

Introduction

Motivation and Image Processing



CSC420

David Lindell

University of Toronto

cs.toronto.edu/~lindell/teaching/420

Slide credit: Babak Taati ← Ahmed Ashraf ← Sanja Fidler

Instructors



David Lindell



Shayan
Shekarforoush



Yun-Chun Chen



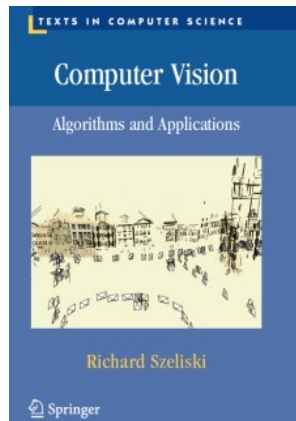
Anagh Malik

Course Email: csc420-2022-09@cs.toronto.edu

Course Info

- Class time: Mondays 2-4 pm GB 248
- Tutorials: Wednesdays 3-4 pm GB 119
- Class Website: <https://www.cs.toronto.edu/~lindell/teaching/420/>
- Quercus: <https://q.utoronto.ca/>
- Course material (lecture notes, reading material, assignments, announcements, etc.) will be posted on Quercus
- Forum: Ed Discussion (link on Quercus)
- Your grade will not depend on your participation on discussions. It's just a good way for asking questions, discussing with your instructor, TAs and your peers

Textbook: We won't directly follow any book, but extra reading in this textbook will be useful:



Rick Szeliski

Computer Vision: Algorithms and Applications

available free online: <http://szeliski.org/Book/>

Links to other material (papers, code, etc.) will be posted on the class webpage

Course Prerequisites

- Data structures
- Linear Algebra
- Vector calculus
- Without this you'll need some serious catching up to do!

Knowing some basics in these is a plus:

- Python
- Machine Learning
- Neural Networks
- (Solving assignments sooner rather than later)

Grading

- Assignment 1: 12%
 - Assignment 2: 20%
 - Assignment 3: 16%
 - Assignment 4: 16%
 - Ethics Module: 1% (2 surveys, 0.5 each)
 - Final Exam: 34%
-
- Assignments: They will consist of problem sets and programming problems with the goal of deepening your understanding of the material covered in class.

Assignments

- Download from Files section on Quercus, Submitted via MarkUs
- Assignments: They will consist of problem sets and programming problems with the goal of deepening your understanding of the material covered in class.
 - Code in python
 - Please comment your code!
- Assignment 1 is out now, due Jan 27 at 11:59 PM

Assignments

Deadline

- The solutions to the assignments / project should be submitted by 11:59 pm on the date they are due.

Lateness

- Each student will be given a total of 3 free late days.
- This means that you can hand in three of the assignments one day late, or one assignment three days late.
- After you have used the 3 day budget, late assignments will not be accepted.

All info on the course website

Schedule and Syllabus

Week	Date	Description	Material	Readings	Event	Deadline
Week 1	Mon 9/1	Lecture 1: Introduction & Linear filters	[slides]	Szeliski 3.2 (optional) Brain mechanisms of early vision (optional) Early vision	Assignment 1 out on Quercus	
	Wed 11/1	Tutorial 1	[code]			
	Thu 12/1	TA Office Hours				
Week 2	Mon 16/1	Lecture 2: Edges	[slides]	Szeliski 4.2 (optional) Fourier Transform (optional) Computer color is broken		
	Wed 18/1	Tutorial 2	[code]			
	Thu 19/1	TA Office Hours				
Week 3	Mon 23/1	Lecture 3: Image pyramids	[slides]	Szeliski 3.5 (optional) Pyramid methods		
	Wed 25/1	Tutorial 3	[code]			
	Thu 26/1	TA Office Hours				
	Fri 27/1					Assignment 1 due at 11:59pm

Let's begin!

Introduction to Intro to Image Understanding

- What is Computer Vision?
- Why study Computer Vision?
- Which cool applications can we do with it? Is vision a hard problem?

What is Computer Vision?

What is computer vision?

- A field trying to develop automatic algorithms that can “see”



What is computer vision?

- What does it mean to see?



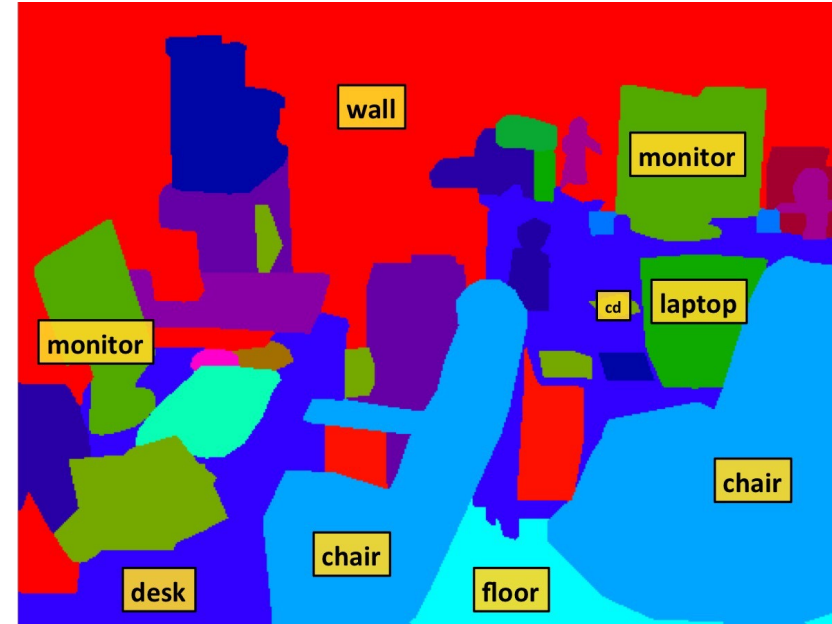
example scene

What is computer vision?

- What does it mean to see?
 - To know what is where by looking – Marr, 1982
 - Understand where things are in the world



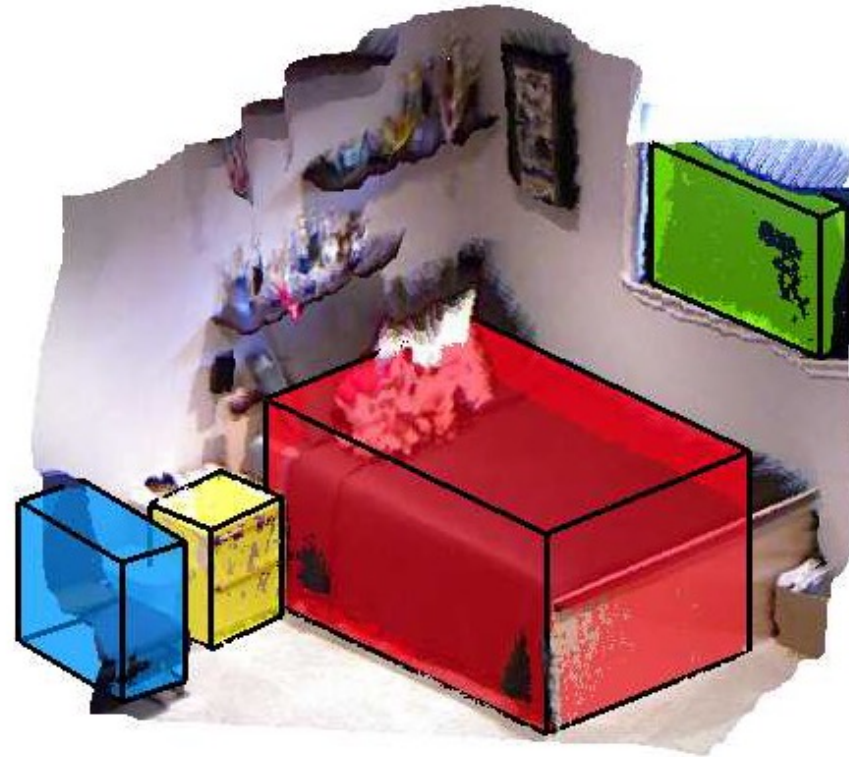
example scene



segmentation

What is computer vision?

- What does it mean to see?
 - To know what is where by looking – Marr, 1982
 - Understand where things are in the world
 - Understand 3D structure



What is computer vision?

- What does it mean to see?
 - To know what is where by looking – Marr, 1982
 - Understand where things are in the world
 - Understand 3D structure
 - Understand physical properties

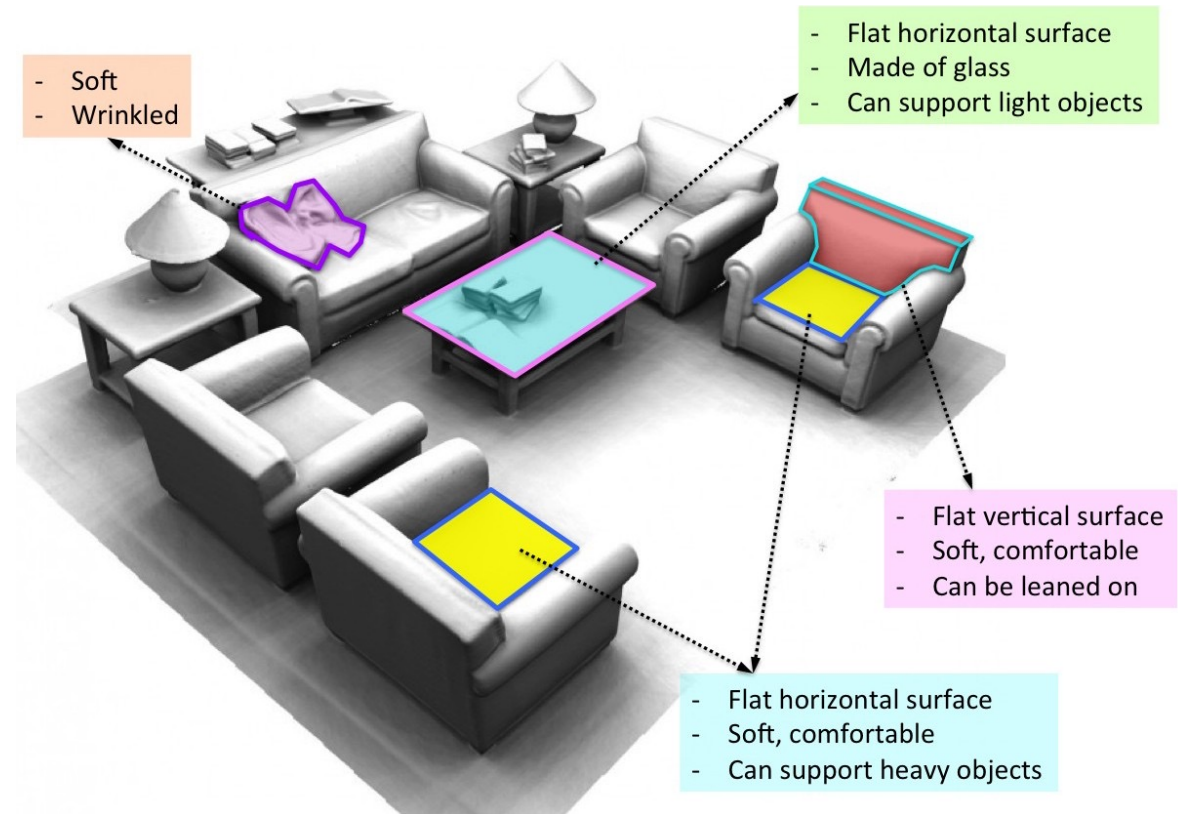


Image: Vladlen Koltun

What is computer vision?

- What does it mean to see?
 - To know what is where by looking – Marr, 1982
 - Understand where things are in the world
 - Understand 3D structure
 - Understand physical properties
 - Understand what actions are taking place

polar bear
eating fish

snake escaping!



boy scaring girl

What is computer vision?

- Full understanding of an image?

What is computer vision?

- Full understanding of an image?
 - Can answer any question about it



Q: What is behind the table?
A: window



Q: What is in front of the toilet?
A: door



Q: What is on the counter in the corner?
A: microwave

What is computer vision?

- Full understanding of an image?
 - Can answer any question about it



Q: What is behind the table?
A: window



Q: What is in front of the toilet?
A: door



Q: What is on the counter in the corner?
A: microwave



Q: What is the shape of the green chair?
A: horse shaped

What is computer vision?

- Full understanding of an image?
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A: microwave



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Q: Where is the oven?

A: on the right side of the fridge

What is computer vision?

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Q: What is on the counter in the corner?
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Q: What is the shape of the green chair?
A: horse shaped



Q: Where is the oven?
A: on the right side of the fridge



Q: What is the largest object?
A: bed

What is computer vision?

- Full understanding of an image?
 - Can answer any question about it



Q: Which object is red?

A: toaster

What is computer vision?

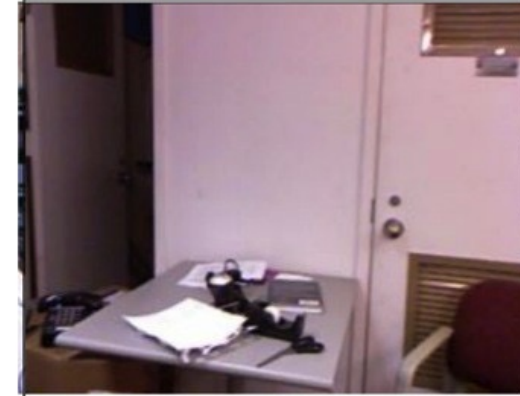
- Full understanding of an image?
 - Can answer any question about it



Q: Which object is red?
A: toaster



Q: How many drawers are there?
A: 6



Q: How many doors are open
A: 1



Q: How many lights are on?
A: 6

What is computer vision?

- Full understanding of an image?
 - Can answer any question about it



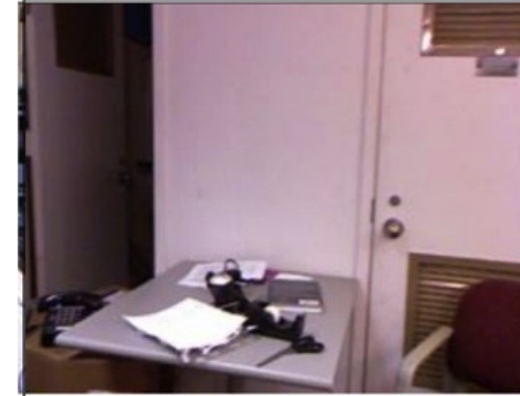
Q: Which object is red?

A: toaster



Q: How many drawers are there?

A: 6



Q: How many doors are open

A: 1



Q: How many lights are on?

A: 6



Q: Can you make pizza in this room?

A: yes



Q: Where can you sit?

A: chairs, table, floor

Why study computer vision?

Why study Computer Vision?

Because you want your robot to fold your laundry



Why study Computer Vision?

And drive you to work



Why study Computer Vision?

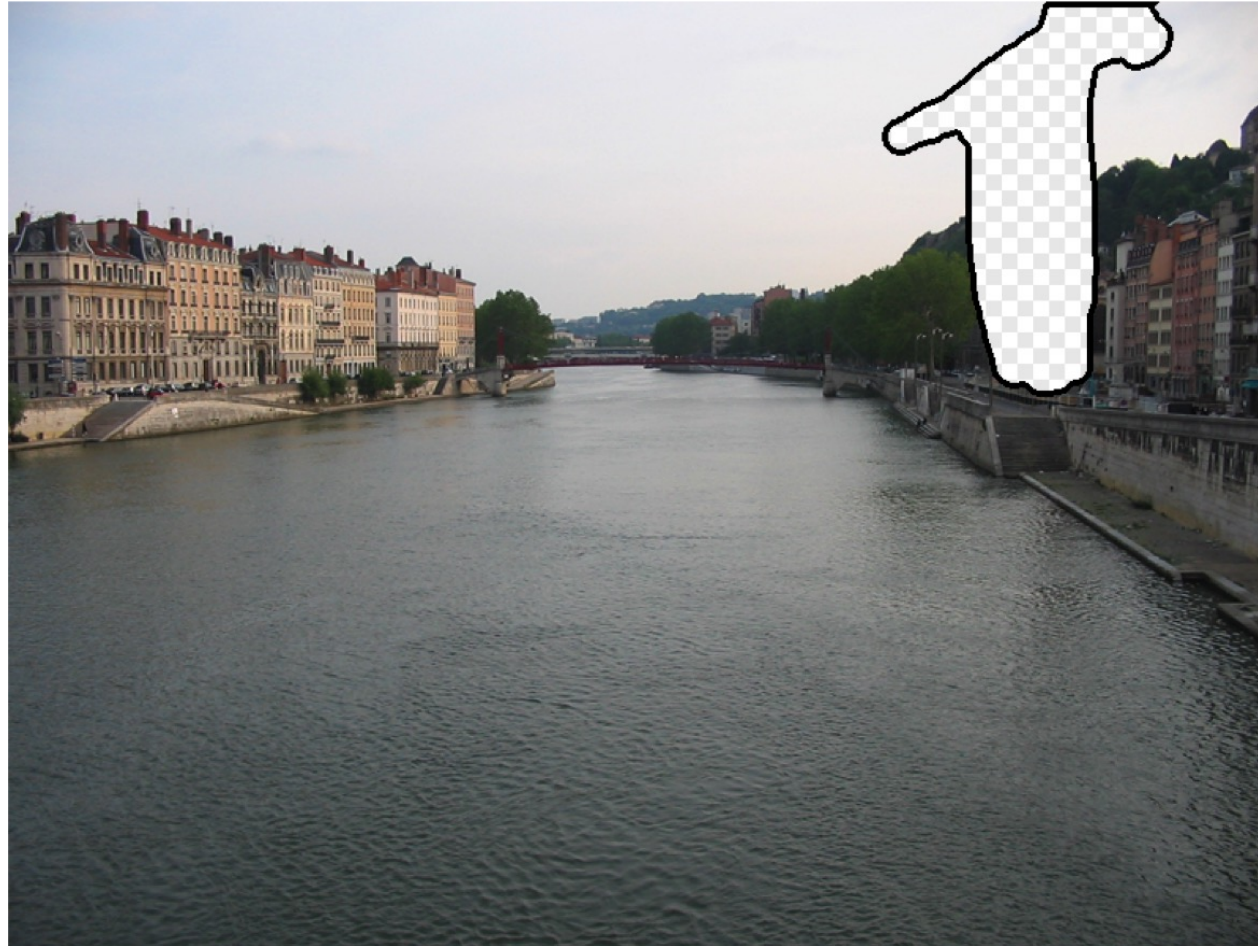
Allows you to manipulate images



Scene Completion using Millions of Photographs, Hays & Efros, SIGGRAPH 2007

Why study Computer Vision?

Allows you to manipulate images



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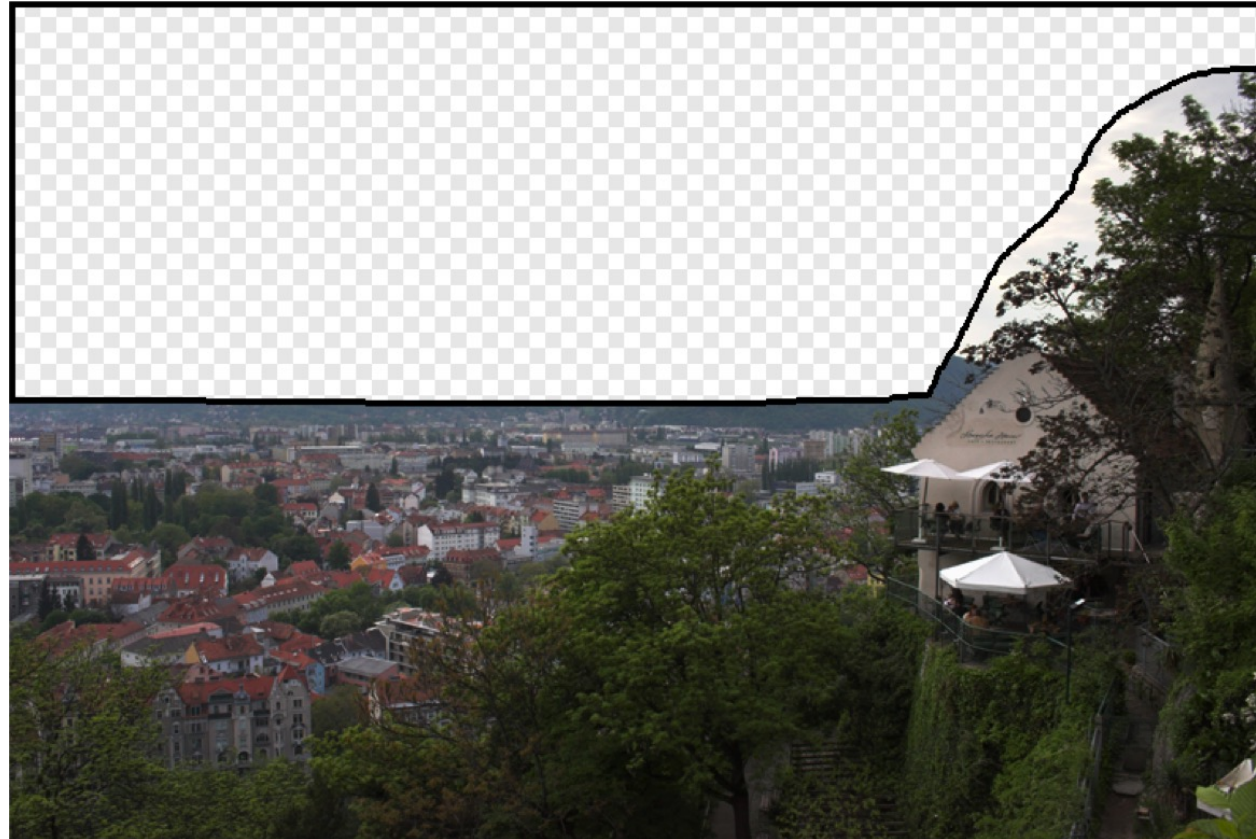
Allows you to manipulate images



Scene Completion using Millions of Photographs, Hays & Efros, SIGGRAPH 2007

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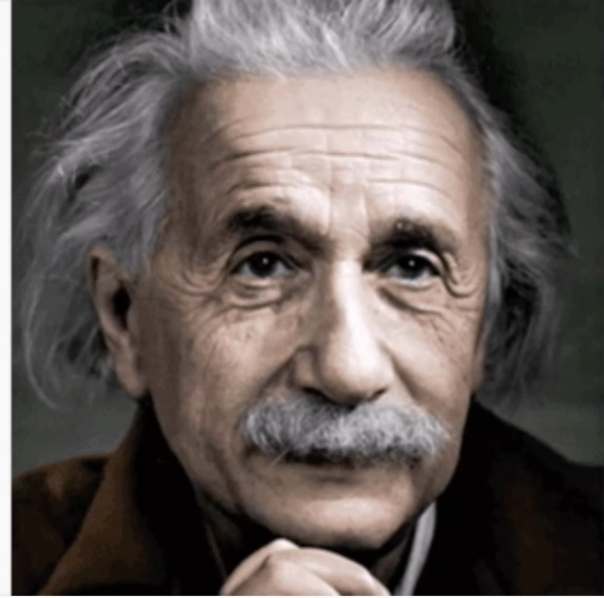
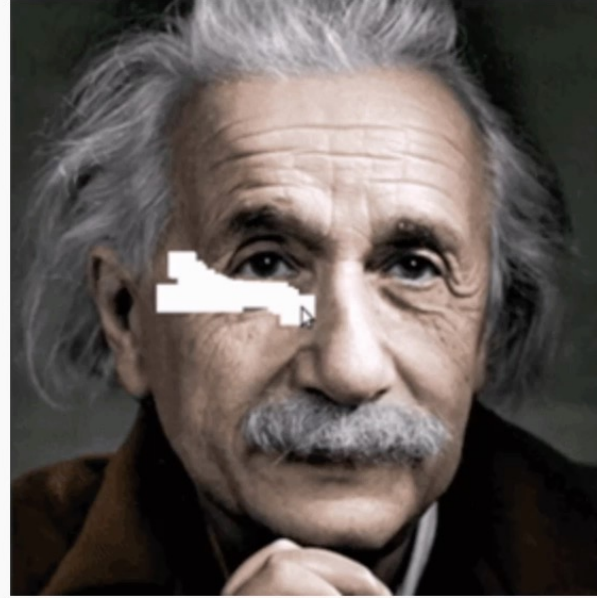
Allows you to manipulate images



Scene Completion using Millions of Photographs, Hays & Efros, SIGGRAPH 2007

Why study Computer Vision?

