechanical: hoods, filters

lens: blocked by lens elements

natural: cosine-fourth falloff

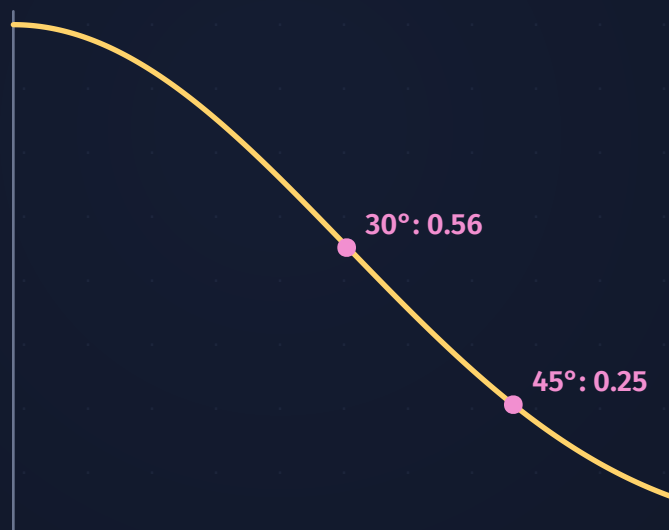
pixel: angle-dependent sensitivity

8 Vignetting and the moon



$\cos^4$ vignetting applied

$$E = L \frac{\pi}{4} \left(\frac{D}{f} \right)^2 \cos^4 \alpha$$



off-axis angle α (degrees)

