Introduction

Motivation and Image Processing

CSC420
David Lindell
University of Toronto
cs.toronto.edu/~lindell/teaching/420
Slide credit: Babak Taati ←Ahmed Ashraf ←Sanja Fidler

TCIG
Toronto Computational Imaging Group

Course staff

Instructor

David Lindell

Teaching Assistants

Shayan Shekarforoush

Vida Adeli

Yun-Chun Chen

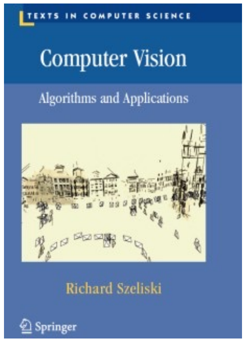
Amirhossein Kazerouni

Victor Rong

Course Info

- Class time: Mondays 1pm-3pm (LEC0101; MP137) and 3pm-5pm (LEC0201; ES B149)
- Tutorials: Wednesdays 1pm-2pm (LEC0101; MP137) and 3pm-4pm (LEC0201; ES B149)
- TA Office Hours: Wednesdays 2pm (MP137)
- Class Website: https://www.cs.toronto.edu/~lindell/teaching/420/
- Quercus: https://q.utoronto.ca/
- Course material (lecture notes, reading material, assignments, announcements, etc.) will be posted on Quercus
- Forum: Piazza (link on Quercus)
- Your grade will not depend on your participation on discussions. It's just a good way for asking questions, discussing with your instructor, TAs and your peers

Textbook: We won't directly follow any book, but extra reading in this textbook will be useful:



Rick Szeliski

Computer Vision: Algorithms and Applications

available free online: http://szeliski.org/Book/

Links to other material (papers, code, etc.) will be posted on the class webpage

Course Prerequisites

- Data structures
- Linear Algebra
- Vector calculus
- Without this you'll need some serious catching up to do!

Knowing some basics in these is a plus:

- Python
- Machine Learning
- Neural Networks
- (Solving assignments sooner rather than later)

Grading

- Assignment 1: 12%
- Assignment 2: 20%
- Assignment 3: 16%
- Assignment 4: 16%
- Ethics Module: 1% (2 surveys, 0.5 each)
- Final Exam: 35%

- Assignments: They will consist of problem sets and programming problems with the goal of deepening your understanding of the material covered in class.

Assignments

- Download from Files section on Quercus, Submitted via MarkUs
- Assignments: They will consist of problem sets and programming problems with the goal of deepening your understanding of the material covered in class.
    - Code in python
    - Please comment your code!
- Assignment 1 is out now, due Jan 24 at 11:59 PM

Assignments

Deadline

- The solutions to the assignments / project should be submitted by 11:59 pm on the date they are due.

Lateness

- Each student will be given a total of 5 free late days.
- This means that you can hand in three of the assignments one day late, or one assignment three days late.
- After you have used the 5-day budget, late assignments will not be accepted.

All info on the course website

Schedule and Syllabus

Week	Date	Description	Material	Readings	Event	Deadline
Week 1	Mon Jan 6	**Lecture 1: Introduction & Linear filters**	[slides]	Szeliski 3.2 (optional) Brain mechanisms of early vision (optional) Early vision	**Assignment 1 out on Quercus**	
	Wed Jan 8	**Tutorial 1**				
Week 2	Mon Jan 13	**Lecture 2: Edges**	[slides]	Szeliski 4.2 (optional) Fourier Transform (optional) Computer color is broken (optional) Fourier Transform Textbook		
	Wed Jan 15	**Tutorial 2**				
Week 3	Mon Jan 20	**Lecture 3: Image pyramids**	[slides]	Szeliski 3.5 (optional) Pyramid methods		
	Wed Jan 22	**Tutorial 3**				
	Fri Jan 24					Assignment 1 due at 11:59pm

Accessibility Services is seeking volunteer note takers for students in this class who are registered in Accessibility Services.

“By volunteering to take notes for students with disabilities, you are making a positive contribution to their academic success. By volunteering as a note-taker, you will benefit as well - It is an excellent way to improve your own note-taking skills and to maintain consistent class attendance. At the end of term, we would be happy to provide a Certificate of Appreciation for your hard work."

See Piazza for details

Let’s begin!

Introduction to Intro to Image Understanding

- What is Computer Vision?
- Why study Computer Vision?
- Which cool applications can we do with it? Is vision a hard problem?

What is Computer Vision?

What is computer vision?

- A field trying to develop automatic algorithms that can "see"

What is computer vision?

- What does it mean to see?

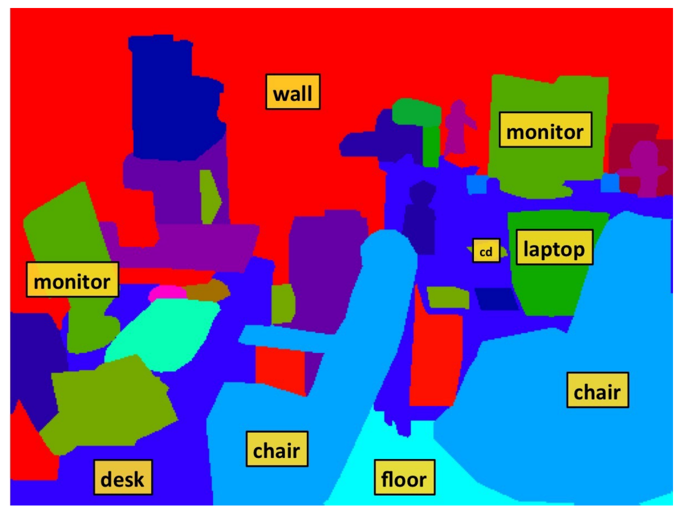
example scene

[adapted from A. Torralba]

What is computer vision?

- What does it mean to see?
  - To know what is where by looking – Marr, 1982
  - Understand where things are in the world

example scene



segmentation

[adapted from A. Torralba]

What is computer vision?

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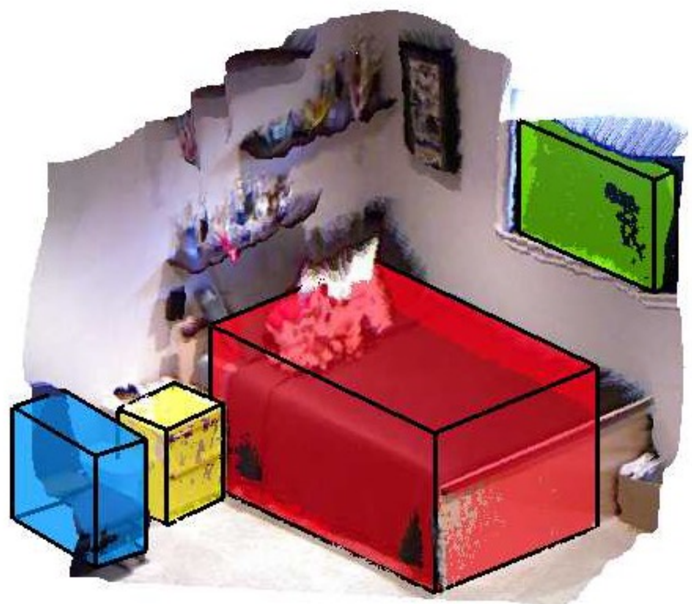
  - Understand where things are in the world
  - Understand 3D structure



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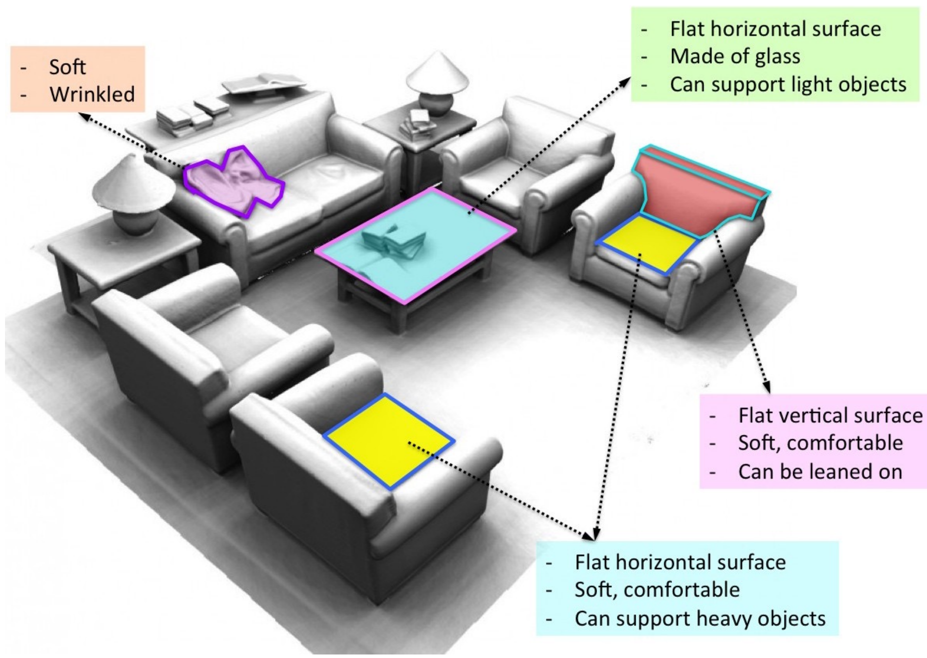


Image: Vladlen Koltun

What is computer vision?

- What does it mean to see?
  - To know what is where by looking – Marr, 1982
  - Understand where things are in the world
  - Understand 3D structure
  - Understand physical properties
  - Understand what actions are taking place

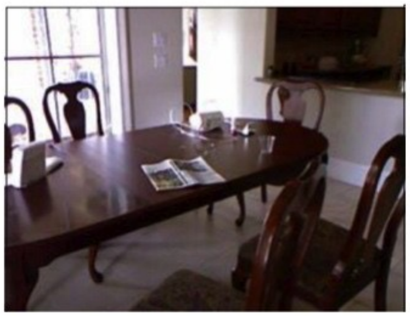
Image: www.cobblehillpuzzles.com

What is computer vision?

- Full understanding of an image?

What is computer vision?

- Full understanding of an image?
  - Can answer any question about it



Q: What is behind the table?
A: window

Q: What is in front of the toilet?
A: door

Q: What is on the counter in the corner? A: microwave

What is computer vision?

- Full understanding of an image?
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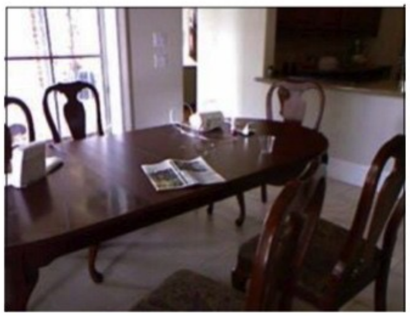

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Q: What is the shape of the green chair? A: horse shaped

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Q: Where is the oven?
A: on the right side of the fridge

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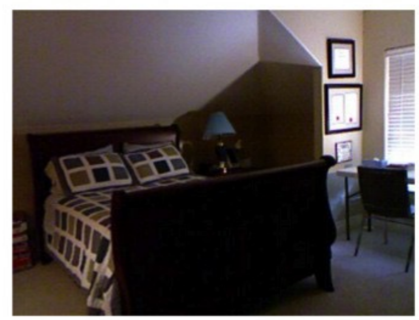

Q: What is the largest object?
A: bed

What is computer vision?

- Full understanding of an image?
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Q: Which object is red?
A: toaster

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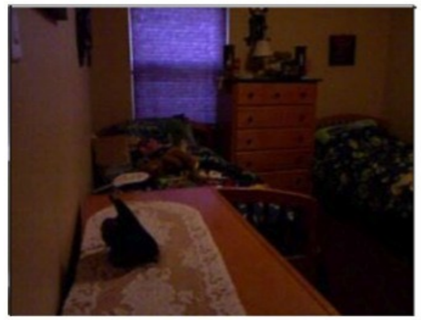
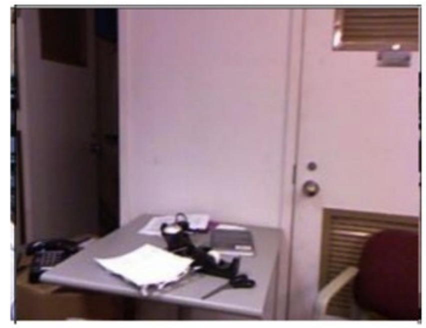


Q: How many drawers are there?
A: 6



Q: How many doors are open
A: 1

Q: How many lights are on?
A: 6

What is computer vision?

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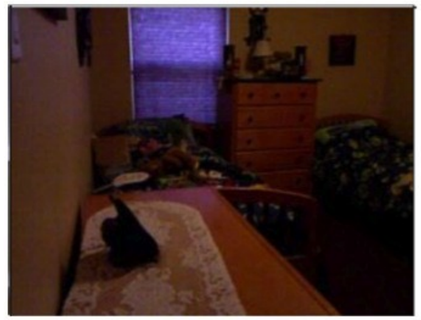

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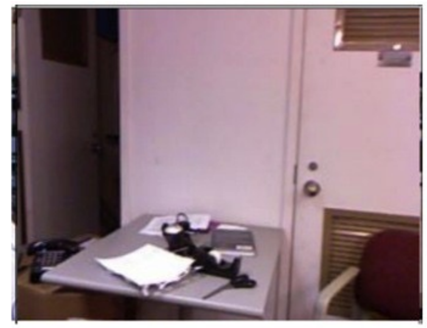

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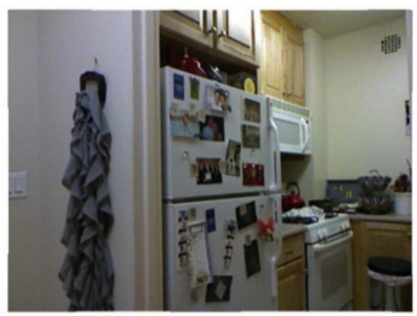


Q: Can you make pizza in this room?
A: yes

Q: Where can you sit?
A: chairs, table, floor

Why study computer vision?

Why study Computer Vision?

Because you want your robot to fold your laundry

Why study Computer Vision?

And drive you to work

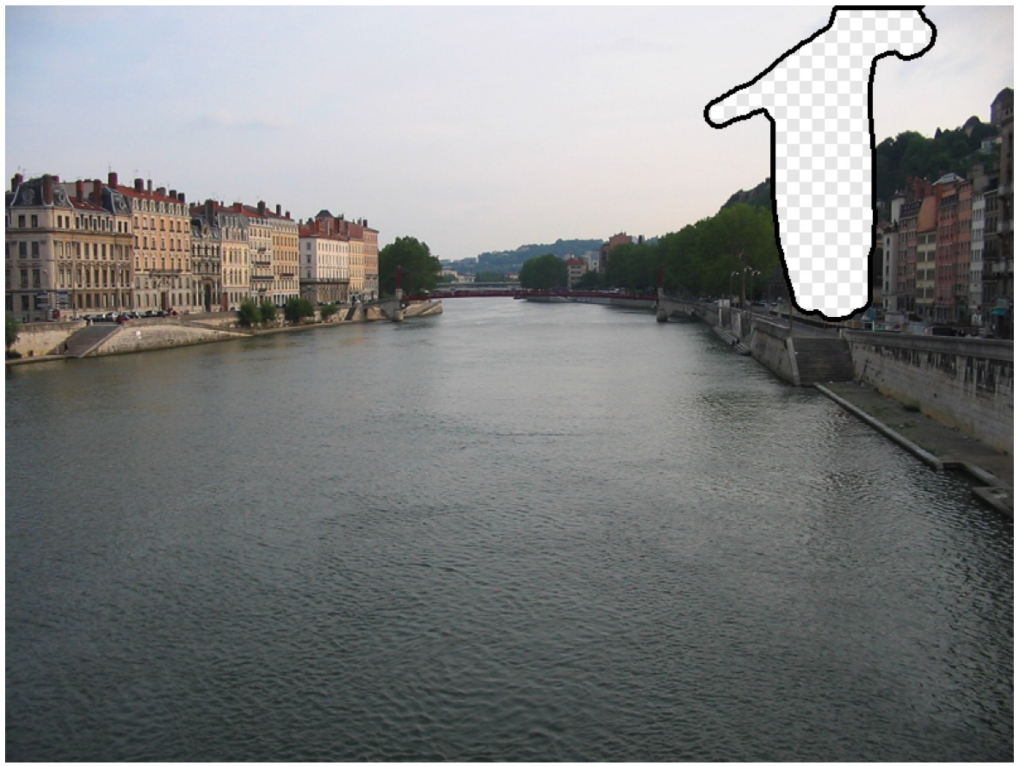
Why study Computer Vision?

Allows you to manipulate images

Scene Completion using Millions of Photographs, Hays & Efros, SIGGRAPH 2007

Why study Computer Vision?

Allows you to manipulate images



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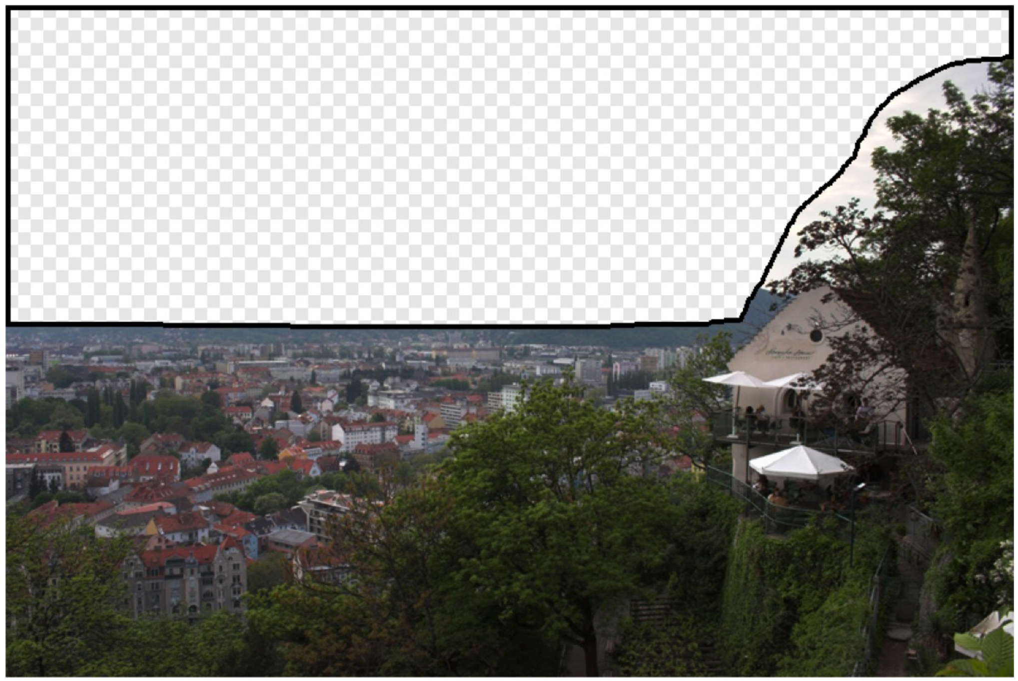

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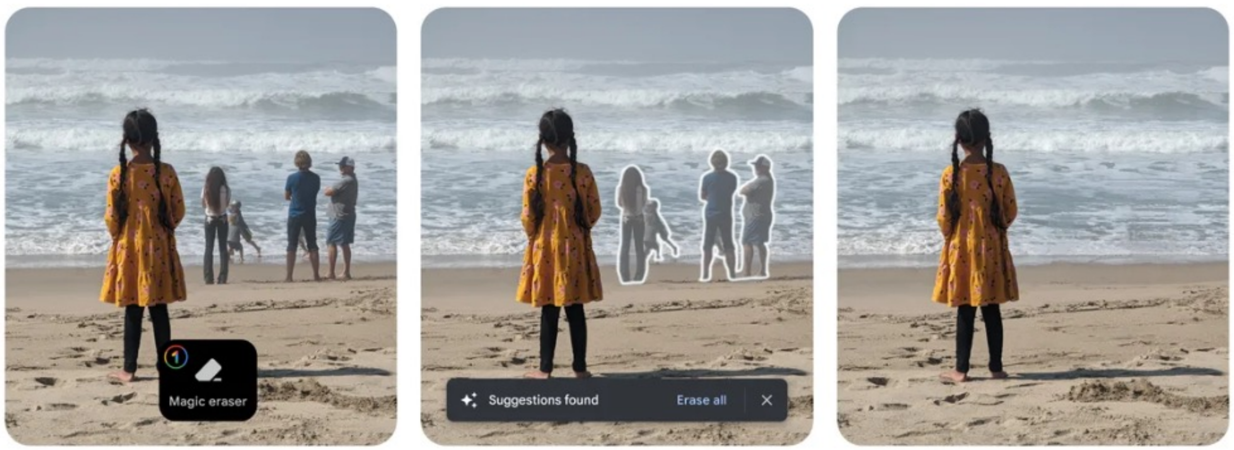

Google Magic Eraser

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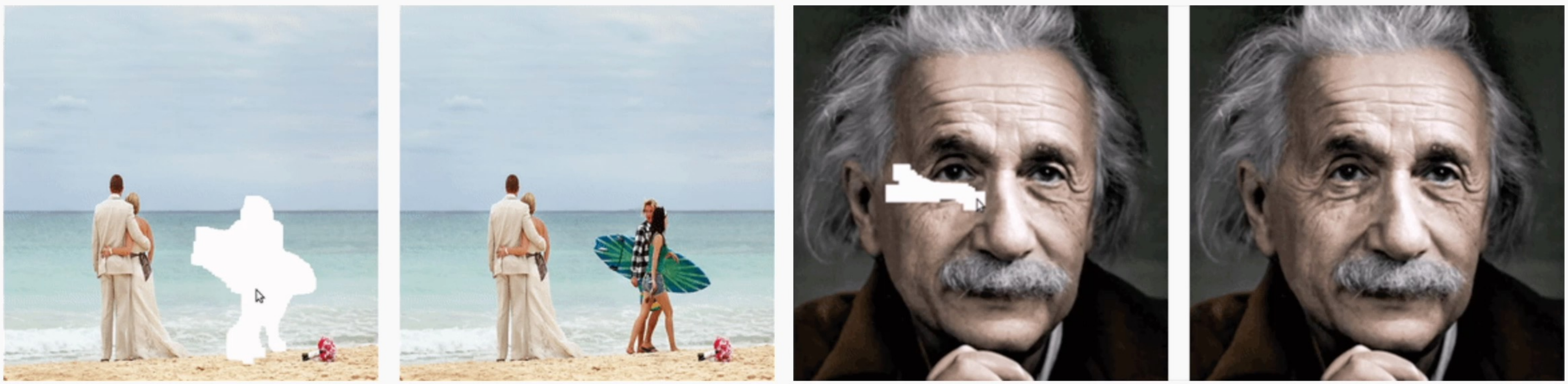


Online demo (NVIDIA inpainting demo)

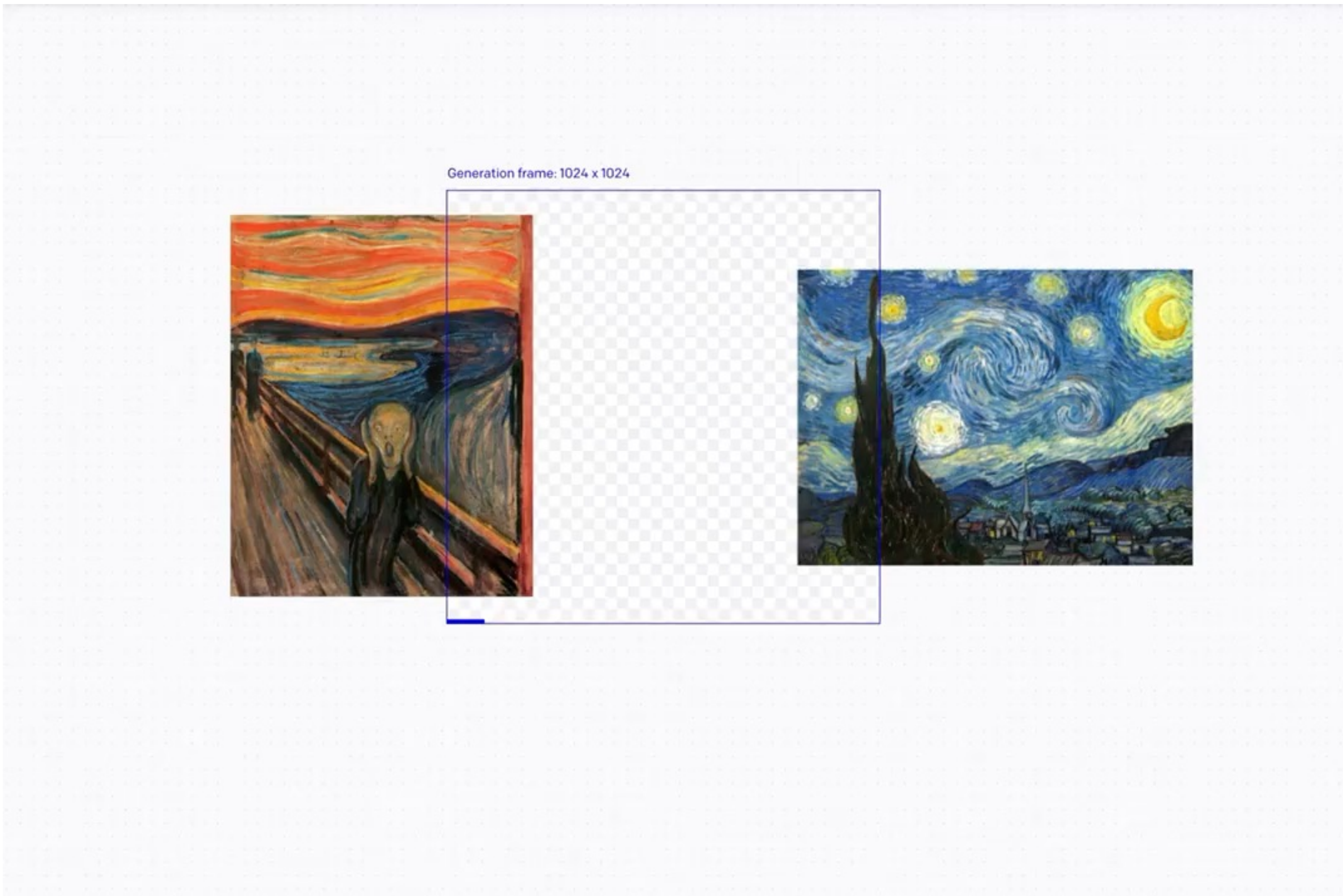
Why study Computer Vision?

Change style of images...

[Gatys, Ecker, Bethge. A Neural Algorithm of Artistic Style. Arxiv'15.]

Why study Computer Vision?