Allows you to manipulate images



[Online demo](#) (NVIDIA inpainting demo)

Why study Computer Vision?

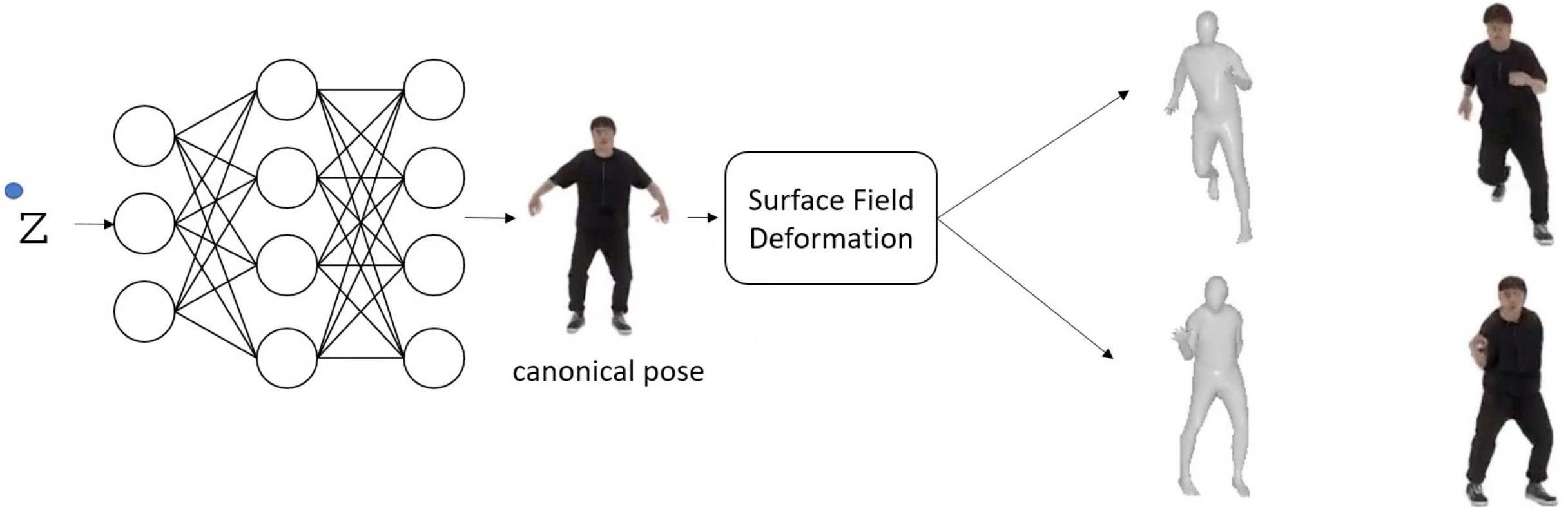
Change style of images...



[Gatys, Ecker, Bethge. A Neural Algorithm of Artistic Style. Arxiv'15.]

Why study Computer Vision?

Synthesize and animate digital humans



[Bergman et al. '22]

Why study Computer Vision?

Synthesize and animate digital humans



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Why study Computer Vision?

See "invisible" changes in a scene...



[Wu et al. SIGGRAPH '12]

Why study Computer Vision?

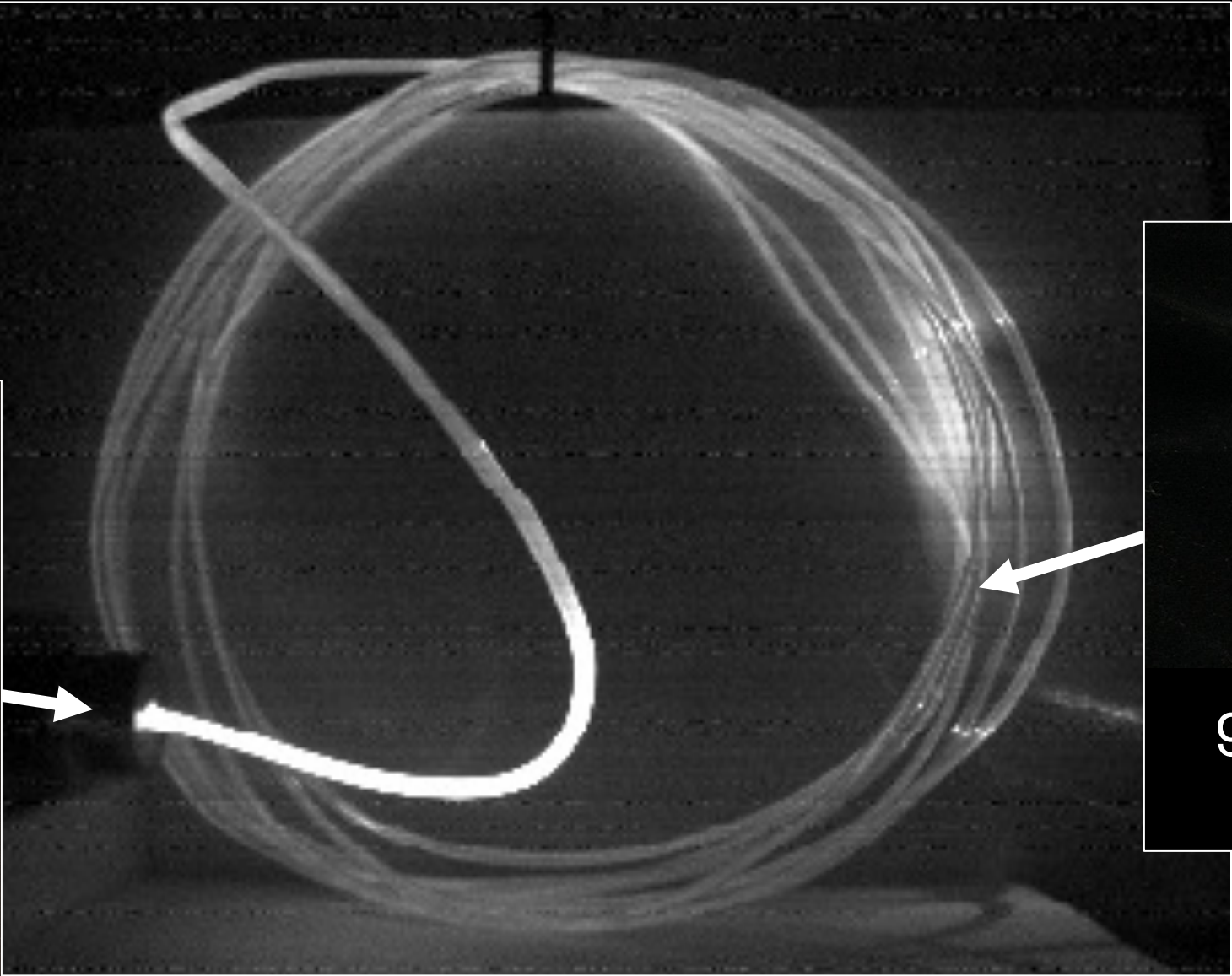
See "invisible" changes in a scene...



[Wu et al. SIGGRAPH '12]



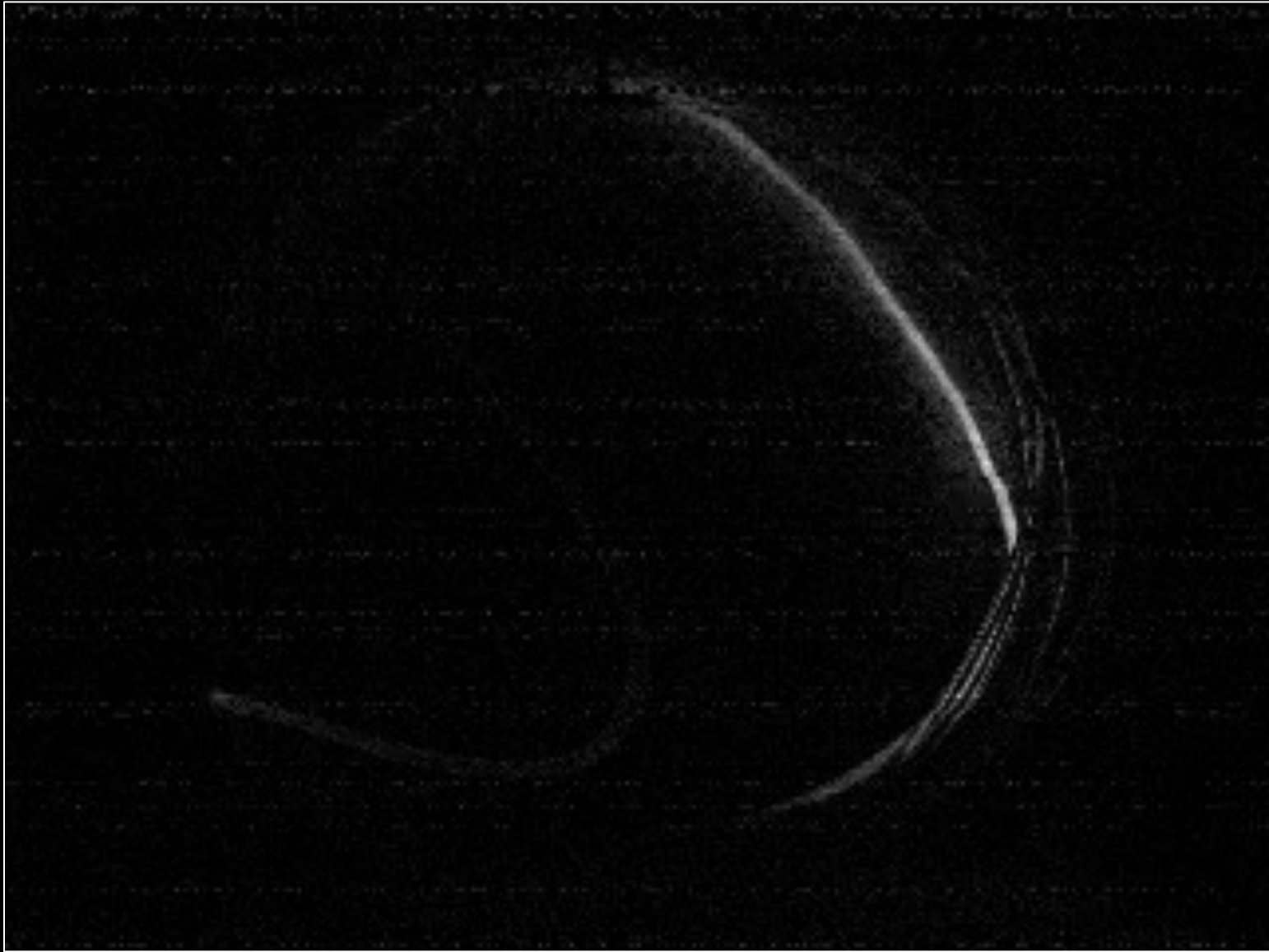
picosecond
laser



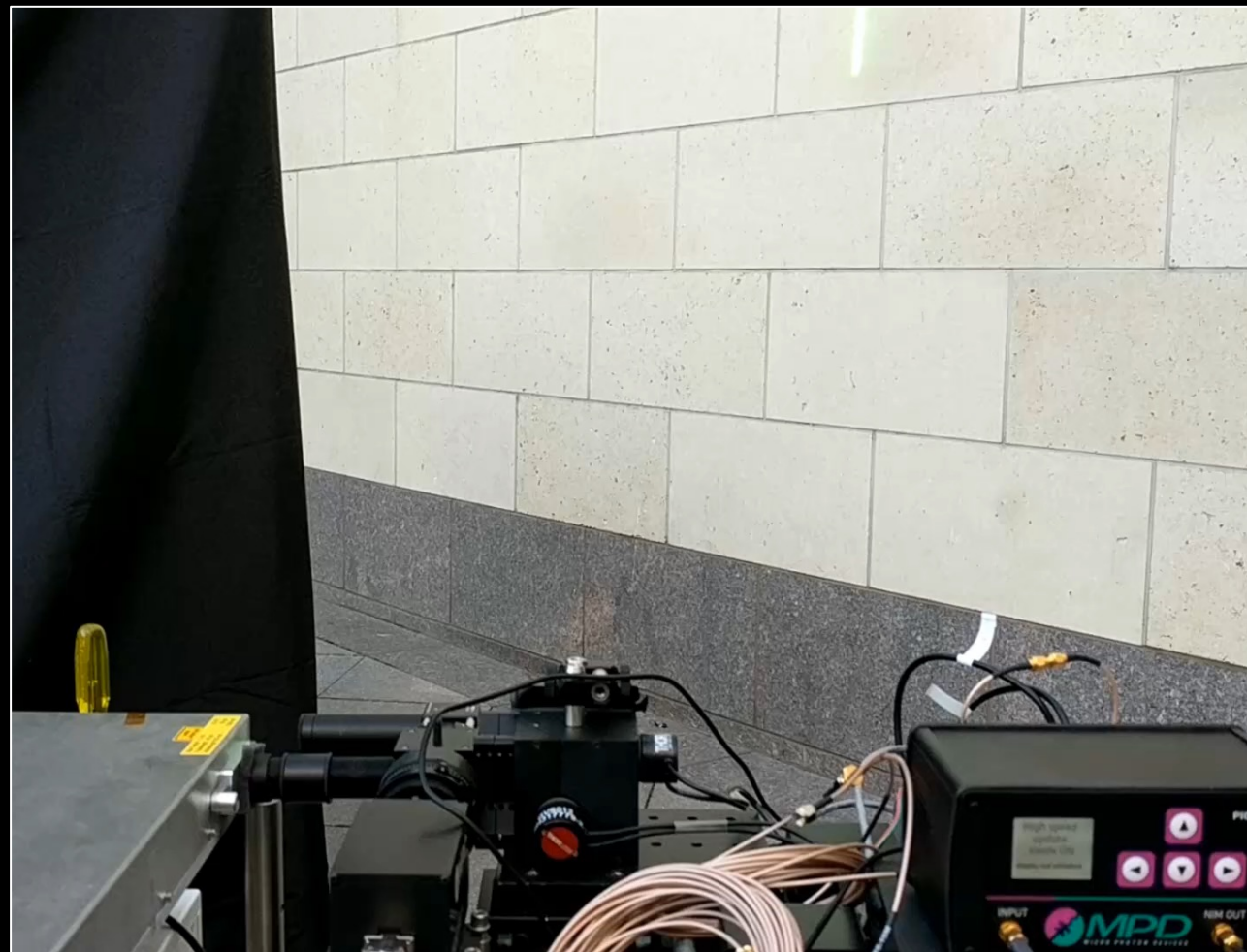
regular image



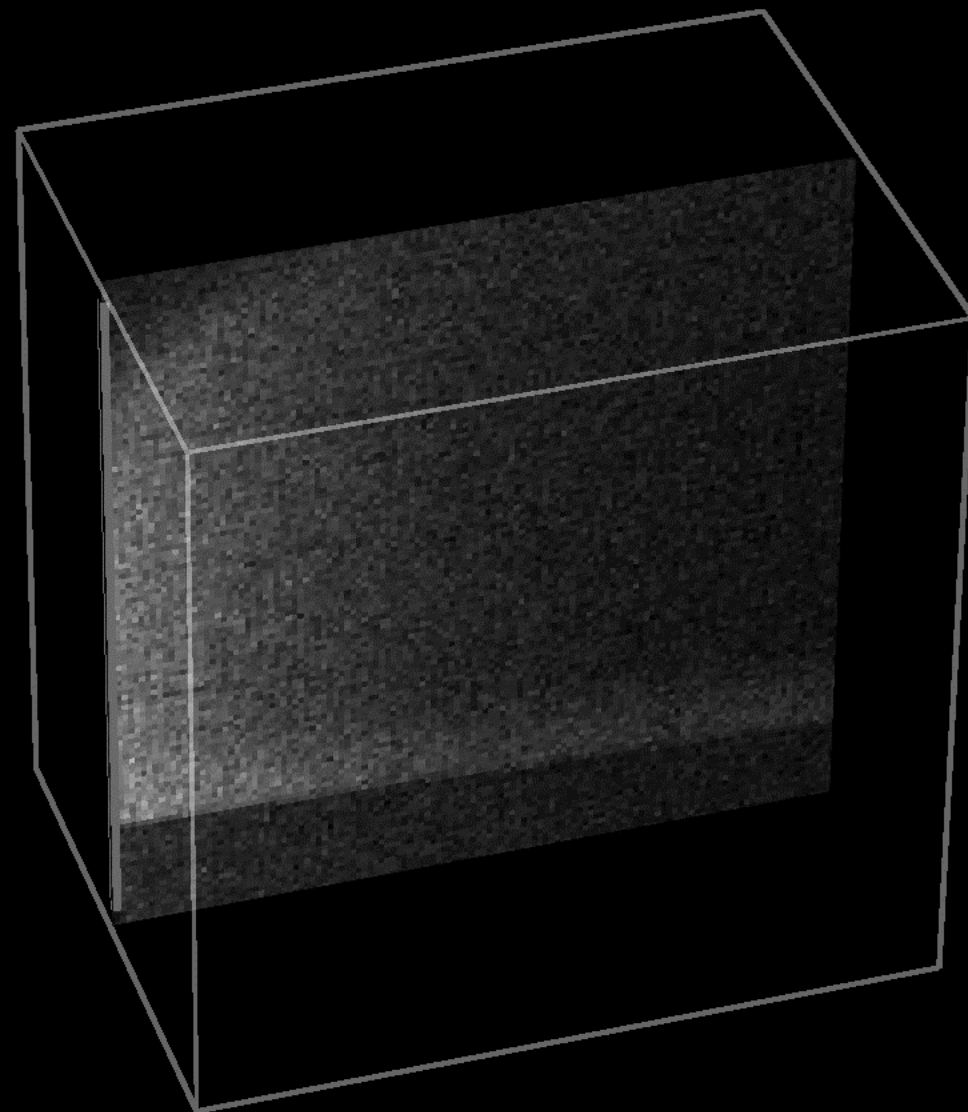
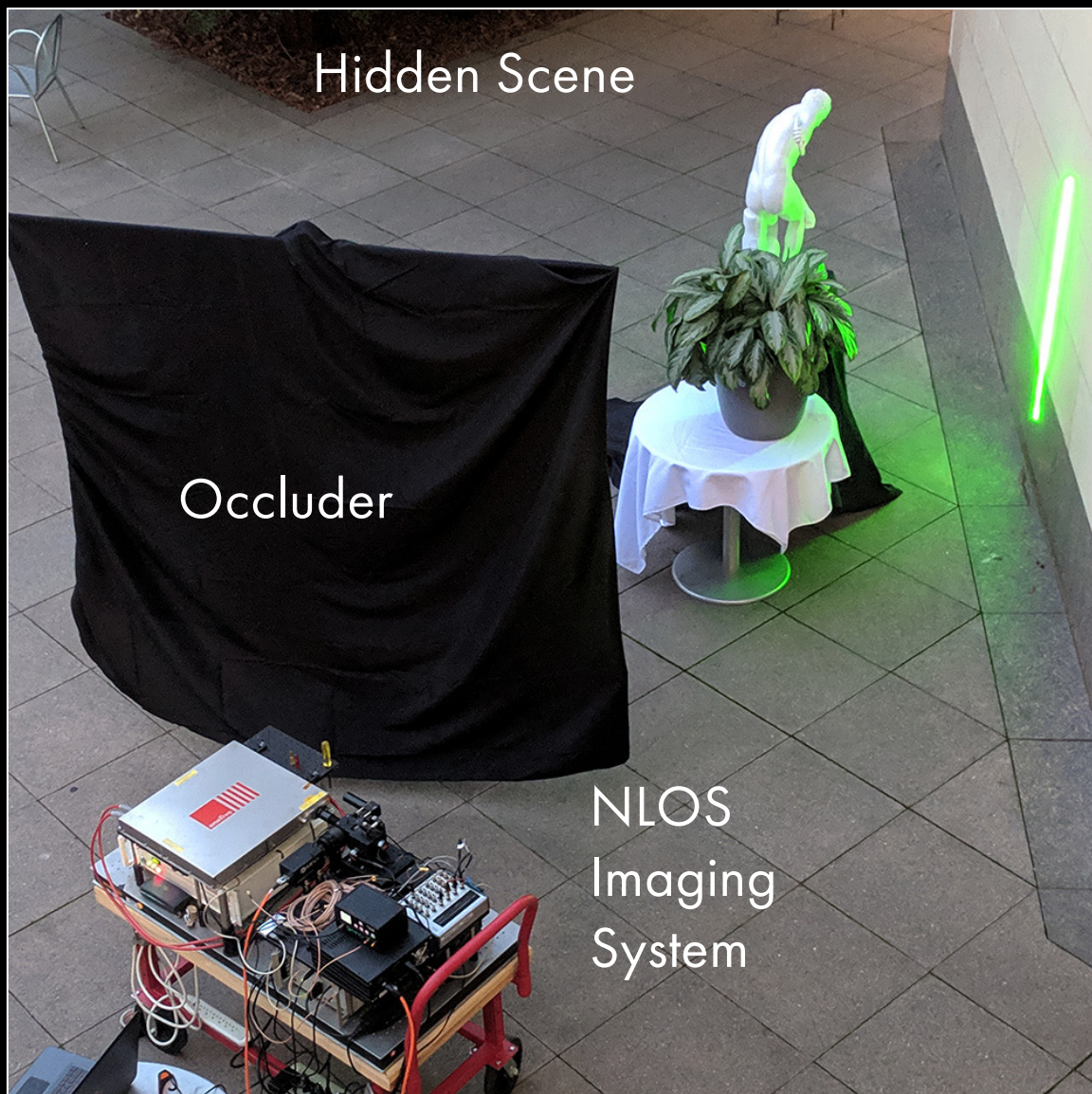
glowing fiber
optic cable