$(D/f)^2 = 1/N^2$: brightness proportional to $1/N^2$

mechanical: hoods, filters

lens: blocked by lens elements

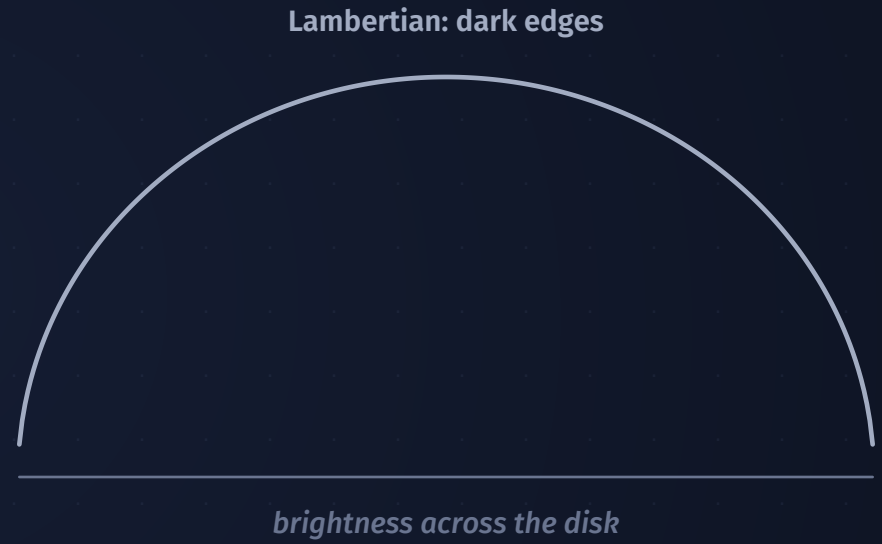
natural: cosine-fourth falloff

pixel: angle-dependent sensitivity

8 Vignetting and the moon



Lambertian sphere, front light



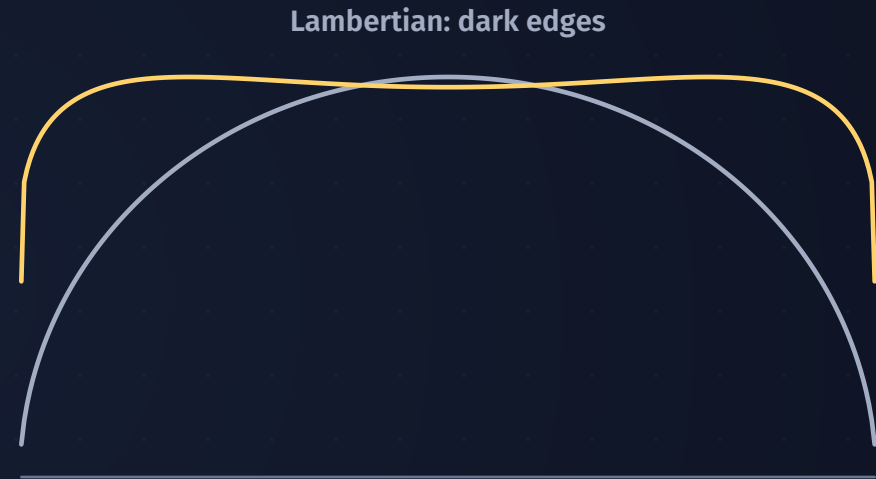
8 Vignetting and the moon



Lambertian sphere, front light



rough diffuse (Oren-Nayar)



brightness across the disk

rough: much flatter, like the moon

9 Shading and surface normals

$\mathbf{n} \cdot \mathbf{l}$

9 Shading and surface normals

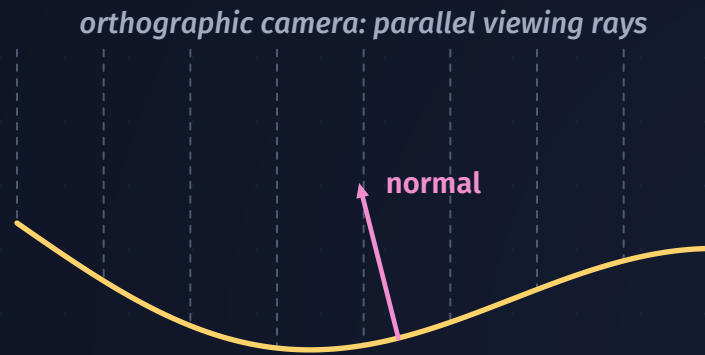


$$I = \rho (\mathbf{n} \cdot \mathbf{s})$$

brightest where the normal faces the light

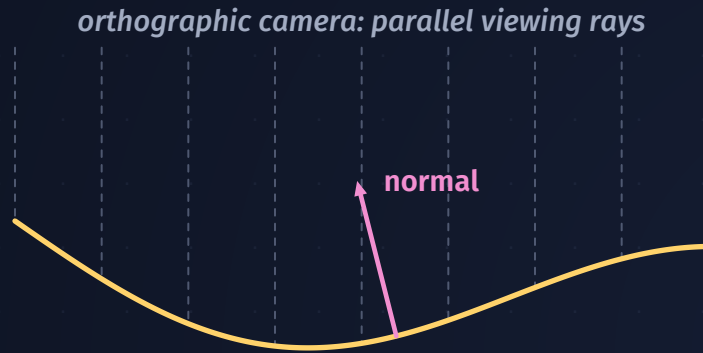
s: light direction scaled by its strength

9 Shading and surface normals



$$\tilde{\mathbf{n}} = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, -1 \right), \quad \mathbf{n} = \frac{\tilde{\mathbf{n}}}{\|\tilde{\mathbf{n}}\|}$$