Inpainting art...



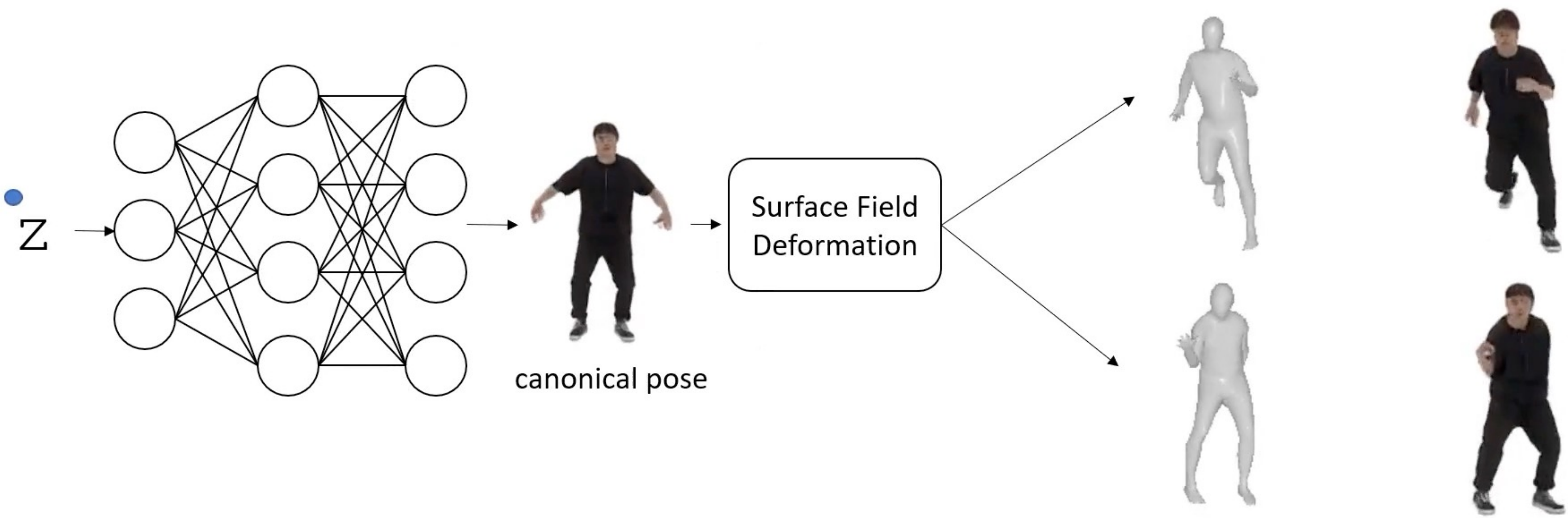
Why study Computer Vision?

Automatically caption images...

[Source: L. Zitnick, NIPS'14 Workshop on Learning Semantics]

Why study Computer Vision?

Synthesize and animate digital humans



[Bergman et al. ‘22]

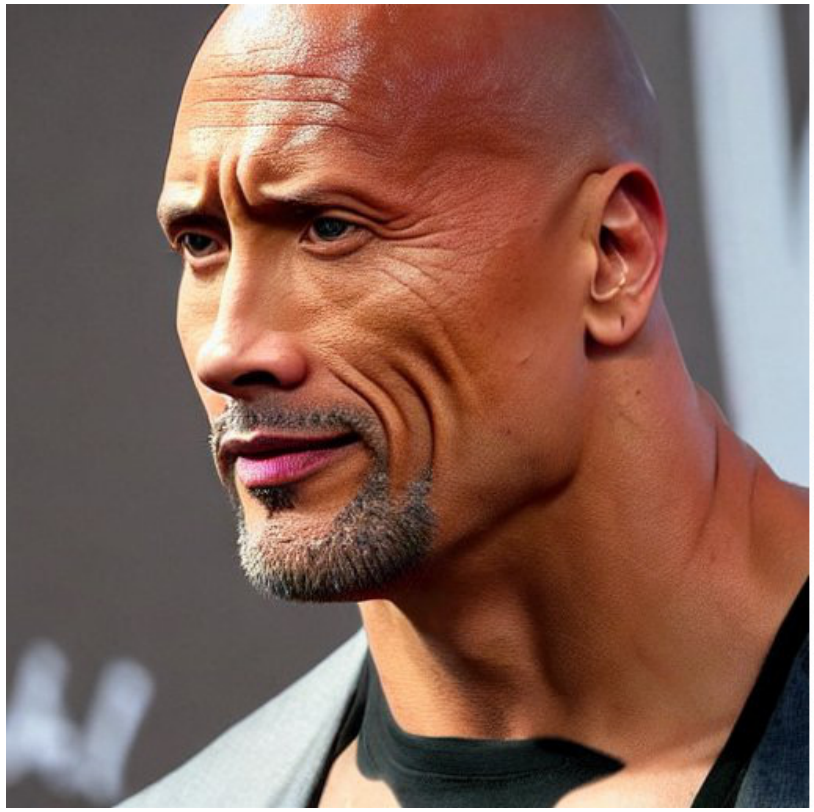
Why study Computer Vision?

Synthesize and animate digital humans

[Bergman et al. '22]

Why study Computer Vision?

Generate an image from a caption (stable diffusion)



"Dwayne Johnson side view"

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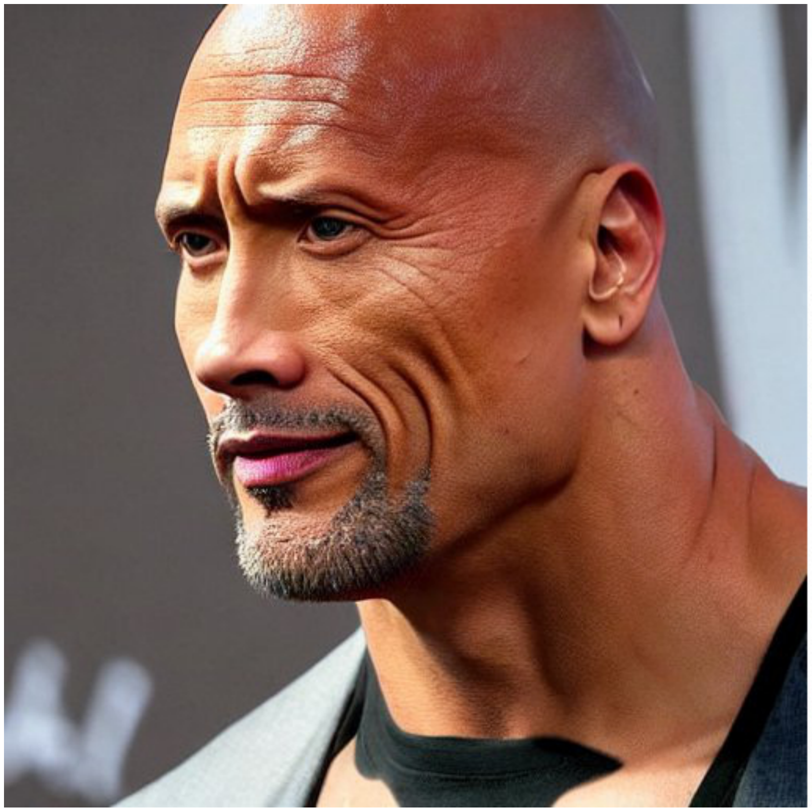

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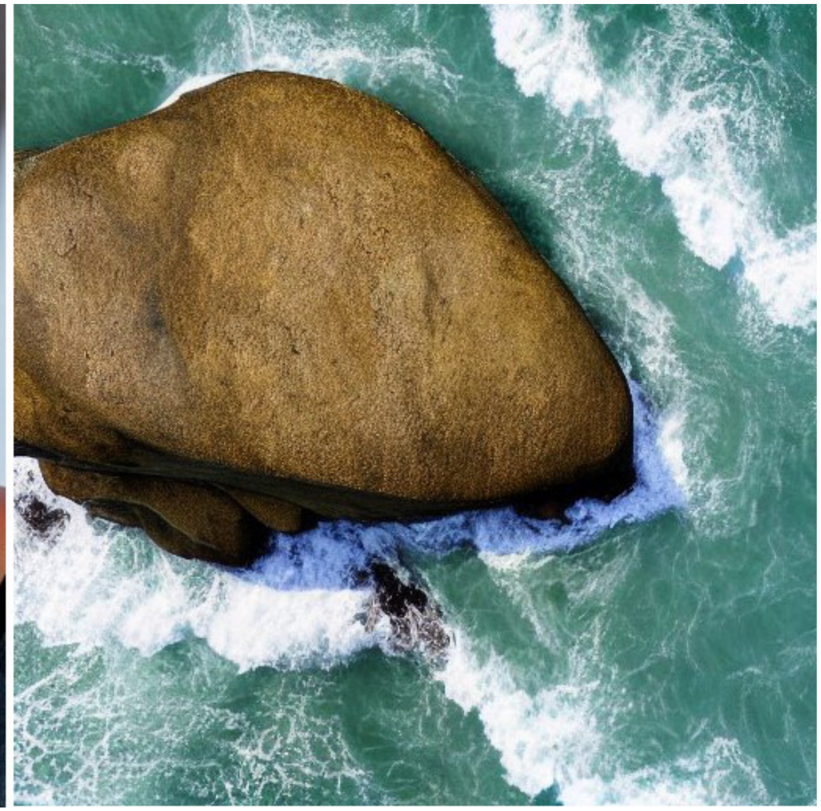


"Dwayne Johnson top view"

Why study Computer Vision?

generate animated 3D models from text

“a panda dancing”

“a space shuttle launching”

“a bear driving a car”

Why study Computer Vision?

See "invisible" changes in a scene...

[Wu et al. SIGGRAPH '12]

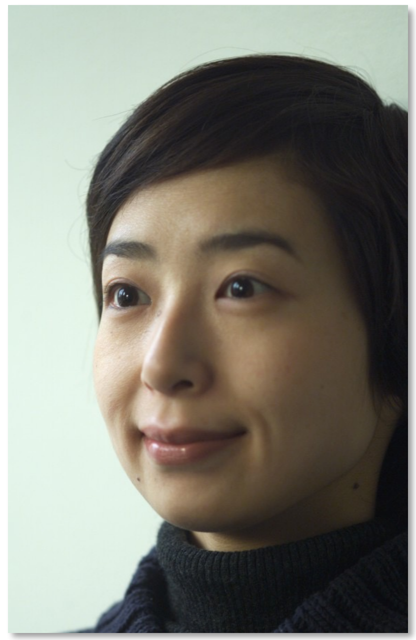
Why study Computer Vision?

See "invisible" changes in a scene...

[Wu et al. SIGGRAPH '12]

Why study Computer Vision?

Movie-like image forensics



[Nayar and Nishino, Eyes for Relighting]

[Slide: N. Snavely]

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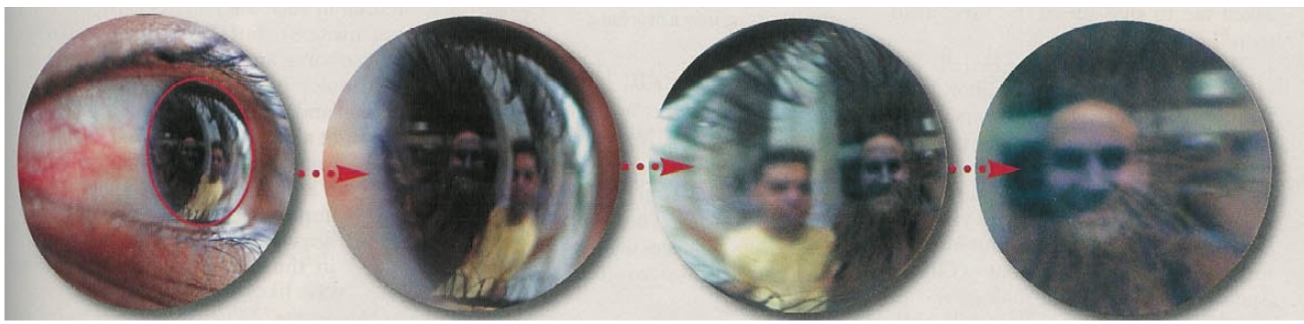


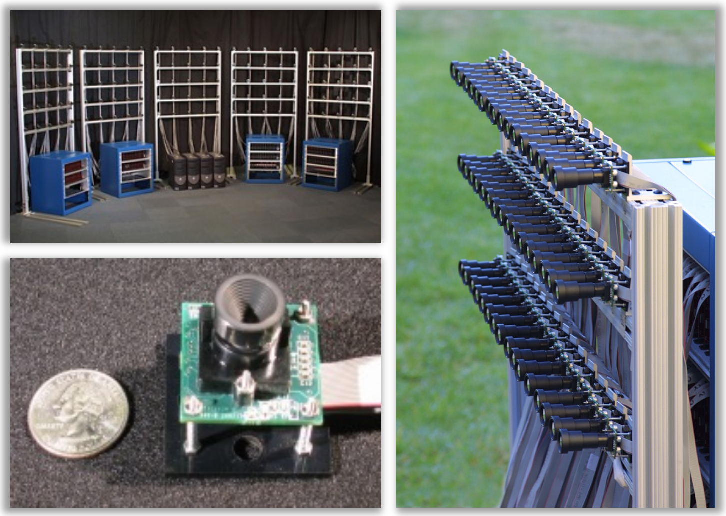
[Nayar and Nishino, Eyes for Relighting]

[Slide: N. Snavely]

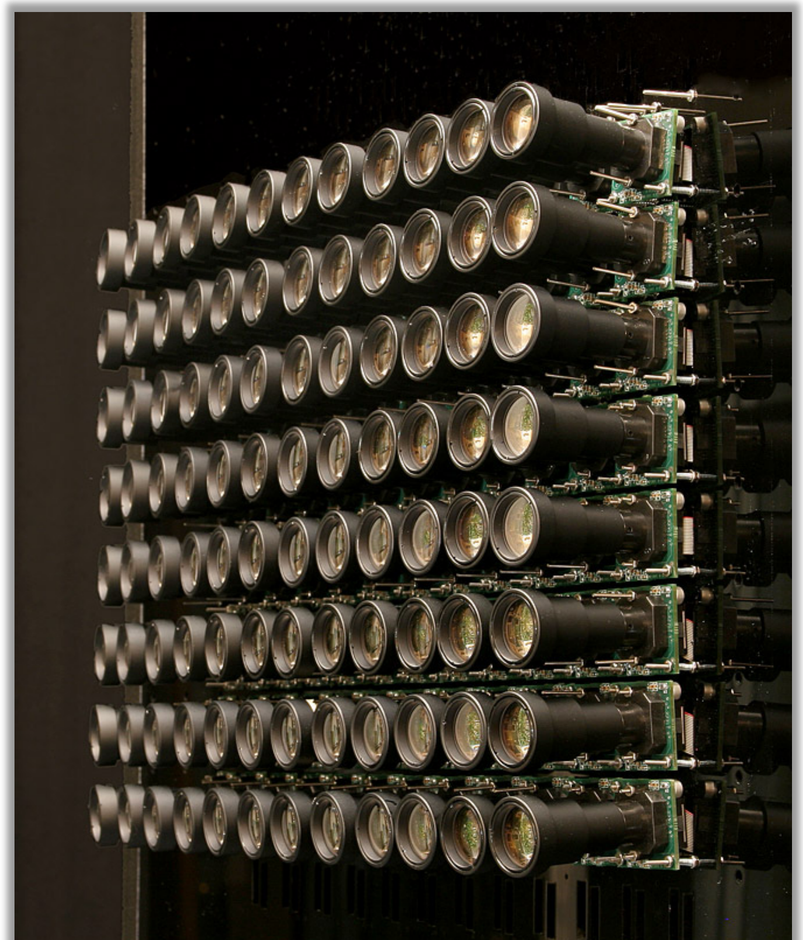
Why study Computer Vision?

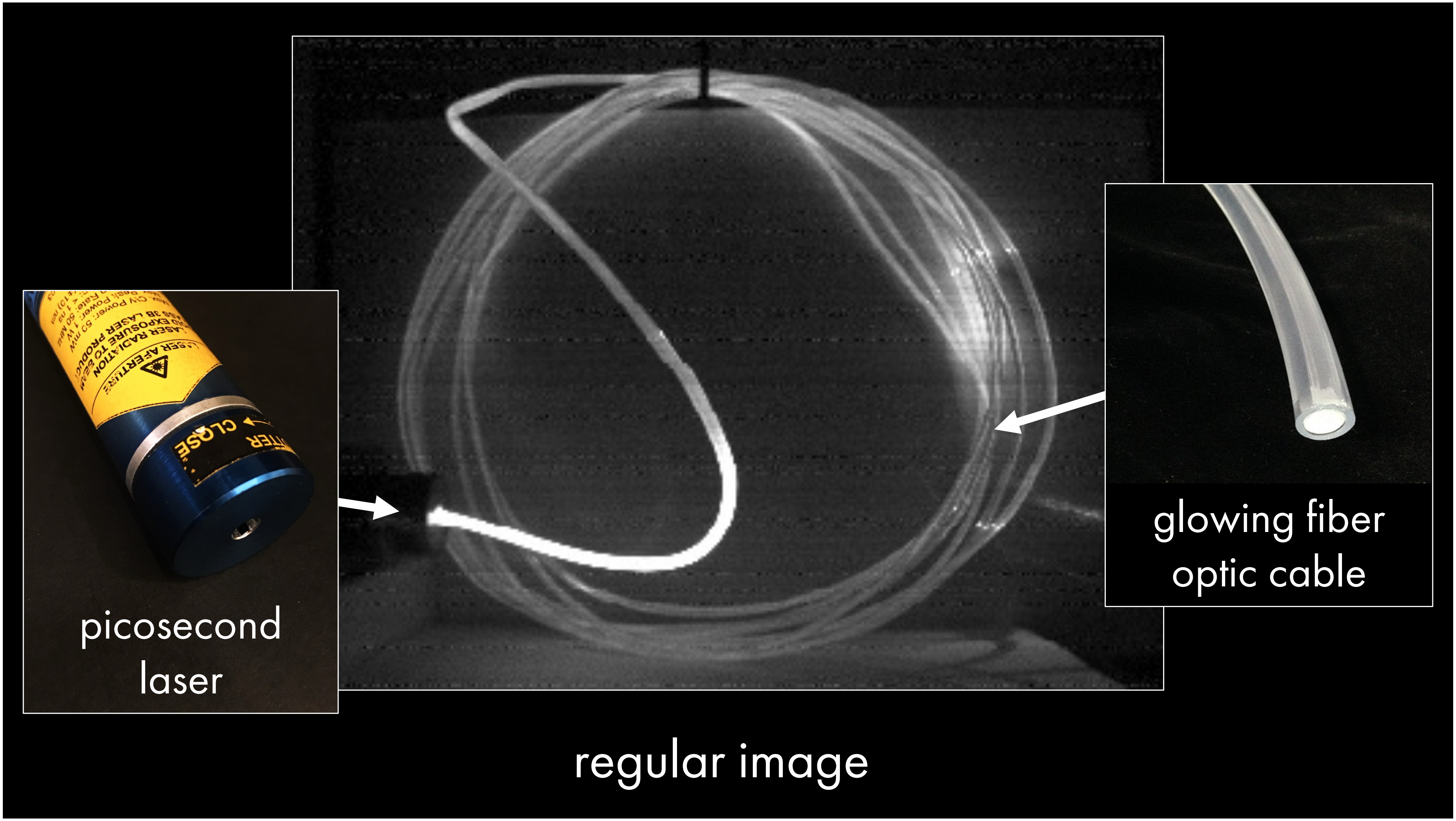
Capture light fields

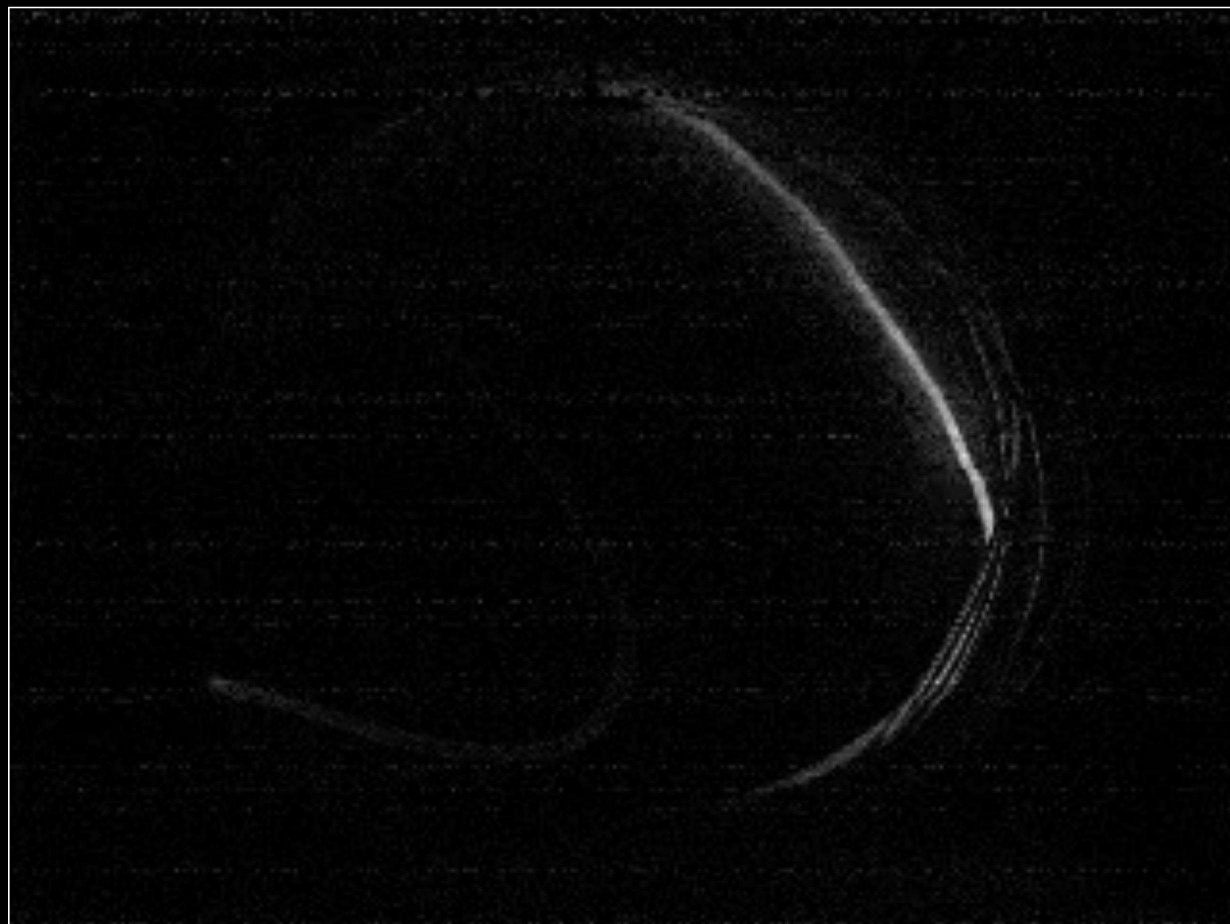
- Stanford Multi-Camera Array



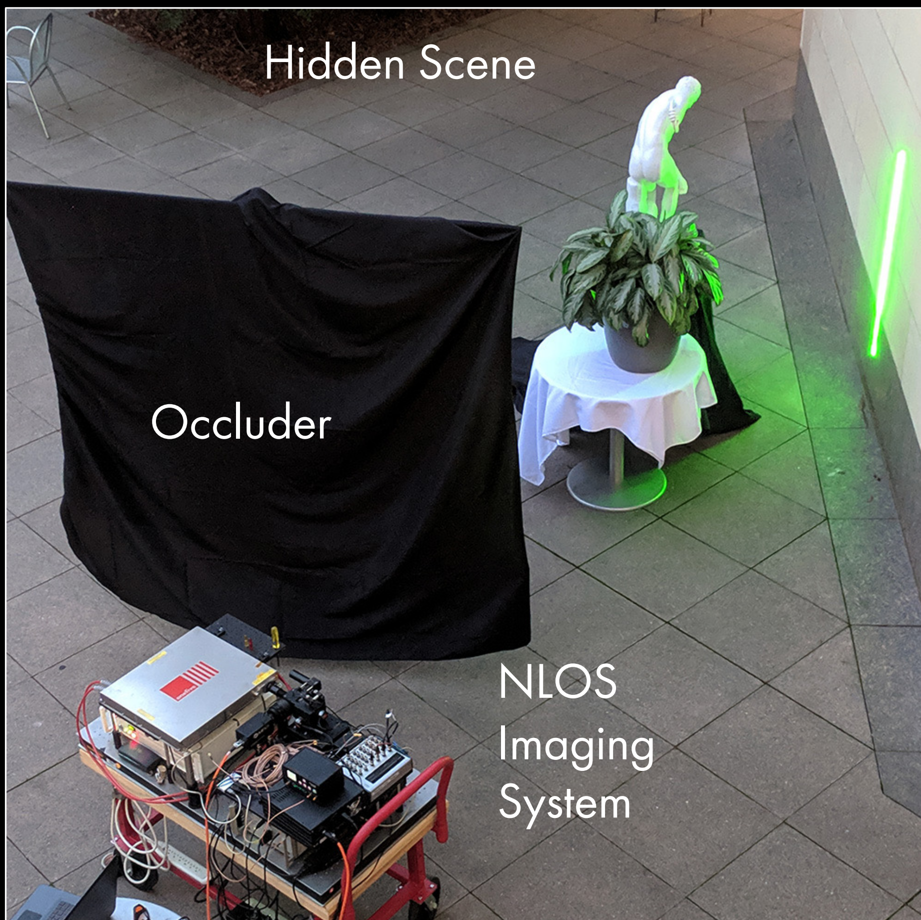
125 cameras using custom hardware
[Wilburn et al. 2002, Wilburn et al. 2005]