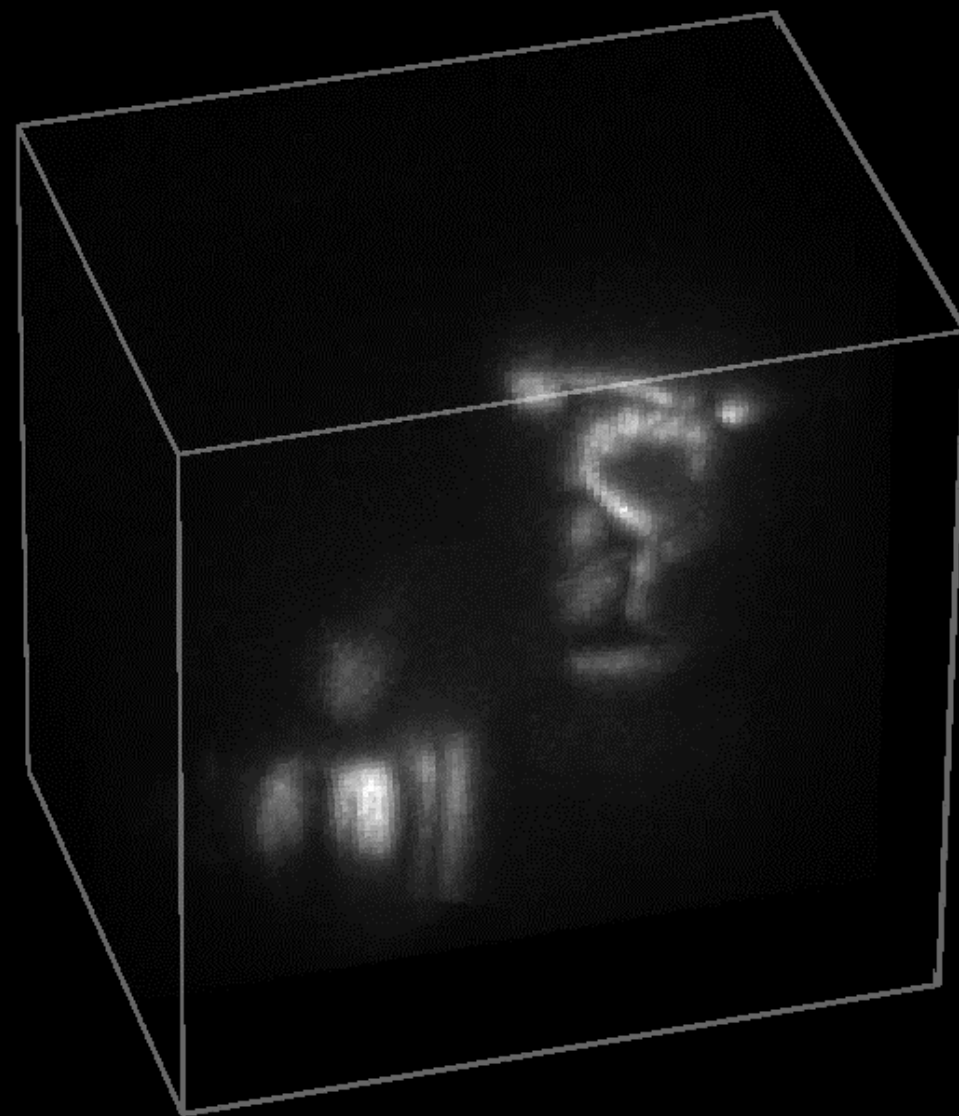
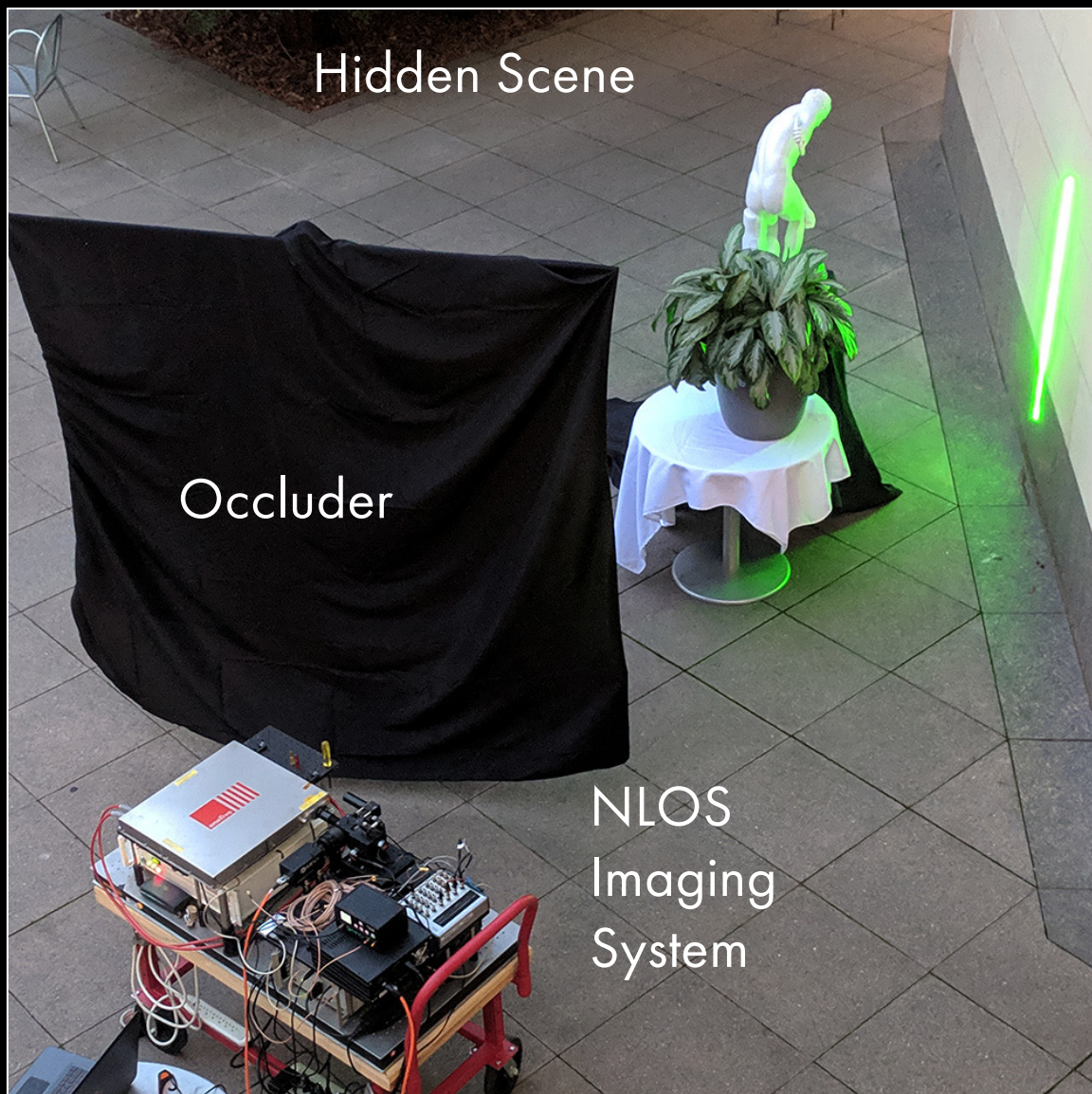
transient image







Time-resolved Measurements



3D Reconstruction

Why study Computer Vision?

Automatically caption images...

A small plane parked in a field with trees in the background.



[Source: L. Zitnick, NIPS'14 Workshop on Learning Semantics]

Why study Computer Vision?

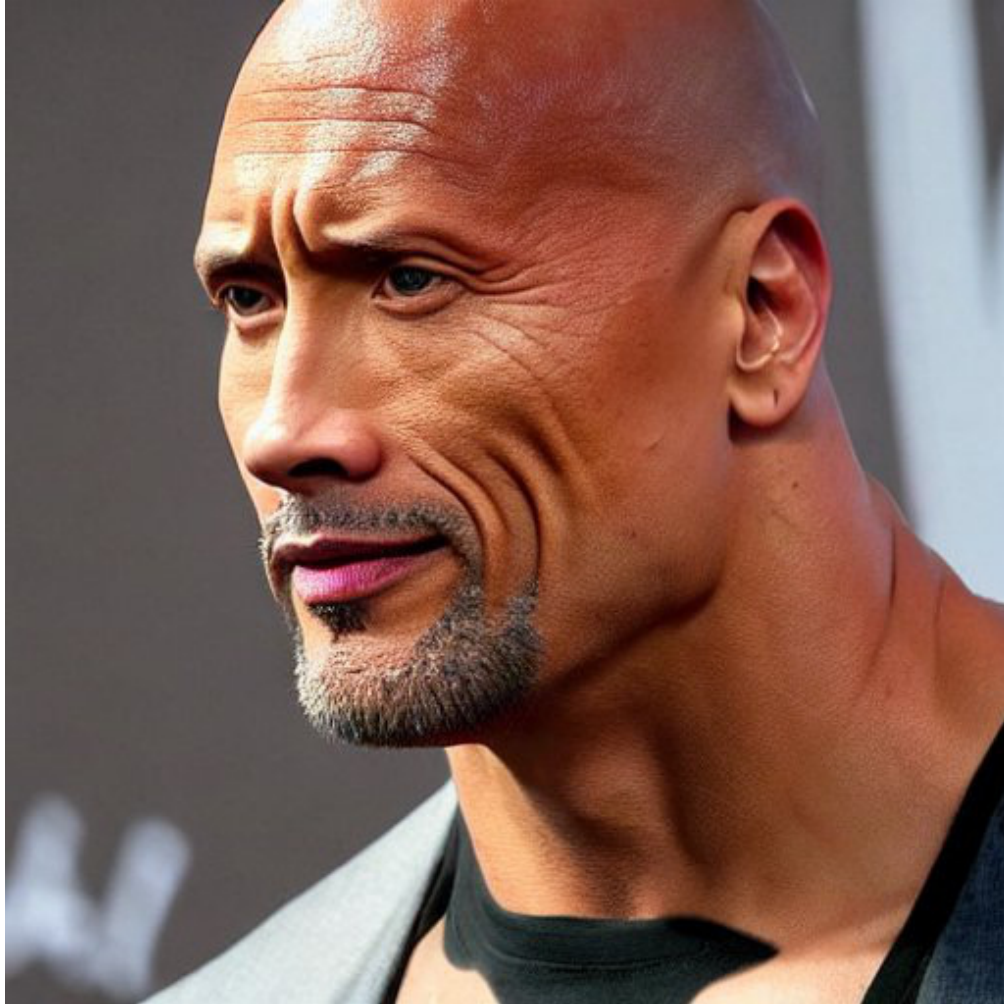
Generate an image from a caption (stable diffusion)



"Dwayne Johnson side view"

Why study Computer Vision?

Generate an image from a caption (stable diffusion)



"Dwayne Johnson side view"



"Dwayne Johnson top view"

Why study Computer Vision?

Inpainting art...



Generation frame: 1024 x 1024



Why study Computer Vision?

Movie-like image forensics



[Nayar and Nishino, Eyes for Relighting]

[Slide: N. Snavely]

Why study Computer Vision?

Movie-like image forensics



[Nayar and Nishino, Eyes for Relighting]

Why study Computer Vision?

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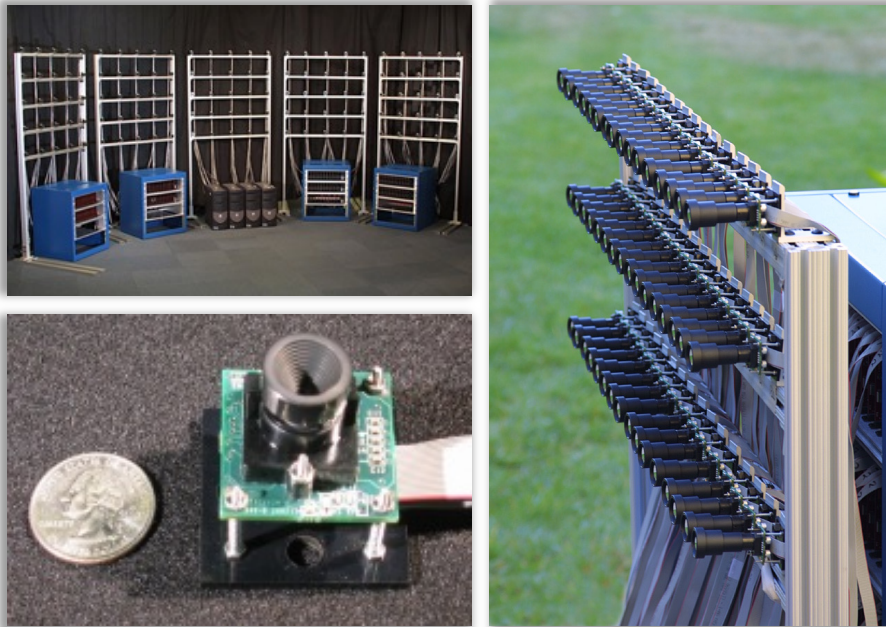
[Nayar and Nishino, Eyes for Relighting]

[Slide: N. Snavely]

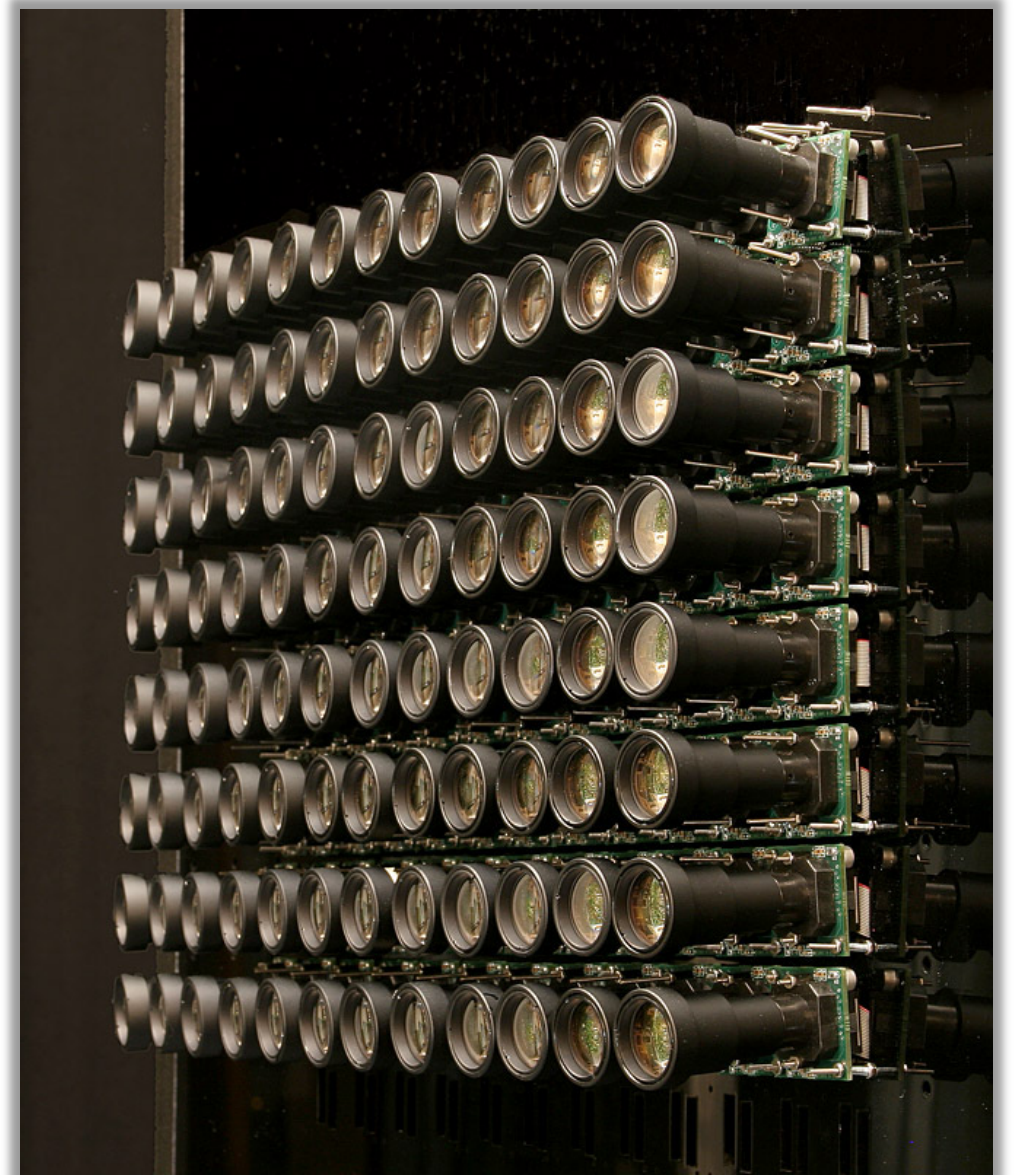
Why study Computer Vision?

Capture light fields

- Stanford Multi-Camera Array



125 cameras using custom hardware
[Wilburn et al. 2002, Wilburn et al. 2005]





How it all began

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

PROJECT MAC

Artificial Intelligence Group
Vision Memo. No. 100.

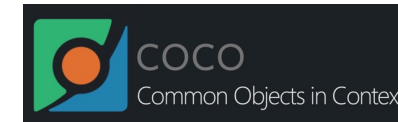
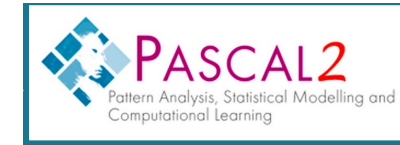
July 7, 1966

THE SUMMER VISION PROJECT

Seymour Papert


The summer vision project is an attempt to use our summer workers effectively in the construction of a significant part of a visual system. The particular task was chosen partly because it can be segmented into sub-problems which will allow individuals to work independently and yet participate in the construction of a system complex enough to be a real landmark in the development of "pattern recognition".

Popular benchmarks:



http://en.wikipedia.org/wiki/List_of_datasets_for_machine_learning_research

Car

	Method	Setting	Code	Moderate	Easy	Hard	Runtime	Environment	Compare
1	DenseBox2			89.32 %	93.94 %	79.81 %	5 s	GPU @ 2.5 Ghz (C/C++)	<input type="checkbox"/>
2	DJML			88.79 %	91.31 %	77.73 %	x s	GPU @ 1.5 Ghz (Matlab + C/C++)	<input type="checkbox"/>
3	3DOP			88.64 %	93.04 %	79.10 %	3s	GPU @ 2.5 Ghz (Matlab + C/C++)	<input type="checkbox"/>

X. Chen, K. Kundu, Y. Zhu, A. Berneshawi, H. Ma, S. Fidler and R. Urtasun: [3D Object Proposals for Accurate Object Class Detection](#). NIPS 2015.

		mean	aero plane	bicycle	bird	boat	bottle	bus	car	cat	chair	cow	dining table	dog	horse	motor bike	person	potted plant	sheep	sofa	train	tv/monitor	submission date
		▼	▼	▼	▼	▼	▼	▼	▼	▼	▼	▼	▼	▼	▼	▼	▼	▼	▼	▼	▼	▼	
►	Fast R-CNN + YOLO ^[?]	70.8	82.7	77.7	74.3	59.1	47.1	78.0	73.1	89.2	49.6	74.3	55.9	87.4	79.8	82.2	75.3	43.1	71.4	67.8	81.9	65.6	05-Jun-2015
►	Fast R-CNN VGG16 extra data ^[?]	68.8	82.0	77.8	71.6	55.3	42.4	77.3	71.7	89.3	44.5	72.1	53.7	87.7	80.0	82.5	72.7	36.6	68.7	65.4	81.1	62.7	18-Apr-2015
►	segDeepM ^[?]	67.2	82.3	75.2	67.1	50.7	49.8	71.1	69.6	88.2	42.5	71.2	50.0	85.7	76.6	81.8	69.3	41.5	71.9	62.2	73.2	64.6	29-Jan-2015
►	BabyLearning ^[?]	63.8	77.7	73.8	62.3	48.8	45.4	67.3	67.0	80.3	41.3	70.8	49.7	79.5	74.7	78.6	64.5	36.0	69.9	55.7	70.4	61.7	12-Nov-2014

- Algorithms work pretty well
- Still some embarrassing mistakes...
- The general vision problem is not yet solved



“panda”

57.7% confidence

+ .007 ×



noise

=



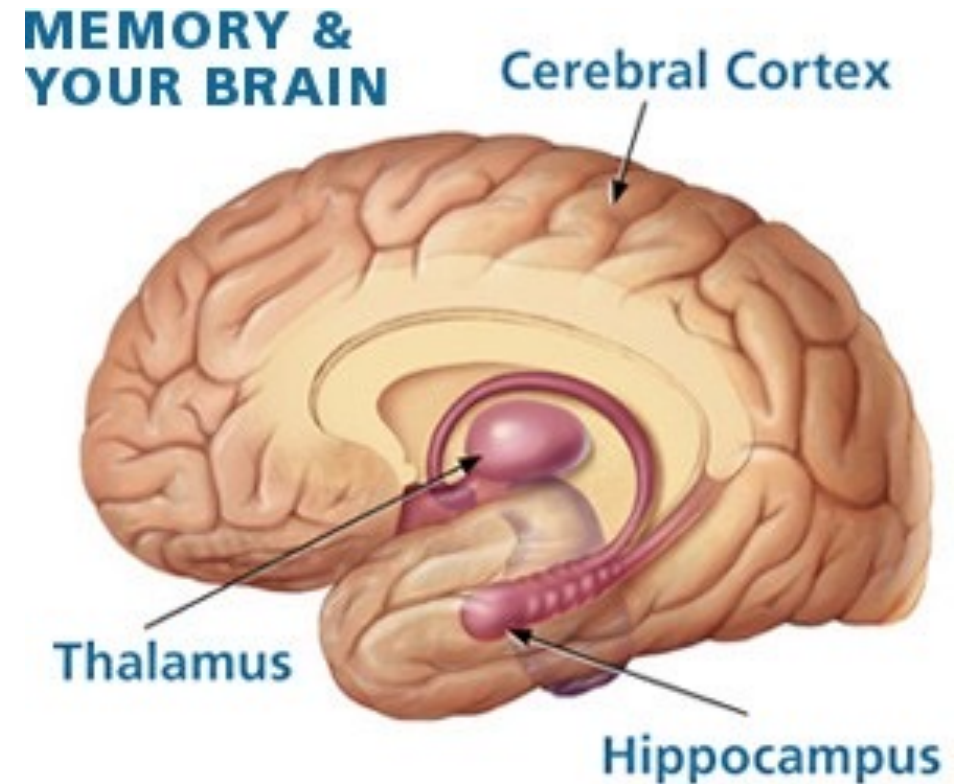
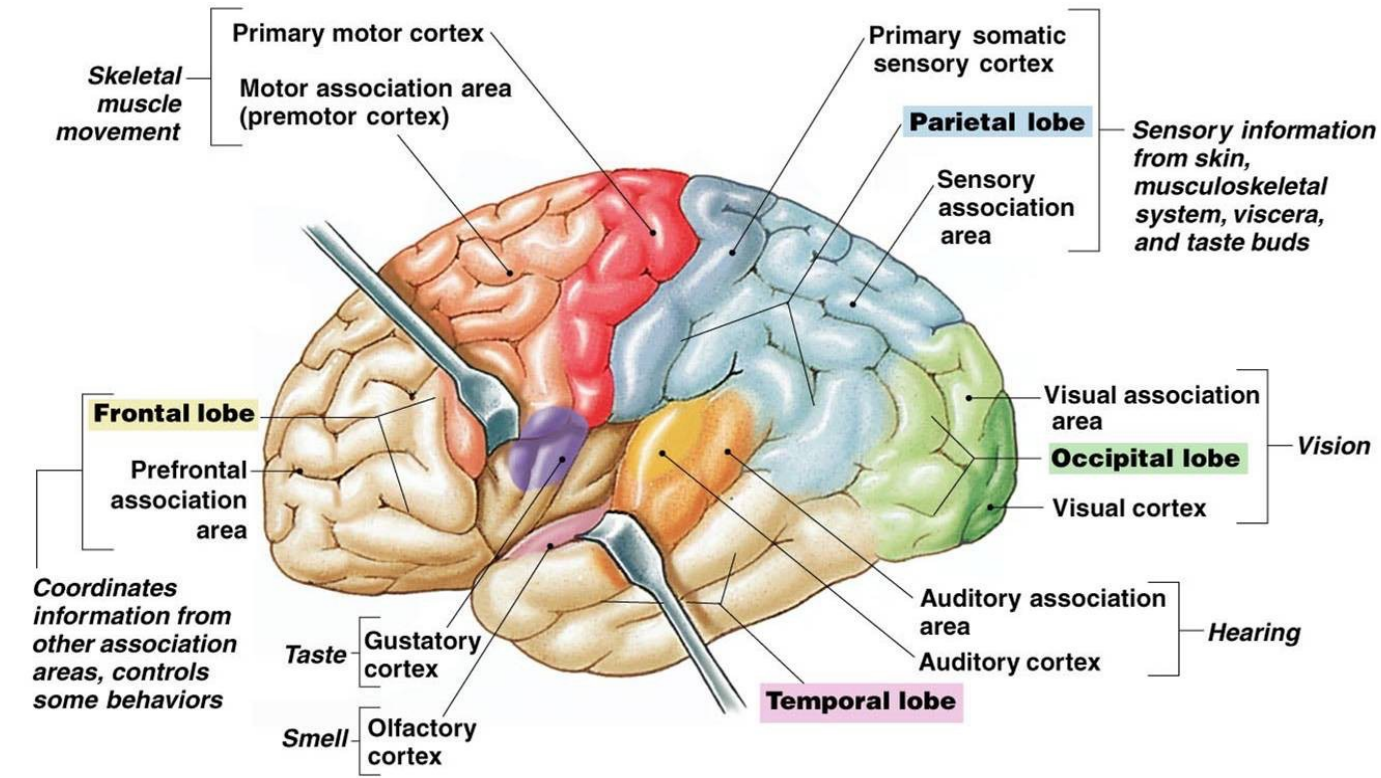
“gibbon”

99.3% confidence

Why is vision hard?