9 Shading and surface normals

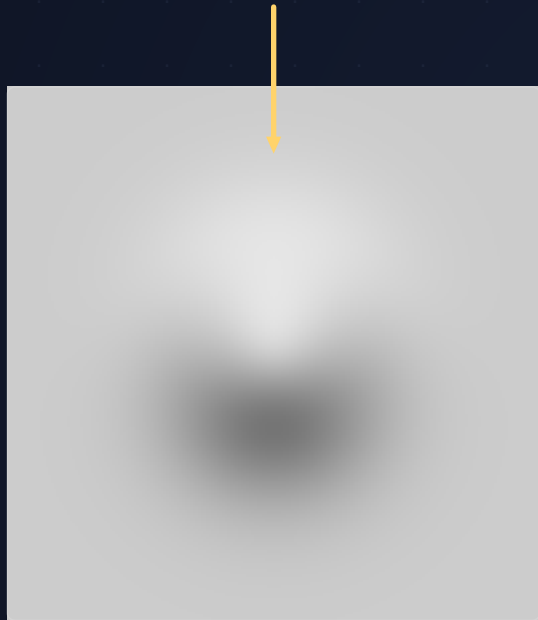


$$\tilde{\mathbf{n}} = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, -1 \right), \quad \mathbf{n} = \frac{\tilde{\mathbf{n}}}{\|\tilde{\mathbf{n}}\|}$$

$$f = x^2 + y: \quad \frac{\partial f}{\partial x} = 2x = 2, \quad \frac{\partial f}{\partial y} = 1$$