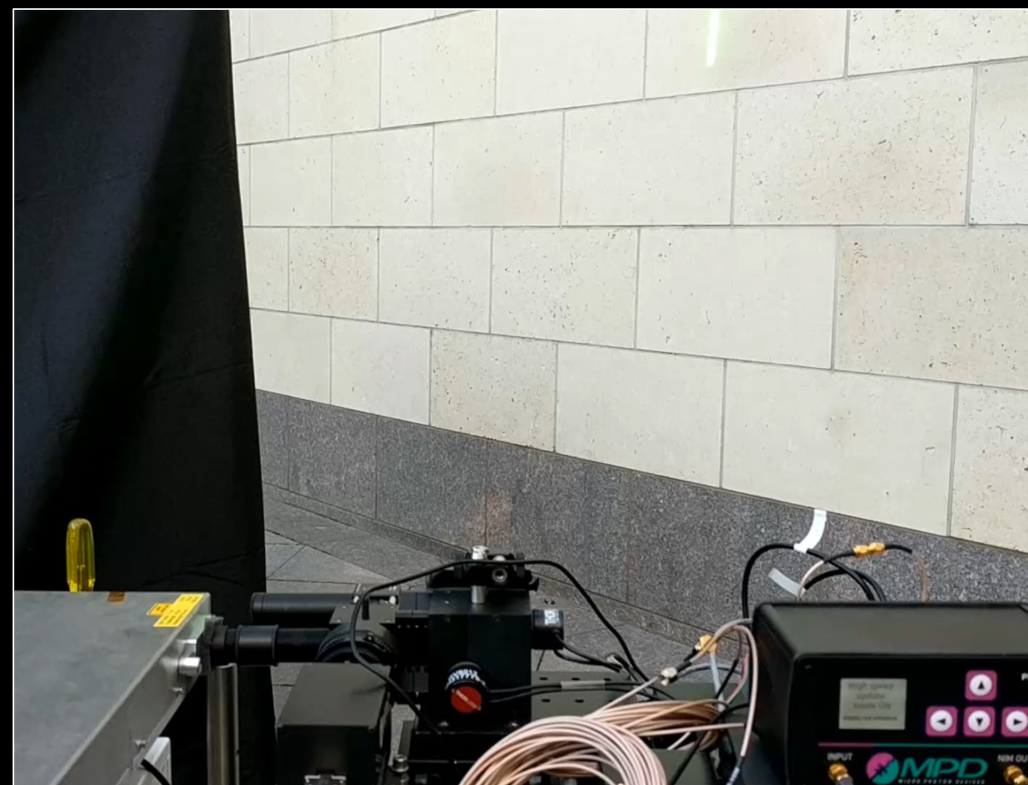



picosecond
laser
glowing fiber
optic cable
regular image

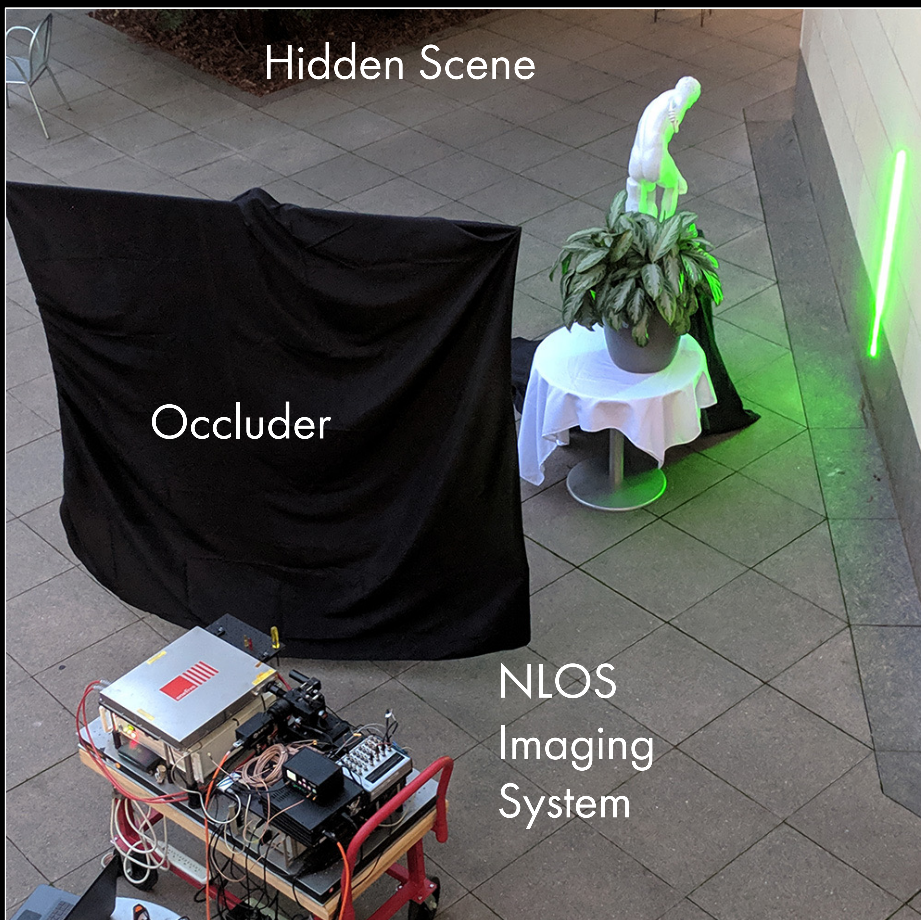


transient image

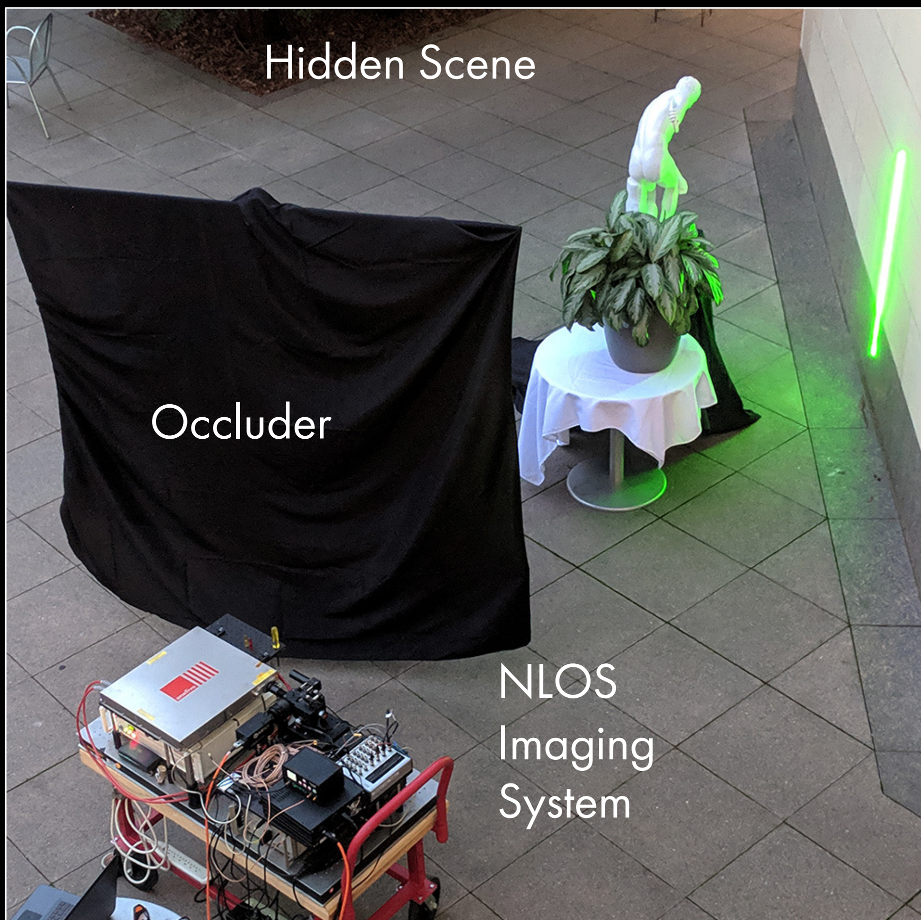




[Lindell et al. SIGGRAPH '19]



[Lindell et al. SIGGRAPH '19]



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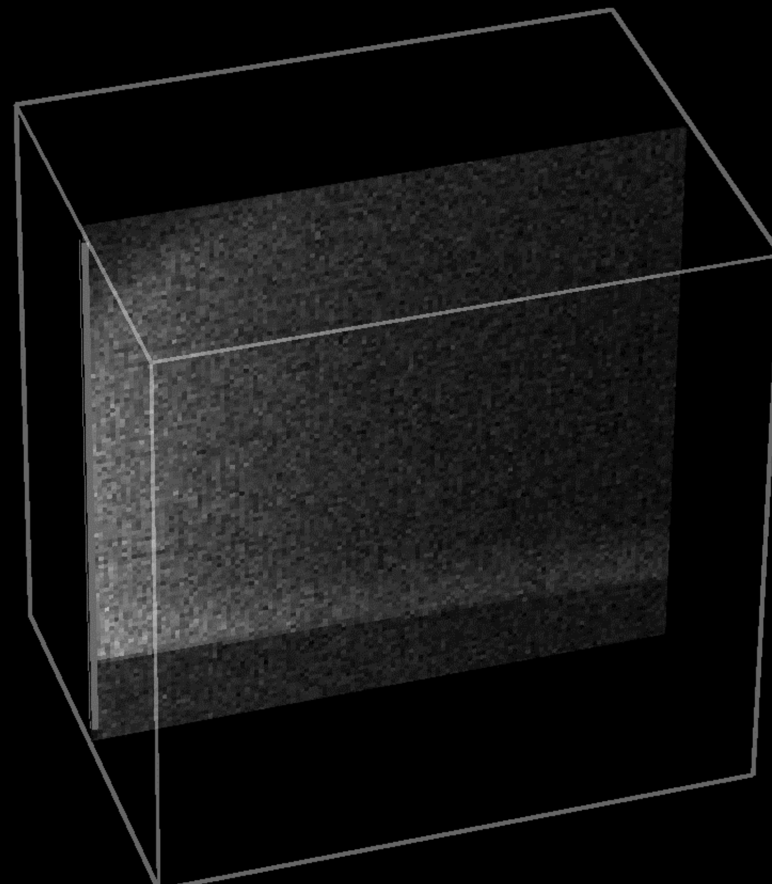
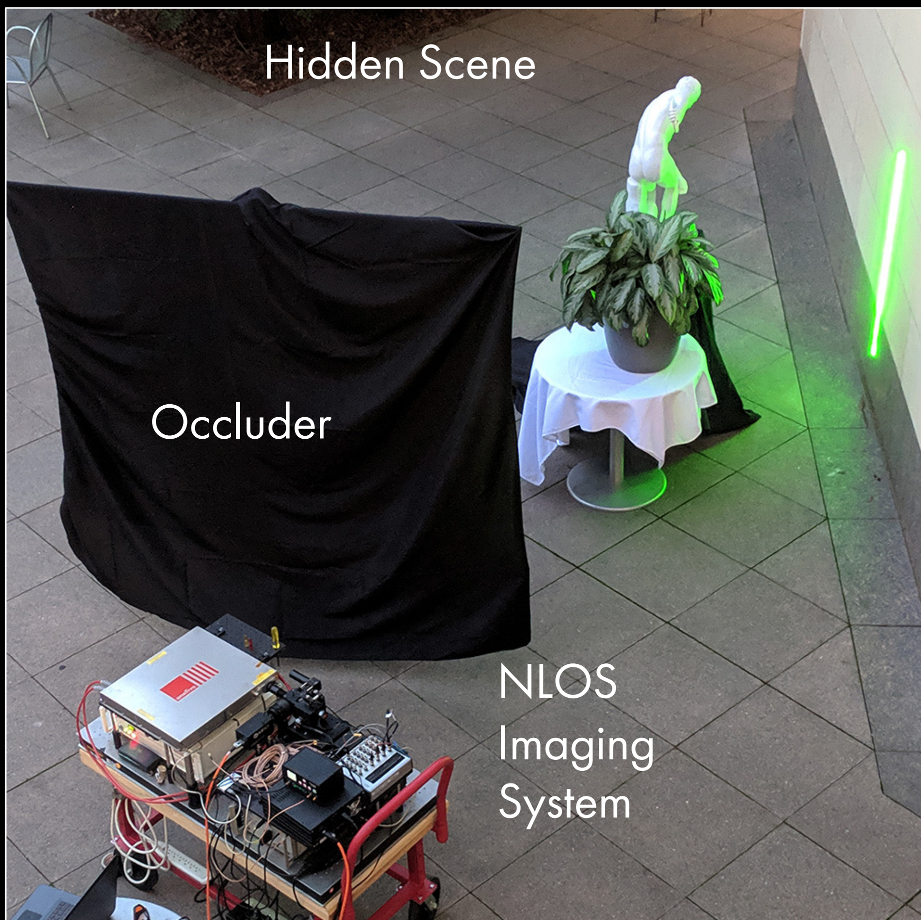
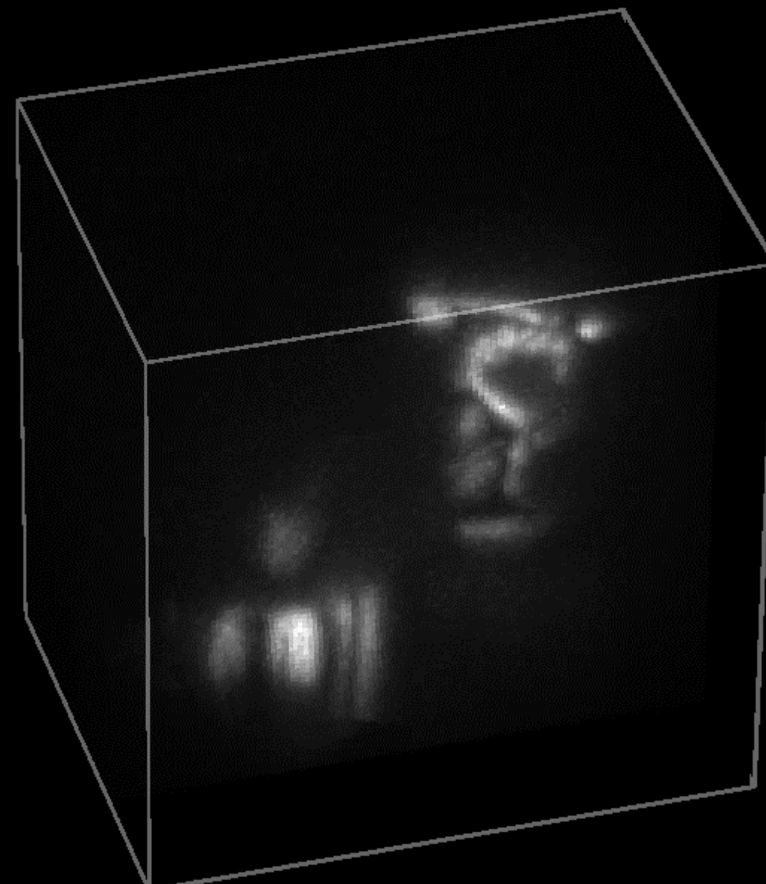


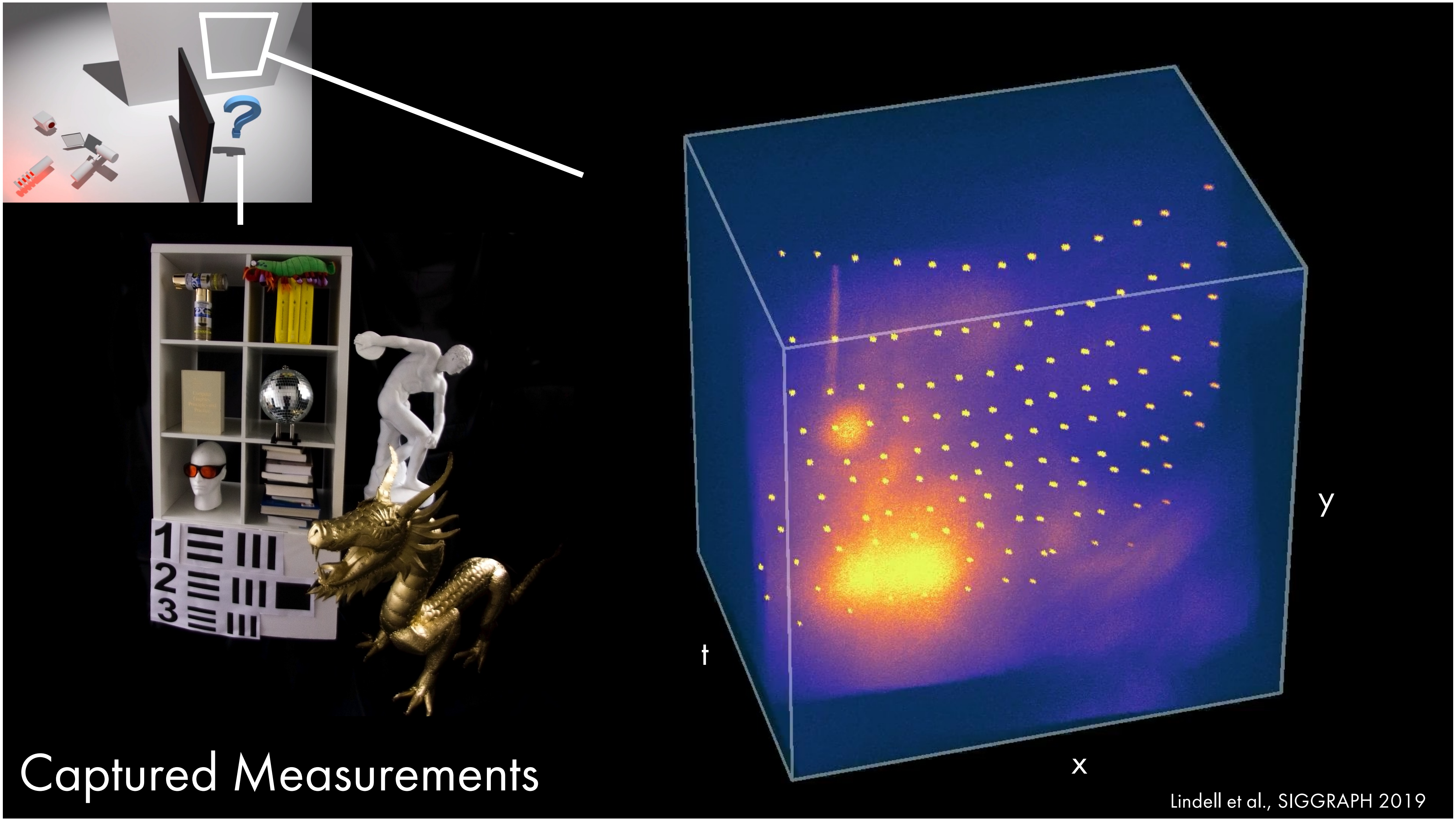
Time-resolved Measurements



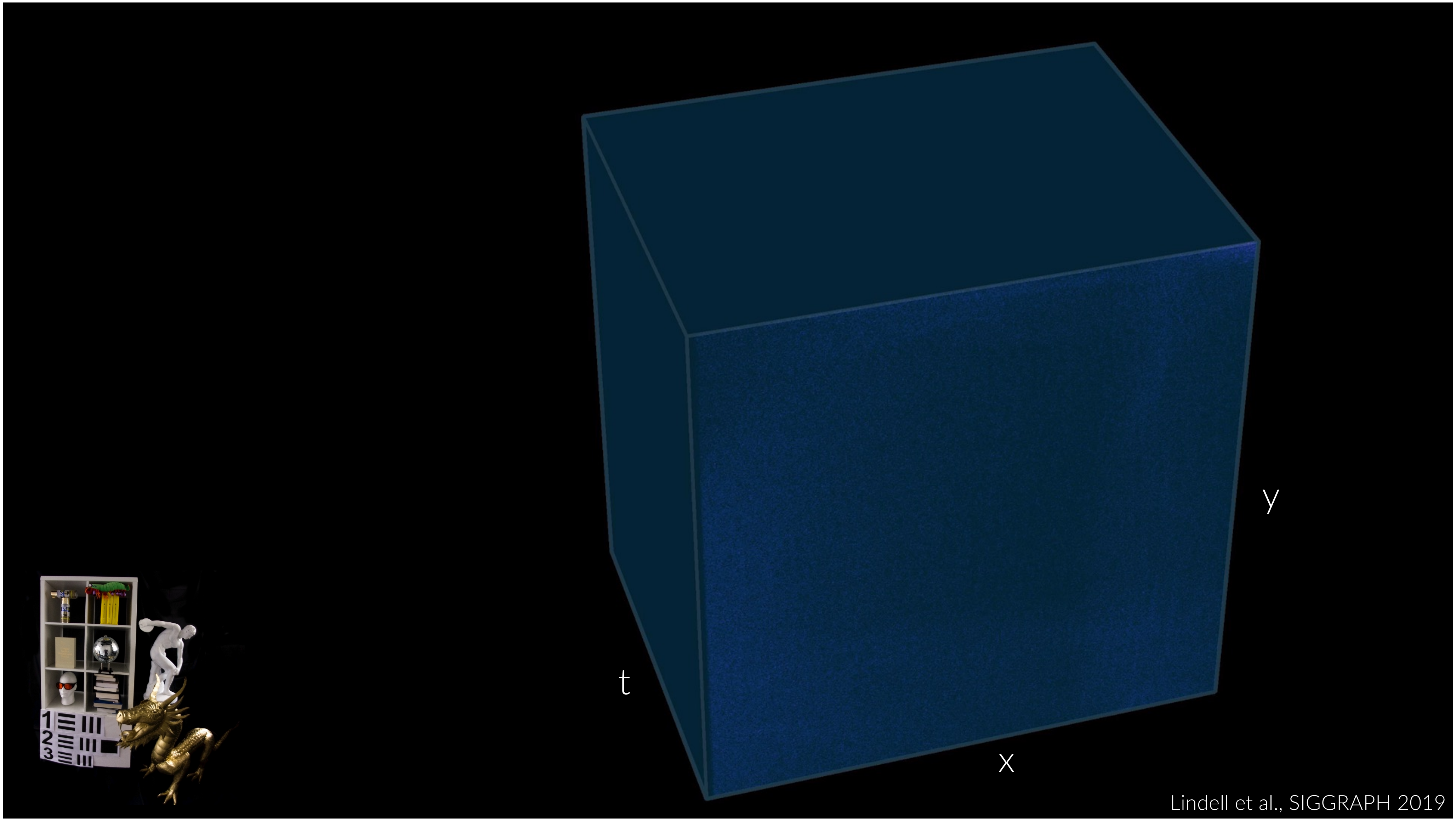


3D Reconstruction

[Lindell et al. SIGGRAPH '19]



Captured Measurements


t
y
x
Lindell et al., SIGGRAPH 2019


240 FPS


Frame rate: 10.0Hz
Intensity tone mapped
Elapsed time: 24s + 500ms

How it all began

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

PROJECT MAC

Artificial Intelligence Group July 7, 1966

Vision Memo. No. 100.

THE SUMMER VISION PROJECT

Seymour Papert

The summer vision project is an attempt to use our summer workers effectively in the construction of a significant part of a visual system. The particular task was chosen partly because it can be segmented into sub-problems which will allow individuals to work independently and yet participate in the construction of a system complex enough to be a real landmark in the development of "pattern recognition".

[Slide: A. Torralba]

Popular benchmarks:

The KITTI Vision Benchmark Suite
A project of Karlsruhe Institute of Technology and Toyota Technological Institute at Chicago

PASCAL2
Pattern Analysis, Statistical Modelling and Computational Learning

IMAGENET

COCO
Common Objects in Context

http://en.wikipedia.org/wiki/List_of_datasets_for_machine_learning_research

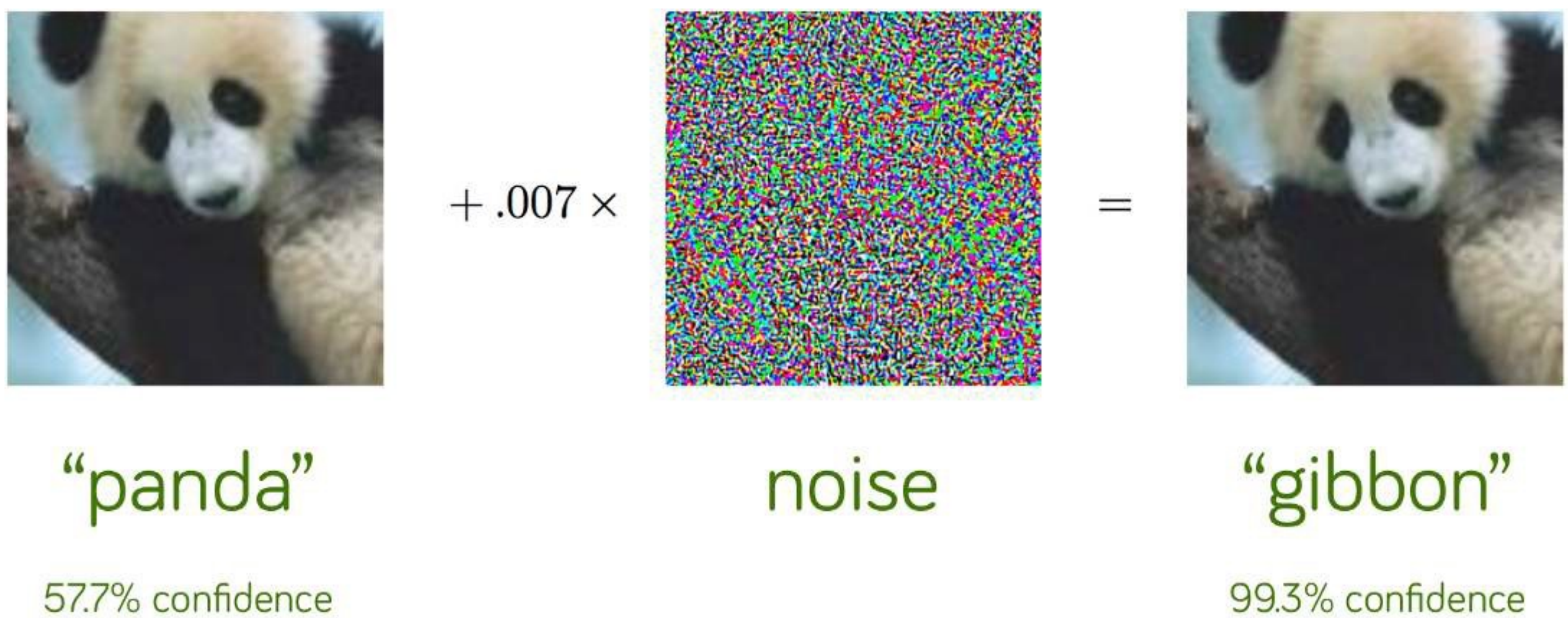
Car

	Method	Setting	Code	Moderate	Easy	Hard	Runtime	Environment	Compare
1	DenseBox2			89.32 %	93.94 %	79.81 %	5 s	GPU @ 2.5 Ghz (C/C++)	
2	DJML			88.79 %	91.31 %	77.73 %	x s	GPU @ 1.5 Ghz (Matlab + C/C++)	
3	3DOP			88.64 %	93.04 %	79.10 %	3s	GPU @ 2.5 Ghz (Matlab + C/C++)	

X. Chen, K. Kundu, Y. Zhu, A. Berneshawi, H. Ma, S. Fidler and R. Urtasun: 3D Object Proposals for Accurate Object Class Detection. NIPS 2015.