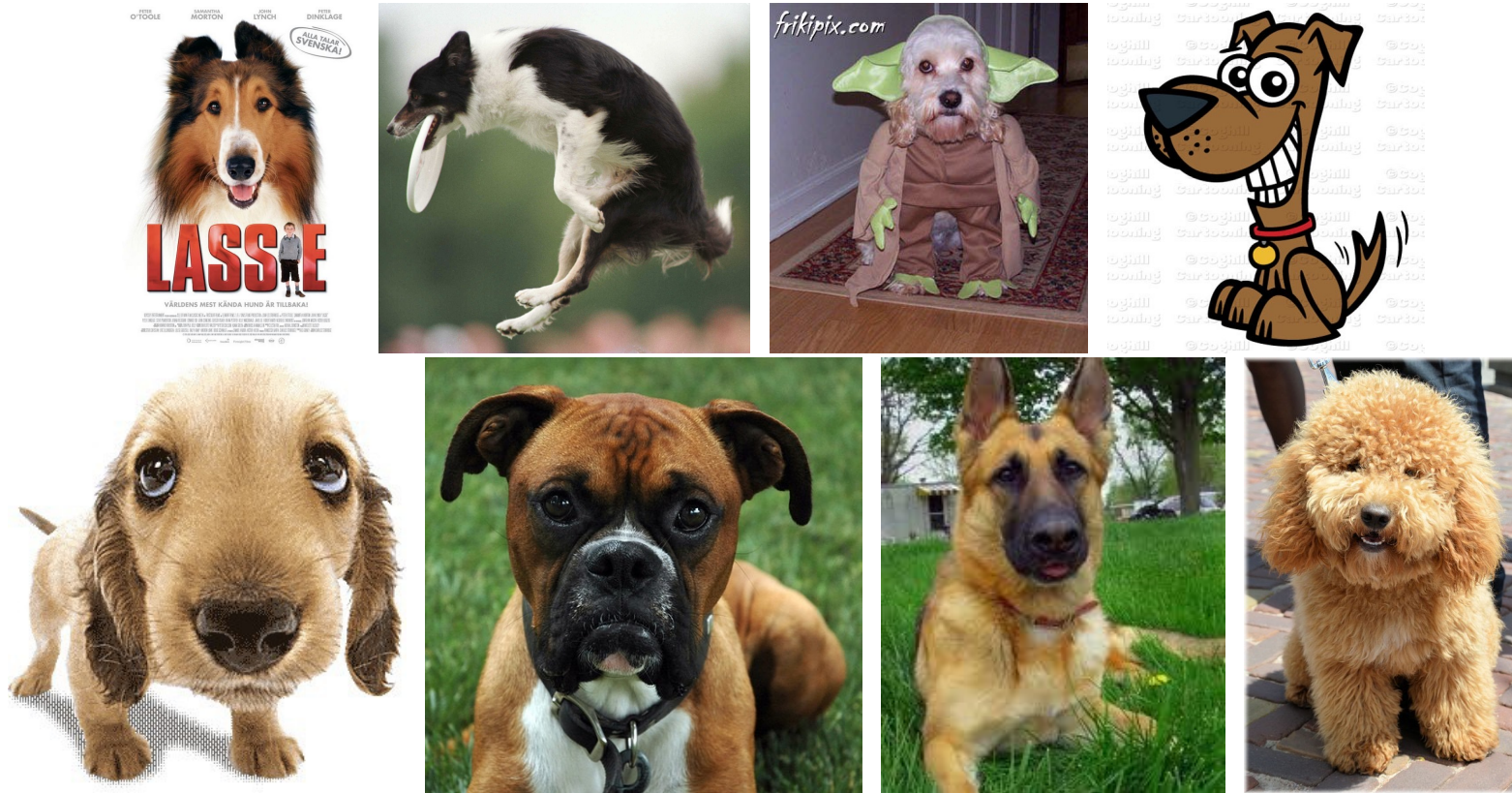
Why is vision hard?

Half of the cerebral cortex in primates is devoted to processing visual information. This is a lot. Means that vision has to be pretty hard!



Why is vision hard?

Visual information is complicated and nuanced...



These are all dogs!

Why is vision hard?



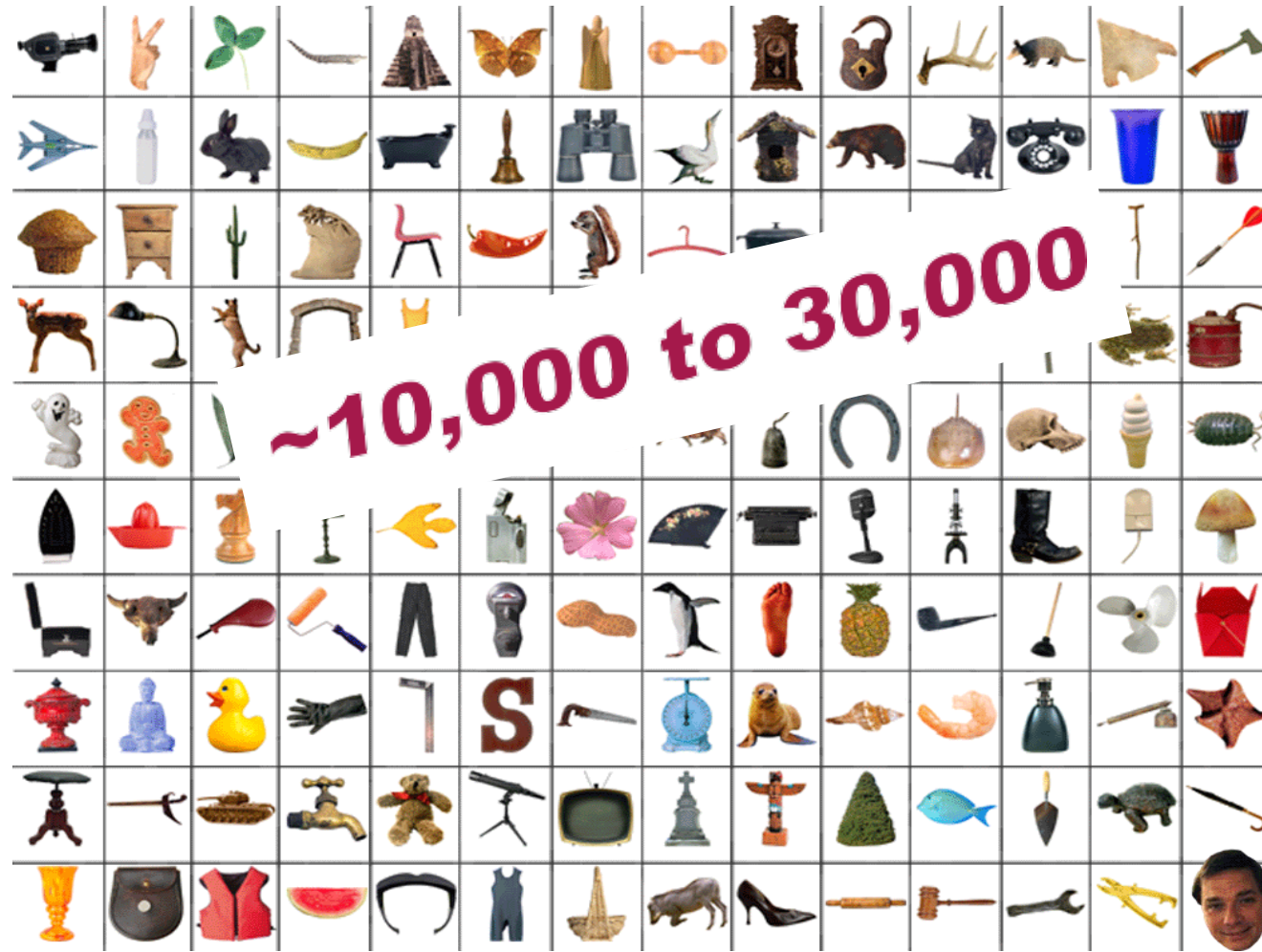
Image: Karen Zack

Why is vision hard?



Image: Karen Zack

Why is vision hard?

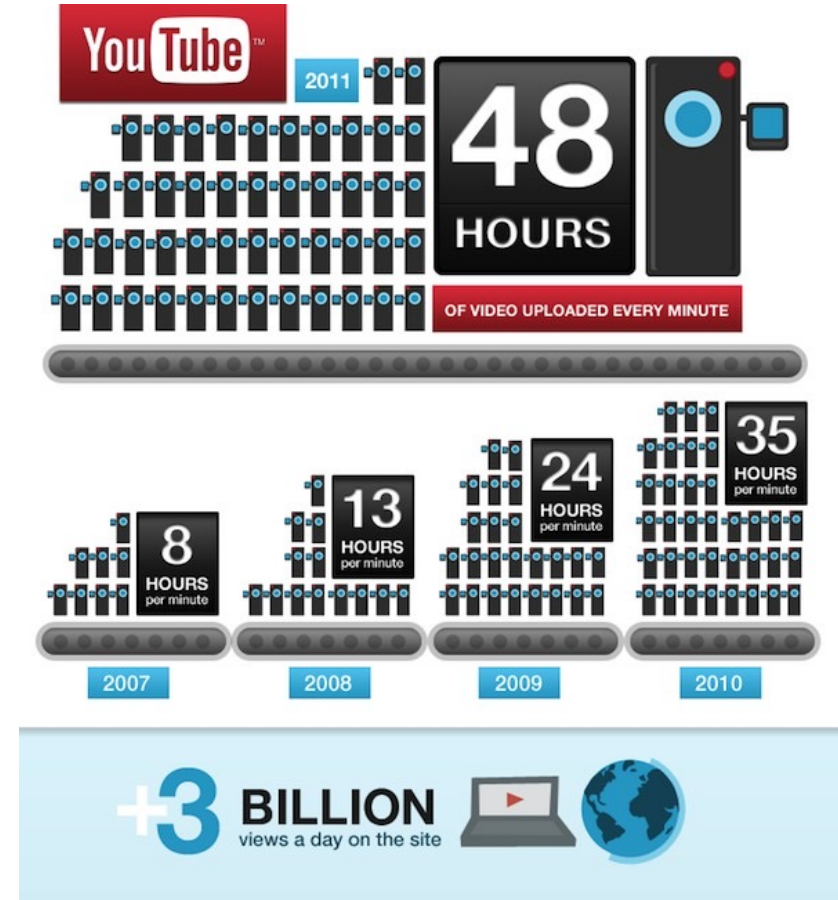


Biederman, 1987

Why is vision hard?

Lots of data to process:

- Thousands to millions of pixels in an image
- 400 hours of video added to YouTube per minute (2022)
- Every day, people watch one billion hours of video on YouTube (2022)
- Much more considering all other platforms



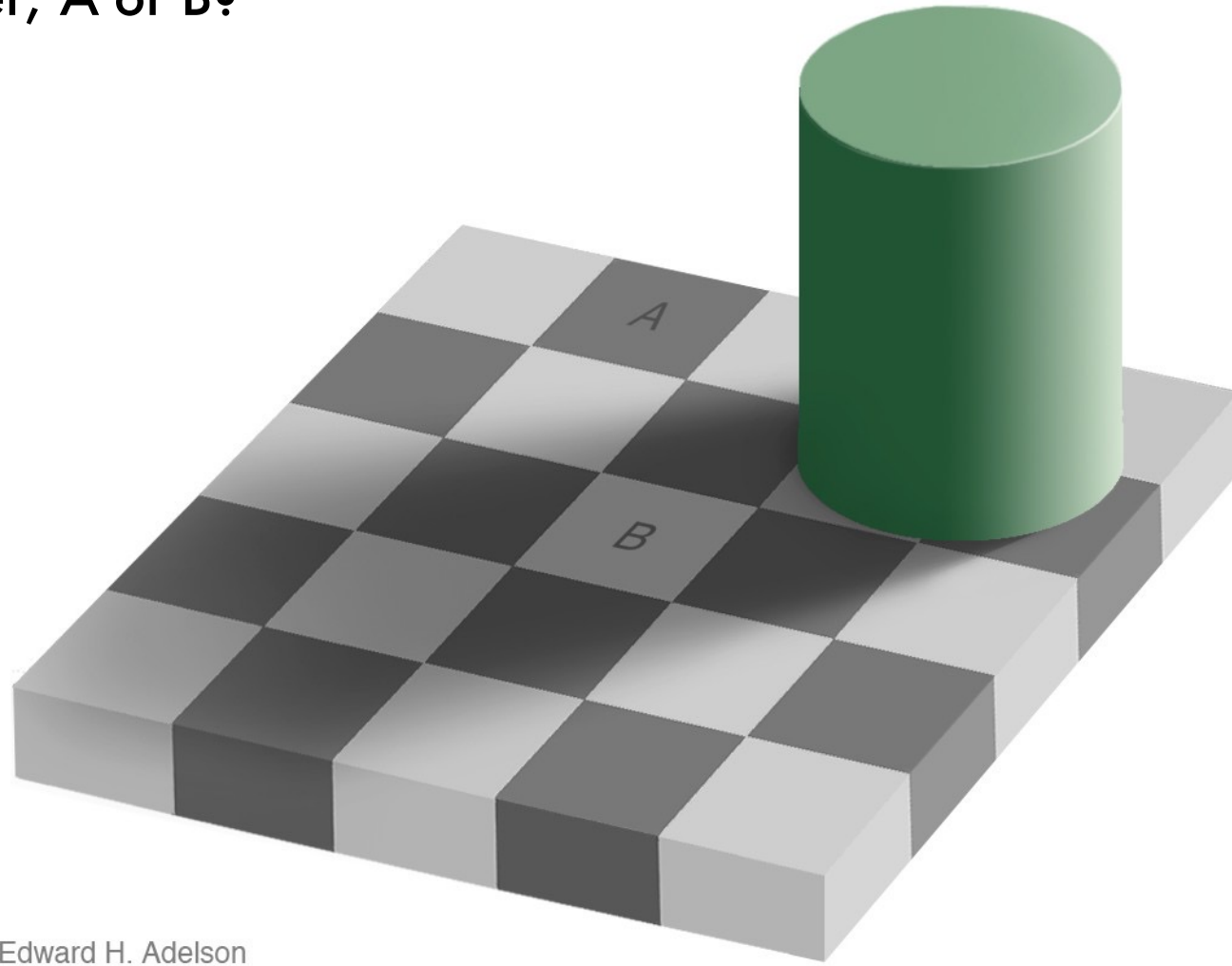
Human vision seems to work quite well.

How well does it really work?

Let's play some games!

How good are humans?

Which square is lighter, A or B?

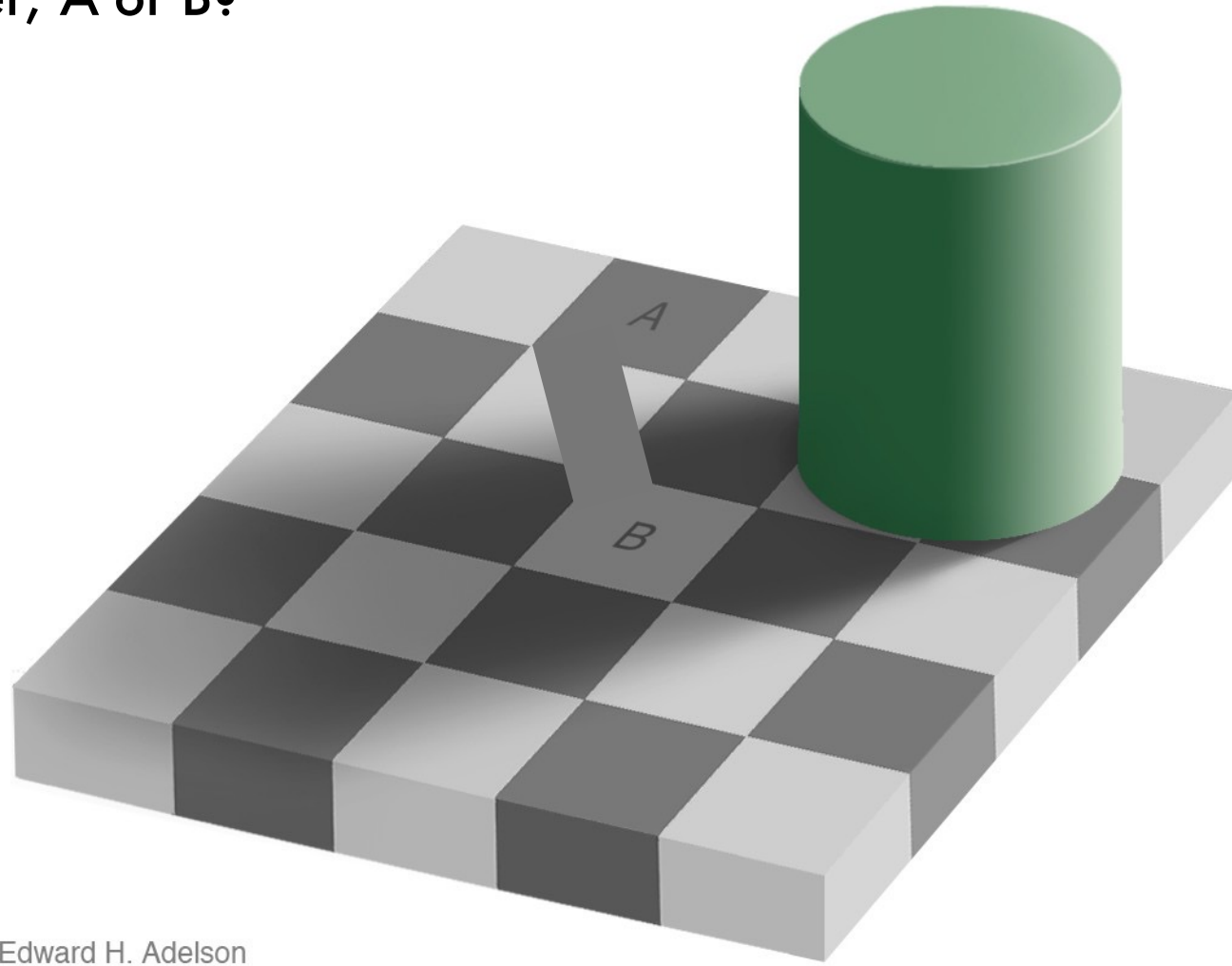


Edward H. Adelson

How good are humans?

Which square is lighter, A or B?

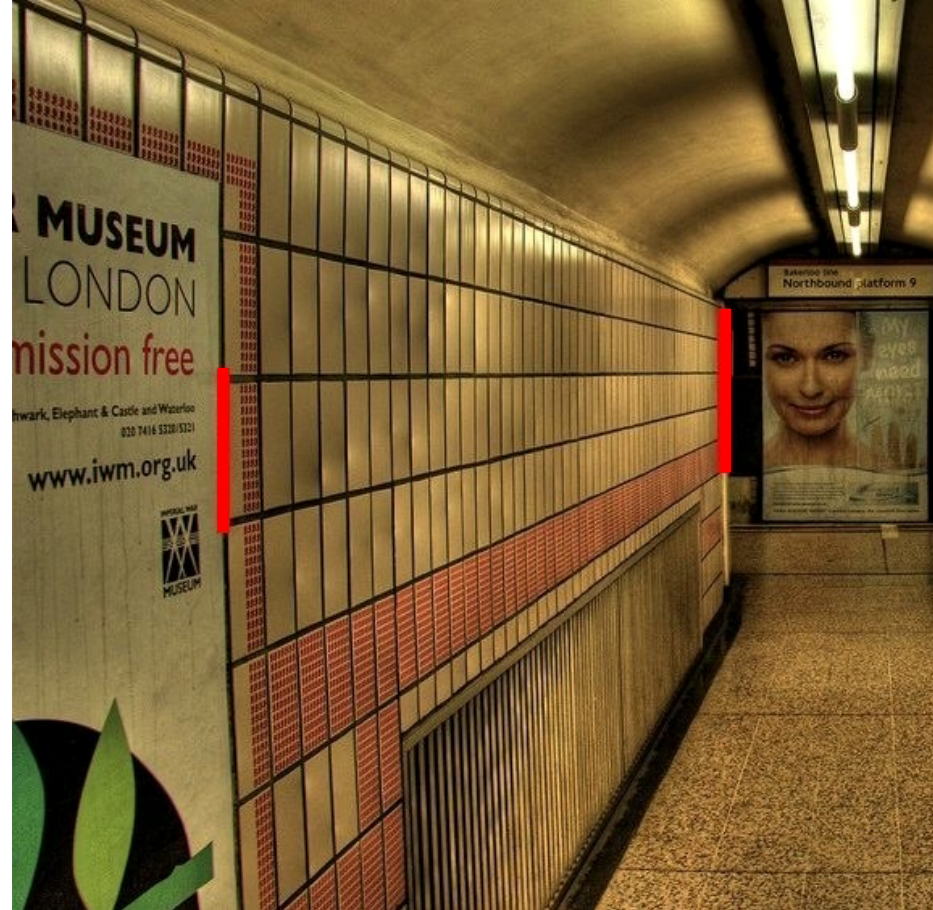
They are the same...



Edward H. Adelson

How good are humans?

Which red line is longer?



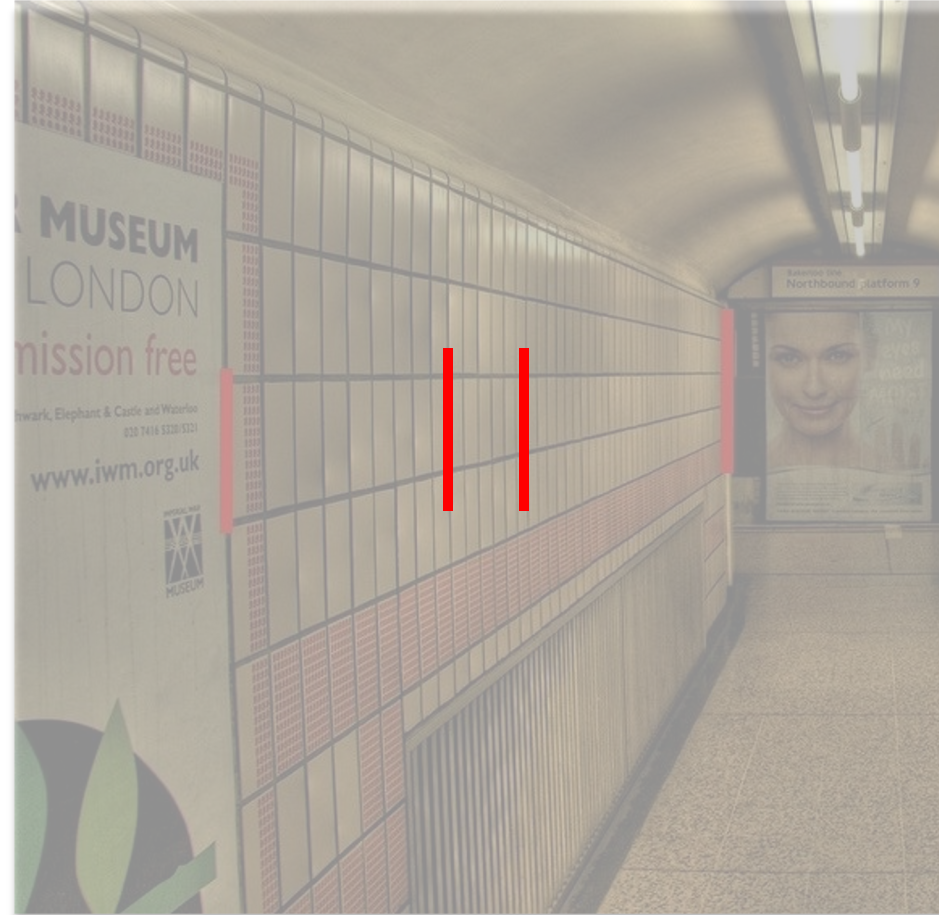
[Walt Anthony 2006]

[Slide: A. Torralba]

How good are humans?