$$\mathbf{n} = \frac{(2, 1, -1)}{\sqrt{6}} \approx (0.82, 0.41, -0.41)$$

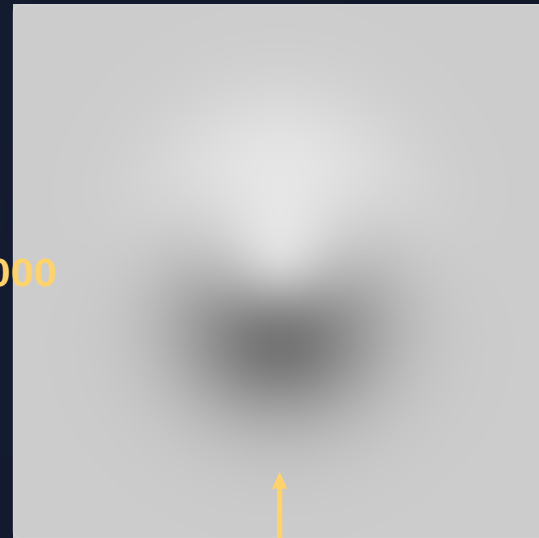
9 Shading and surface normals



bump, light from above

pixel difference: 0.000

identical images

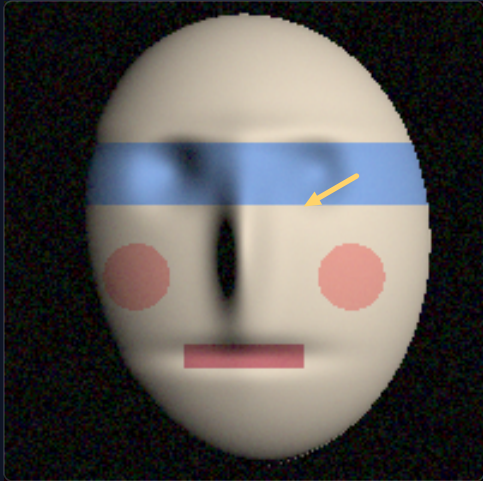


dent, light from below

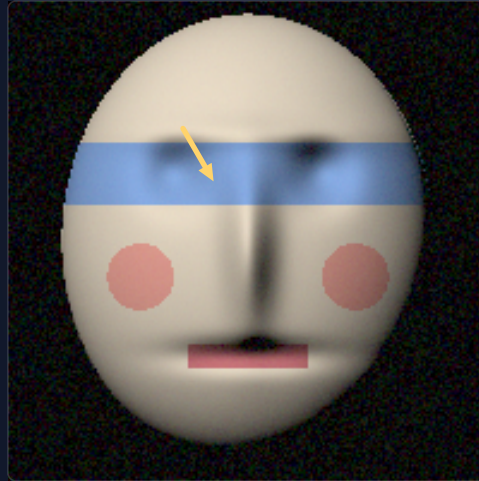
10 Photometric stereo

Shape from several lights

10 Photometric stereo



light 1



light 2



light 3



light 4

$$I_k = \mathbf{s}_k^T \mathbf{b}, \quad \mathbf{b} = \rho \mathbf{n}$$

$$\mathbf{I} = S \mathbf{b}$$

$$(K \times 1) = (K \times 3)(3 \times 1)$$

$$I_k = \mathbf{s}_k^T \mathbf{b}, \quad \mathbf{b} = \rho \mathbf{n}$$

$$\mathbf{I} = S \mathbf{b}$$

$$(K \times 1) = (K \times 3)(3 \times 1)$$

$$K = 3: \quad \mathbf{b} = S^{-1} \mathbf{I}$$

$$\rho = \|\mathbf{b}\|, \quad \mathbf{n} = \frac{\mathbf{b}}{\|\mathbf{b}\|}$$

$$I_k = \mathbf{s}_k^T \mathbf{b}, \quad \mathbf{b} = \rho \mathbf{n}$$

$$\mathbf{I} = S \mathbf{b}$$

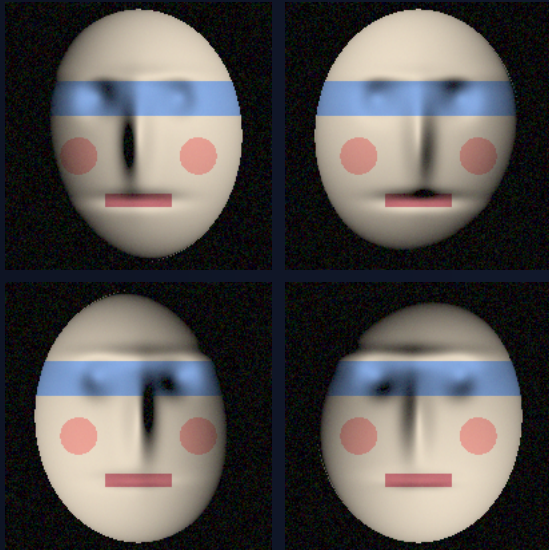
$$(K \times 1) = (K \times 3)(3 \times 1)$$

$$K > 3: \quad \mathbf{b} = (S^T S)^{-1} S^T \mathbf{I}$$

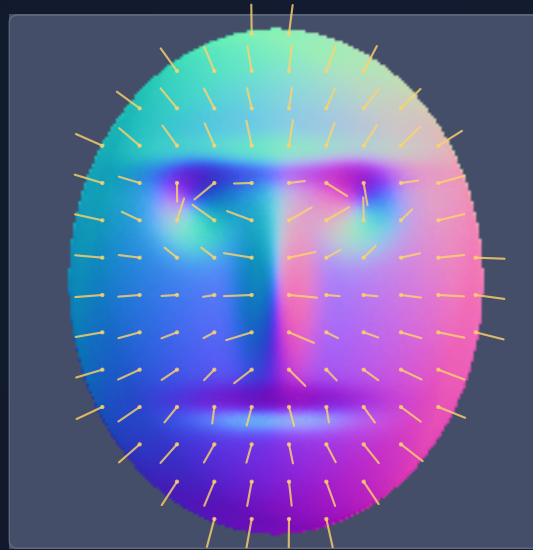
the Moore–Penrose pseudo-inverse: a least-squares fit