	mean	aero plane	bicycle	bird	boat	bottle	bus	car	cat	chair	cow	dining table	dog	horse	motor bike	person	potted plant	sheep	sofa	train	tv/ monitor	submission date
Fast R-CNN + YOLO [?]	**70.8**	**82.7**	77.7	**74.3**	**59.1**	47.1	**78.0**	**73.1**	89.2	**49.6**	**74.3**	**55.9**	87.4	79.8	82.2	**75.3**	**43.1**	71.4	**67.8**	**81.9**	**65.6**	05-Jun-2015
Fast R-CNN VGG16 extra data [?]	68.8	82.0	**77.8**	71.6	55.3	42.4	77.3	71.7	**89.3**	44.5	72.1	53.7	**87.7**	**80.0**	**82.5**	72.7	36.6	68.7	65.4	81.1	62.7	18-Apr-2015
segDeepM [?]	67.2	82.3	75.2	67.1	50.7	**49.8**	71.1	69.6	88.2	42.5	71.2	50.0	85.7	76.6	81.8	69.3	41.5	**71.9**	62.2	73.2	64.6	29-Jan-2015
BabyLearning [?]	63.8	77.7	73.8	62.3	48.8	45.4	67.3	67.0	80.3	41.3	70.8	49.7	79.5	74.7	78.6	64.5	36.0	69.9	55.7	70.4	61.7	12-Nov-2014

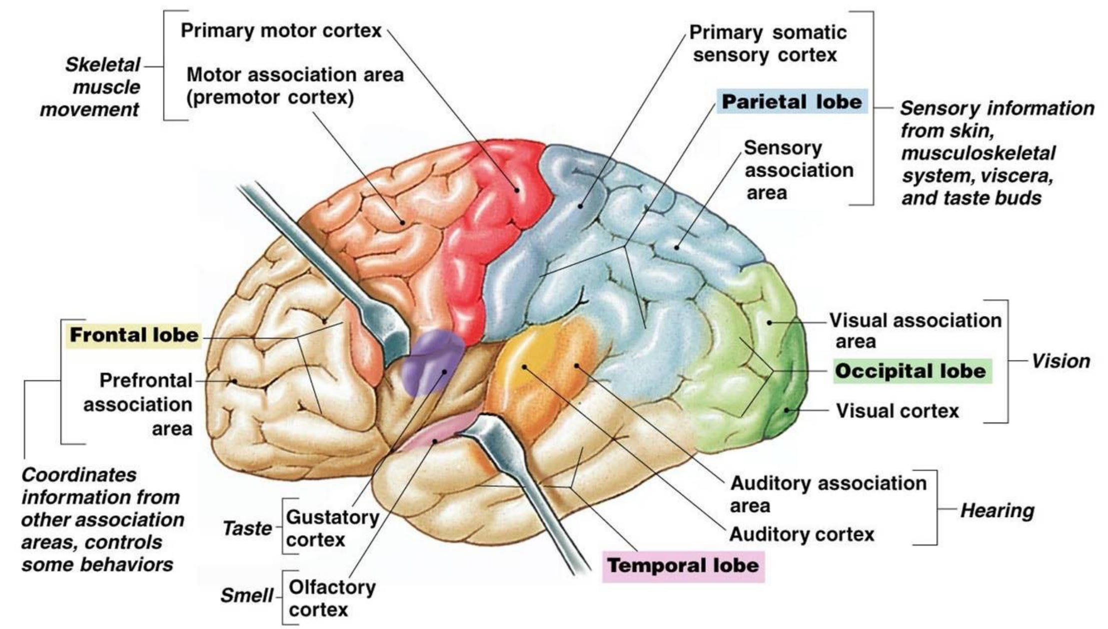
- Algorithms work pretty well
- Still some embarrassing mistakes...
- The general vision problem is not yet solved

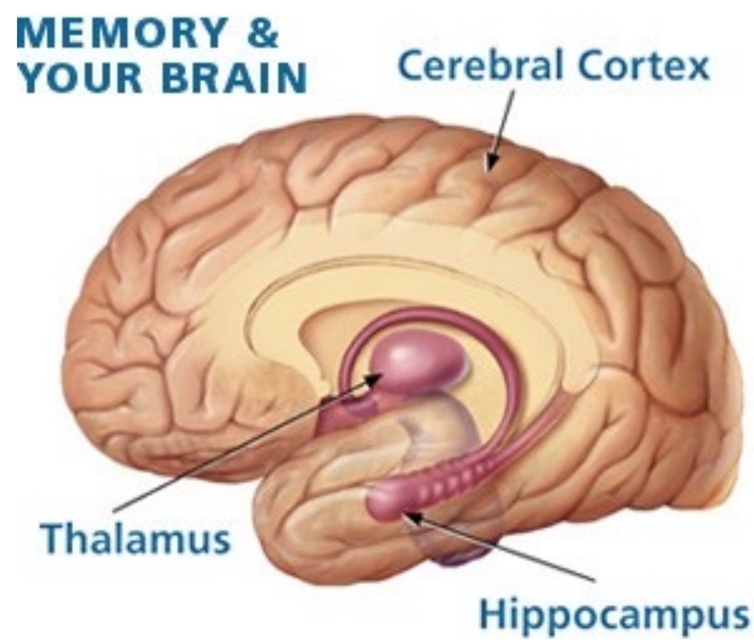


Why is vision hard?

Why is vision hard?

Half of the cerebral cortex in primates is devoted to processing visual information. This is a lot. Means that vision has to be pretty hard!





Why is vision hard?

Visual information is complicated and nuanced...

These are all dogs!

[Slide: R. Urtasun]

Why is vision hard?

Image: Karen Zack

Why is vision hard?

Image: Karen Zack

Why is vision hard?

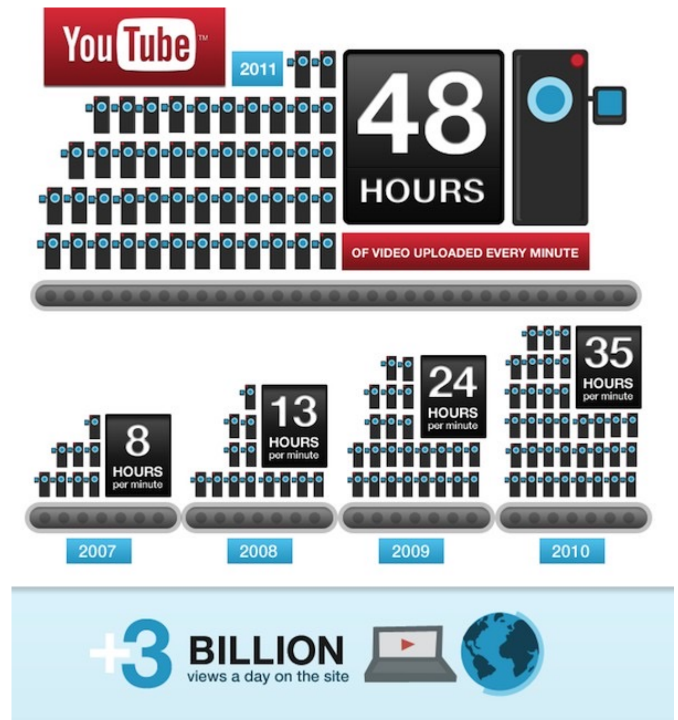
Biederman, 1987

[Slide: R. Urtasun]

Why is vision hard?

Lots of data to process:

- Thousands to millions of pixels in an image
- 400 hours of video added to YouTube per minute (2022)
- Every day, people watch one billion hours of video on YouTube (2022)
- <u>Much</u> more considering all other platforms



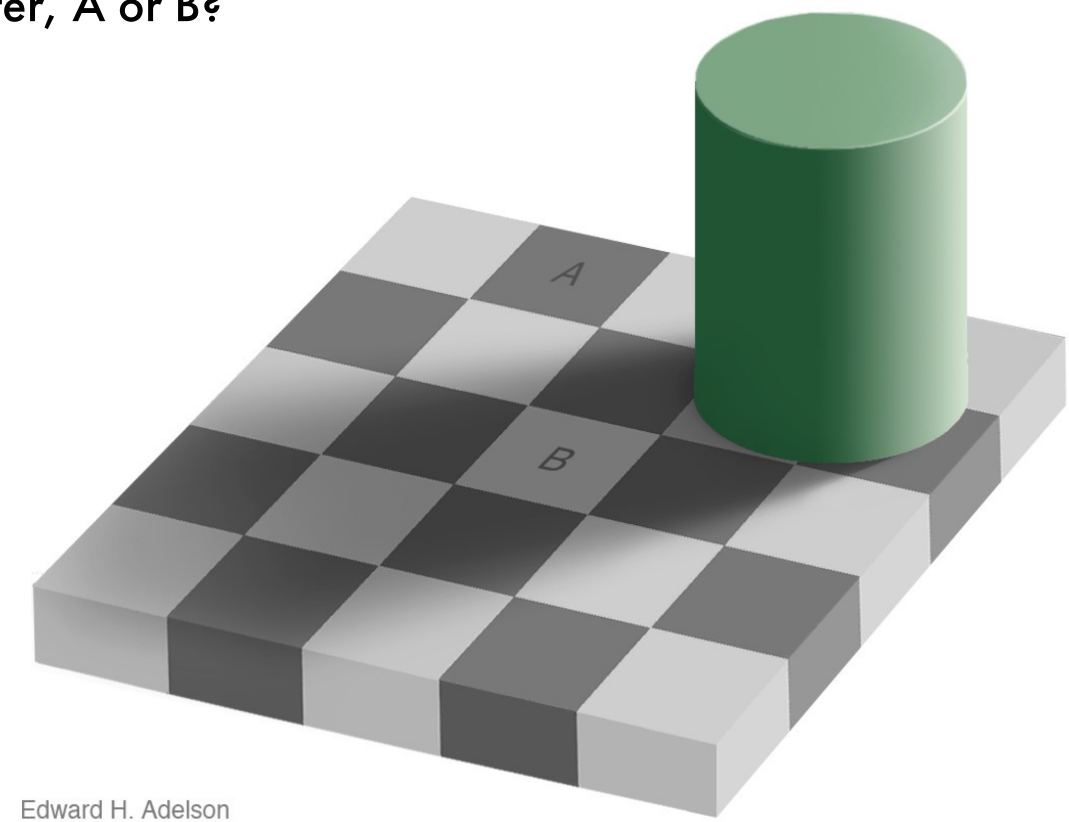
Human vision seems to work quite well.

How well does it really work?

Let’s play some games!

How good are humans?

Which square is lighter, A or B?



[Slide: A. Torralba]

How good are humans?

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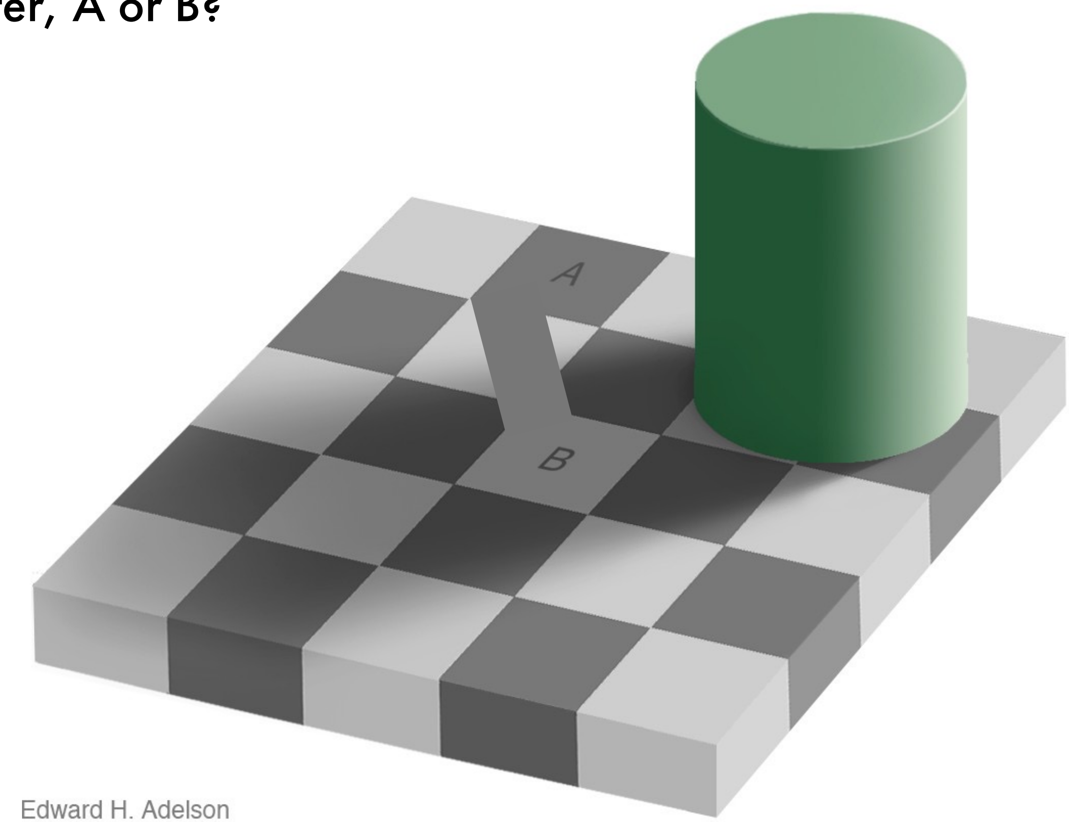
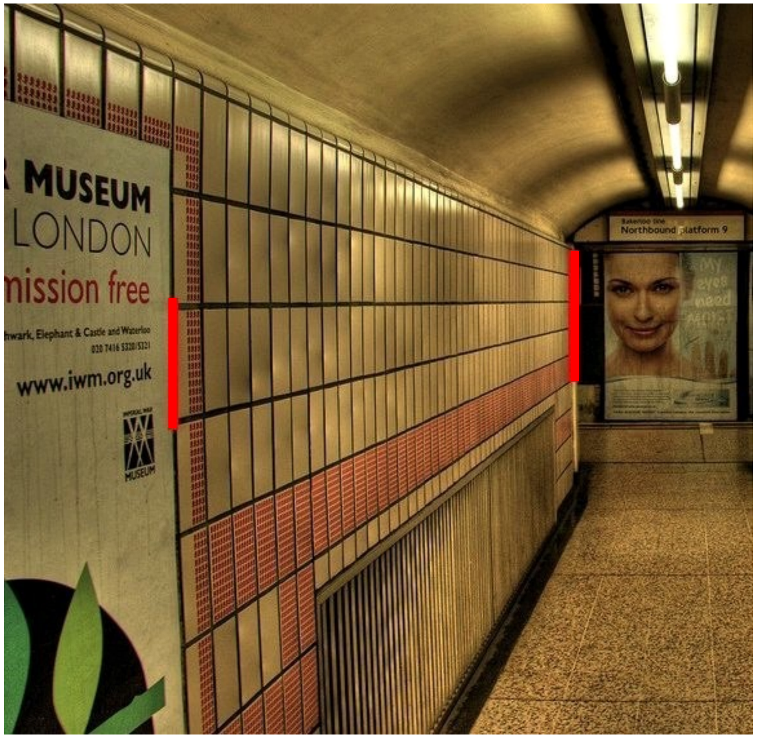
They are the same...



[Slide: A. Torralba]

How good are humans?

Which red line is longer?



[Walt Anthony 2006]

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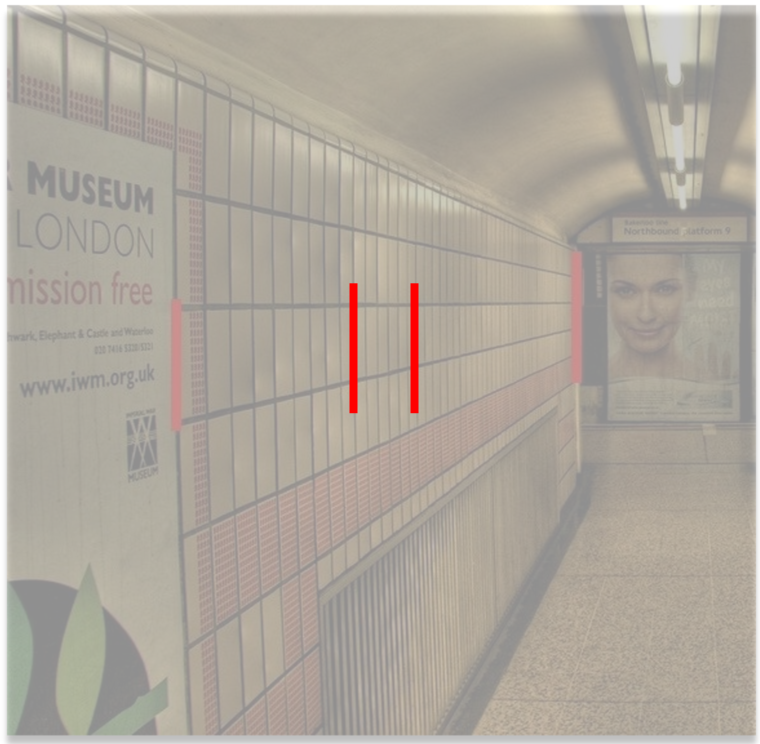


[Walt Anthony 2006]

[Slide: A. Torralba]

How good are humans?

- Count the number of times the white team pass the ball
- Concentrate, it's difficult!

[Chabris & Simons]

How good are humans?

Can you describe what this is?

[Torralba et al.]

How good are humans?

Can you describe what this is?

[Torralba et al.]

Humans can tell a lot from a little information...
we have prior knowledge that can (usually) fill in the right information

What do I need to become a good Computer Vision researcher?

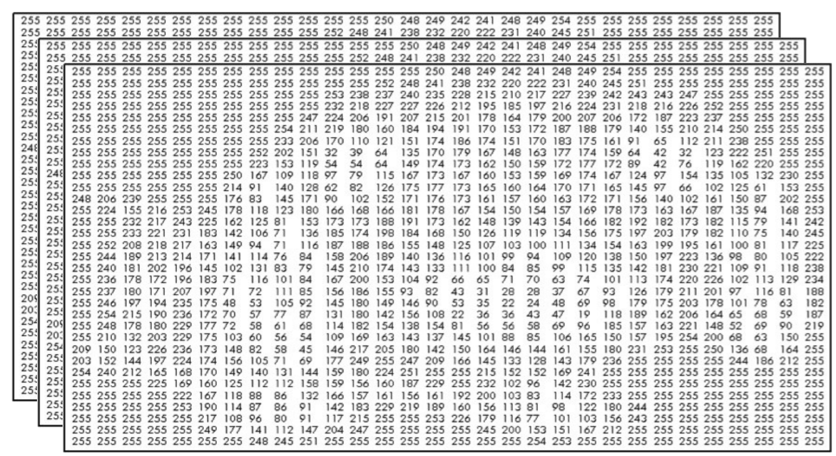
- Some math knowledge
- Good programming skills
- Imagination
- Even better intuition
- Lots of persistence
- Some luck always helps

Images

Digital Image

An image is a matrix with (typically) integer values

- We will typically denote the image as I
- Pixel values in the image are given by $I(i, j)$, the intensity value at each pixel
- For a grayscale image we have $I \in \mathbb{R}^{m \times n}$, color is $I \in \mathbb{R}^{m \times n \times 3}$



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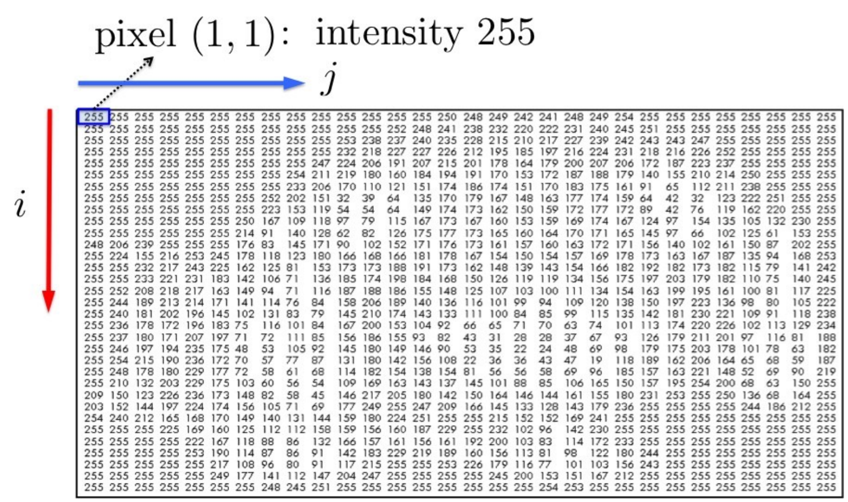

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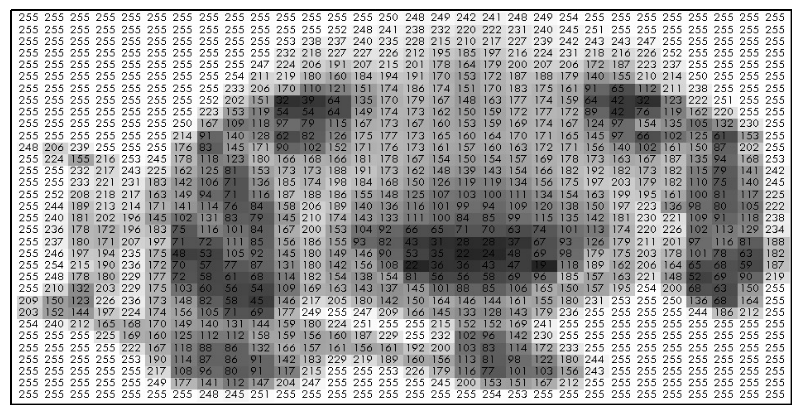

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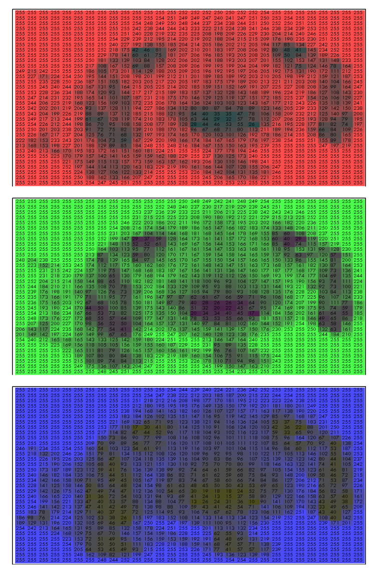
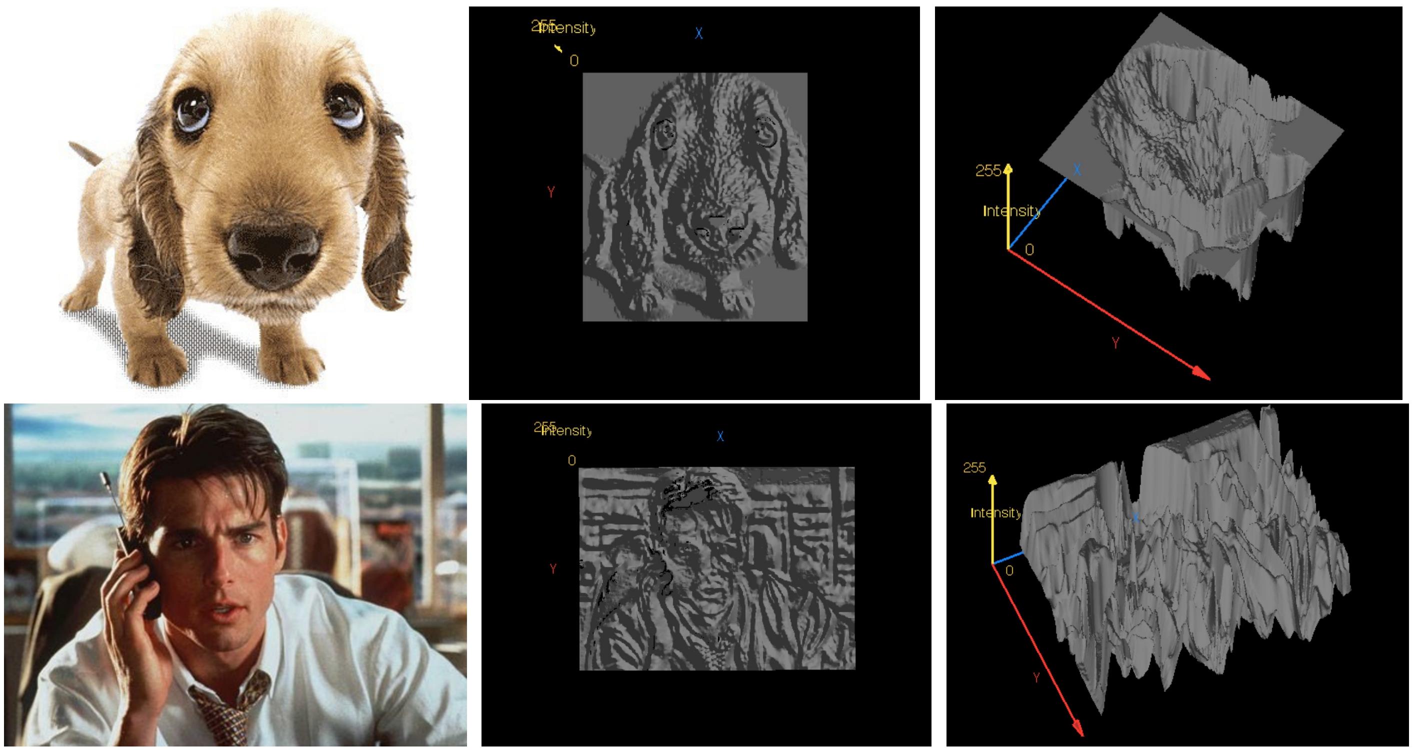


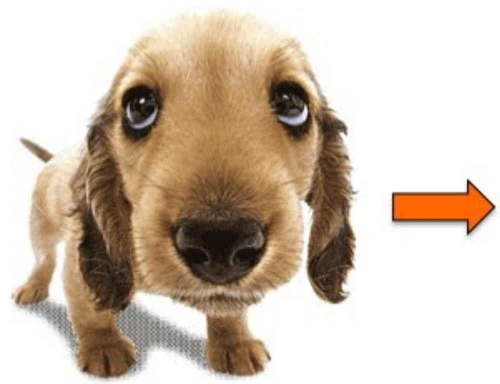
Digital Image



- We can think of a (grayscale) image as a function $f : \mathbb{R}^2 \mapsto \mathbb{R}$ giving the intensity at position (i, j)
- Intensity 0 is black and 255 is white

Digital Image

As with any function, we can apply operators to an image, e.g.:



$I(i, j)$ $\quad\quad$ $J(i, j) = I(i, j) + 50$

We'll talk about special kinds of operators, correlation and convolution (linear filtering)

[Slide: N. Snavely]

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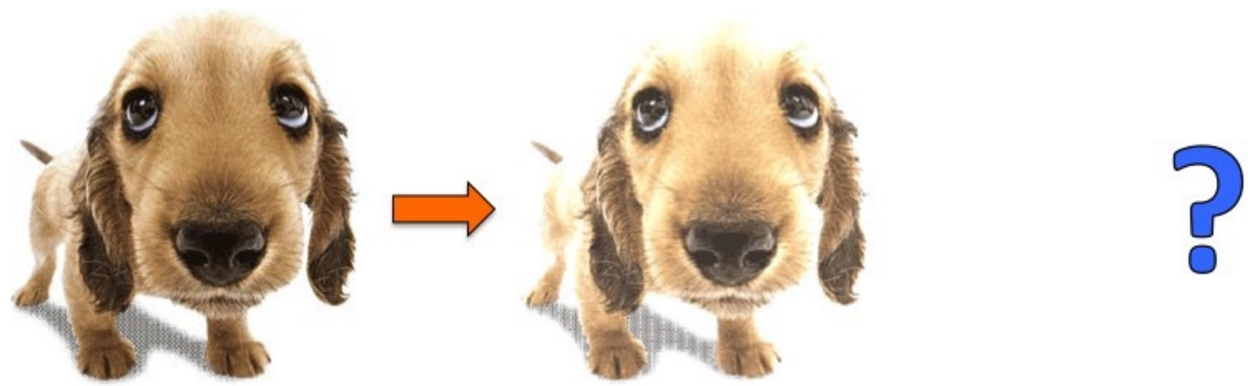

$I(i,j)$ $\qquad$ $J(i,j) = I(i,j) + 50$ $\qquad$ $J(i,j) = I(i,-j)$

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We'll talk about special kinds of operators, correlation and convolution (linear filtering)

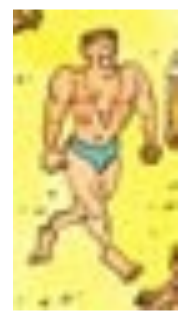
[Slide: N. Snavely]

Linear Filters

Reading: Szeliski book, Chapter 3.2

Motivation: Finding Waldo

How can we find Waldo?