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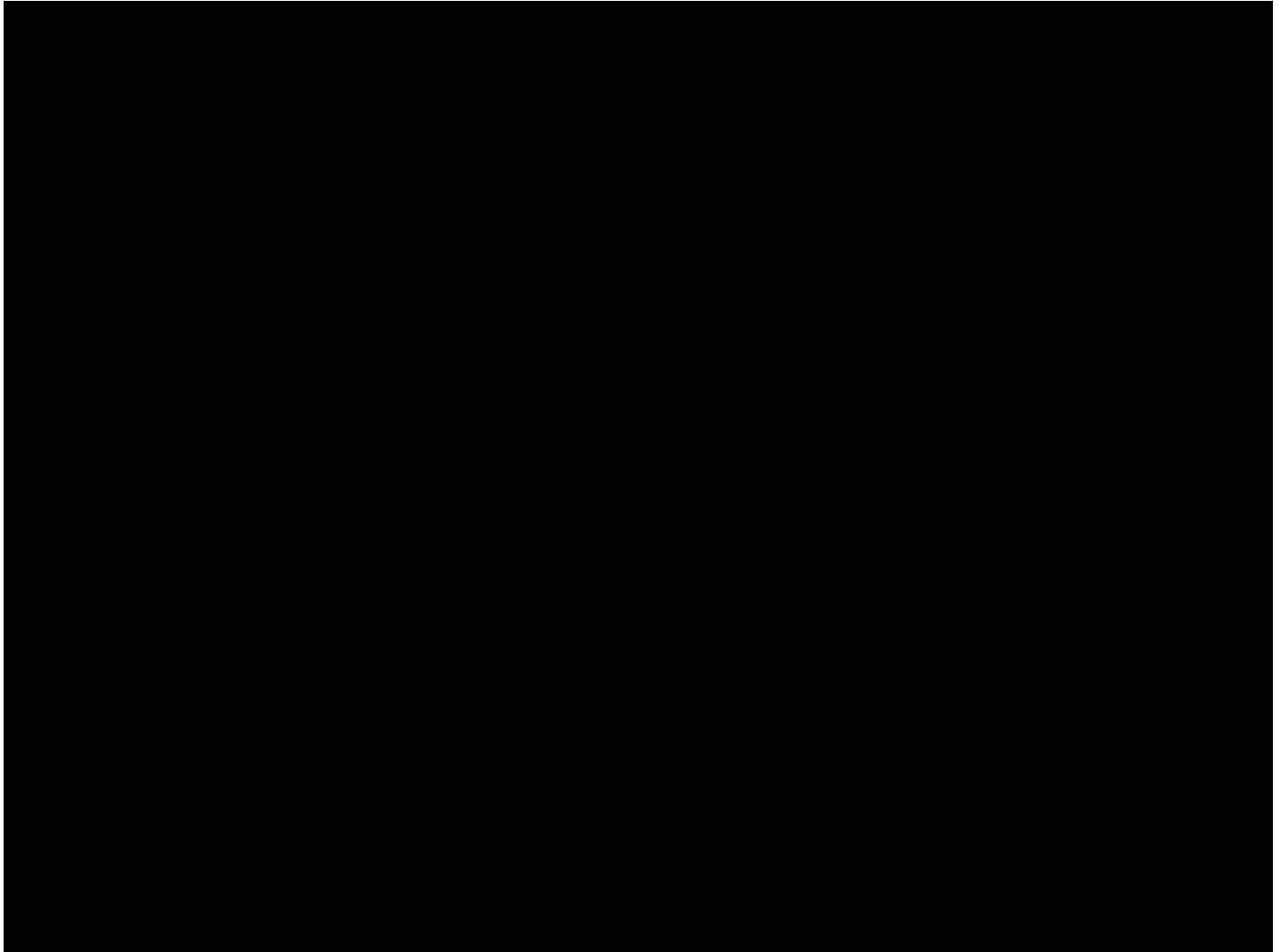
They are the same...



[Walt Anthony 2006]

How good are humans?

- Count the number of times the white team pass the ball
- Concentrate, it's difficult!



[Chabris & Simons]

How good are humans?

Can you describe what this is?



[Torralba et al.]

How good are humans?

Can you describe what this is?



[Torralba et al.]

Humans can tell a lot from a little information...
we have prior knowledge that can (usually) fill in the right information

What do I need to become a good Computer Vision researcher?

- Some math knowledge
- Good programming skills
- Imagination
- Even better intuition
- Lots of persistence
- Some luck always helps

Images

Digital Image

An image is a matrix with (typically) integer values

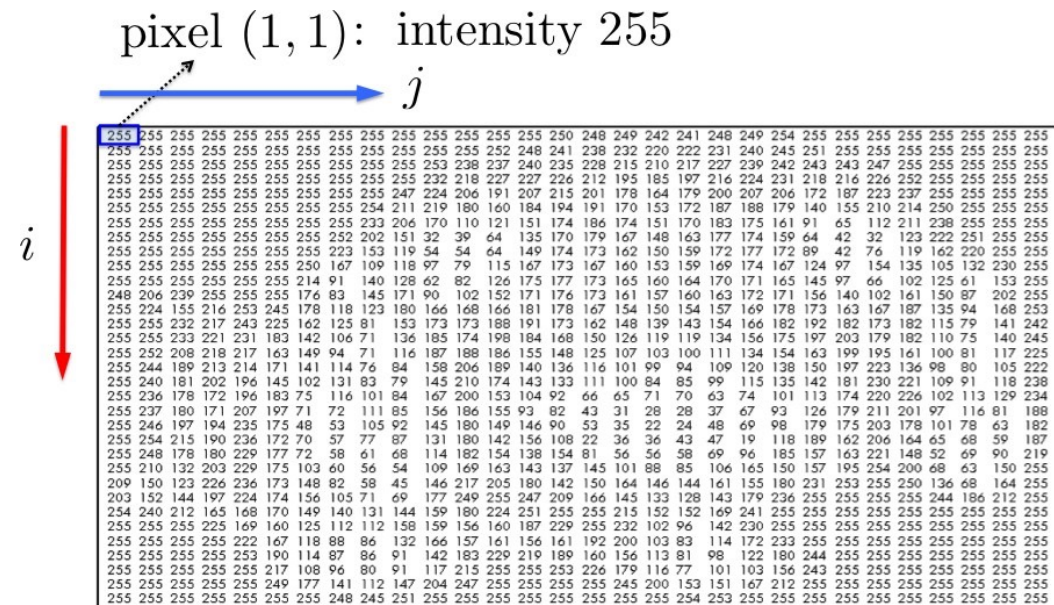
- We will typically denote the image as I
- Pixel values in the image are given by $I(i, j)$, the intensity value at each pixel
- For a grayscale image we have $I \in \mathbb{R}^{m \times n}$, color is $I \in \mathbb{R}^{m \times n \times 3}$

[illegible]

Digital Image

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Digital Image

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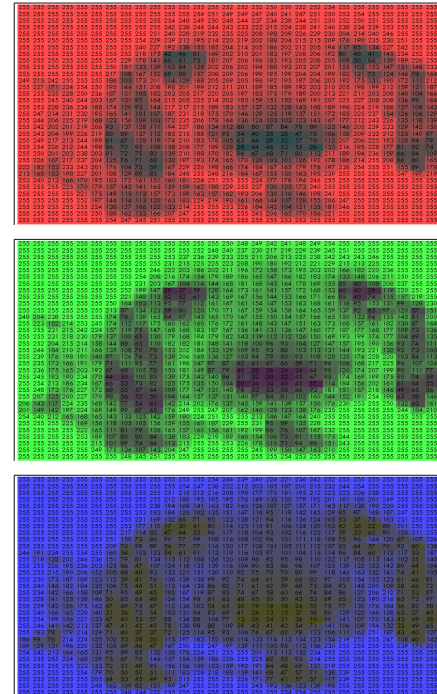
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[illegible]

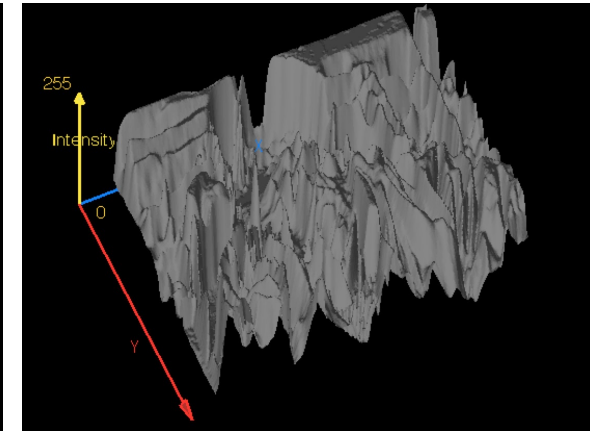
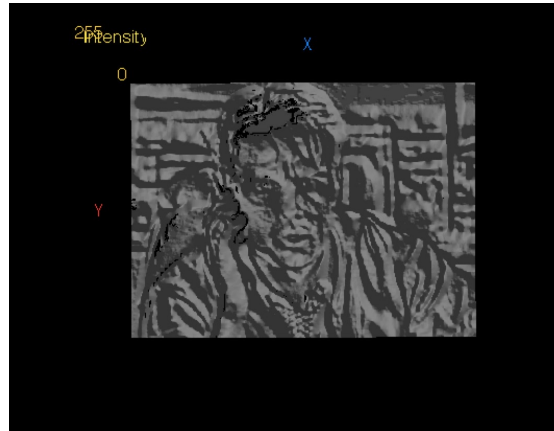
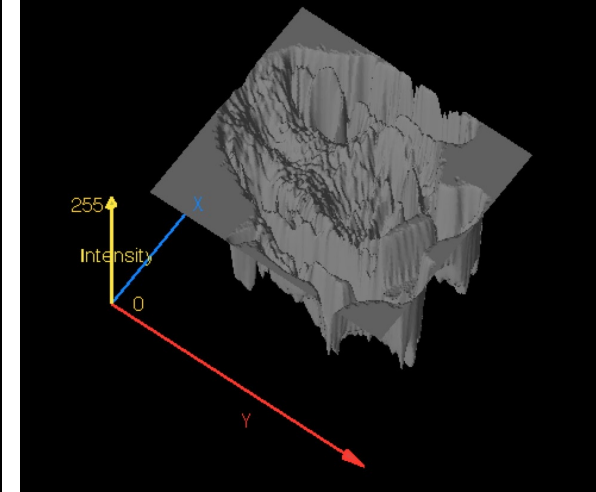
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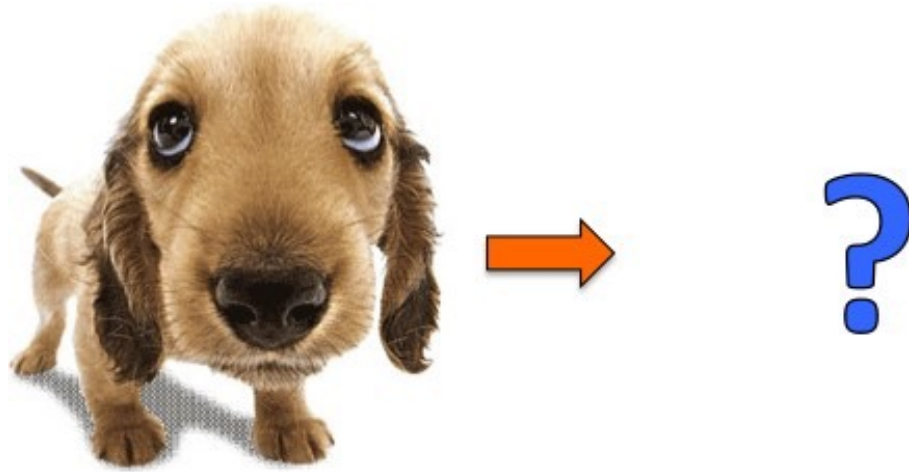
Digital Image



- We can think of a (grayscale) image as a function $f : \mathbb{R}^2 \mapsto \mathbb{R}$ giving the intensity at position (i, j)
- Intensity 0 is black and 255 is white

Digital Image

As with any function, we can apply operators to an image, e.g.:



$$I(i, j)$$

$$J(i, j) = I(i, j) + 50$$

We'll talk about special kinds of operators, correlation and convolution (linear filtering)

Digital Image

As with any function, we can apply operators to an image, e.g.:



$$I(i, j)$$

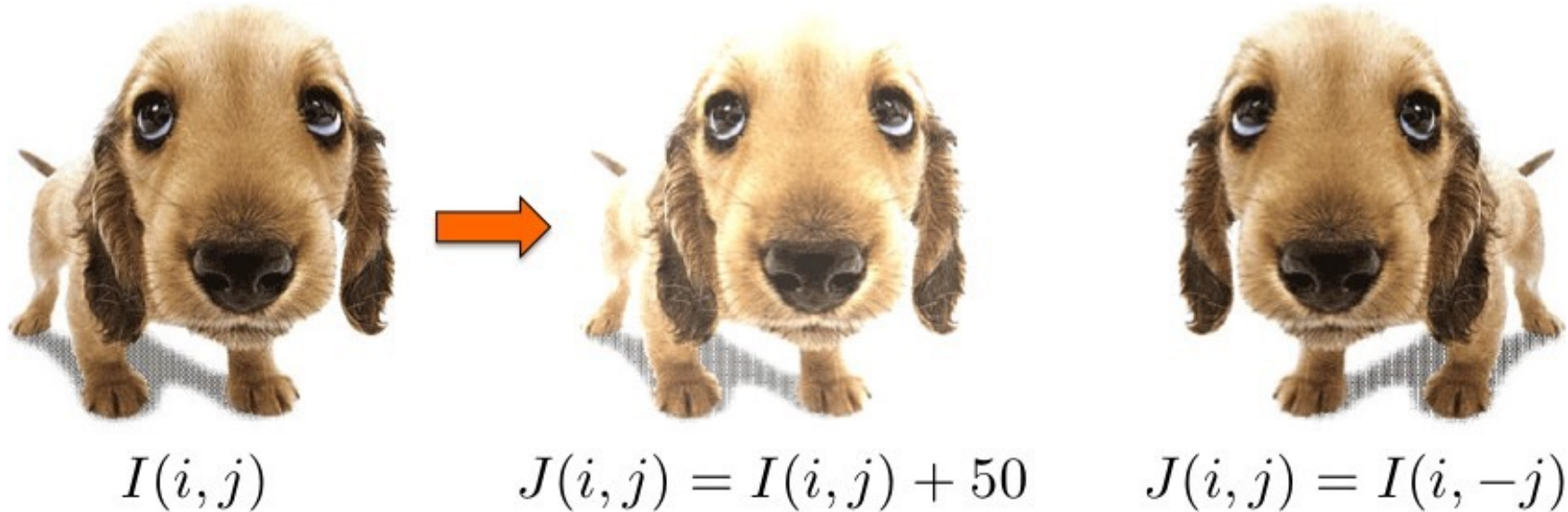
$$J(i, j) = I(i, j) + 50$$

$$J(i, j) = I(i, -j)$$

We'll talk about special kinds of operators, correlation and convolution (linear filtering)

Digital Image

As with any function, we can apply operators to an image, e.g.:



We'll talk about special kinds of operators, correlation and convolution (linear filtering)

Linear Filters

Reading: Szeliski book, Chapter 3.2

Motivation: Finding Waldo

How can we find Waldo?



[Source: R. Urtasun]

Motivation: Finding Waldo

Slide and compare! In formal language: filtering



Image Filtering

- Modify the pixels in an image based on some function of a local neighborhood of each pixel
- In other words, filtering

10	5	3
4	5	1
1	1	7

Local image data



Some function

	7	

Modified image data

Applications of Filtering

- Enhance an image, e.g., denoise.
- Detect patterns, e.g., template matching.
- Extract information, e.g., texture, edges.

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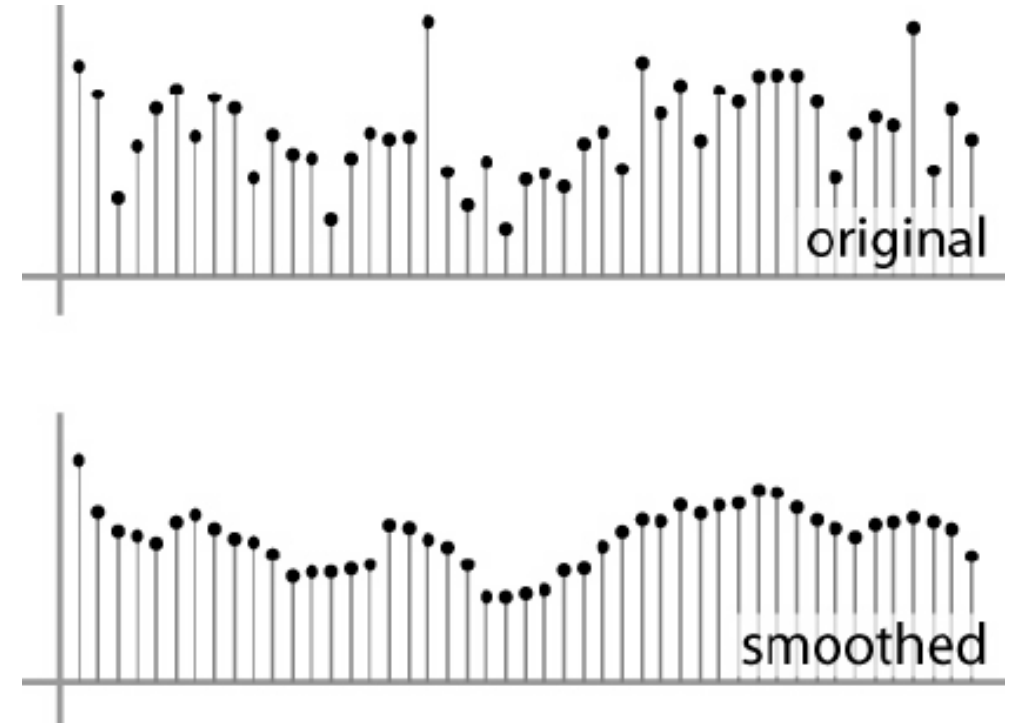
Noise reduction

Given a camera and a still scene, how can you reduce noise?



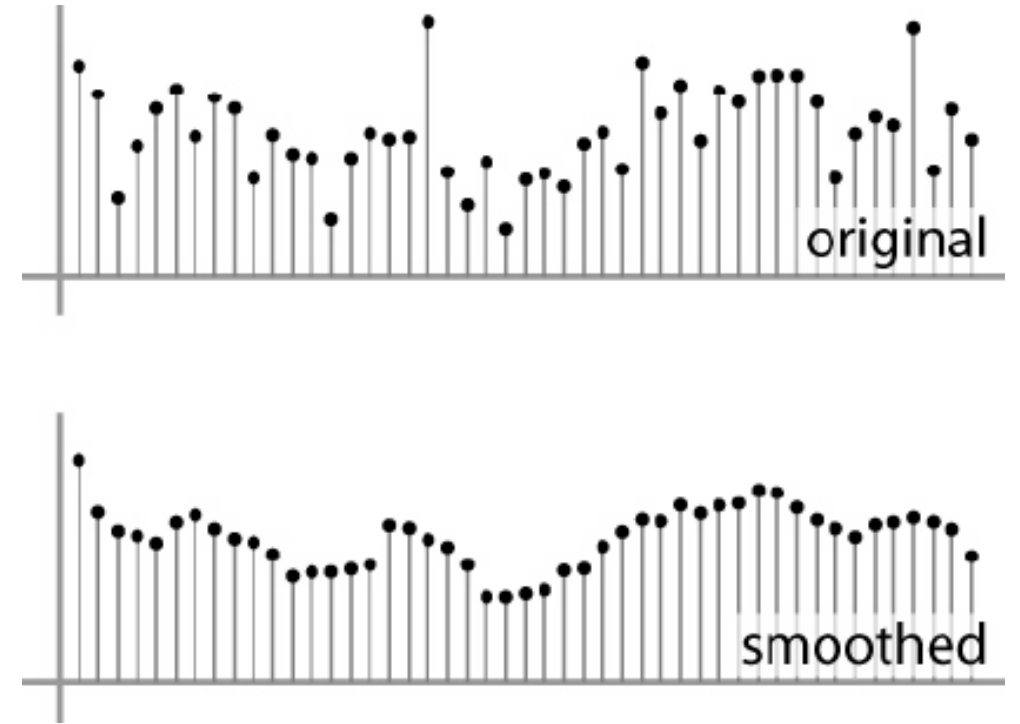
Noise reduction

- Simplest thing: replace each pixel by the average of its neighbors.



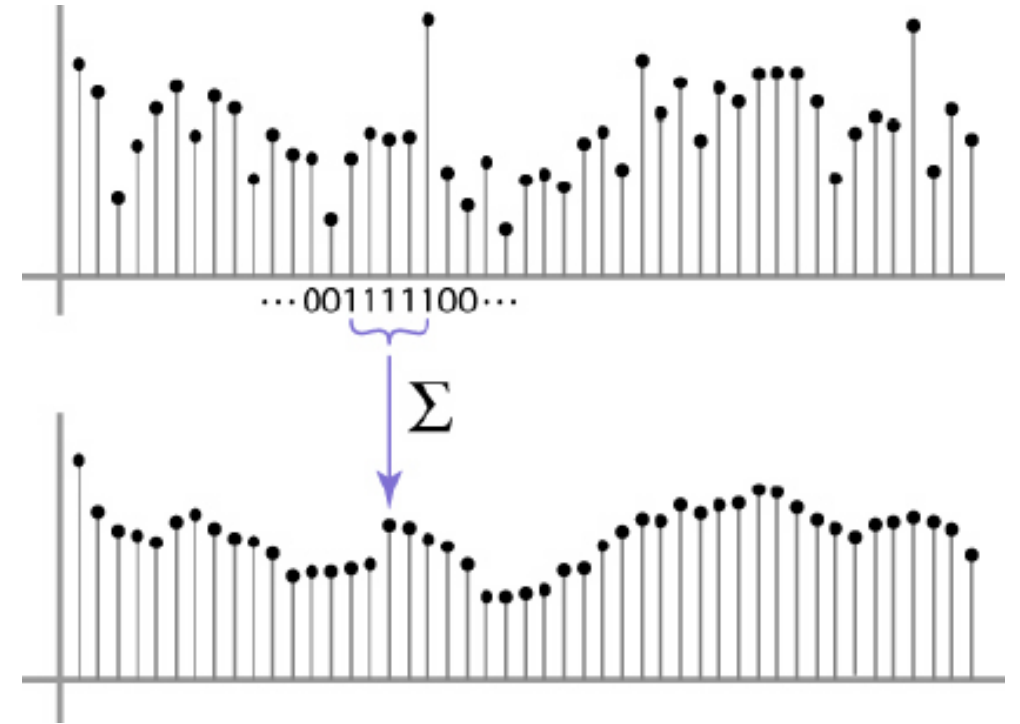
Noise reduction

- Simplest thing: replace each pixel by the average of its neighbors.
- This assumes that neighboring pixels are similar, and the noise to be independent from pixel to pixel.