more lights: less noise, and shadowed or shiny measurements can be dropped

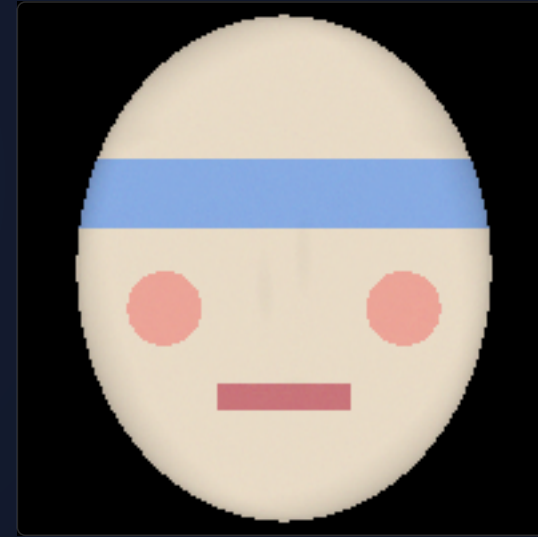
10 Photometric stereo



4 input images

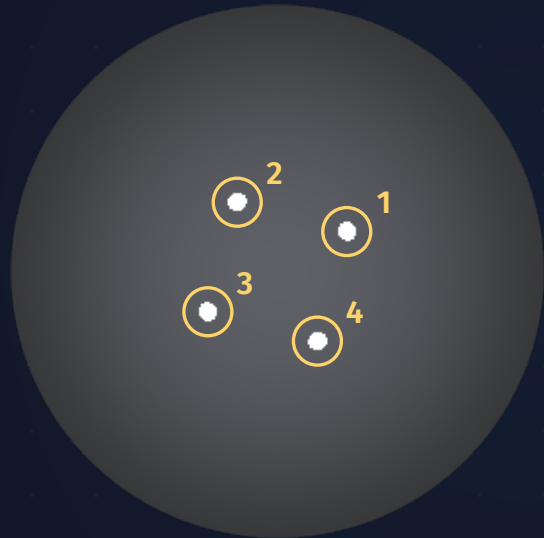


normals (RGB = x, y, z)