[Source: R. Urtasun]

Motivation: Finding Waldo

Slide and compare! In formal language: filtering

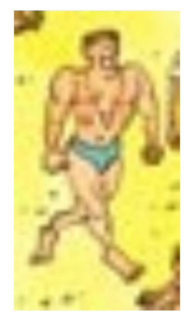
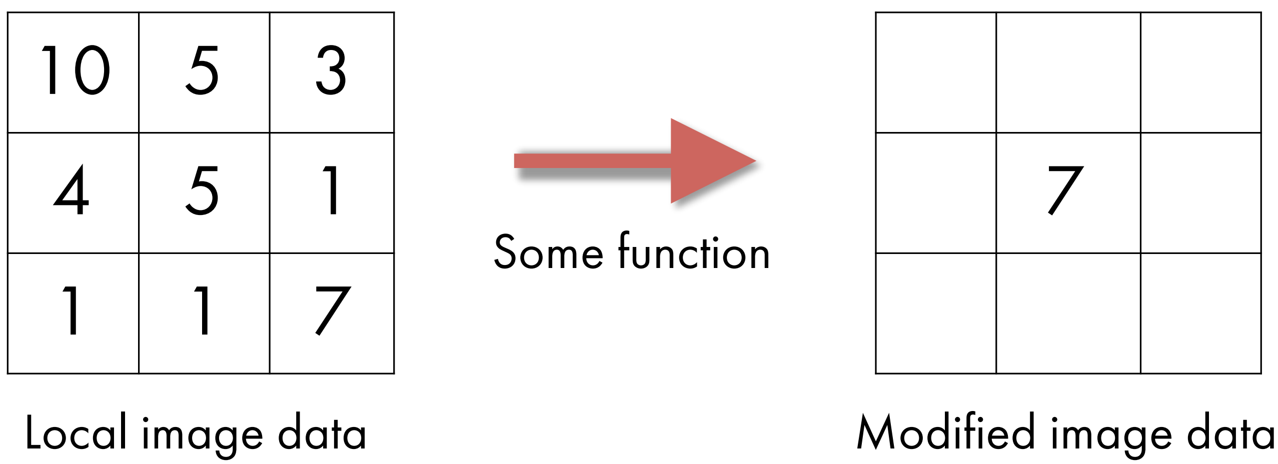


Image Filtering

- Modify the pixels in an image based on some function of a local neighborhood of each pixel
- In other words, filtering



[Source: L. Zhang]

Applications of Filtering

- Enhance an image, e.g., denoise.
- Detect patterns, e.g., template matching.
- Extract information, e.g., texture, edges.

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- Enhance an image, e.g., denoise.
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- Extract information, e.g., texture, edges.

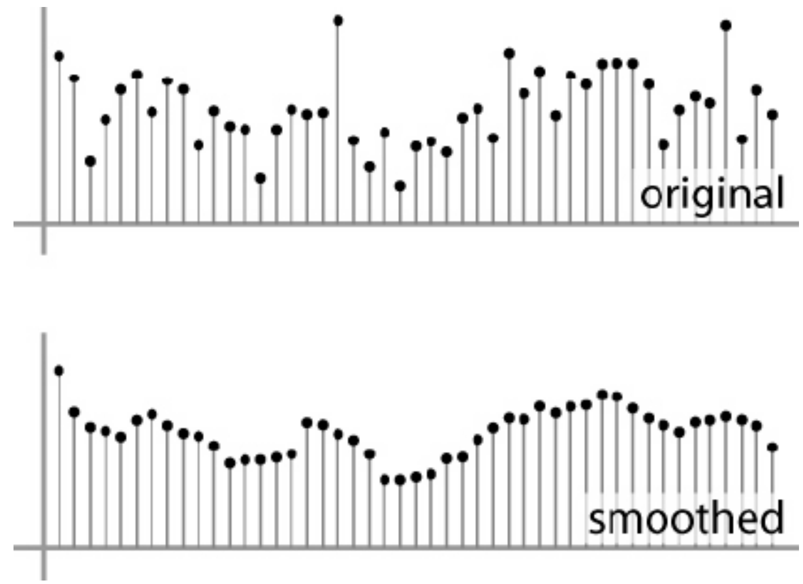
Noise reduction

Given a camera and a still scene, how can you reduce noise?

[Source: S. Seitz]

Noise reduction

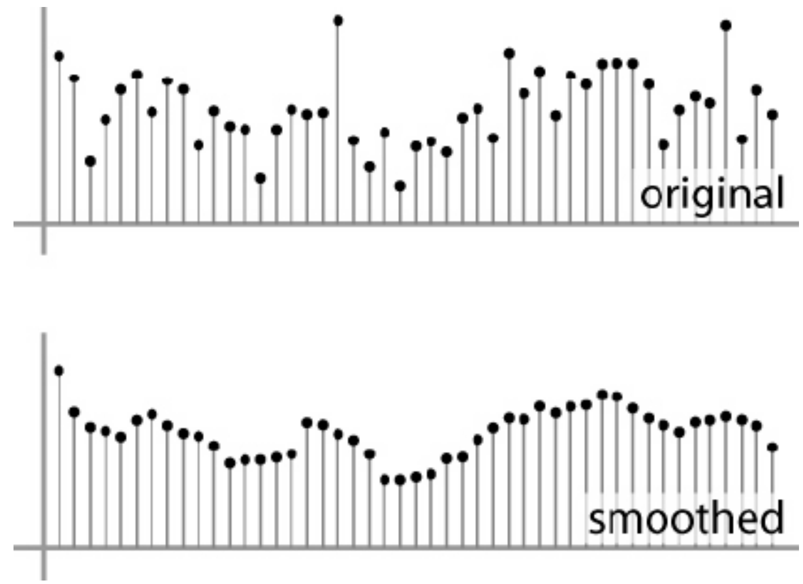
- Simplest thing: replace each pixel by the average of its neighbors.



[Source: S. Marschner]

Noise reduction

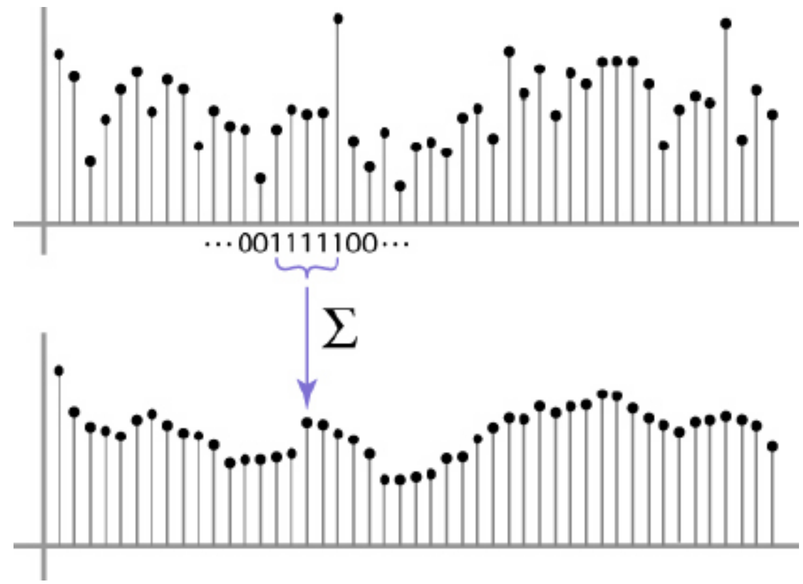
- Simplest thing: replace each pixel by the average of its neighbors.
- This assumes that neighboring pixels are similar, and the noise to be independent from pixel to pixel.



[Source: S. Marschner]

Noise reduction

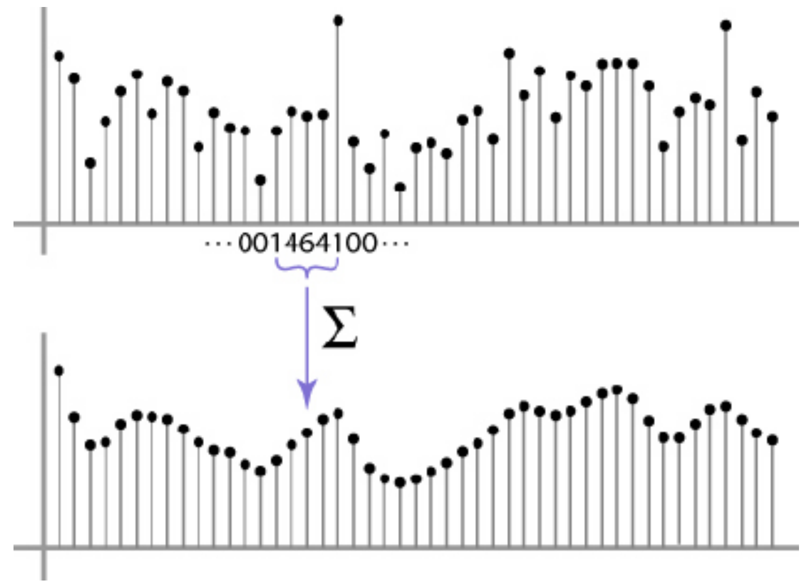
- Simplest thing: replace each pixel by the average of its neighbors.
- This assumes that neighboring pixels are similar, and the noise to be independent from pixel to pixel.
- Moving average in 1D: [1, 1, 1, 1, 1]/5



[Source: S. Marschner]

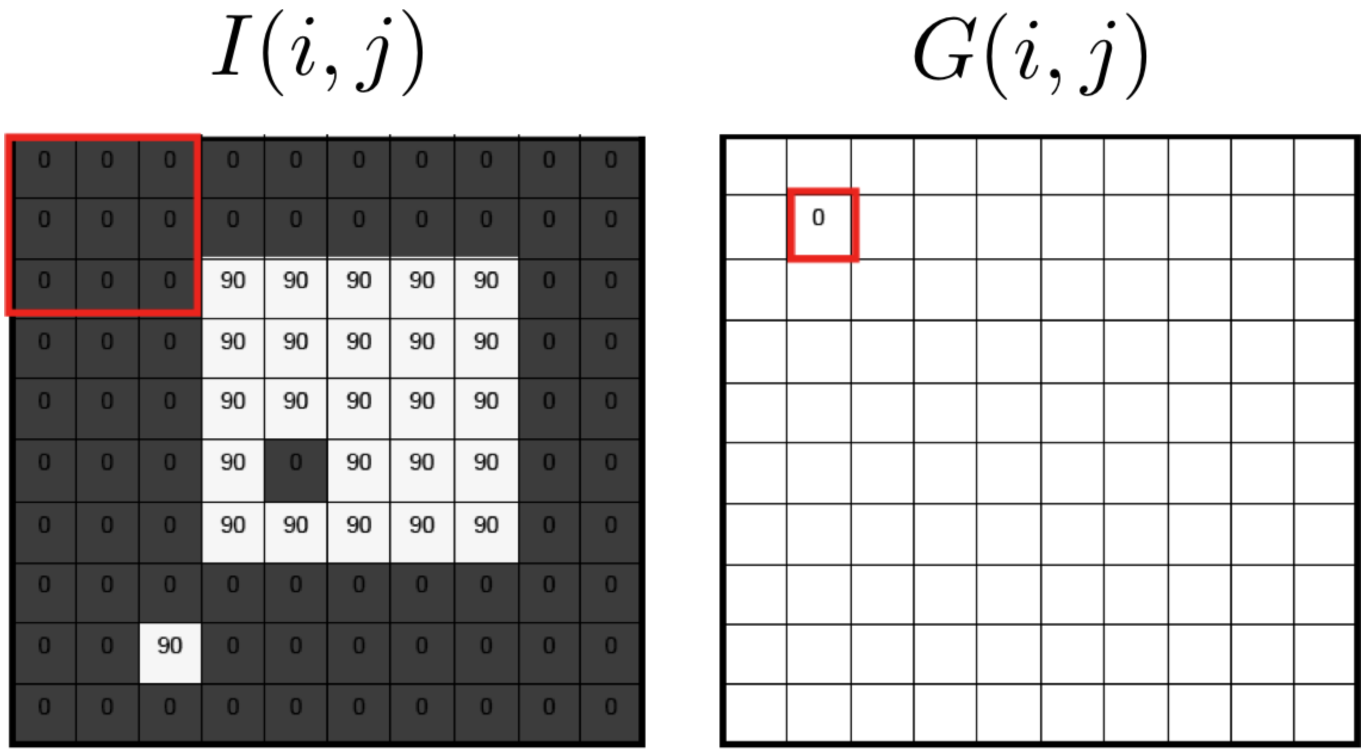
Noise reduction

- Simplest thing: replace each pixel by the average of its neighbors.
- This assumes that neighboring pixels are similar, and the noise to be independent from pixel to pixel.
- Non-uniform weights [1, 4, 6, 4, 1] / 16



[Source: S. Marschner]

Moving average in 2D



[Source: S. Seitz]

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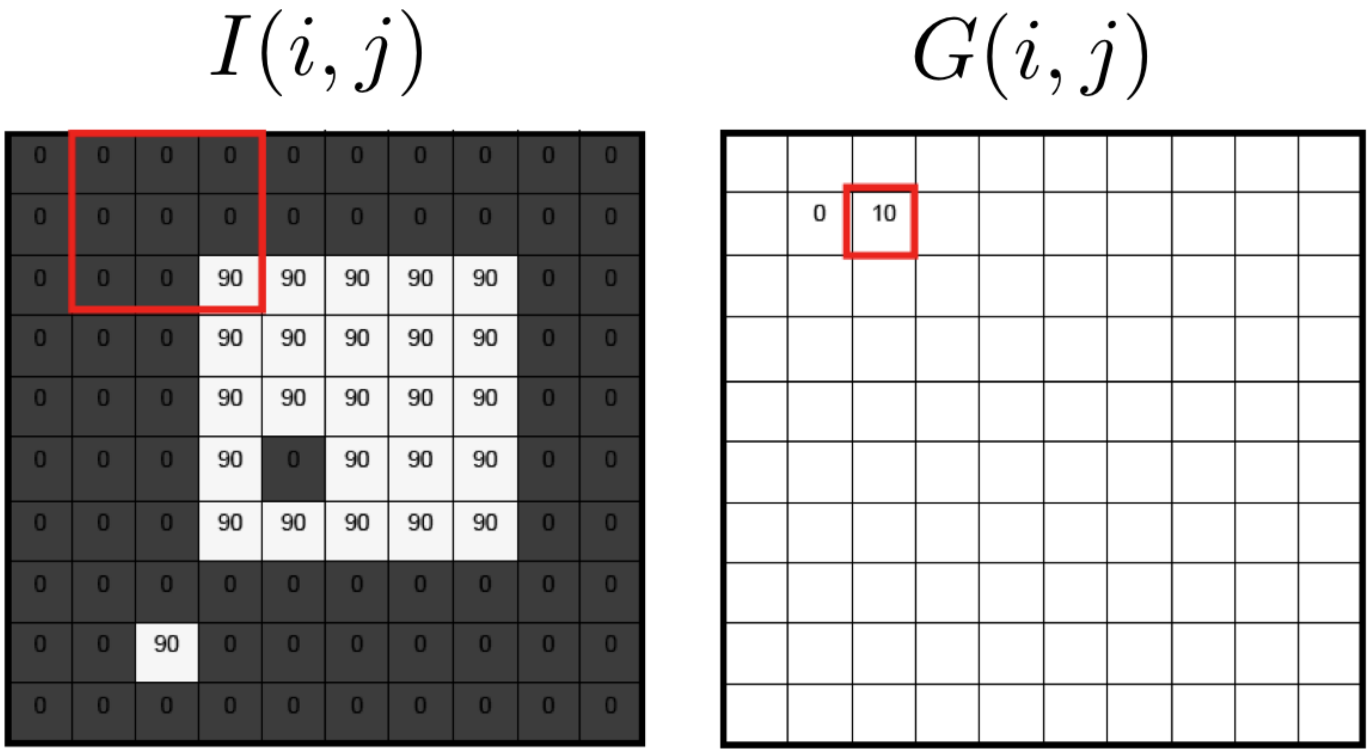


[Source: S. Seitz]

Moving average in 2D

$I(i,j)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G(i,j)$

	0	10	20						

[Source: S. Seitz]

Moving average in 2D

$I(i,j)$

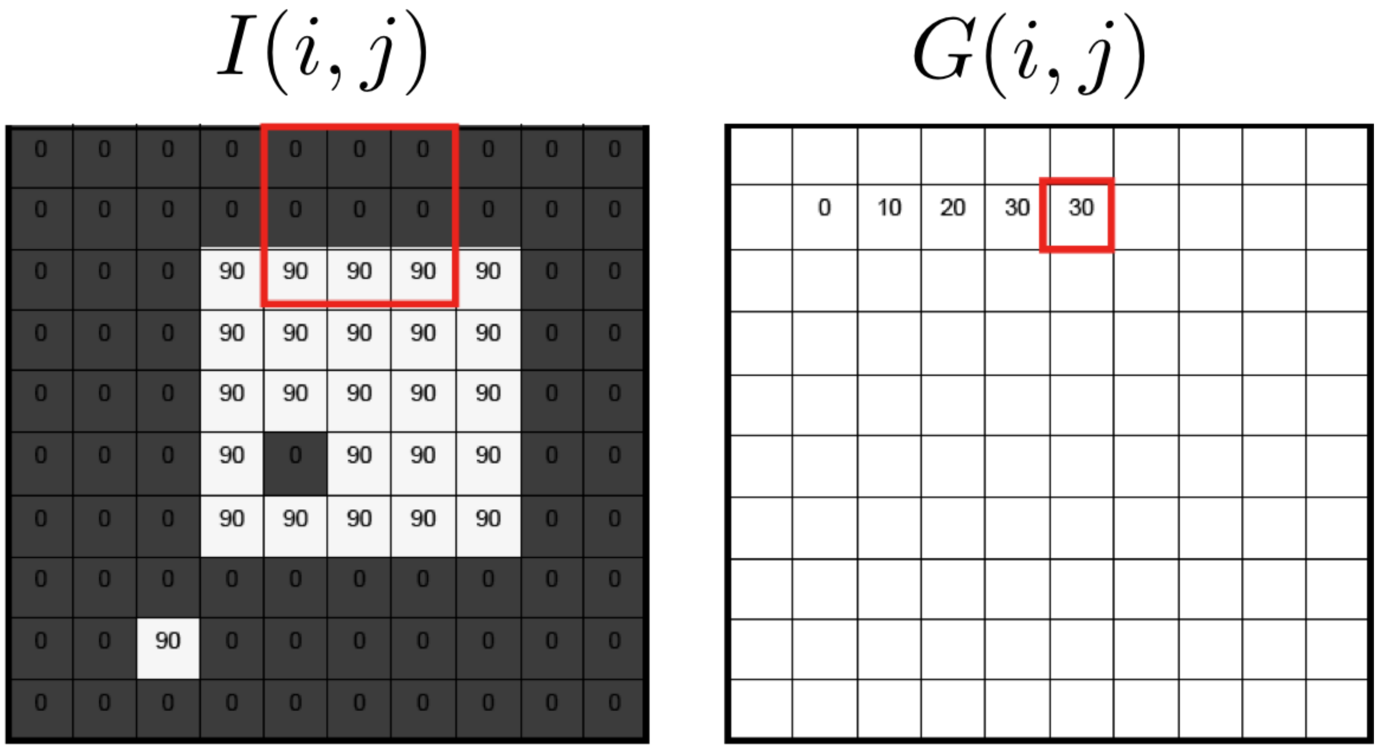
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G(i,j)$

	0	10	20	30					

[Source: S. Seitz]

Moving average in 2D



[Source: S. Seitz]

Moving average in 2D

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0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$G(i, j)$$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

[Source: S. Seitz]

Linear Filtering: Correlation

Involves weighted combinations of pixels in small neighborhoods (avg. filter):

$$G(i,j) = \frac{1}{(2k+1)^2} \sum_{u=-k}^{k} \sum_{v=-k}^{k} I(i+u, j+v)$$

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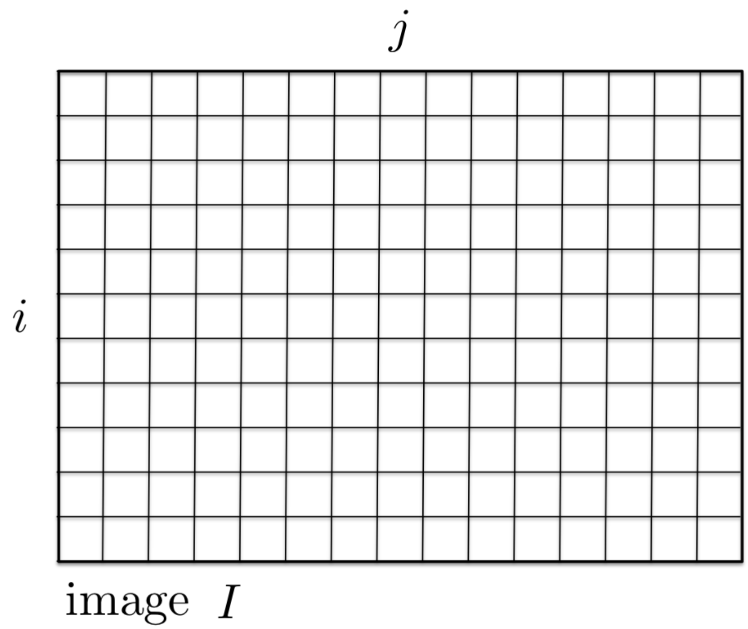
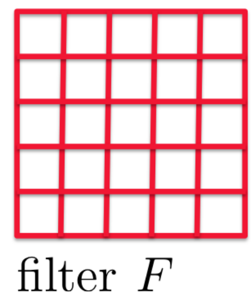
The entries of the **weight kernel** or **mask** are often called the **filter coefficients**

This operator is called the **correlation operator**

$$G = F \bigotimes I$$

Linear Filtering: Correlation





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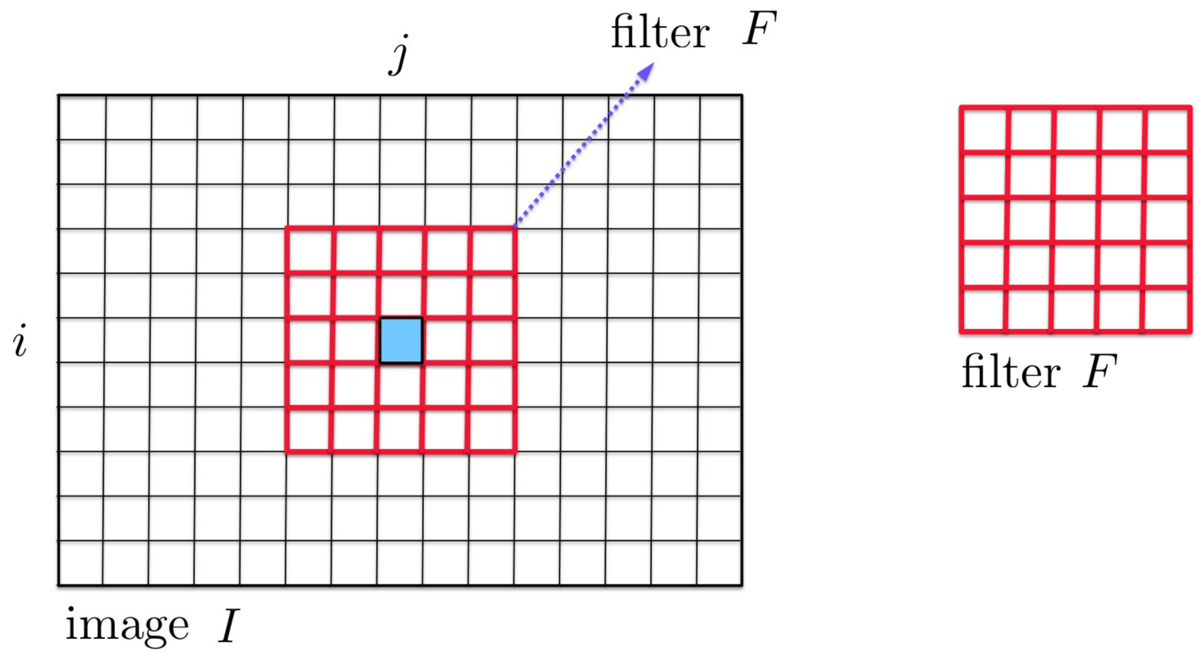