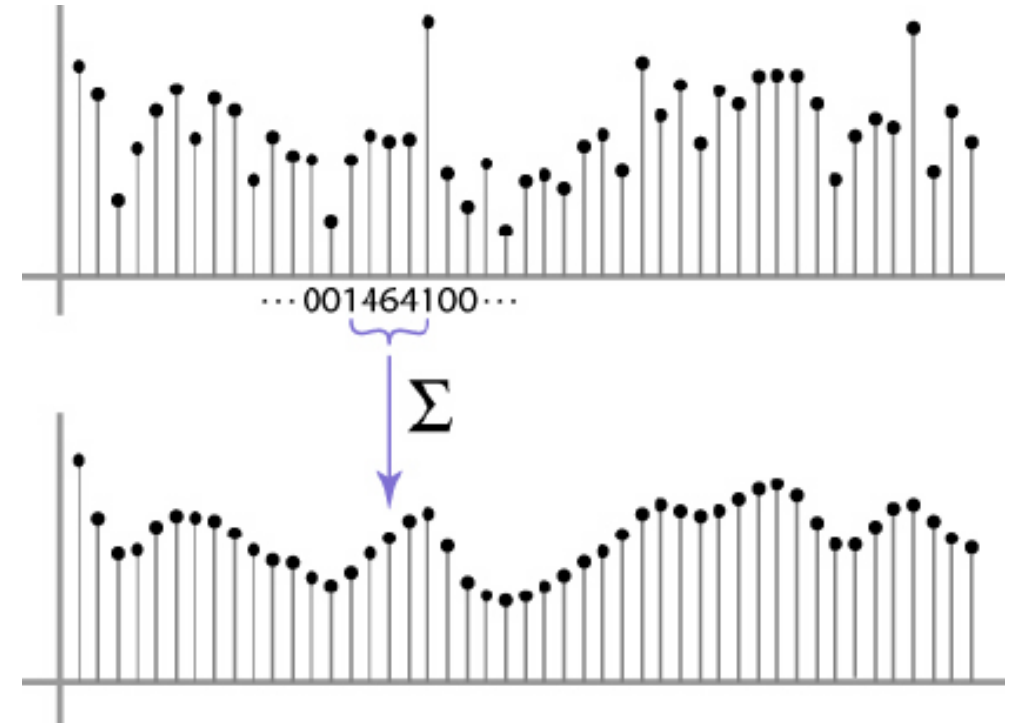
Noise reduction

- Simplest thing: replace each pixel by the average of its neighbors.
- This assumes that neighboring pixels are similar, and the noise to be independent from pixel to pixel.
- Moving average in 1D: $[1, 1, 1, 1, 1]/5$



Noise reduction

- Simplest thing: replace each pixel by the average of its neighbors.
- This assumes that neighboring pixels are similar, and the noise to be independent from pixel to pixel.
- Non-uniform weights $[1, 4, 6, 4, 1] / 16$



Moving average in 2D

$$I(i, j)$$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$G(i, j)$$

	0								

Moving average in 2D

$$I(i, j)$$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$G(i, j)$$

	0	10							

Moving average in 2D

$$I(i, j)$$

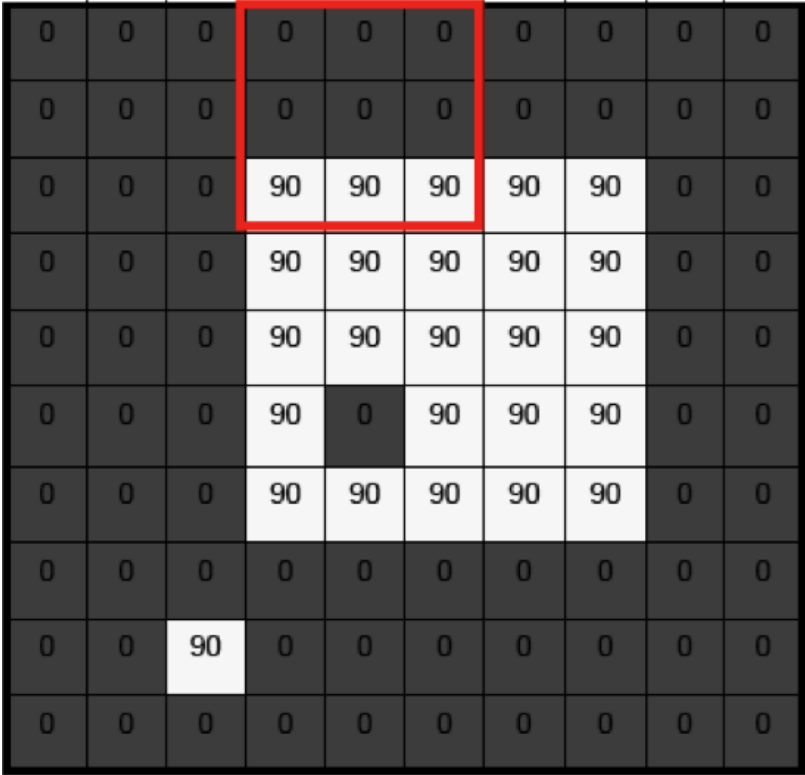
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$G(i, j)$$

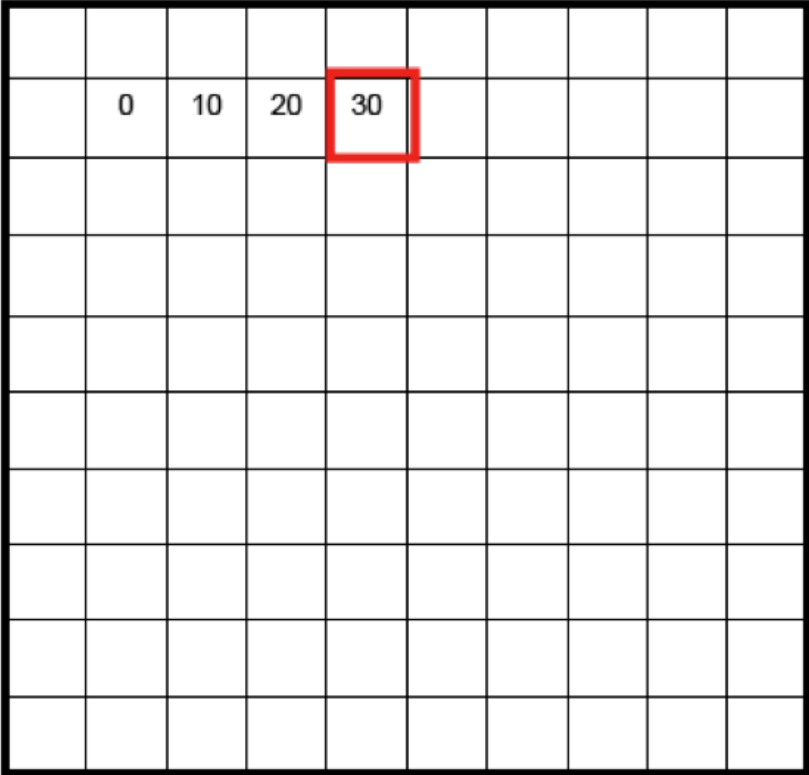
	0	10	20						

Moving average in 2D

$$I(i, j)$$



$$G(i, j)$$



Moving average in 2D

$$I(i, j)$$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$G(i, j)$$

	0	10	20	30	30				

Moving average in 2D

$$I(i, j)$$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$G(i, j)$$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

Linear Filtering: Correlation

Involves weighted combinations of pixels in small neighborhoods (avg. filter):

$$G(i, j) = \frac{1}{(2k+1)^2} \sum_{u=-k}^k \sum_{v=-k}^k I(i+u, j+v)$$

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This operator is called the **correlation operator**

$$G = F \otimes I$$

Linear Filtering: Correlation

j

i

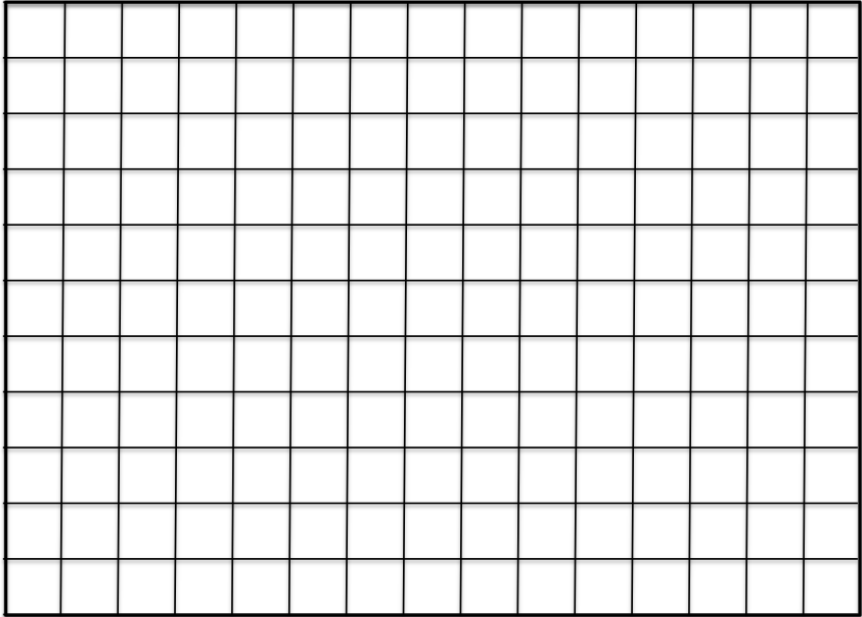
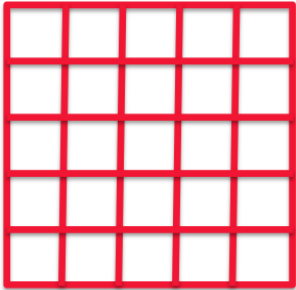
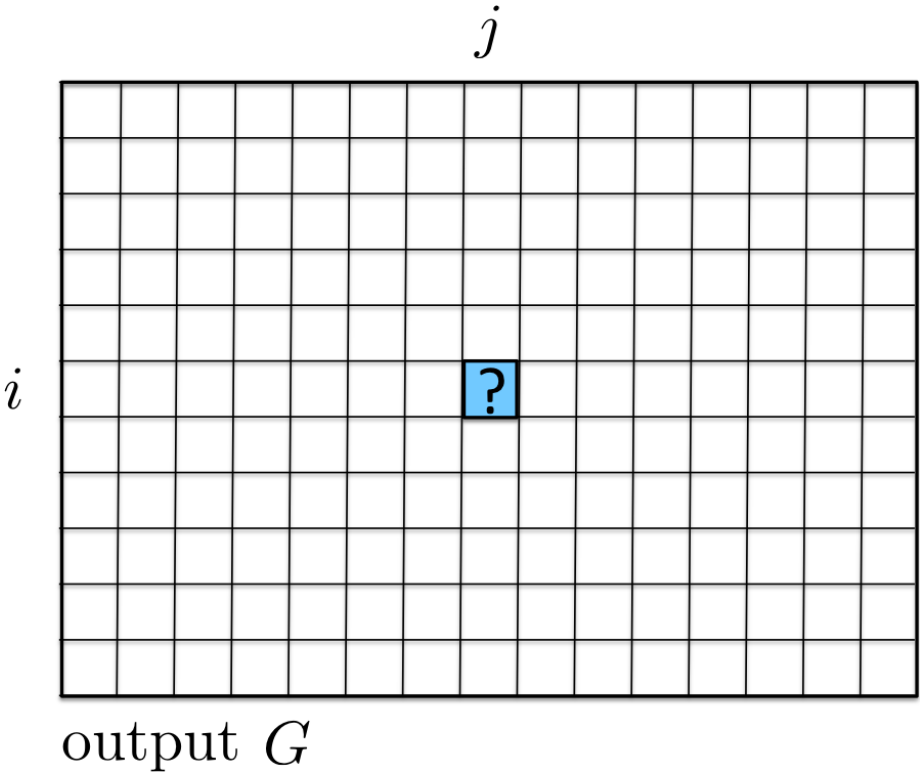
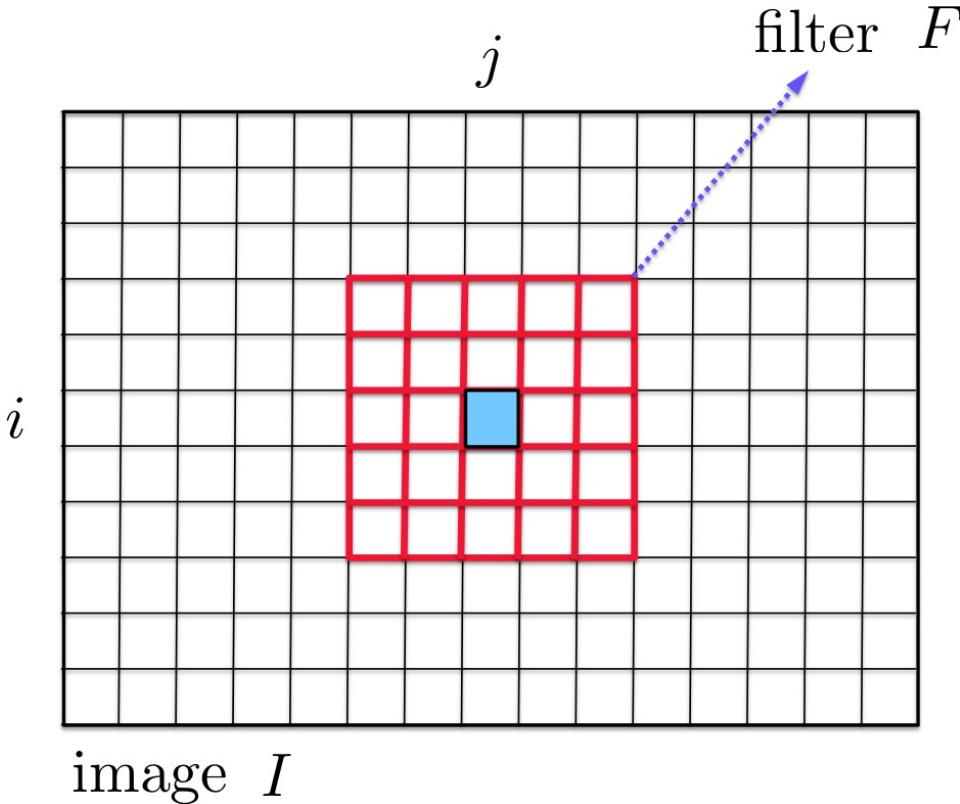


image I

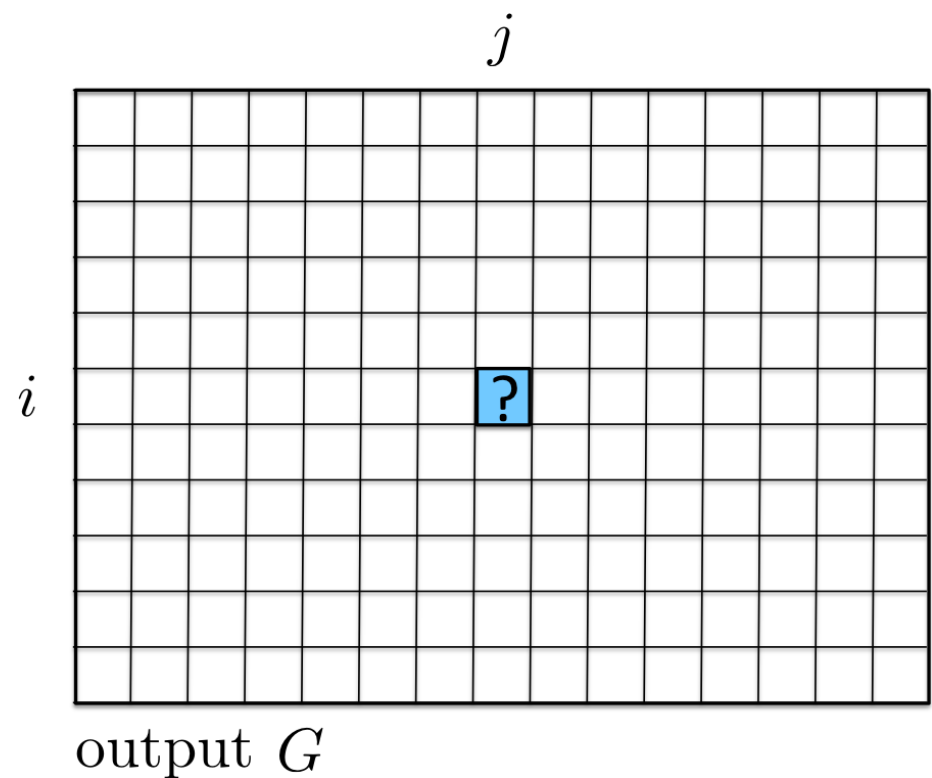
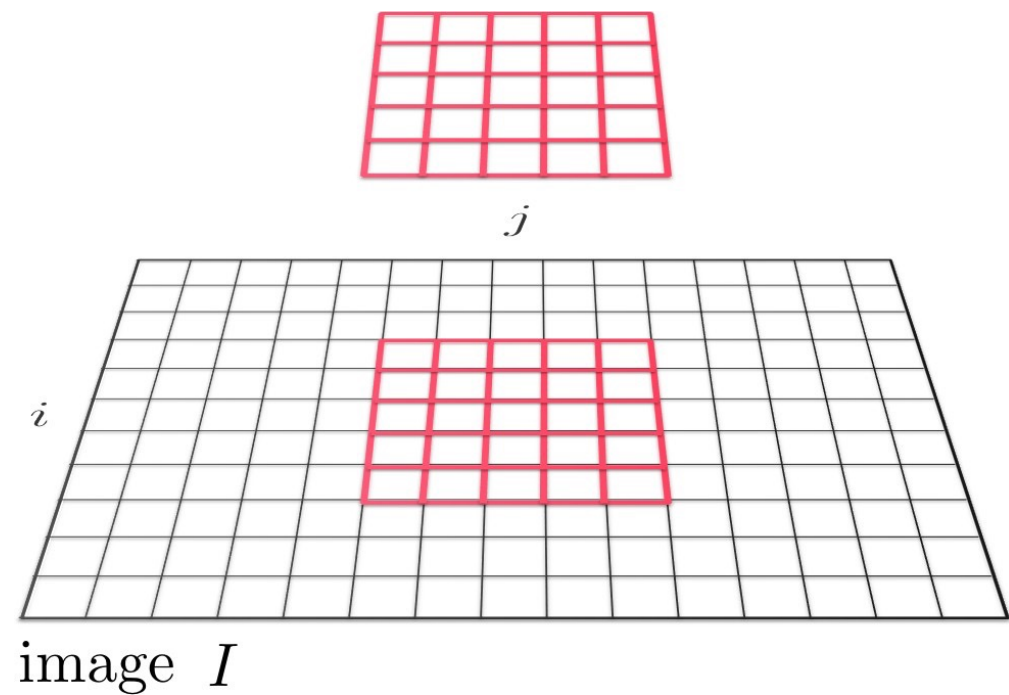


filter F

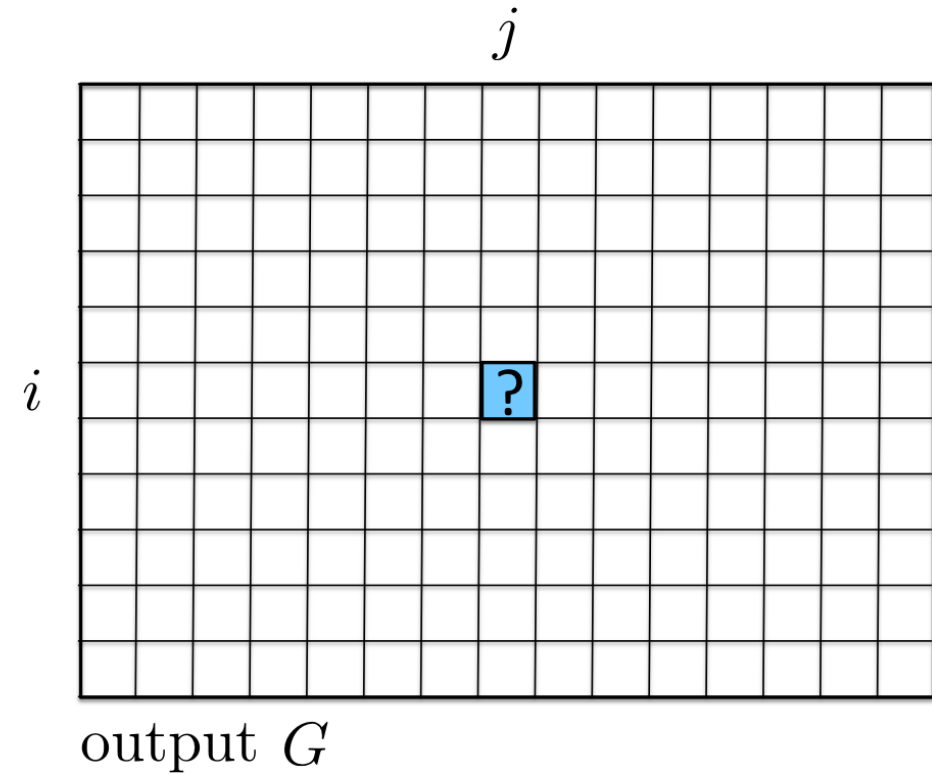
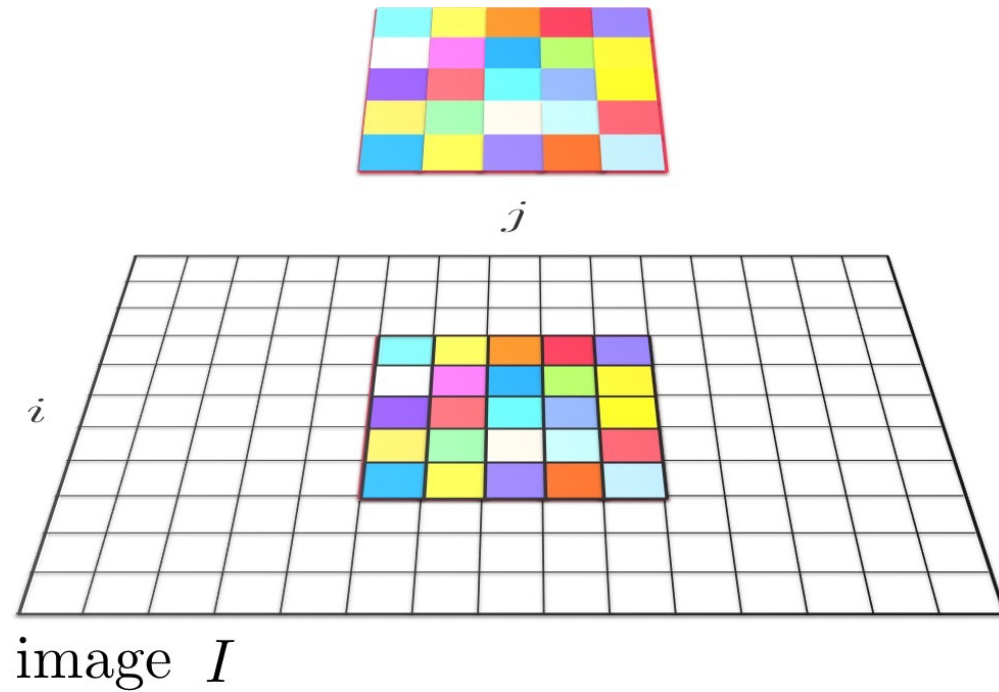
Linear Filtering: Correlation



Linear Filtering: Correlation



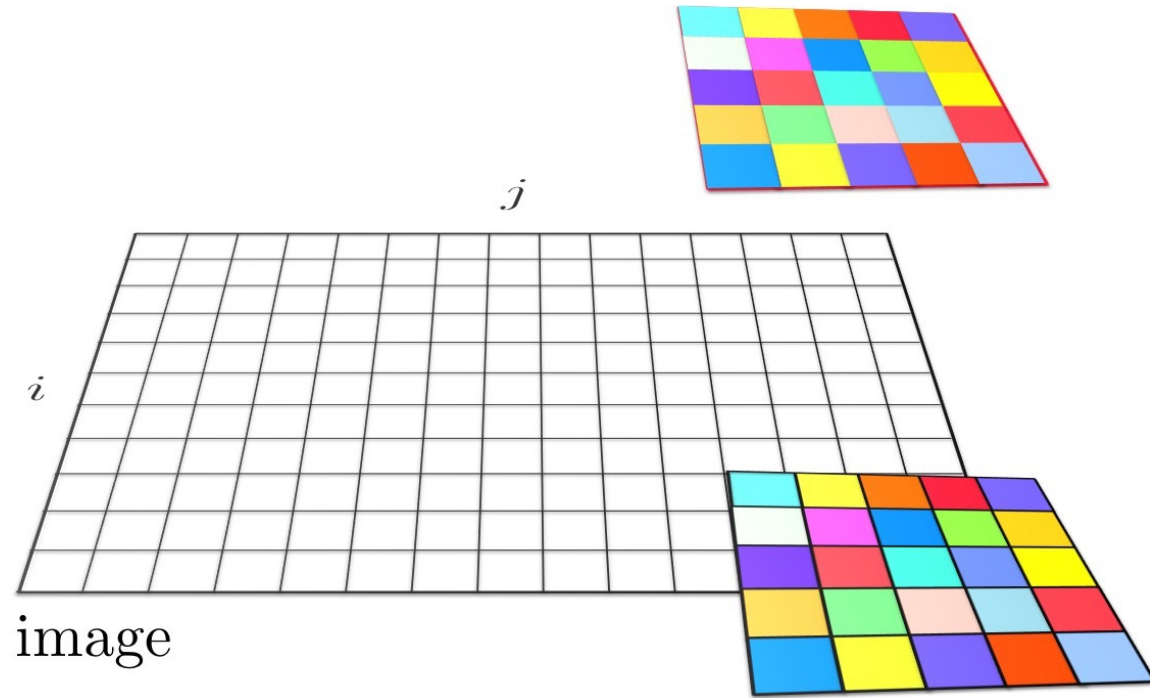
Linear Filtering: Correlation



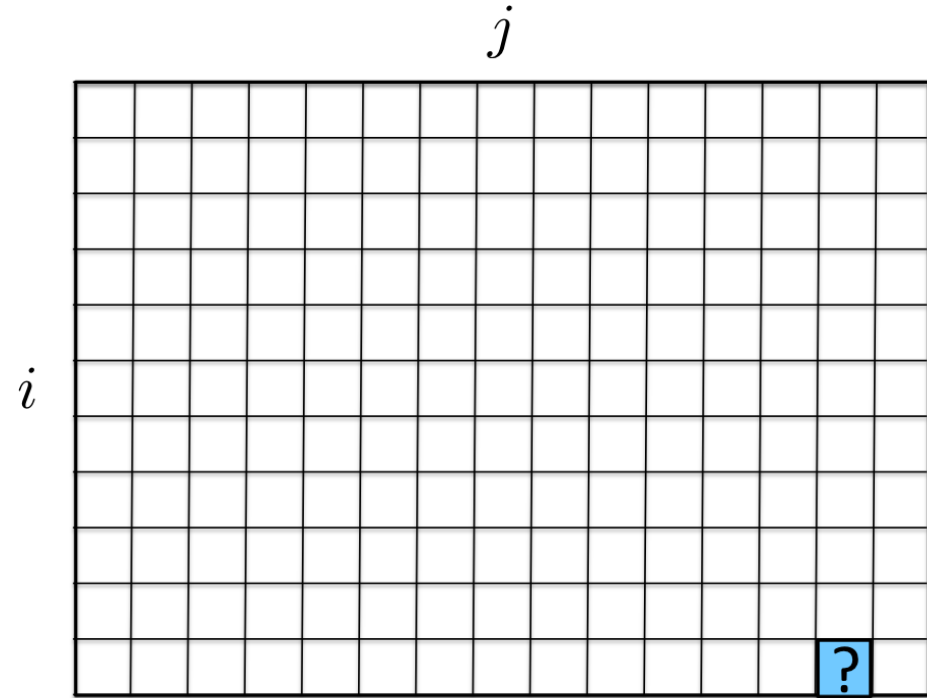
$$G(i, j) = \sum_{u=-k}^k \sum_{v=-k}^k F(u, v) \cdot I(i + u, j + v)$$

$$G(i, j) = F(\text{cyan}) \cdot I(\text{cyan}) + F(\text{yellow}) \cdot I(\text{yellow}) + F(\text{orange}) \cdot I(\text{orange}) + \dots + F(\text{light blue}) \cdot I(\text{light blue})$$