recovered albedo

10 Photometric stereo



highlight position $\rightarrow$ light direction

$$\mathbf{s} = 2(\mathbf{n} \cdot \mathbf{v}) \mathbf{n} - \mathbf{v}$$

11 From normals to depth

Integrating the surface

11 From normals to depth

$$n_x + n_z (z_{x+1,y} - z_{x,y}) = 0$$

$$n_y + n_z (z_{x,y+1} - z_{x,y}) = 0$$

the tangent between neighbouring pixels must be perpendicular to the normal

11 From normals to depth

$$n_x + n_z (z_{x+1,y} - z_{x,y}) = 0$$

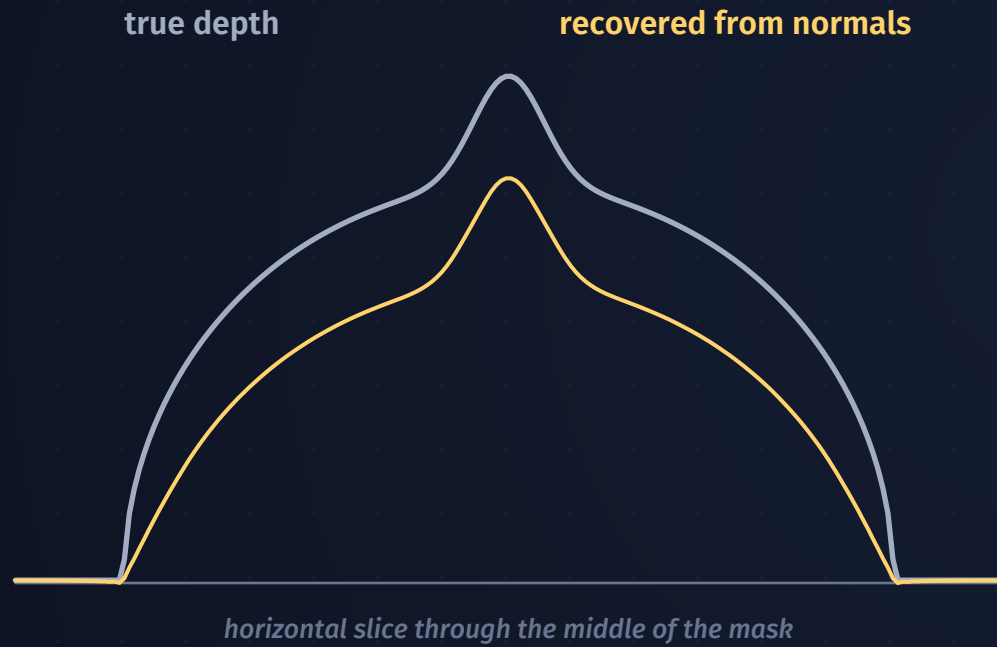
$$n_y + n_z (z_{x,y+1} - z_{x,y}) = 0$$

the tangent between neighbouring pixels must be perpendicular to the normal

stack all constraints: one big sparse linear system

least squares (or a fast Fourier-domain solver) → depth map

11 From normals to depth



relit: estimated albedo



relit: uniform albedo

11 From normals to depth

shiny or translucent materials

the Lambertian model fails

shadows and interreflections

extra or missing light

calibration

light directions, intensities, camera response

distant lights and camera

assumed by the model

12 Practice problems

Try each one before looking at the answer

Problem 1



What solid angle does a hemisphere subtend at its center?

try it first

Problem 1



What solid angle does a hemisphere subtend at its center?

Answer

2π sr

$$\frac{4\pi}{2} = 2\pi \text{ sr}$$

Problem 2



A 2 cm × 2 cm patch is 2 m away, tilted 60° from facing you. What solid angle does it subtend?

try it first

Problem 2



A 2 cm × 2 cm patch is 2 m away, tilted 60° from facing you. What solid angle does it subtend?

Answer

$5 \times 10^{-5} \text{ sr}$

$$d\omega = \frac{4 \times 10^{-4} \cdot \cos 60^\circ}{2^2} = 5 \times 10^{-5} \text{ sr}$$

Problem 3



What are the units of irradiance and of radiance?

try it first

Problem 3



What are the units of irradiance and of radiance?

Answer

Irradiance: W/m^2 . Radiance: $\text{W}/(\text{m}^2 \cdot \text{sr})$.

Problem 4



You move a point light twice as far from a surface. What happens to the irradiance?

try it first

Problem 4



You move a point light twice as far from a surface. What happens to the irradiance?

Answer

It drops to **one quarter**.

$$E \propto \frac{1}{d^2} : \quad \frac{1}{2^2} = \frac{1}{4}$$

Problem 5



Why does a white wall look equally bright from 1 m and from 10 m away?

try it first

Problem 5



Why does a white wall look equally bright from 1 m and from 10 m away?

Answer

Radiance is constant along rays; a pixel measures radiance.

Problem 6



What is the BRDF of a Lambertian surface with albedo 0.6?

try it first

Problem 6



What is the BRDF of a Lambertian surface with albedo 0.6?

Answer

$$f = 0.6 / \pi \approx 0.19 \text{ sr}^{-1}$$

$$f = \frac{\rho}{\pi} = \frac{0.6}{\pi} \approx 0.19 \text{ sr}^{-1}$$

Problem 7



Why can the albedo of a Lambertian surface not exceed 1?

try it first

Problem 7



Why can the albedo of a Lambertian surface not exceed 1?

Answer

Energy conservation: reflected fraction = ρ .

$$\int_{\Omega} \frac{\rho}{\pi} \cos \theta \, d\omega = \rho \leq 1$$

Problem 8



Albedo 0.5, light strength 2, light 60° from the normal. What is the $\mathbf{n} \cdot \mathbf{l}$ intensity?

try it first

Problem 8



Albedo 0.5, light strength 2, light 60° from the normal. What is the n-dot-l intensity?

Answer

$I = 0.5$

$$I = \rho |s| \cos \theta = 0.5 \cdot 2 \cdot 0.5 = 0.5$$

Problem 9



From natural vignetting alone, how bright is a pixel 45° off axis compared with the center?

try it first

Problem 9



From natural vignetting alone, how bright is a pixel 45° off axis compared with the center?

Answer

0.25: a quarter as bright.

$$\cos^4 45^\circ = \left(\frac{1}{\sqrt{2}}\right)^4 = 0.25$$

Problem 10



The full moon looks about as bright at its edge as at its center. Is its surface Lambertian?

try it first

Problem 10



The full moon looks about as bright at its edge as at its center. Is its surface Lambertian?