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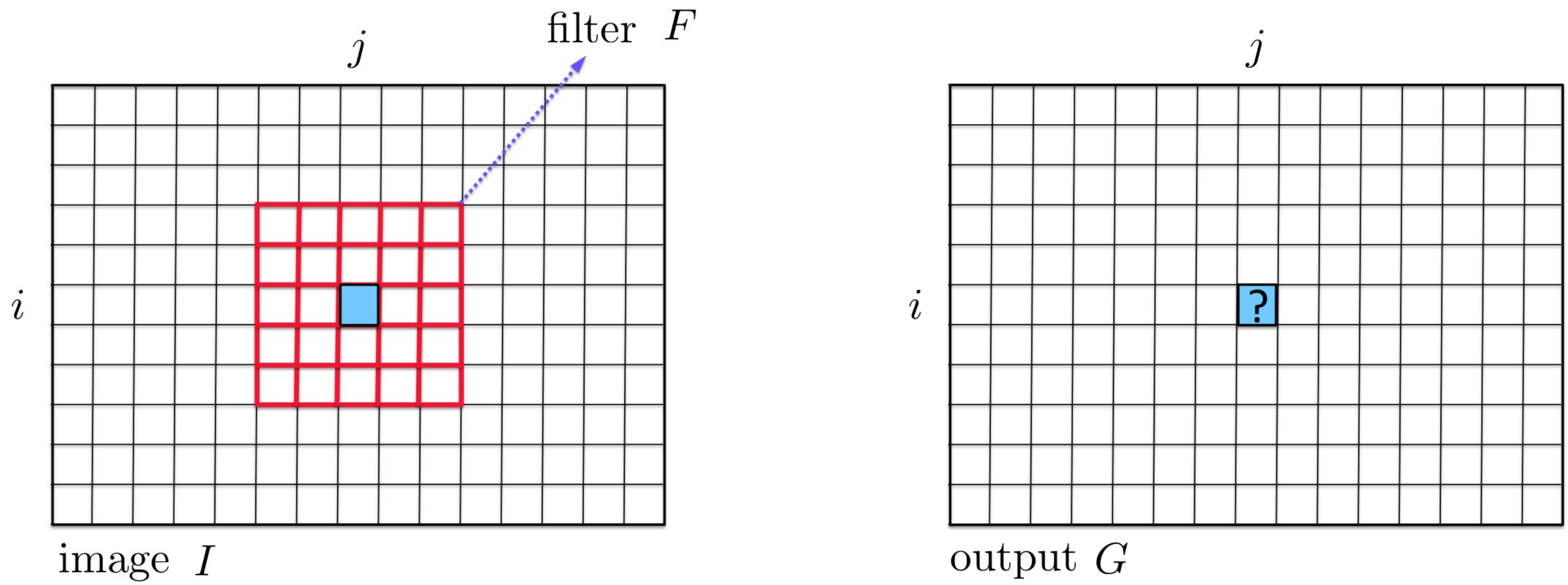

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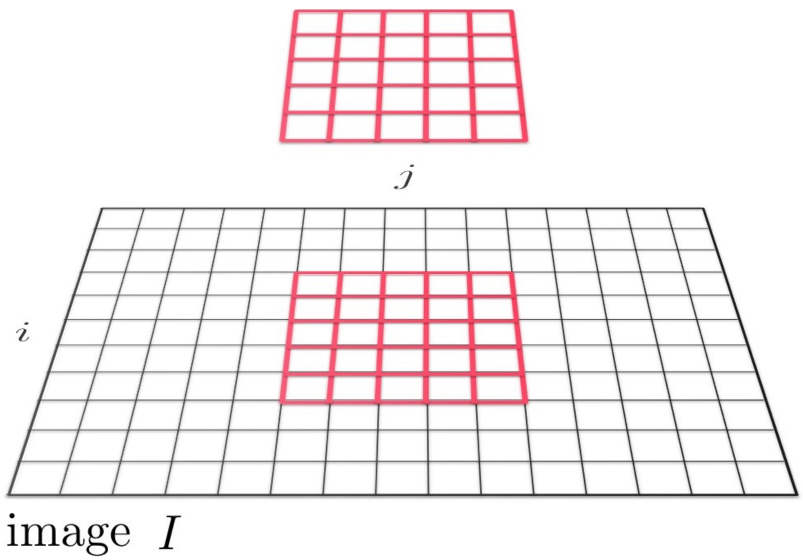

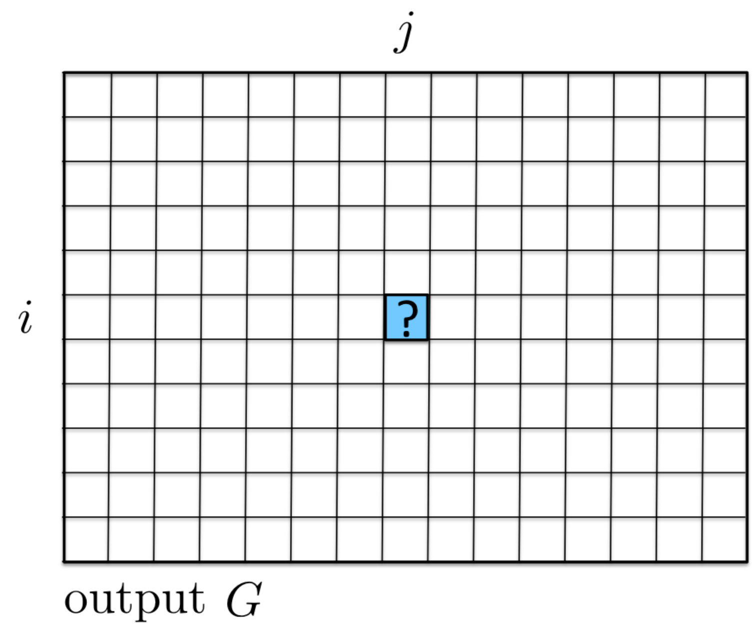

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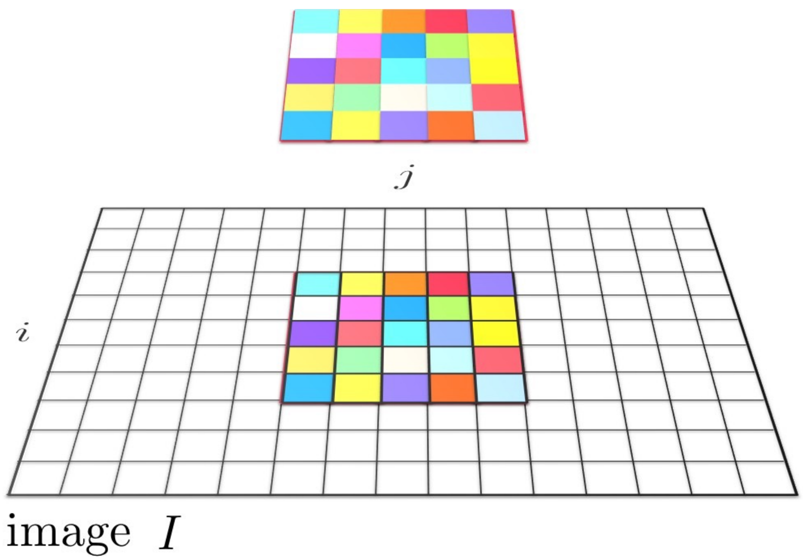

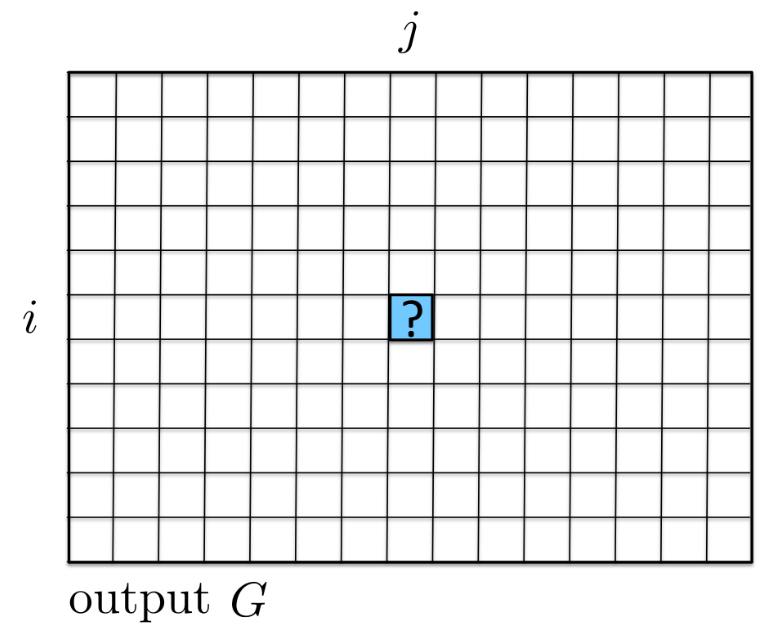

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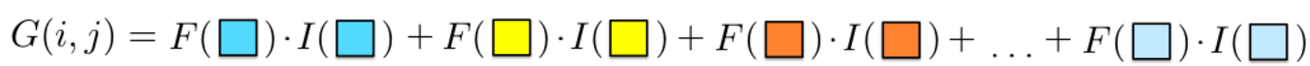

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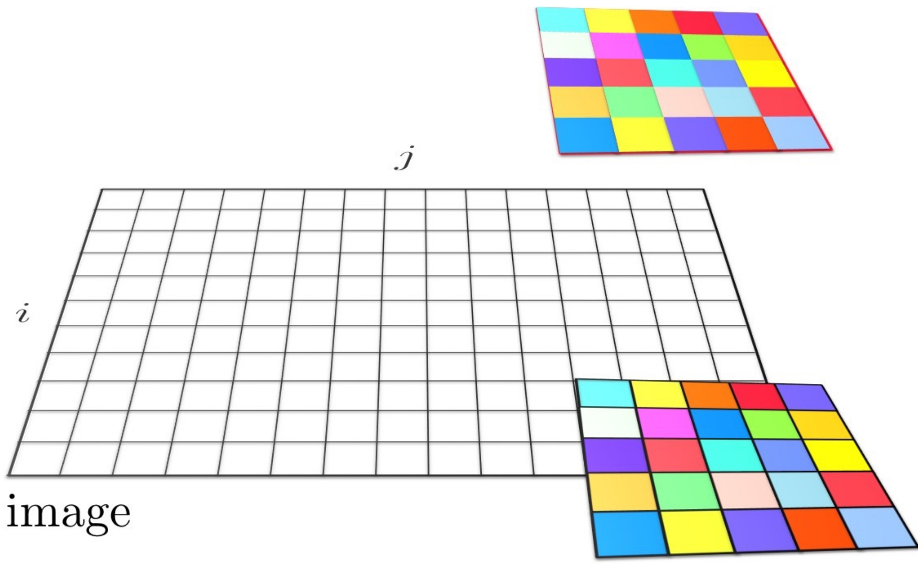
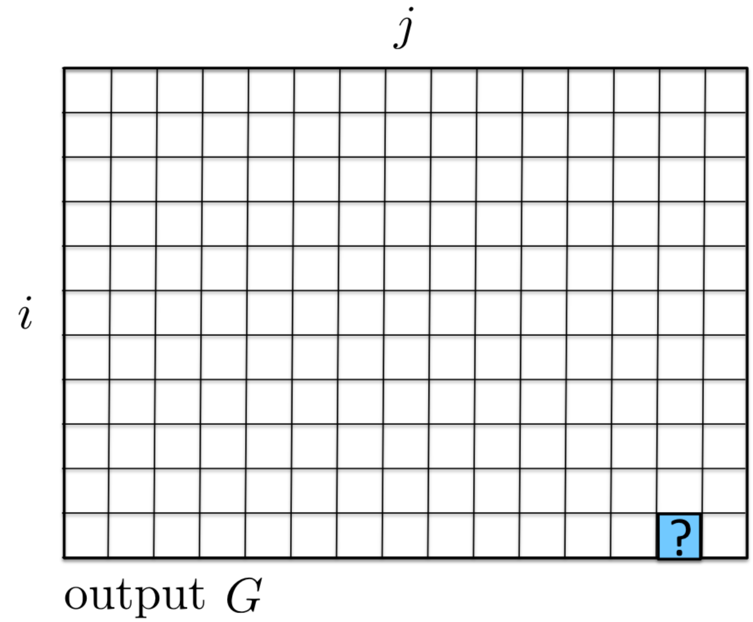


What happens at the borders?



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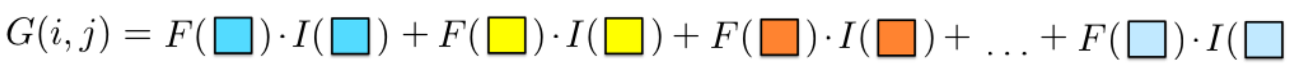

Boundary Effects

- What happens at the border of the image? What's the size of the output matrix?
    - depends on how you implement it

- Scipy: scipy.signal.convolve2d
    - mode = 'full' output size is bigger than the image
    - mode = 'same': output size is same as I
    - mode = 'valid': output size is smaller than the image

[Source: S. Lazebnik]

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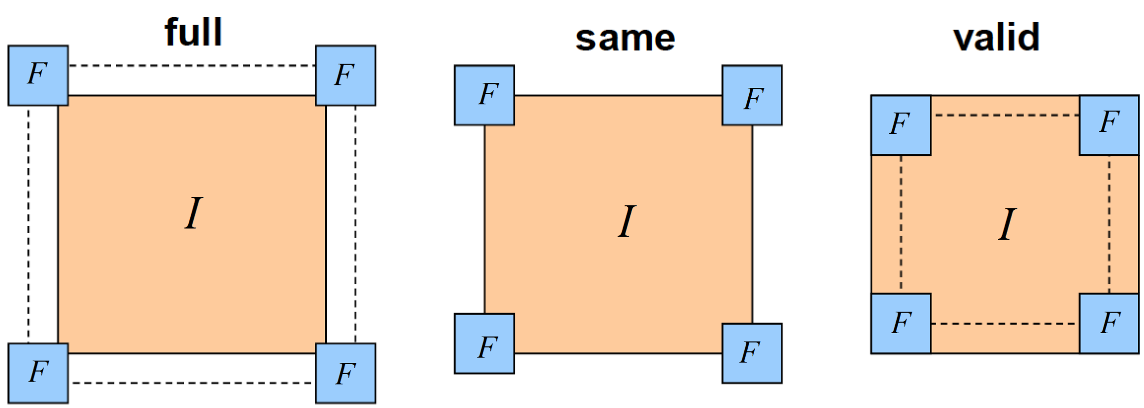


[Source: S. Lazebnik]

Correlation Example

What's the result?

Original

0	0	0
0	1	0
0	0	0

?

[Source: D. Lowe]

Correlation Example

What's the result?

Original

0	0	0
0	1	0
0	0	0

Filtered
(no change)

[Source: D. Lowe]

Correlation Example

What’s the result?

Original

0	0	0
0	0	1
0	0	0

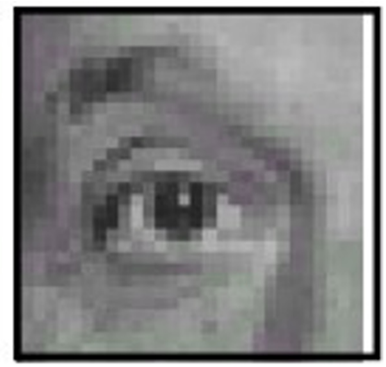
?

[Source: D. Lowe]

Correlation Example

What's the result?

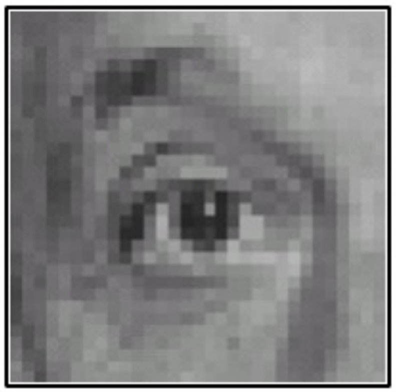
0	0	0
0	0	1
0	0	0



[Source: D. Lowe]

Correlation Example

What's the result?

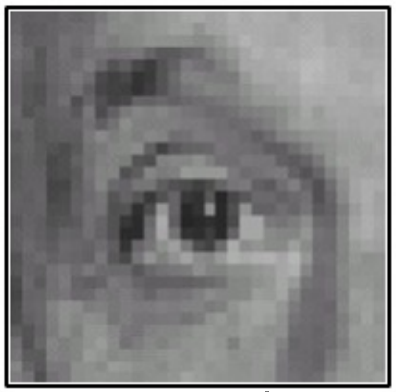


$$* \left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right) =$$

[Source: D. Lowe]

Correlation Example

What's the result?



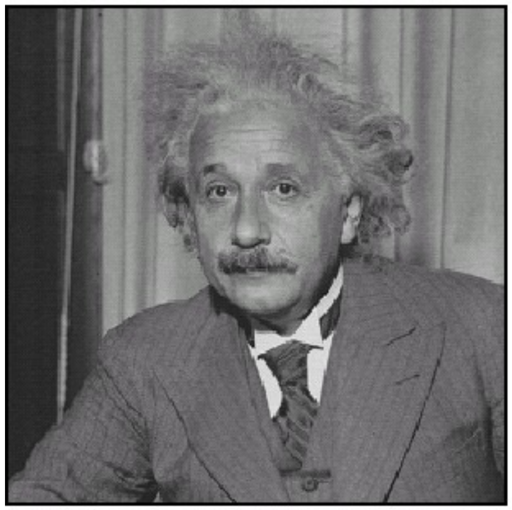
Original

$$* \left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right) =$$

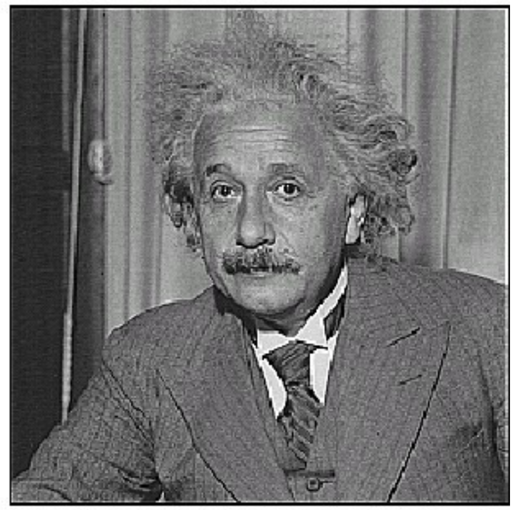
Sharpening Filter

[Source: D. Lowe]

Sharpening



before



after

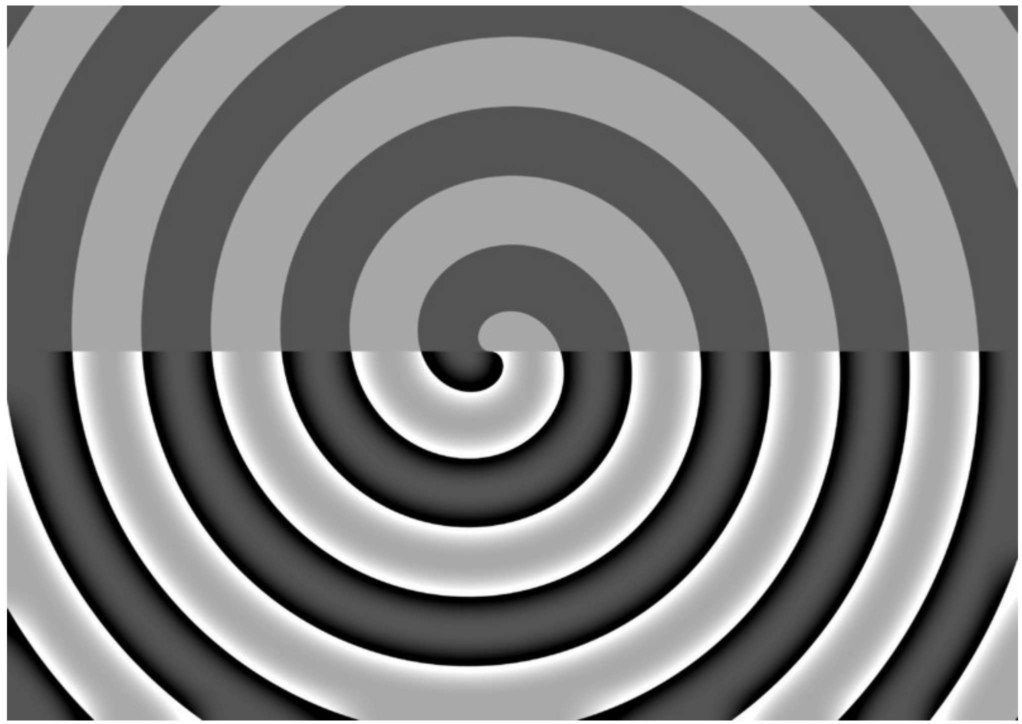
This is a prelude to edge detection (next time)!

[Source: D. Lowe]

Sharpening

unfiltered

filtered

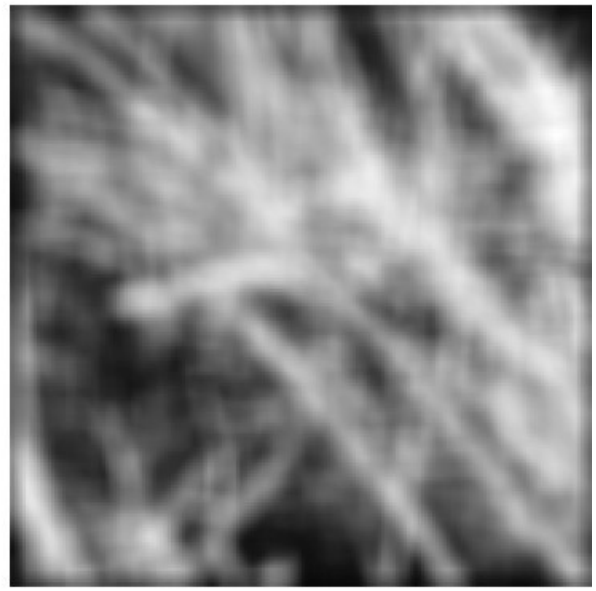


[Source: N. Snavely]

Smoothing by averaging

depicts box filter:
white = high value, black = low value

original



filtered

What if the filter size was 5 x 5 instead of 3 x 3?

[Source: K. Grauman]

Gaussian filter

What if we want nearest neighboring pixels to have the most influence on the output?

Removes high-frequency components from the image (low-pass filter).

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

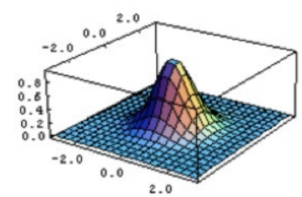
$$I(i,j)$$

$$\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

$$F(i,j)$$

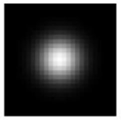
This kernel is an approximation of a 2d Gaussian function:

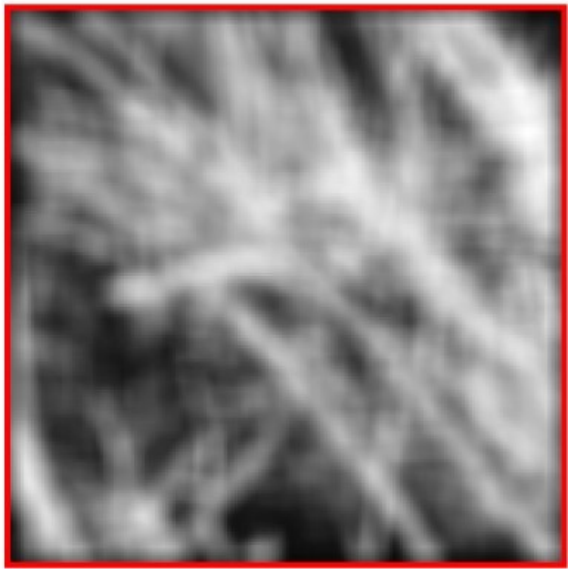
$$h(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$



[Source: S. Seitz]

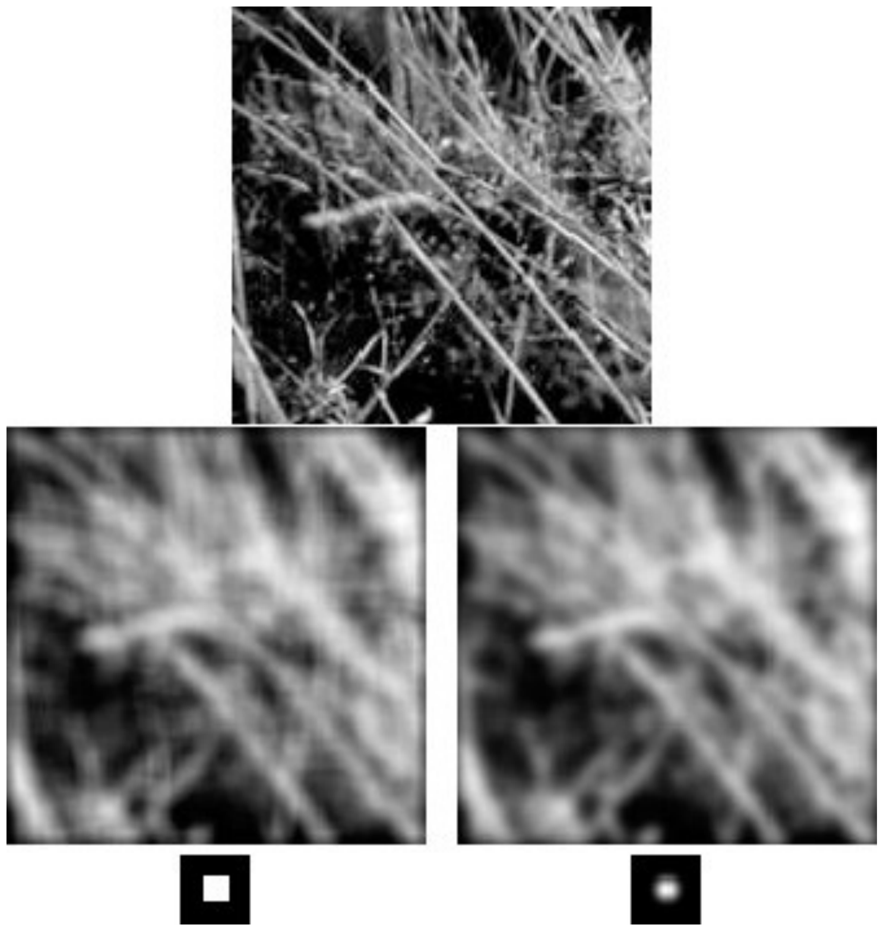
Gaussian filter





[Source: K. Grauman]

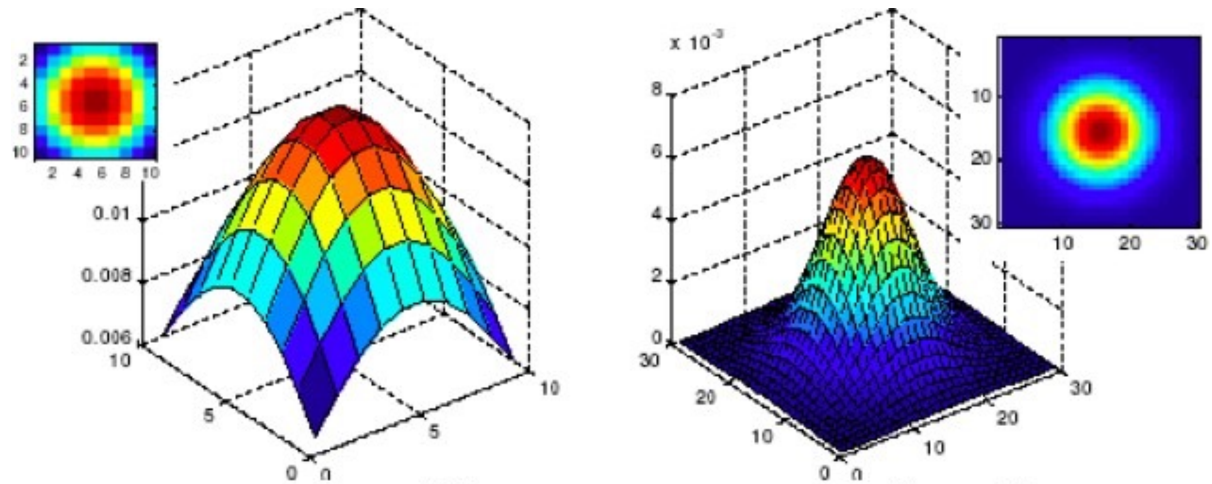
Mean vs. Gaussian filter



[Source: K. Grauman]

Gaussian filter parameters

Size of filter or mask: Gaussian function has infinite support, but discrete filters use finite kernels.