Linear Filtering: Correlation



What happens at the borders?



output G

$$G(i, j) = \sum_{u=-k}^k \sum_{v=-k}^k F(u, v) \cdot I(i + u, j + v)$$

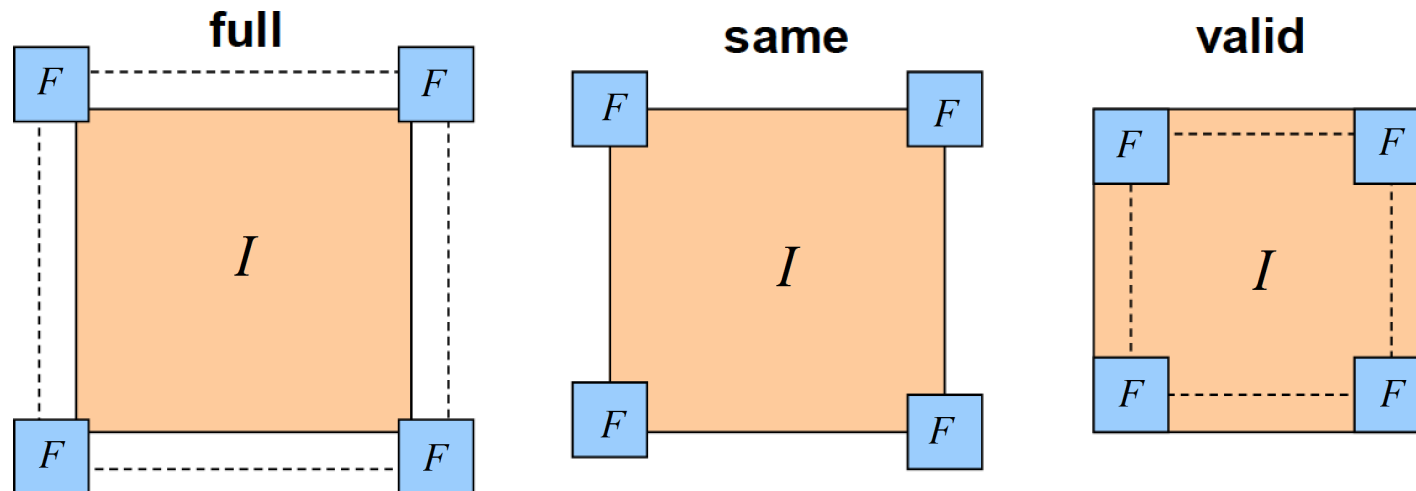
$$G(i, j) = F(\text{cyan}) \cdot I(\text{cyan}) + F(\text{yellow}) \cdot I(\text{yellow}) + F(\text{orange}) \cdot I(\text{orange}) + \dots + F(\text{light blue}) \cdot I(\text{light blue})$$

Boundary Effects

- What happens at the border of the image? What's the size of the output matrix?
 - depends on how you implement it
- Scipy: `scipy.signal.convolve2d`
 - `mode = 'full'` output size is bigger than the image
 - `mode = 'same'`: output size is same as I
 - `mode = 'valid'`: output size is smaller than the image

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Correlation Example

What's the result?



Original

0	0	0
0	1	0
0	0	0

?

Correlation Example

What's the result?



Original

0	0	0
0	1	0
0	0	0



**Filtered
(no change)**

Correlation Example

What's the result?



Original

0	0	0
0	0	1
0	0	0

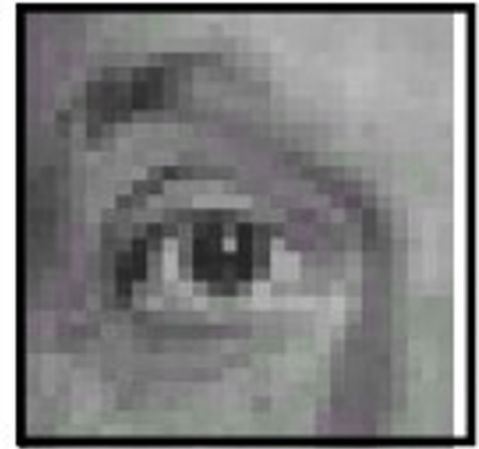
?

Correlation Example

What's the result?



0	0	0
0	0	1
0	0	0



Correlation Example

What's the result?



$$* \left(\begin{array}{|c|c|c|} \hline 0 & 0 & 0 \\ \hline 0 & 2 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array} - \frac{1}{9} \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \right) =$$

Correlation Example

What's the result?



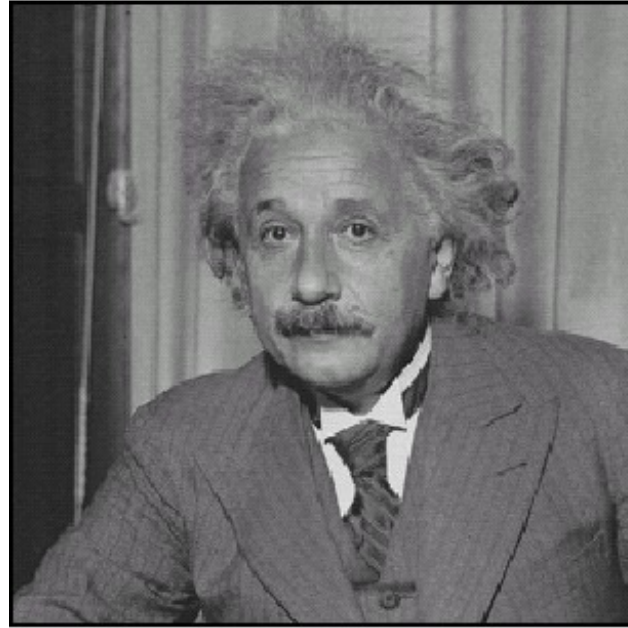
Original

$$* \left(\begin{array}{|c|c|c|} \hline 0 & 0 & 0 \\ \hline 0 & 2 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array} - \frac{1}{9} \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \right) =$$

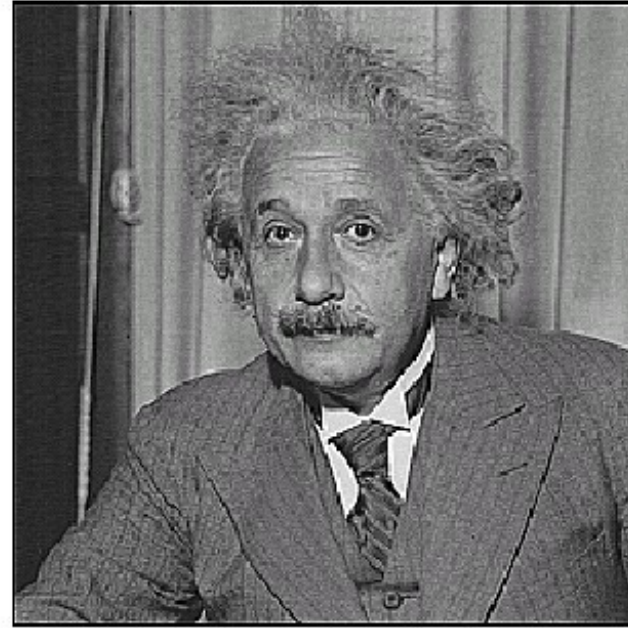


Sharpening Filter

Sharpening



before



after

This is a prelude to edge detection (next time)!

Sharpening



Smoothing by averaging



depicts box filter:
white = high value, black = low value



original



filtered

What if the filter size was 5×5 instead of 3×3 ?

Gaussian filter

What if we want nearest neighboring pixels to have the most influence on the output?

Removes high-frequency components from the image (low-pass filter).

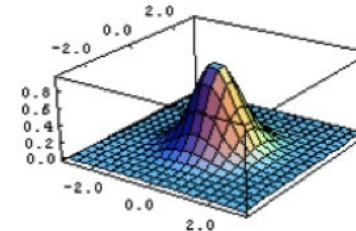
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$I(i, j)$

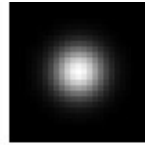
$$\frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} F(i, j)$$

This kernel is an approximation of a 2d Gaussian function:

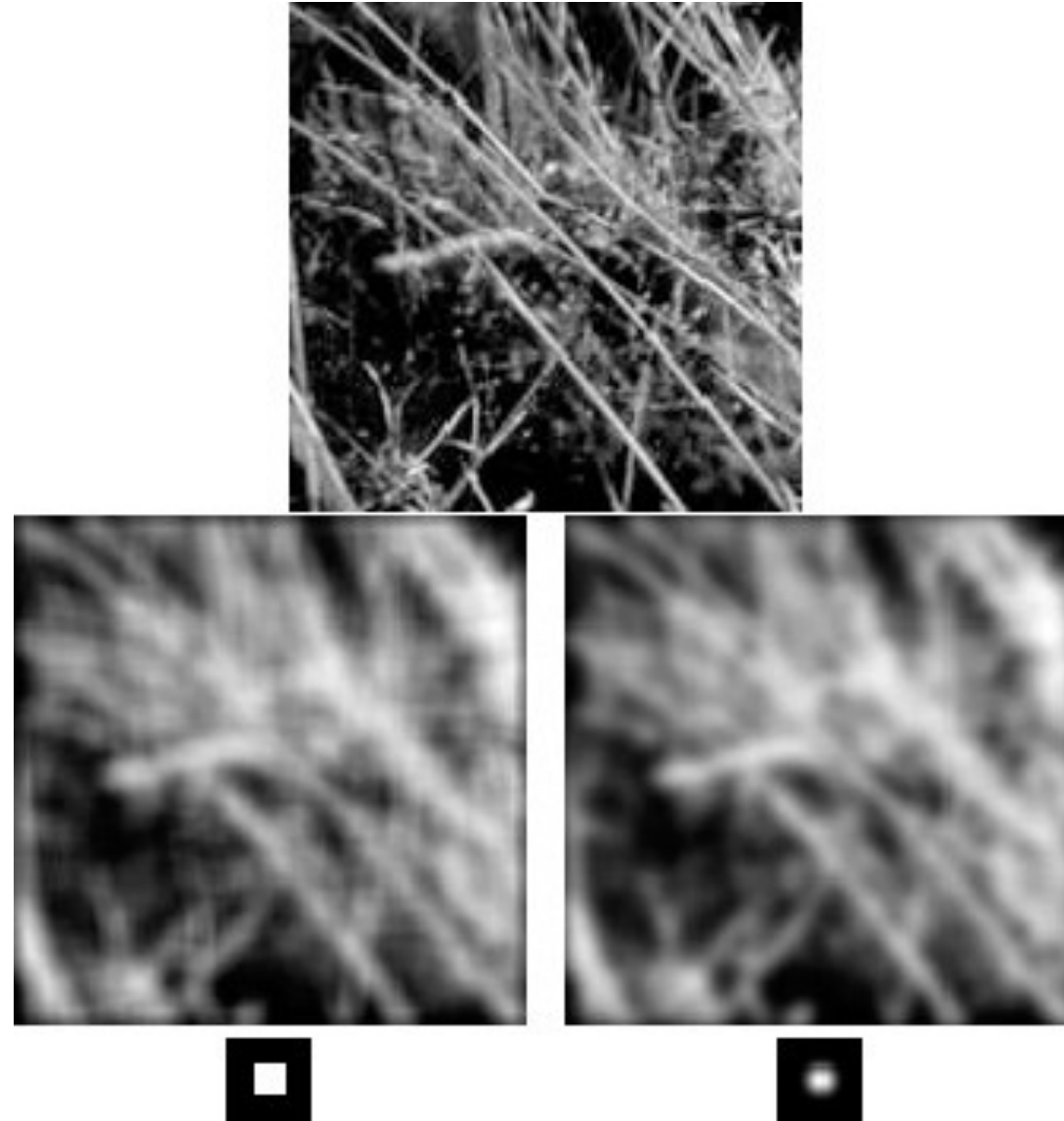
$$h(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$



Gaussian filter



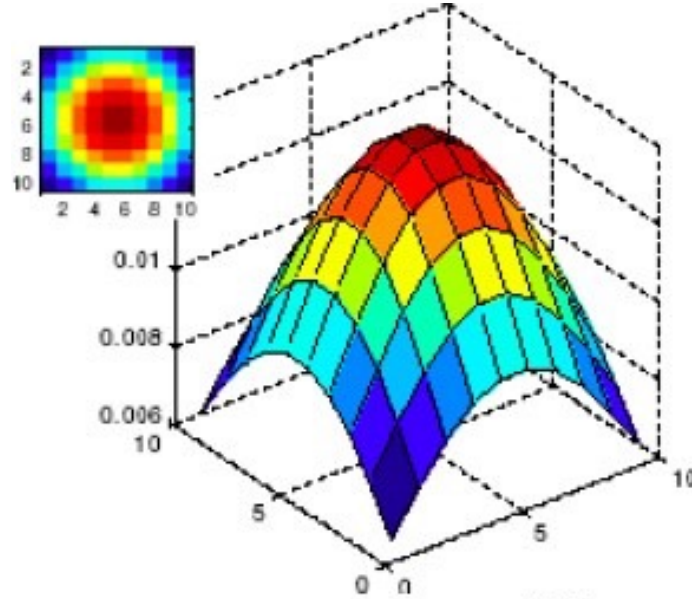
Mean vs. Gaussian filter



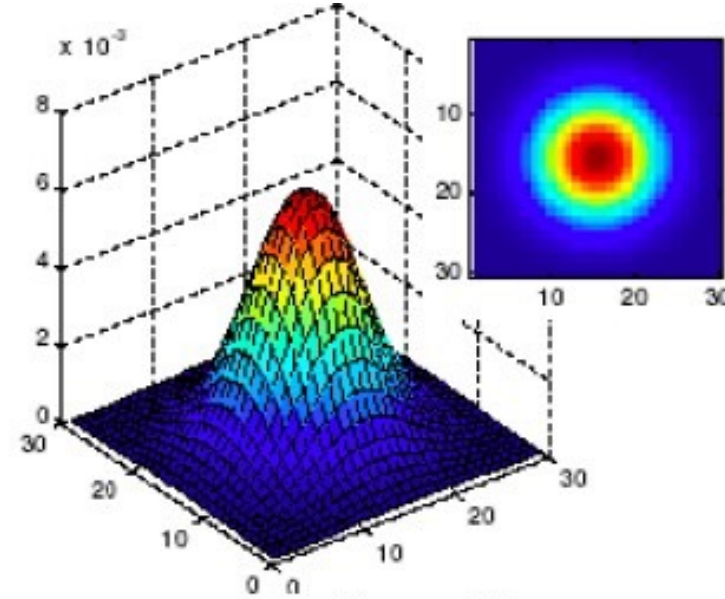
[Source: K. Grauman]

Gaussian filter parameters

Size of filter or mask: Gaussian function has infinite support, but discrete filters use finite kernels.



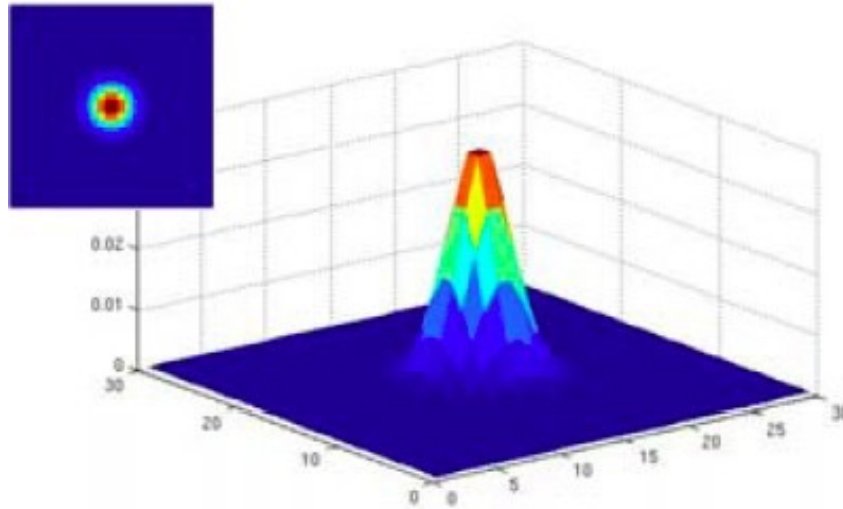
$\sigma = 5$ with
10 x 10
kernel



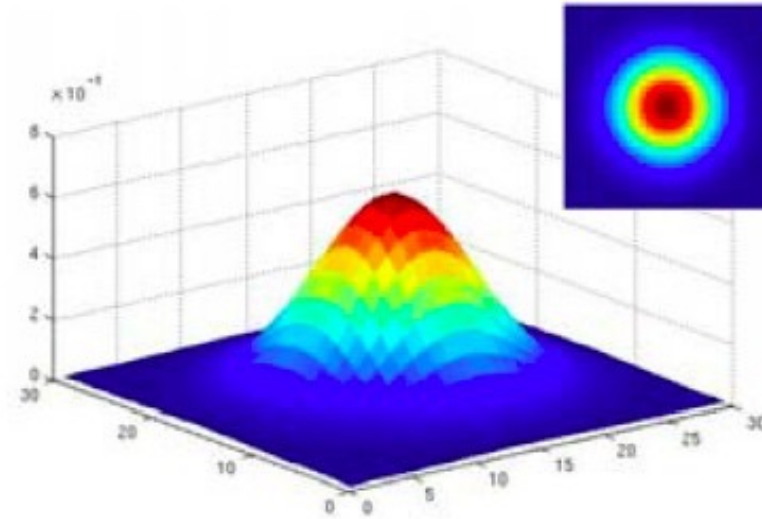
$\sigma = 5$ with
30 x 30
kernel

Gaussian filter parameters

Variance of the Gaussian: determines extent of smoothing.

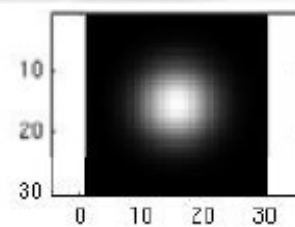
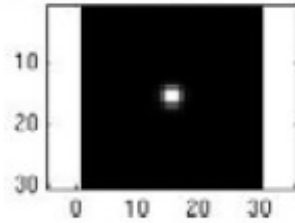


$\sigma = 2$ with
30 x 30
kernel

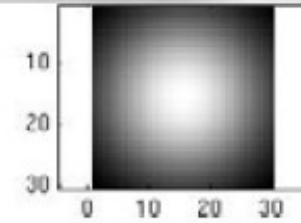


$\sigma = 5$ with
30 x 30
kernel

Gaussian filter parameters



...

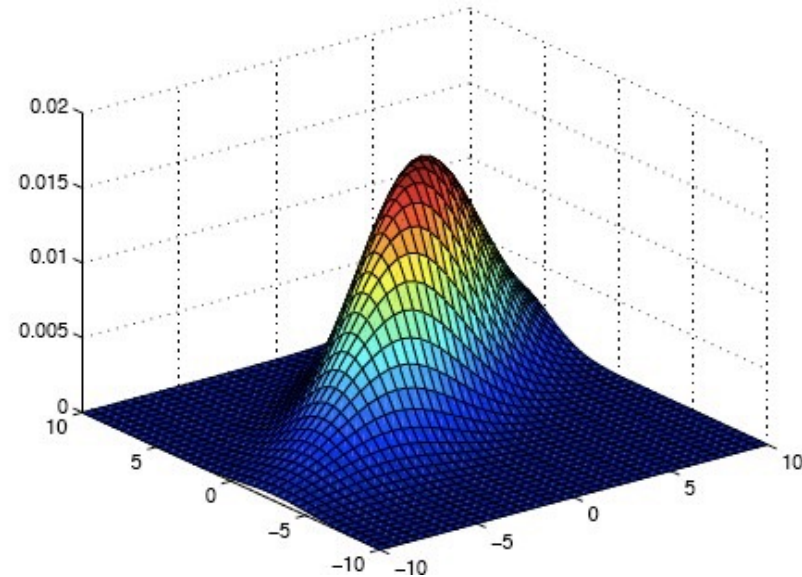
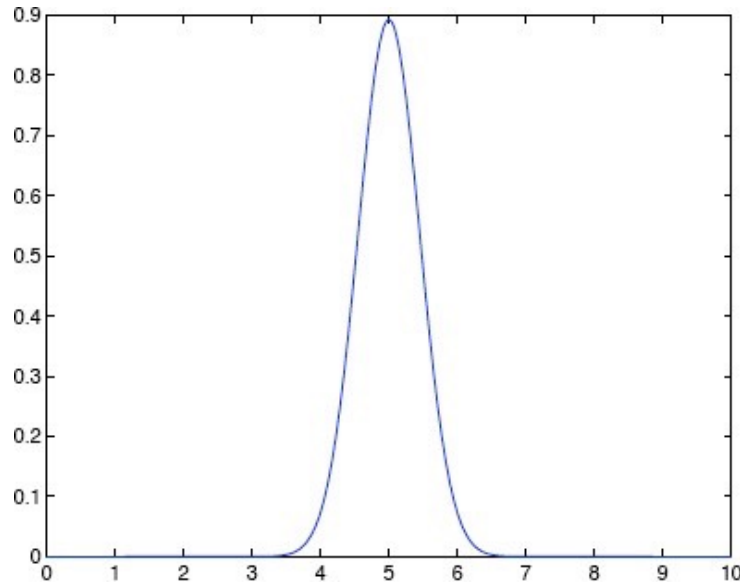


```
for sigma=1:3:10
    h = fspecial('gaussian', fsize, sigma);
    out = imfilter(im, h);
    imshow(out);
    pause;
end
```

Is this the most general Gaussian?

No, the most general form is anisotropic (i.e., not symmetric) $\mathbf{x} \in \mathbb{R}^d$

$$\mathcal{N}(\mathbf{x}; \mu, \Sigma) = \frac{1}{(2\pi)^{d/2} |\Sigma|^{1/2}} \exp \left(-\frac{1}{2} (\mathbf{x} - \mu)^T \Sigma^{-1} (\mathbf{x} - \mu) \right)$$



But the simplified version is typically used for filtering.

Properties of smoothing kernels

- All values are positive.
- They all sum to 1 to prevent re-scaling of the image.