Answer

No: Lambertian would darken toward the edge.

Problem 11



The depth is $f = x^2 + y$. What is the unit normal at the point $(1, 0)$?

try it first

Problem 11



The depth is $f = x^2 + y$. What is the unit normal at the point $(1, 0)$?

Answer

$$\mathbf{n} = (2, 1, -1) / \sqrt{6}$$

$$\tilde{\mathbf{n}} = (2x, 1, -1) = (2, 1, -1)$$

Problem 12



At minimum, how many lights does Lambertian photometric stereo need for a unique solution?

try it first

Problem 12



At minimum, how many lights does Lambertian photometric stereo need for a unique solution?

Answer

Three lights.

$$\mathbf{b} = \rho \mathbf{n} \in \mathbb{R}^3$$

Problem 13



Lights $s_1 = (0, 0, 1)$, $s_2 = (0.6, 0, 0.8)$, $s_3 = (0, 0.6, 0.8)$. A pixel measures 0.4, 0.428, 0.464.
Find the albedo and normal.

try it first

Problem 13



Lights $s_1 = (0, 0, 1)$, $s_2 = (0.6, 0, 0.8)$, $s_3 = (0, 0.6, 0.8)$. A pixel measures 0.4, 0.428, 0.464. Find the albedo and normal.

Answer

$$\rho = 0.5, n = (0.36, 0.48, 0.80)$$

$$b_z = 0.4, \quad b_x = \frac{0.428 - 0.32}{0.6} = 0.18, \quad b_y = \frac{0.464 - 0.32}{0.6} = 0.24$$

Problem 14



From a **single shaded image**, can you always tell a bump from a dent?

try it first

Problem 14



From a single shaded image, can you always tell a bump from a dent?

Answer

No: bump + light from above = dent + light from below.

13 Recap

Everything on one page

Measuring light

solid angle; flux, irradiance, radiance; radiance is constant along rays; RAW
proportional to radiance

13 Recap

Measuring light

solid angle; flux, irradiance, radiance; radiance is constant along rays; RAW proportional to radiance

Reflectance

BRDF and its properties; reflectance equation; Lambertian ρ/π ; mirror, glossy, dichromatic

Measuring light

solid angle; flux, irradiance, radiance; radiance is constant along rays; RAW proportional to radiance

Reflectance

BRDF and its properties; reflectance equation; Lambertian ρ/π ; mirror, glossy, dichromatic

Shape

$\mathbf{n} \cdot \mathbf{l}$ shading; normals from depth; photometric stereo; integrating normals to depth

13 Recap

Solid angle

$$d\omega = \frac{dA \cos \theta}{r^2}$$

Irradiance, radiance

$$E = \frac{d\Phi}{dA}, \quad L = \frac{d^2\Phi}{dA \cos \theta d\omega}$$

BRDF

$$f(\omega_i, \omega_r) = \frac{dL_r}{dE_i}$$

Reflectance equation

$$L_r = \int_{\Omega} L_i f \cos \theta_i d\omega_i$$

Lambertian

$$f = \frac{\rho}{\pi}$$

n-dot-l shading

$$I = \rho (\mathbf{n} \cdot \mathbf{s})$$

Photometric stereo

$$\mathbf{b} = (S^T S)^{-1} S^T \mathbf{I}, \quad \rho = \|\mathbf{b}\|$$

Vignetting

$$E \propto \frac{\cos^4 \alpha}{N^2}$$

Further reading

Szeliski, Section 2.2

Computer Vision: Algorithms and Applications

Gortler, Chapter 21

Foundations of 3D Computer Graphics

Woodham (1980)

Photometric method for determining surface orientation from multiple images

Oren and Nayar (1995)

Generalization of the Lambertian model

Debevec (1998)

Rendering Synthetic Objects into Real Scenes

CSC420 Tutorial 6: radiometry and photometric stereo, University of Toronto

Presented by Javad Rajabi, Teaching Assistant

cs.toronto.edu/~lindell/teaching/420