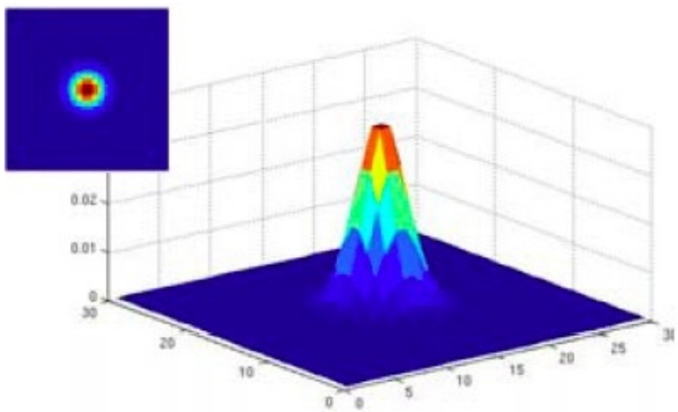
σ = 5 with 10 x 10 kernel

σ = 5 with 30 x 30 kernel

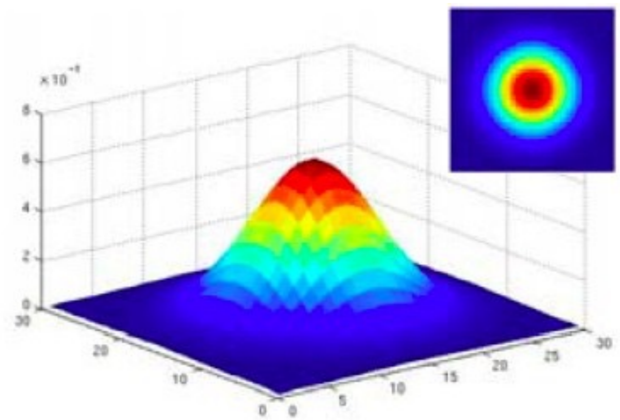
[Source: K. Grauman]

Gaussian filter parameters

Variance of the Gaussian: determines extent of smoothing.



σ = 2 with
30 x 30
kernel



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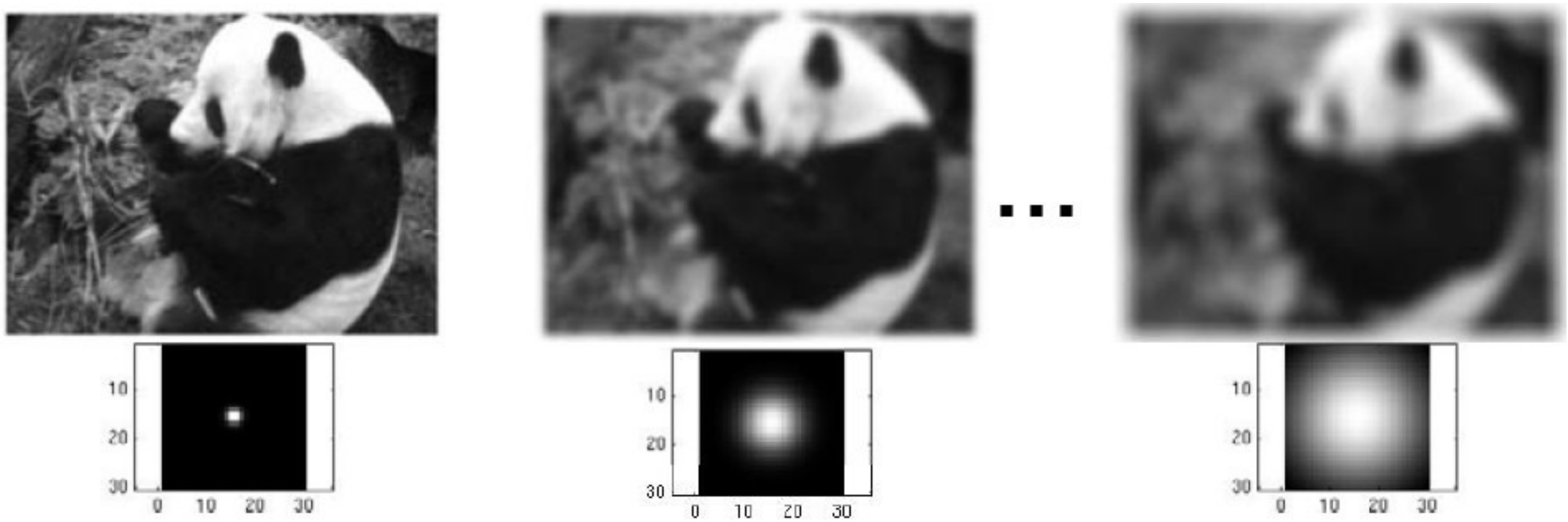
[Source: K. Grauman]

Gaussian filter parameters



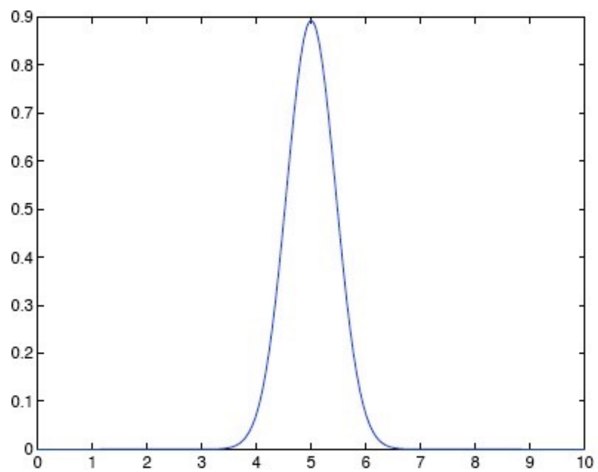

```
for sigma=1:3:10
   h = fspecial('gaussian`, fsize, sigma);
   out = imfilter(im, h);
   imshow(out);
   pause;
end
```

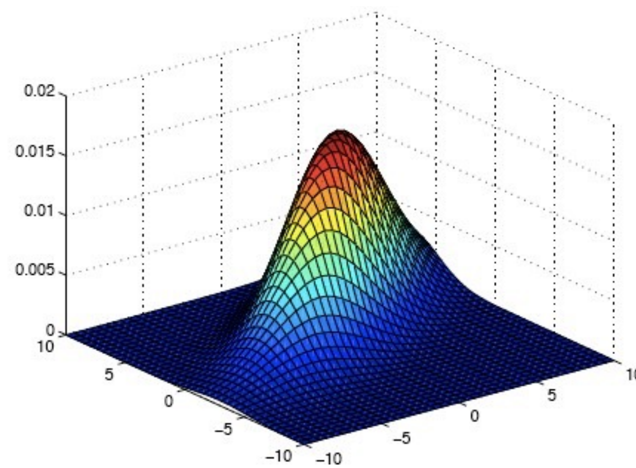
[Source: K. Grauman]

Is this the most general Gaussian?

No, the most general form is anisotropic (i.e., not symmetric) $x \in \Re^d$

$$\mathcal{N}(\mathbf{x}; \mu, \Sigma) = \frac{1}{(2\pi)^{d/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(\mathbf{x}-\mu)^T \Sigma^{-1}(\mathbf{x}-\mu)\right)$$





But the simplified version is typically used for filtering.

Properties of smoothing kernels

- All values are positive.
- They all sum to 1 to prevent re-scaling of the image.

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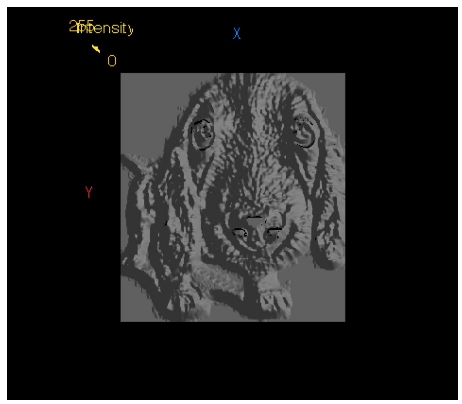
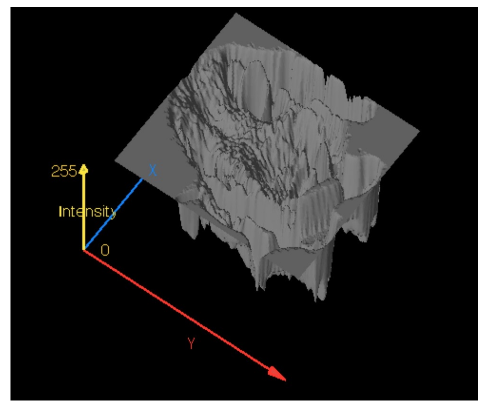
- Edges!





Finding Waldo

How can we use what we just learned to find Waldo?

Finding Waldo

Correlation?

Filter F

Interlude: Correlation in Matrix form

Remember correlation:

$$G(i,j) = \sum_{u=-k}^{k} \sum_{v=-k}^{k} F(u,v) \cdot I(i+u, j+v)$$

Can we write that in a more compact form (with vectors)?

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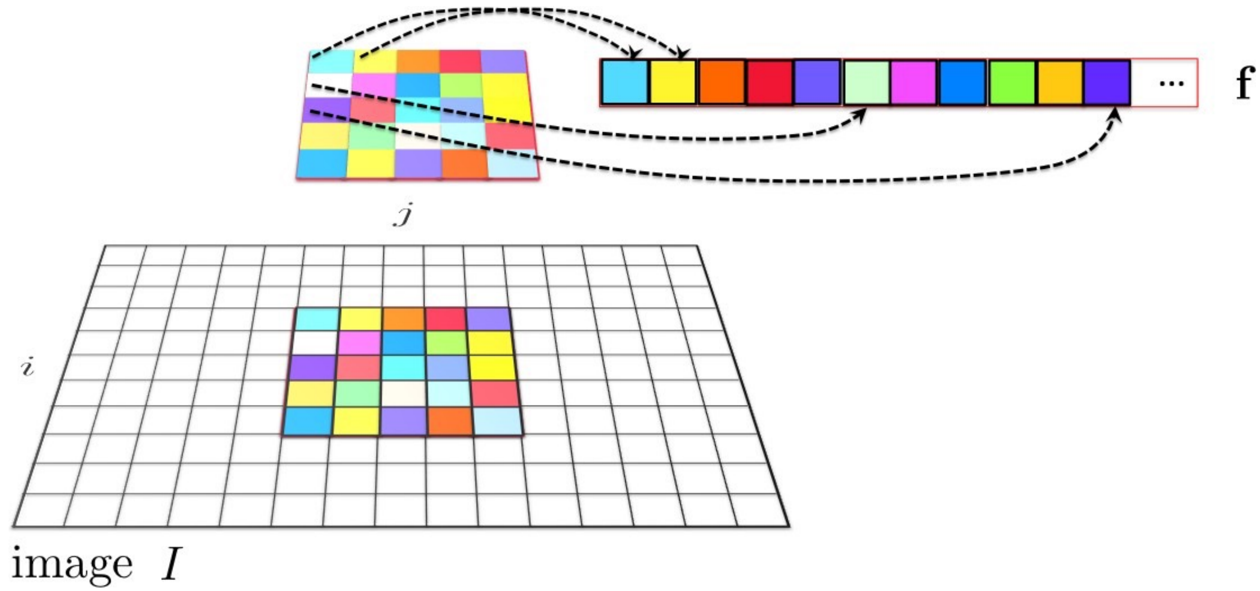

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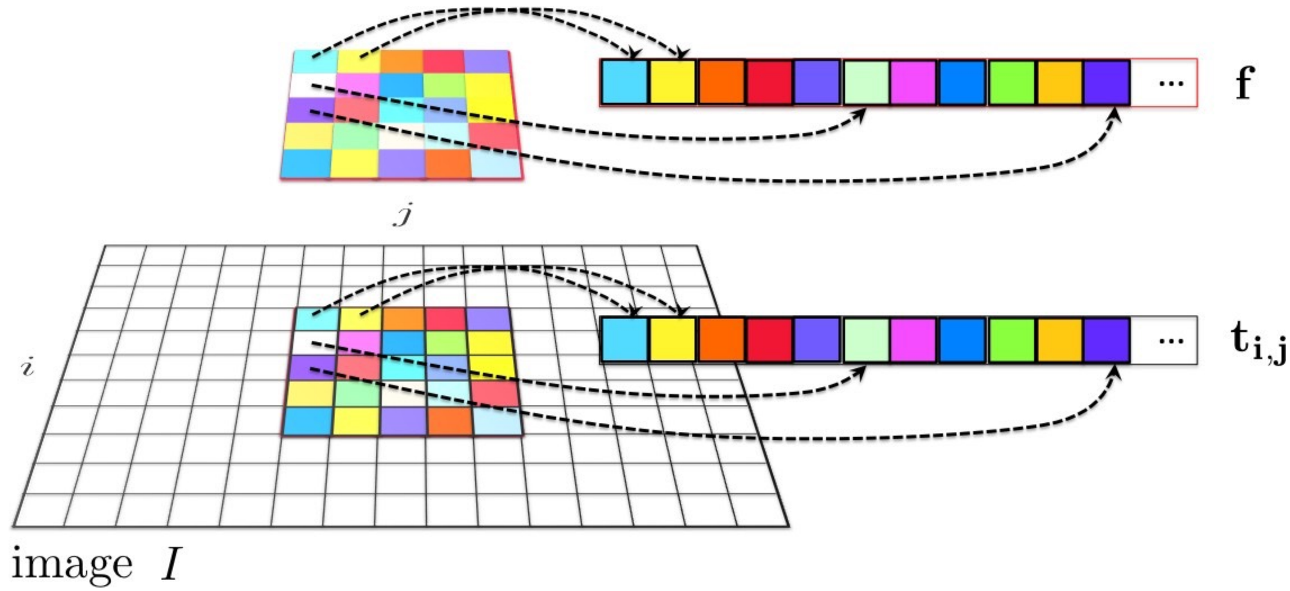

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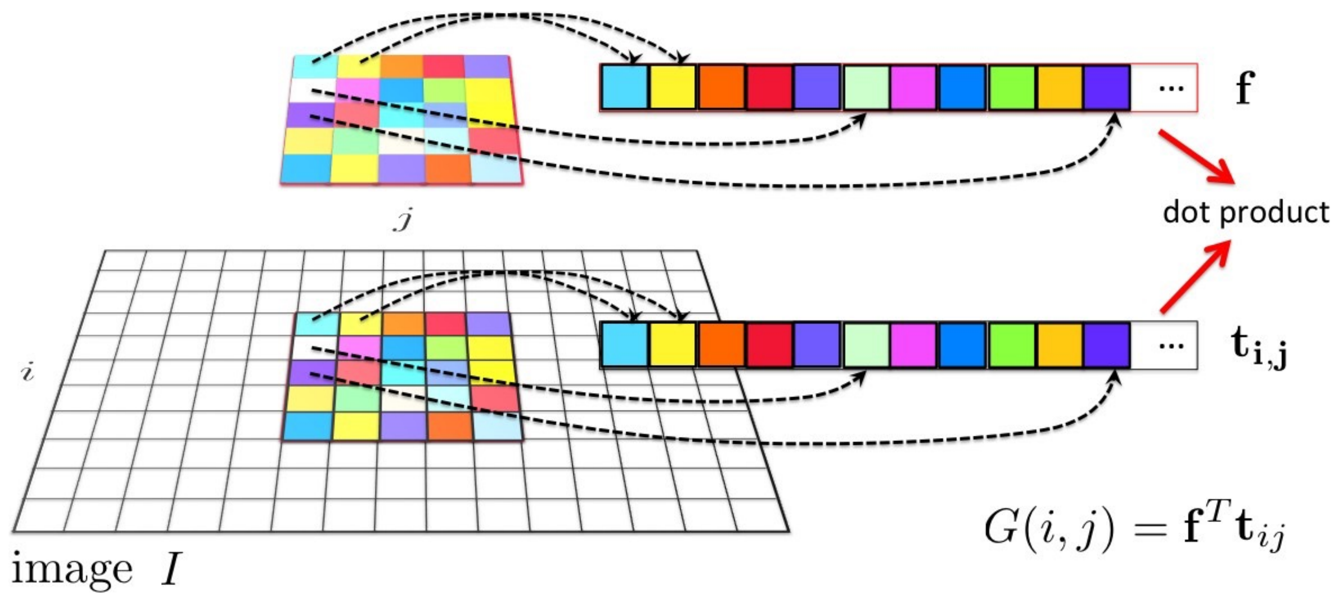

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Can we write that in a more compact form (with vectors)?

Define $\mathbf{f} = F(:), \quad T_{ij} = I(i-k : i+k, j-k : j+k), \quad \mathbf{t}_{ij} = T_{ij}(:)$

$$G(i,j) = \mathbf{f} \cdot \mathbf{t}_{ij}$$

Where $\cdot$ is a dot product

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Can we write full correlation G = F ⊗ I in matrix form?

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Finding Waldo: How could we ensure to get the best “score” (e.g. 1) for an image crop that looks exactly like our filter?

Interlude: Correlation in Matrix form

Finding Waldo: How could we ensure to get the best "score" (e.g. 1) for an image crop that looks exactly like our filter?

Normalized cross-correlation:

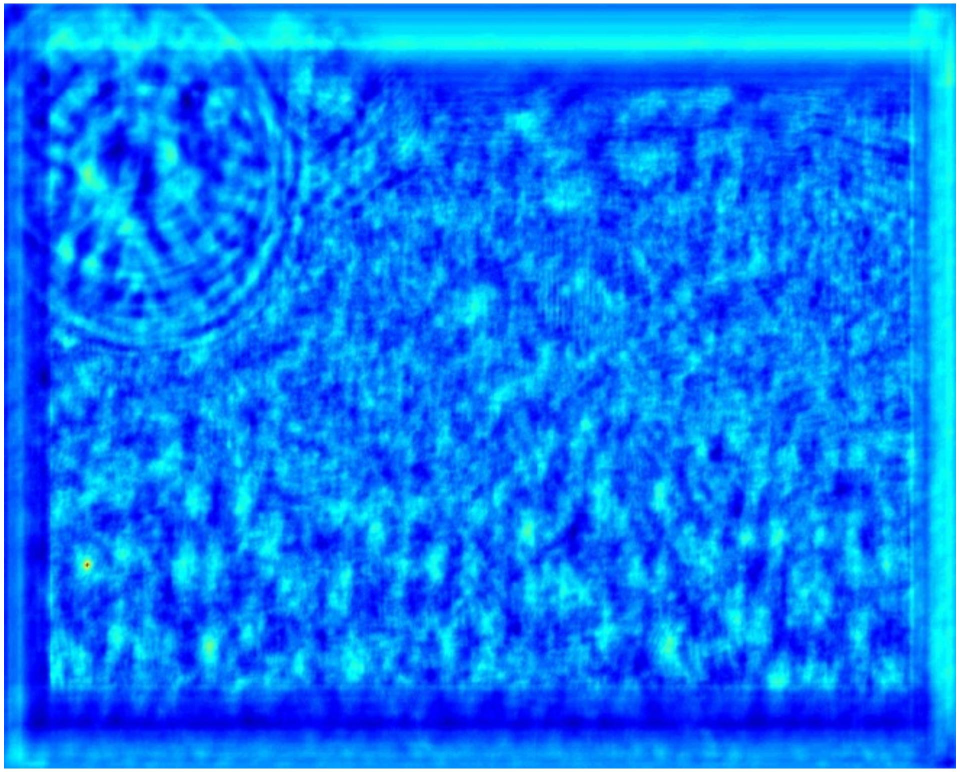
$$G(i,j) = \frac{\mathbf{f}^T \mathbf{t}_{ij}}{\|\mathbf{f}\| \|\mathbf{t}_{ij}\|}$$

Back to Finding Waldo

Image

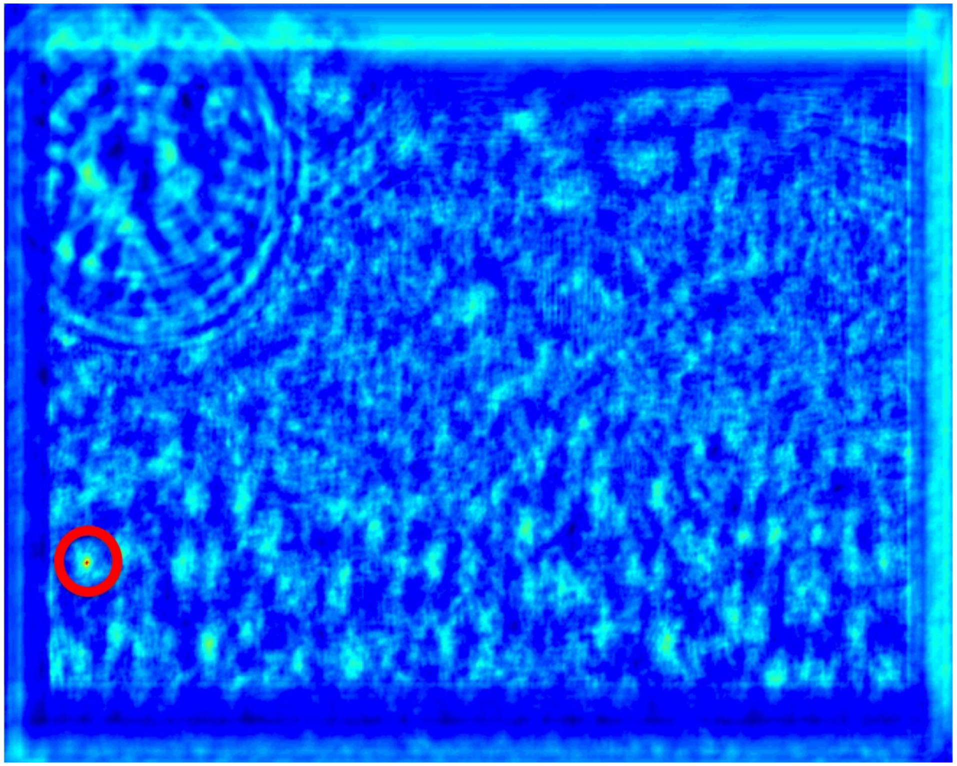
Filter

Back to Finding Waldo



Result of normalized cross-correlation

Back to Finding Waldo



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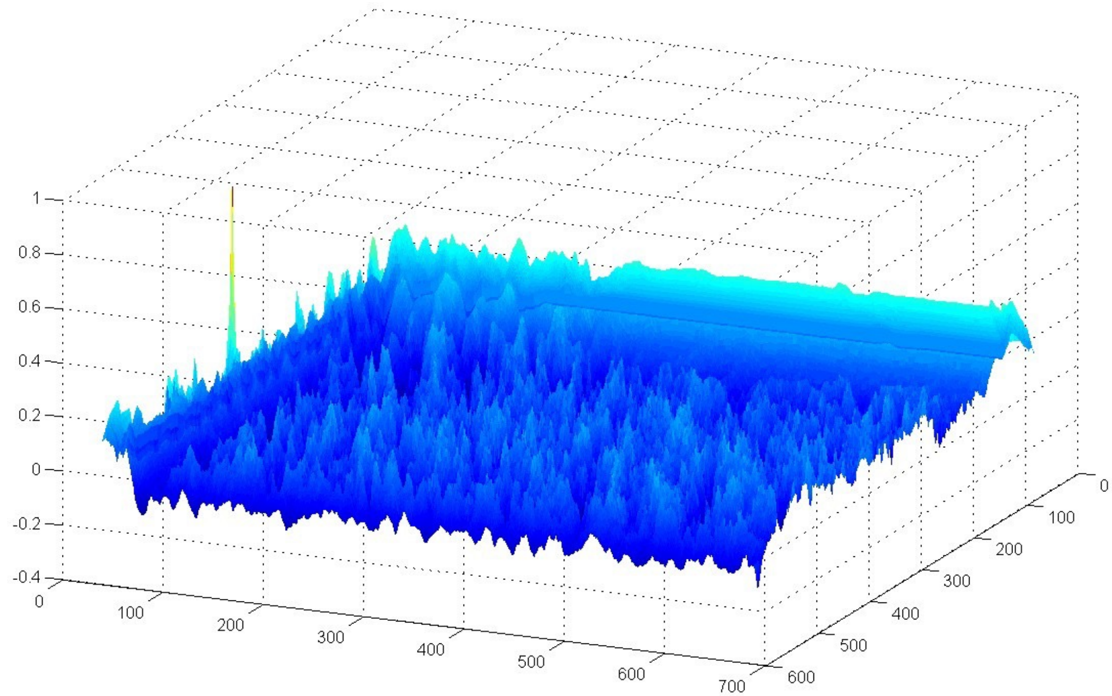
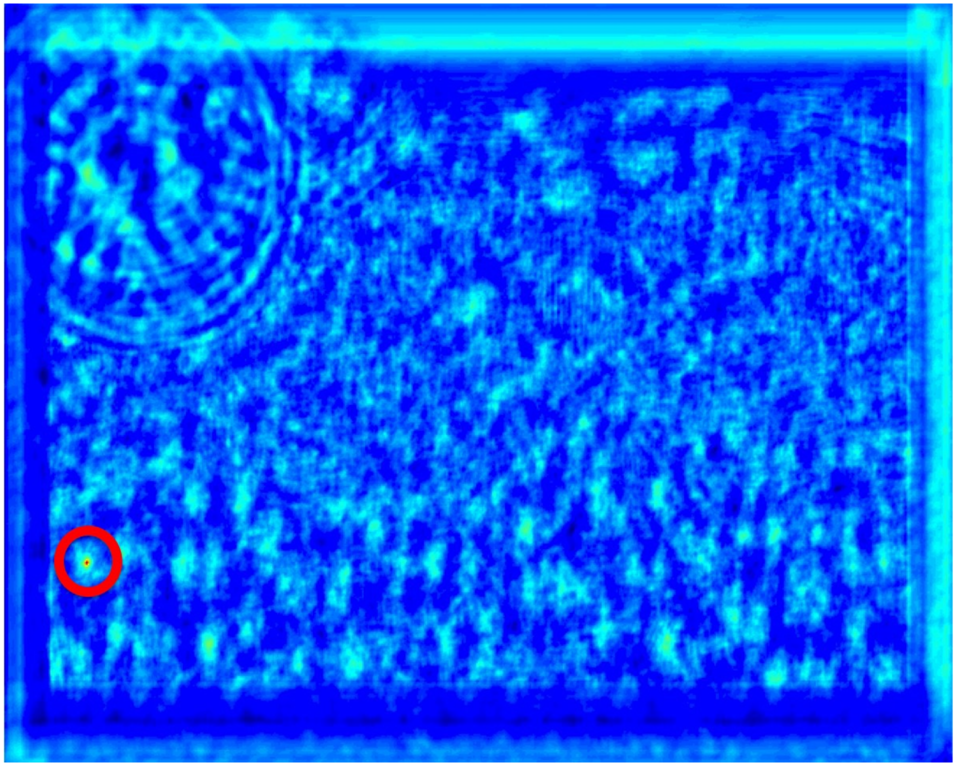


Find the highest peak

Back to Finding Waldo



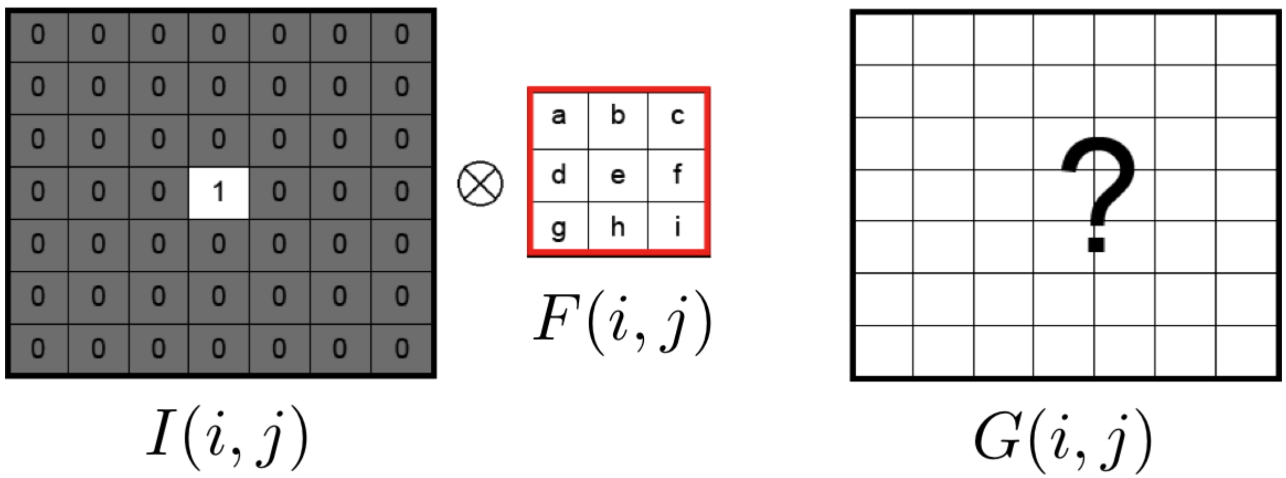
Find the highest peak

Back to Finding Waldo

With a bounding box (rectangle the size of the template) at the point...

Correlation example

What is the result of filtering the impulse signal (image) I with an arbitrary filter F?



[Source: K. Grauman]

Convolution

Convolution operator

$$G(i,j) = \sum_{u=-k}^{k} \sum_{v=-k}^{k} F(u,v) \cdot I(i-u, j-v)$$

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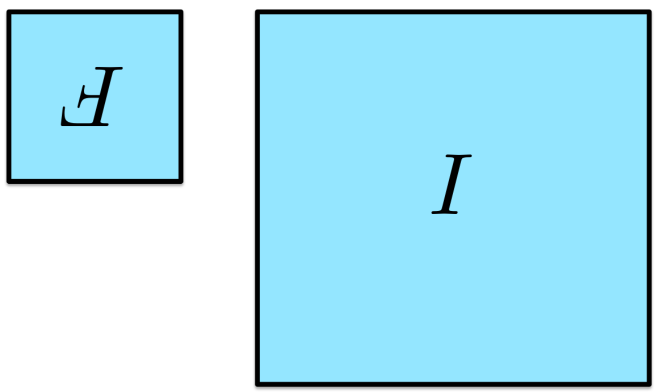
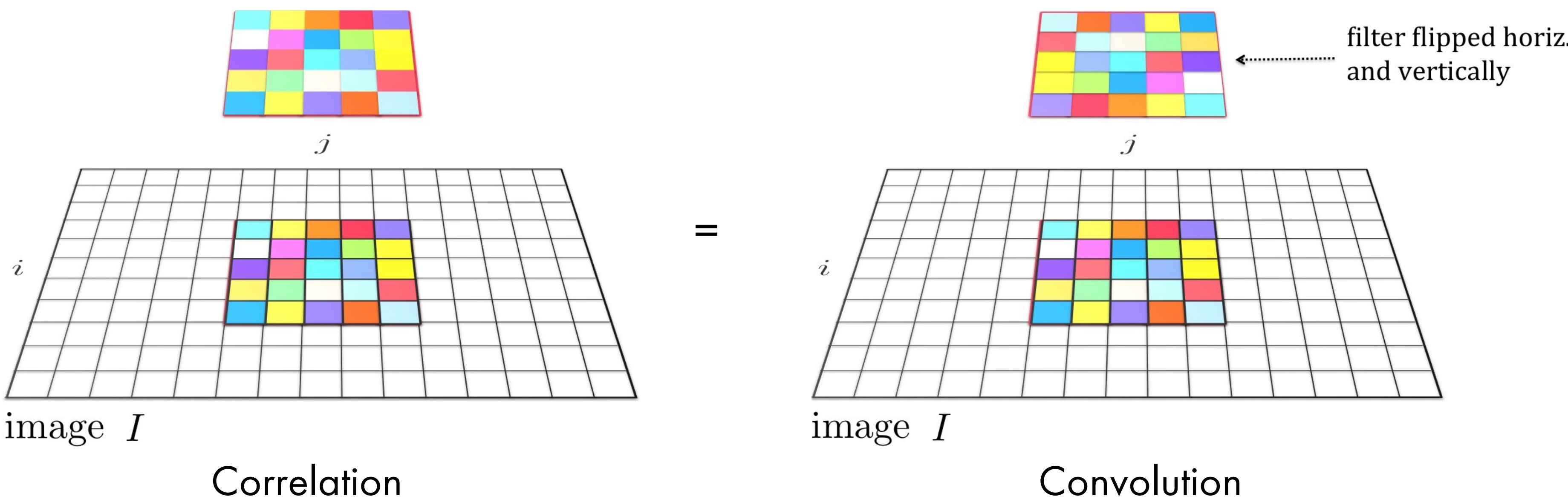
Equivalent to flipping the filter in both dimensions (bottom to top, right to left) and apply correlation.



Correlation vs Convolution



Correlation vs Convolution

For a Gaussian or box filter, how will the outputs $F * I$ and $F \otimes I$ differ?

Correlation vs Convolution

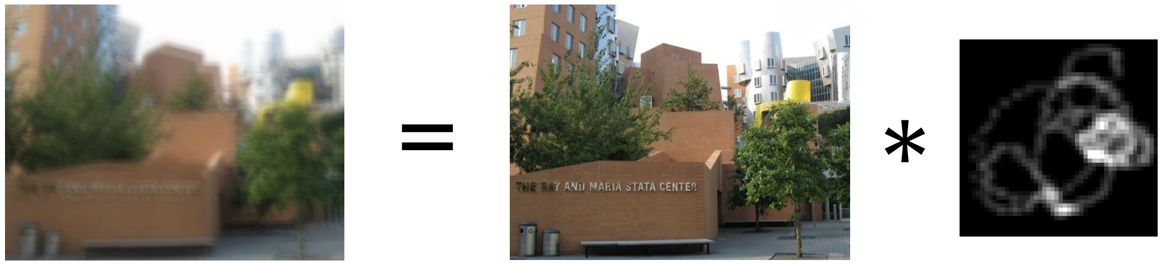
For a Gaussian or box filter, how will the outputs $F * I$ and $F \otimes I$ differ?

How will the outputs differ for:

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

“Optical” Convolution

Camera Shake



[Fergus et al. ,SIGGRAPH 2006]

Blur in out-of-focus regions of an image

Bokeh: http://lullaby.homepage.dk/diy-camera/bokeh.html

[Source: N. Snavely]

Properties of Convolution

Commutative: $f * g = g * f$

Associative: $f * (g * h) = (f * g) * h$

Distributive: $f * (g + h) = f * g + f * h$

Assoc. with scalar multiplier: $\lambda \cdot (f * g) = (\lambda \cdot f) * g$

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The Fourier transform of two convolved images is the product of their individual Fourier transforms:

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Why is this good news?

- Hint: Think of complexity of convolution and Fourier Transform
- What if we wanted to undo the result of convolution?

Separable Filters: Speed-up Trick

- The process of performing a convolution requires K^2 operations per pixel, where K is the size (width or height) of the convolution filter

[Source: R. Urtasun]

Separable Filters: Speed-up Trick

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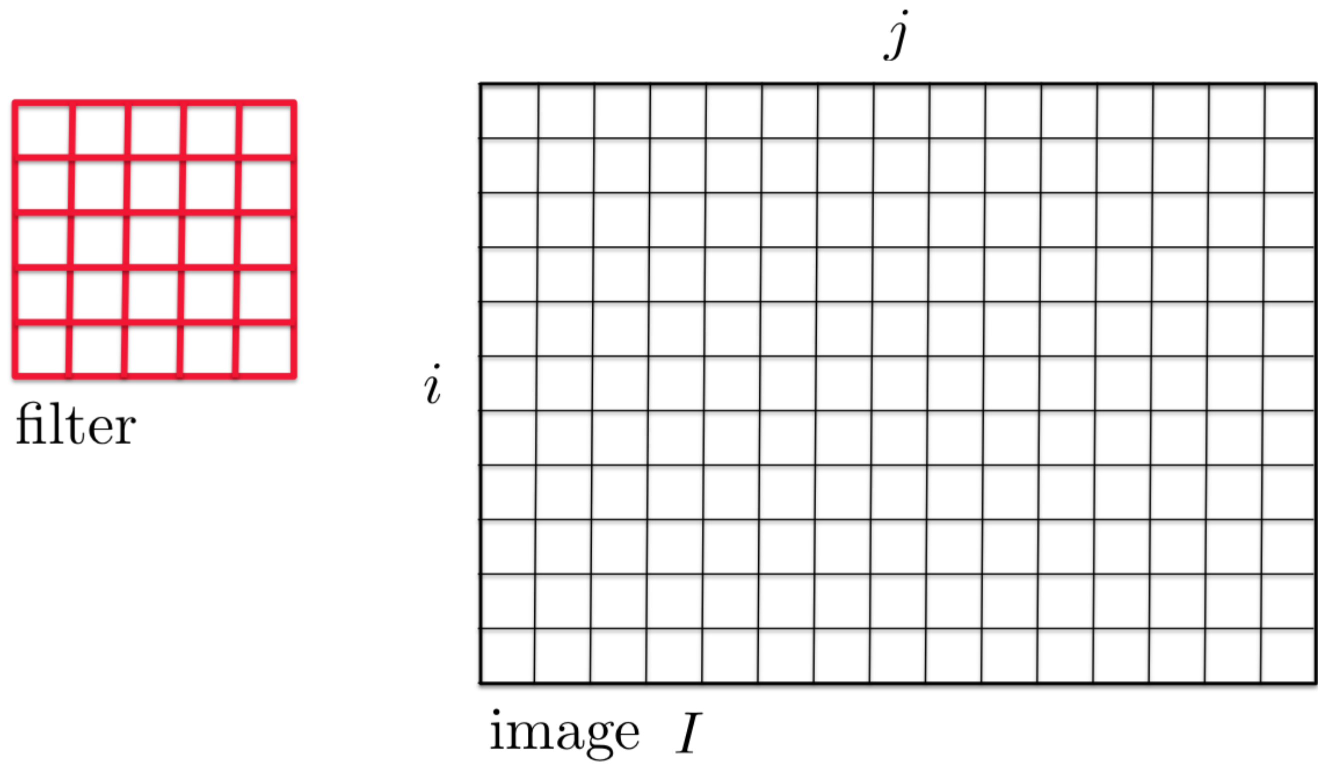
- In many cases (**not all!**), this operation can be sped up by first performing a 1D horizontal convolution followed by a 1D vertical convolution, **requiring only 2K operations**
- If this is possible, then the convolutional filter is called **separable**
- And it is the outer product of two filters:

$$\mathbf{F} = \mathbf{v}\mathbf{h}^T$$

[Source: R. Urtasun]

How it works



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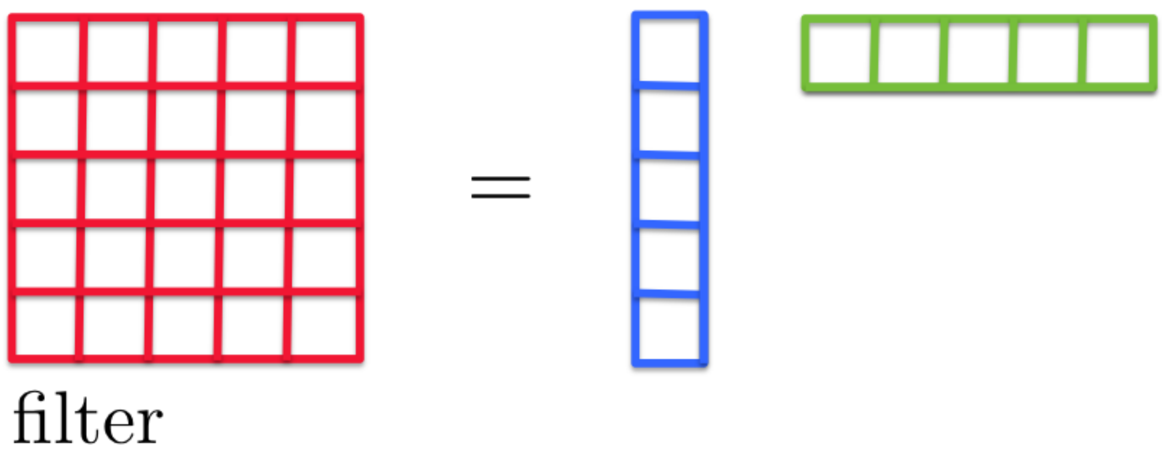

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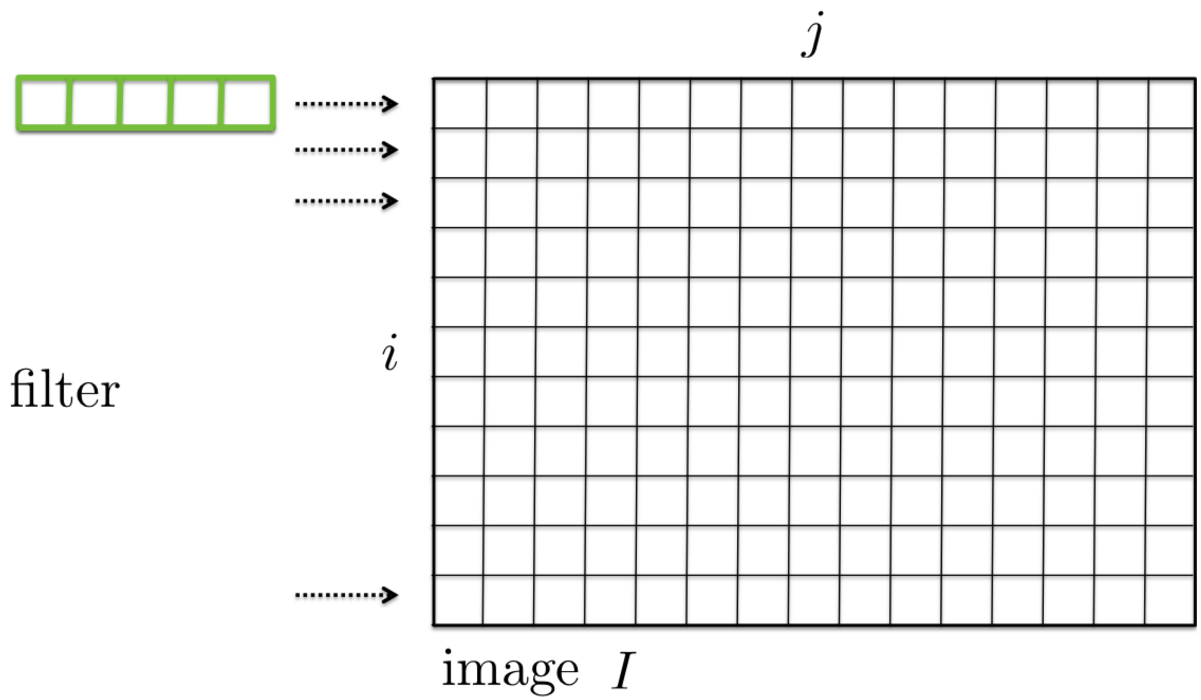

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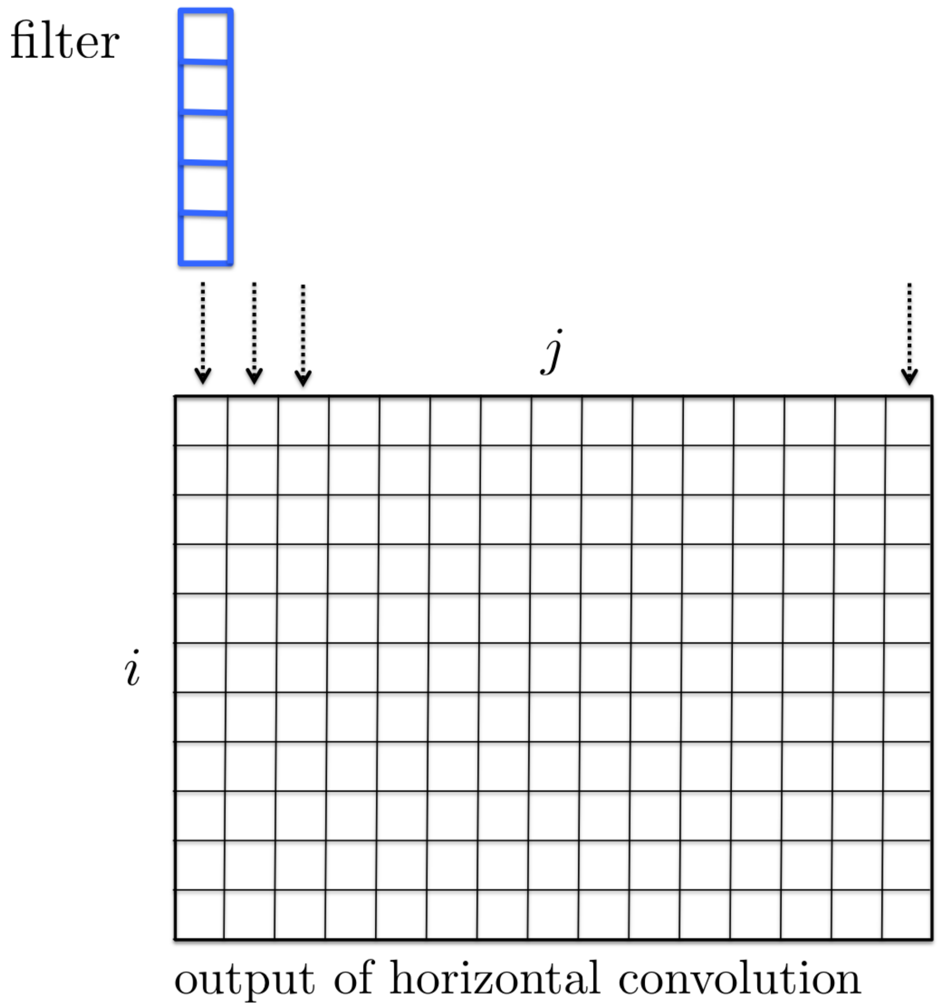
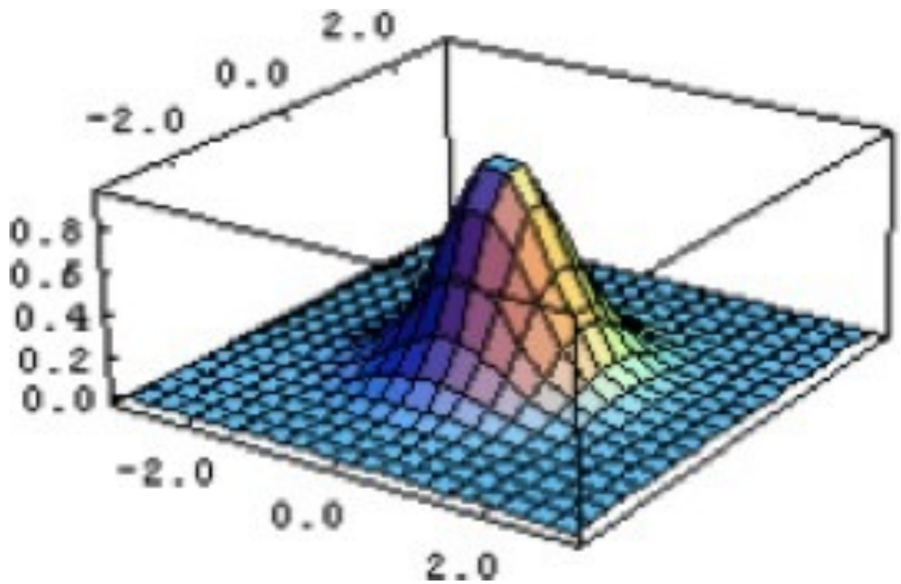


How it works

One famous separable filter we already know:

Gaussian: $f(x,y) = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2+y^2}{\sigma^2}\right)$



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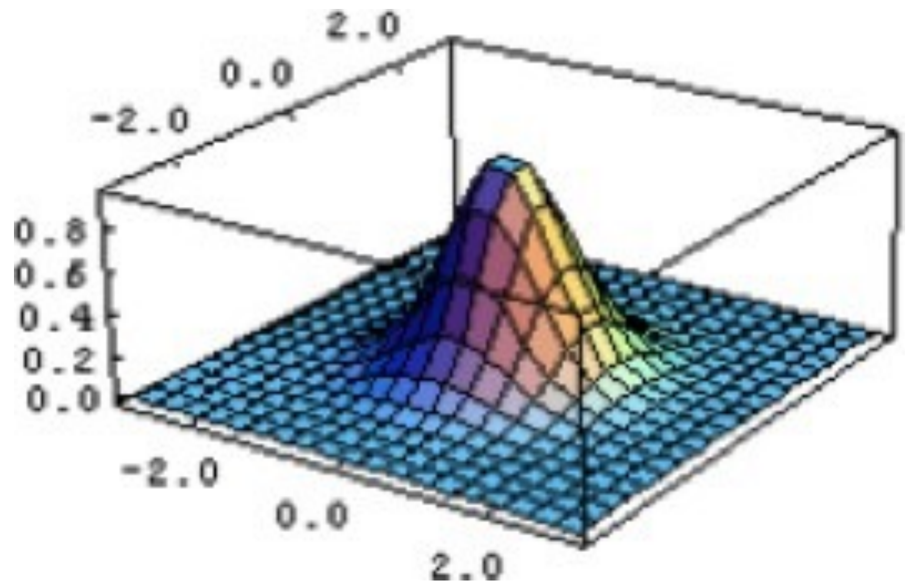

How it works

Is this separable? If yes, what's the separable version?

$$\frac{1}{K^2}\begin{array}{|c|c|c|c|}\hline 1 & 1 & \cdots & 1 \\ \hline 1 & 1 & \cdots & 1 \\ \hline \vdots & \vdots & 1 & \vdots \\ \hline 1 & 1 & \cdots & 1 \\ \hline \end{array}$$

[Source: R. Urtasun]

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What does this filter do?

[Source: R. Urtasun]

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$$\frac{1}{16}\begin{bmatrix}1 & 2 & 1\\ 2 & 4 & 2\\ 1 & 2 & 1\end{bmatrix}$$

[Source: R. Urtasun]

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[Source: R. Urtasun]

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$$\frac{1}{8}\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

[Source: R. Urtasun]

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[Source: R. Urtasun]

How can we tell if a given filter F is indeed separable?

- Inspection... this is what we were doing.

[Source: R. Urtasun]

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- Look at the singular value decomposition (SVD), and if only one singular value is non-zero, then it is separable

$$F = \mathbf{U}\Sigma\mathbf{V}^T = \sum_i \sigma_i u_i v_i^T$$

with $\Sigma = \mathrm{diag}(\sigma_i)$

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- Python: `np.linalg.svd`

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with $\Sigma = \mathrm{diag}(\sigma_i)$

- Python: `np.linalg.svd`
- $\sqrt{\sigma_1}\mathbf{u}_1$ and $\sqrt{\sigma_1}\mathbf{v}_1$ are the vertical and horizontal filters

[Source: R. Urtasun]

Summary – Stuff You Should Know

- **Correlation:** Slide a filter across image and compare (via dot product)
- **Convolution:** Flip the filter to the right and down and do correlation
- **Smooth** image with a Gaussian kernel: bigger σ means more blurring
- **Some** filters (like Gaussian) are separable: you can filter faster. First apply 1D convolution to each row, followed by another 1D conv. to each column

<u>OpenCV</u>:

- `Filter2D` (or `sepFilter2D`): can do both correlation and convolution
- `GaussianBlur`: create a Gaussian kernel
- `medianBlur, medianBlur, bilateralFilter`

Edges

- What does blurring take away?

[Source: S. Lazebnik]

Next time:

Edge Detection