Properties of smoothing kernels

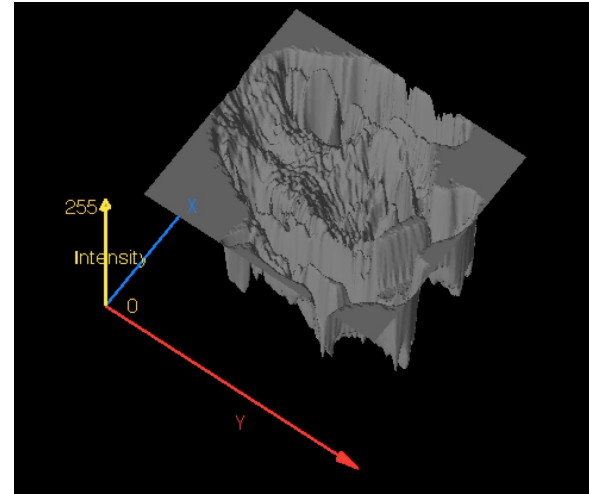
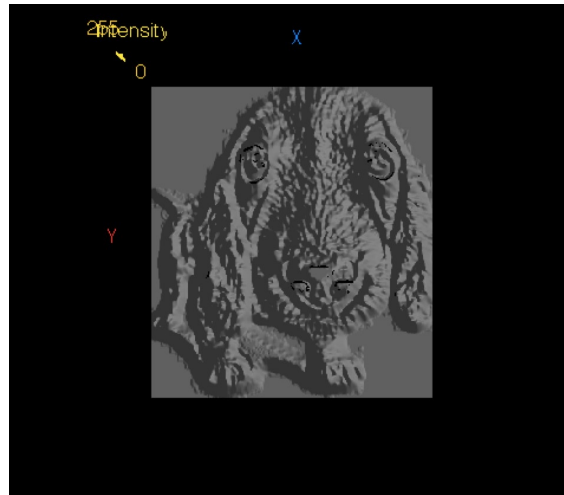
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- What is frequency in this context?

Properties of smoothing kernels

- All values are positive.
- They all sum to 1 to prevent re-scaling of the image.
- Remove high-frequency components; low-pass filter.
- What is frequency in this context?
- Edges!



Finding Waldo



How can we use what we just learned to find Waldo?

Finding Waldo



Correlation?



Filter F

Interlude: Correlation in Matrix form

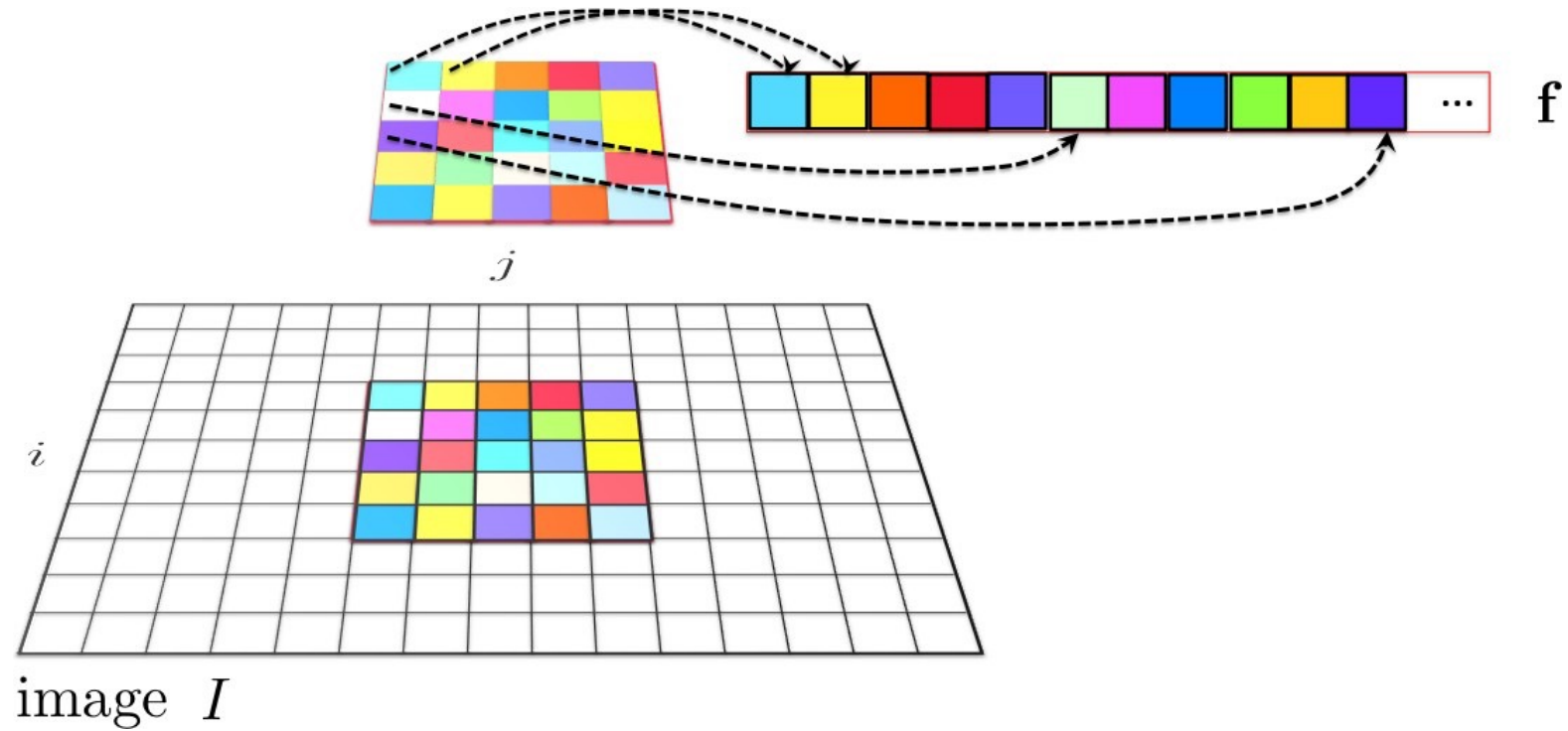
Remember correlation:

$$G(i, j) = \sum_{u=-k}^k \sum_{v=-k}^k F(u, v) \cdot I(i + u, j + v)$$

Can we write that in a more compact form (with vectors)?

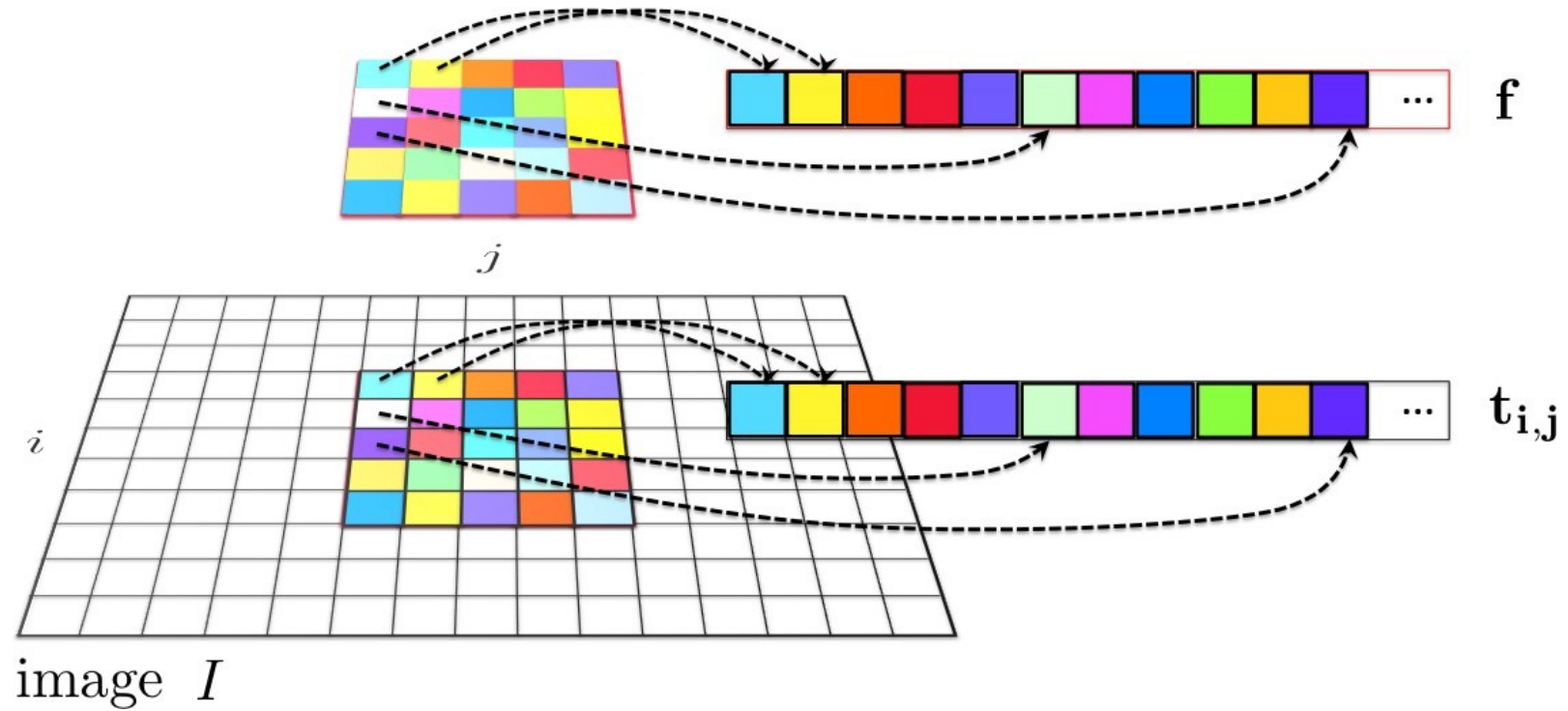
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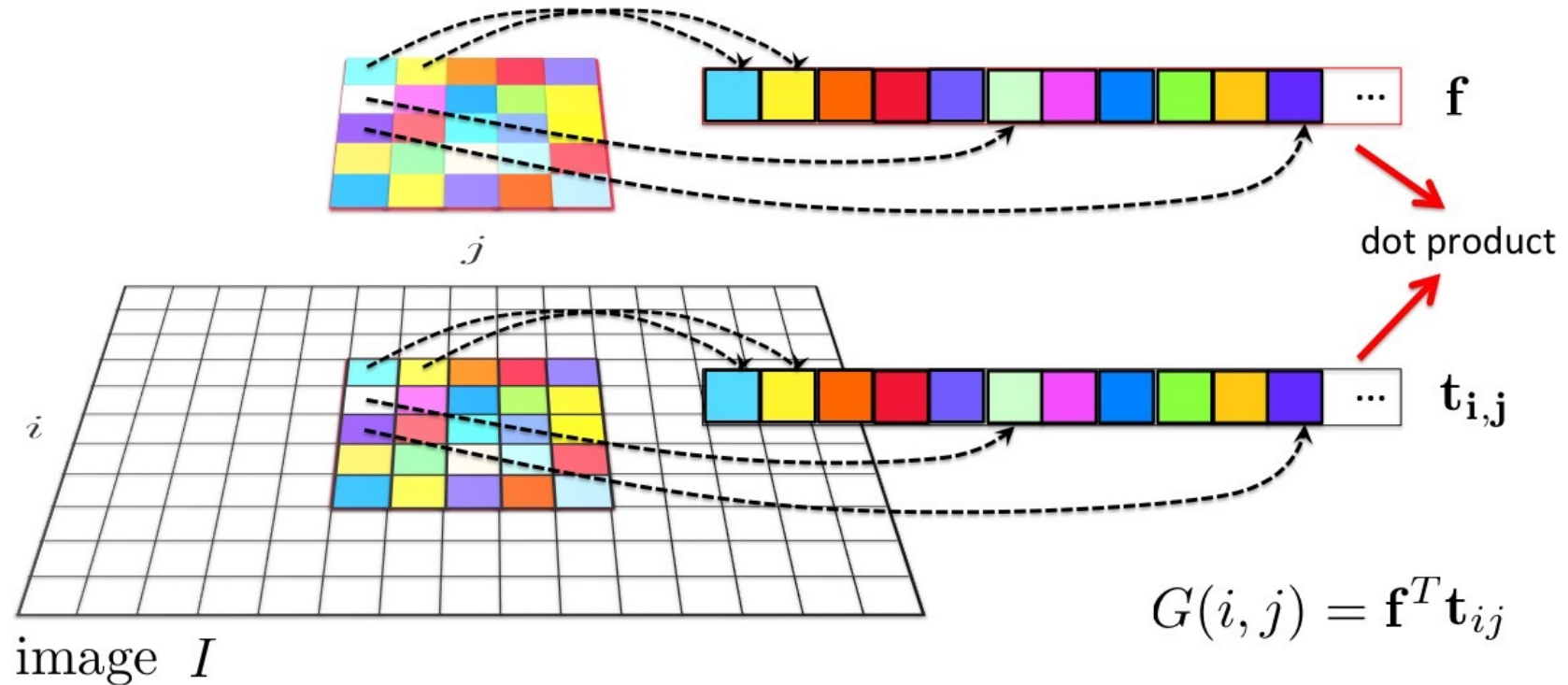
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Can we write that in a more compact form (with vectors)?

Define $\mathbf{f} = F(:)$, $T_{ij} = I(i - k : i + k, j - k : j + k)$, $\mathbf{t}_{ij} = T_{ij}(:)$

$$G(i, j) = \mathbf{f} \cdot \mathbf{t}_{ij}$$

Where \cdot is a dot product

Interlude: Correlation in Matrix form

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Where \cdot is a dot product

Can we write full correlation $G = F \otimes I$ in matrix form?

Interlude: Correlation in Matrix form

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Finding Waldo: How could we ensure to get the best “score” (e.g. 1) for an image crop that looks exactly like our filter?

Interlude: Correlation in Matrix form

Finding Waldo: How could we ensure to get the best “score” (e.g. 1) for an image crop that looks exactly like our filter?

Normalized cross-correlation:

$$G(i, j) = \frac{\mathbf{f}^T \mathbf{t}_{ij}}{\|\mathbf{f}\| \|\mathbf{t}_{ij}\|}$$

Back to Finding Waldo

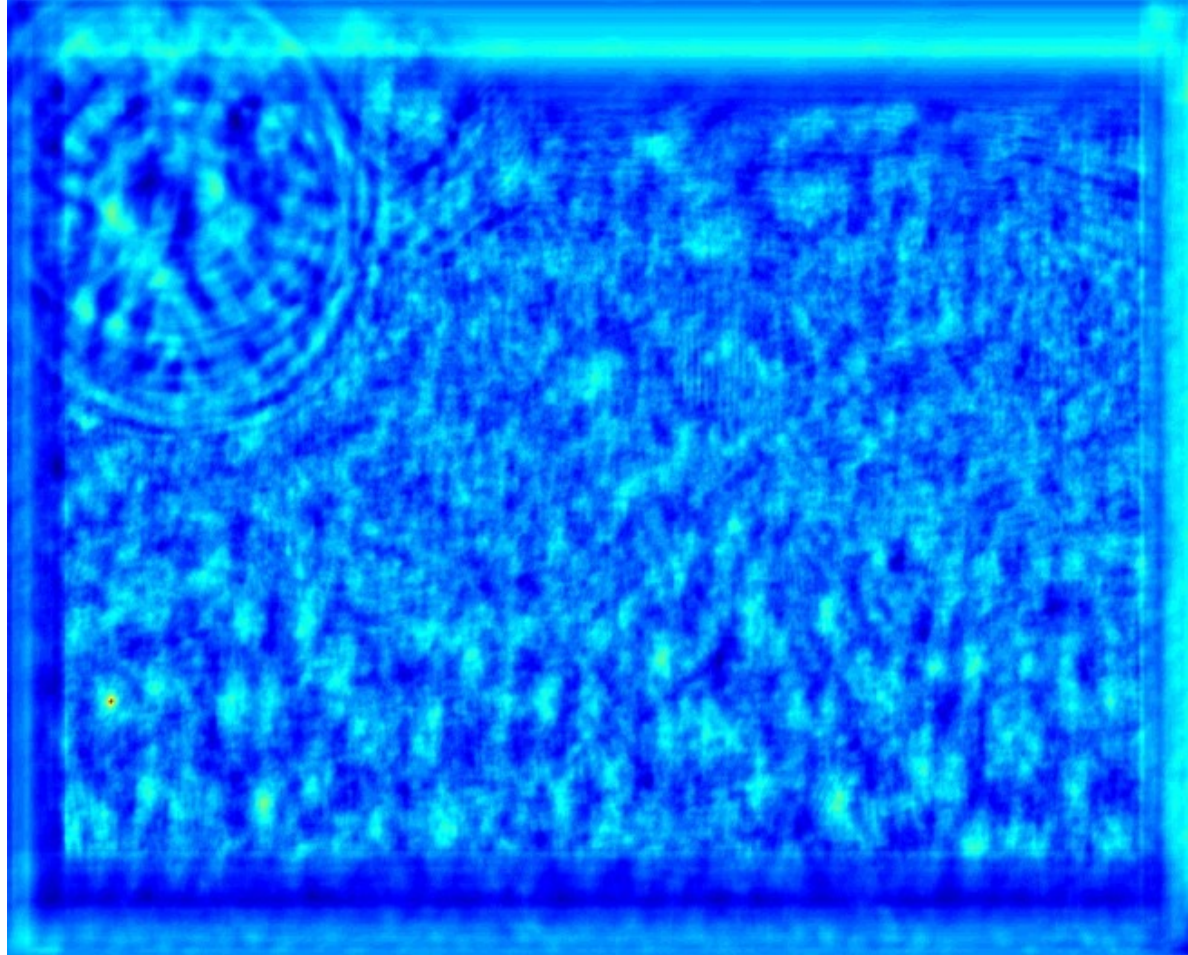


Image



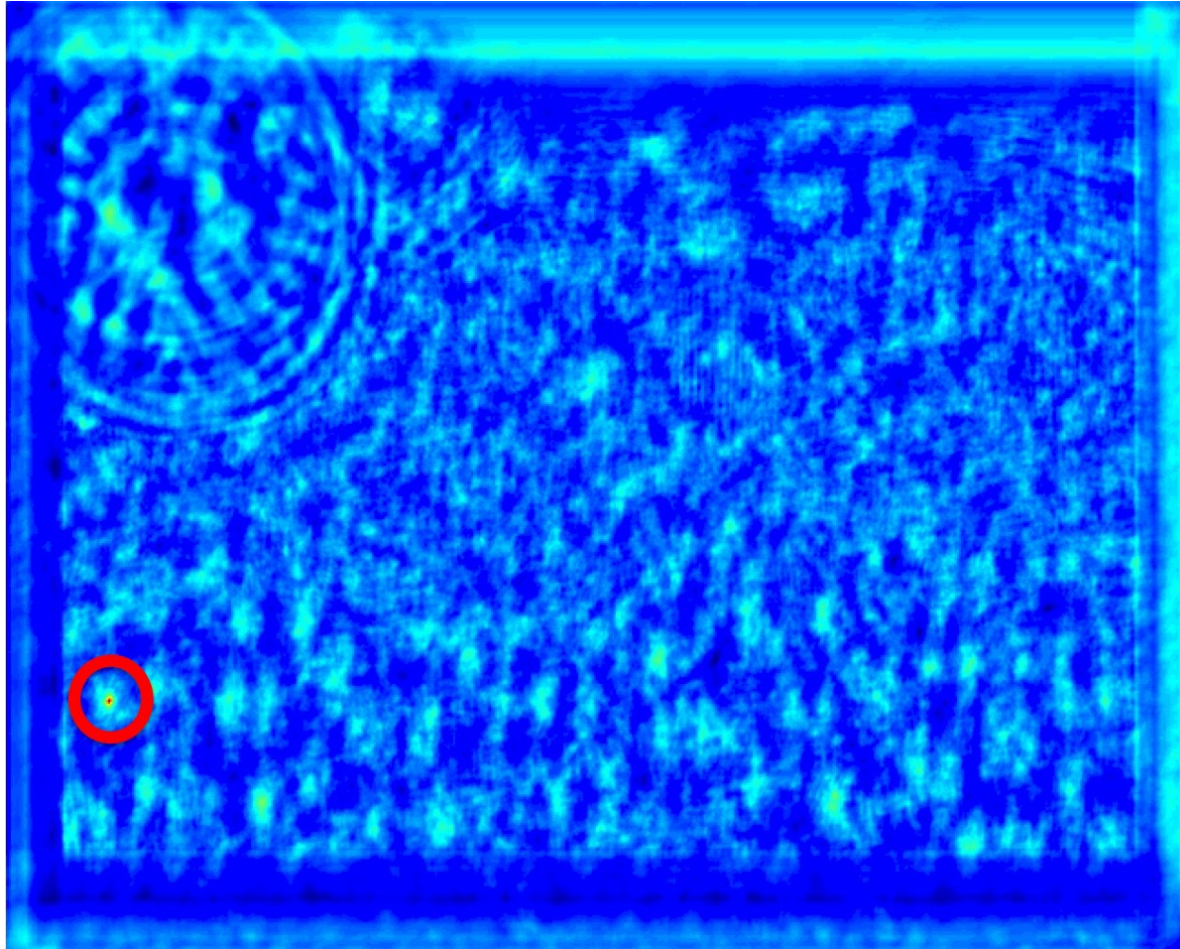
Filter

Back to Finding Waldo



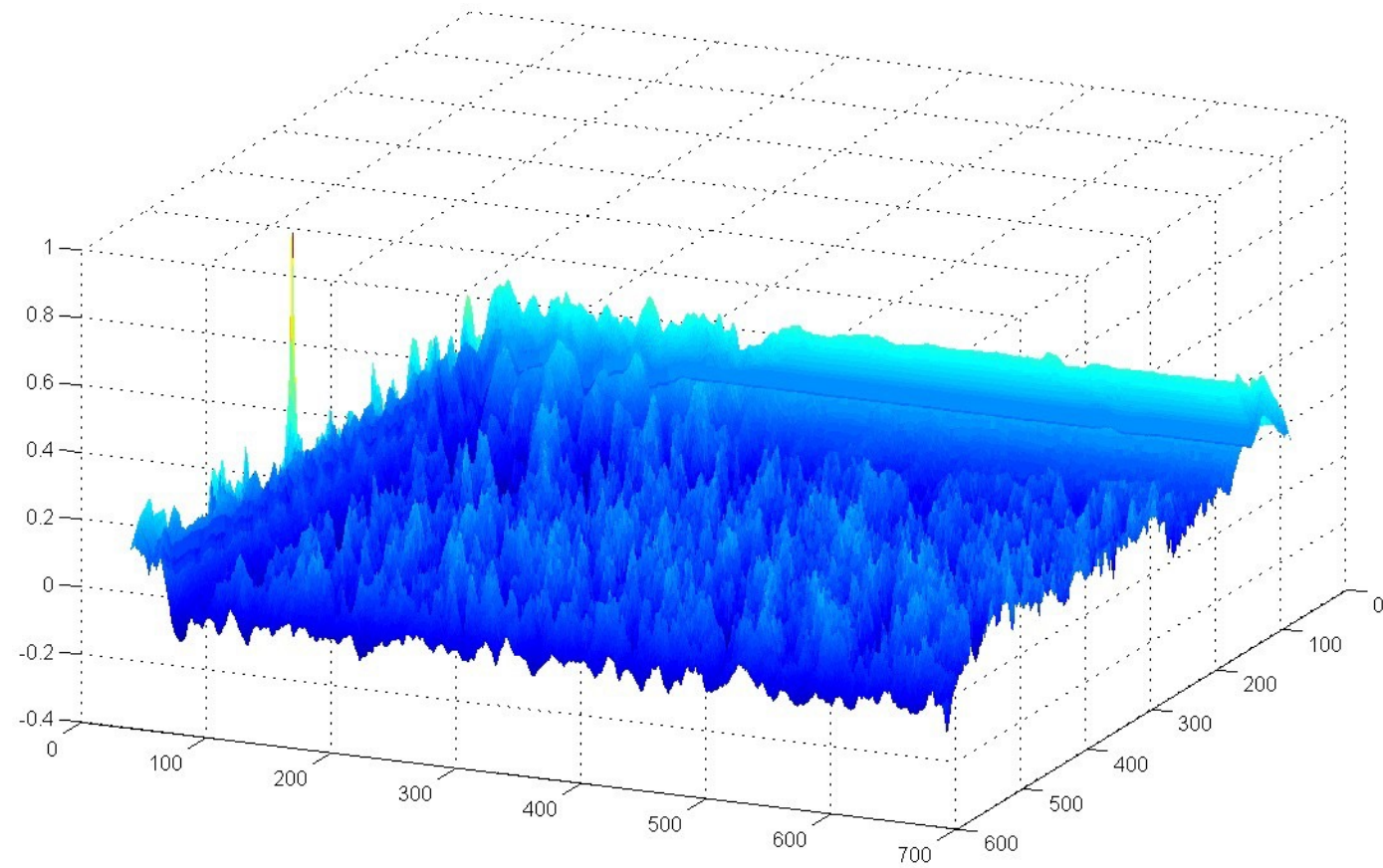
Result of normalized cross-correlation

Back to Finding Waldo



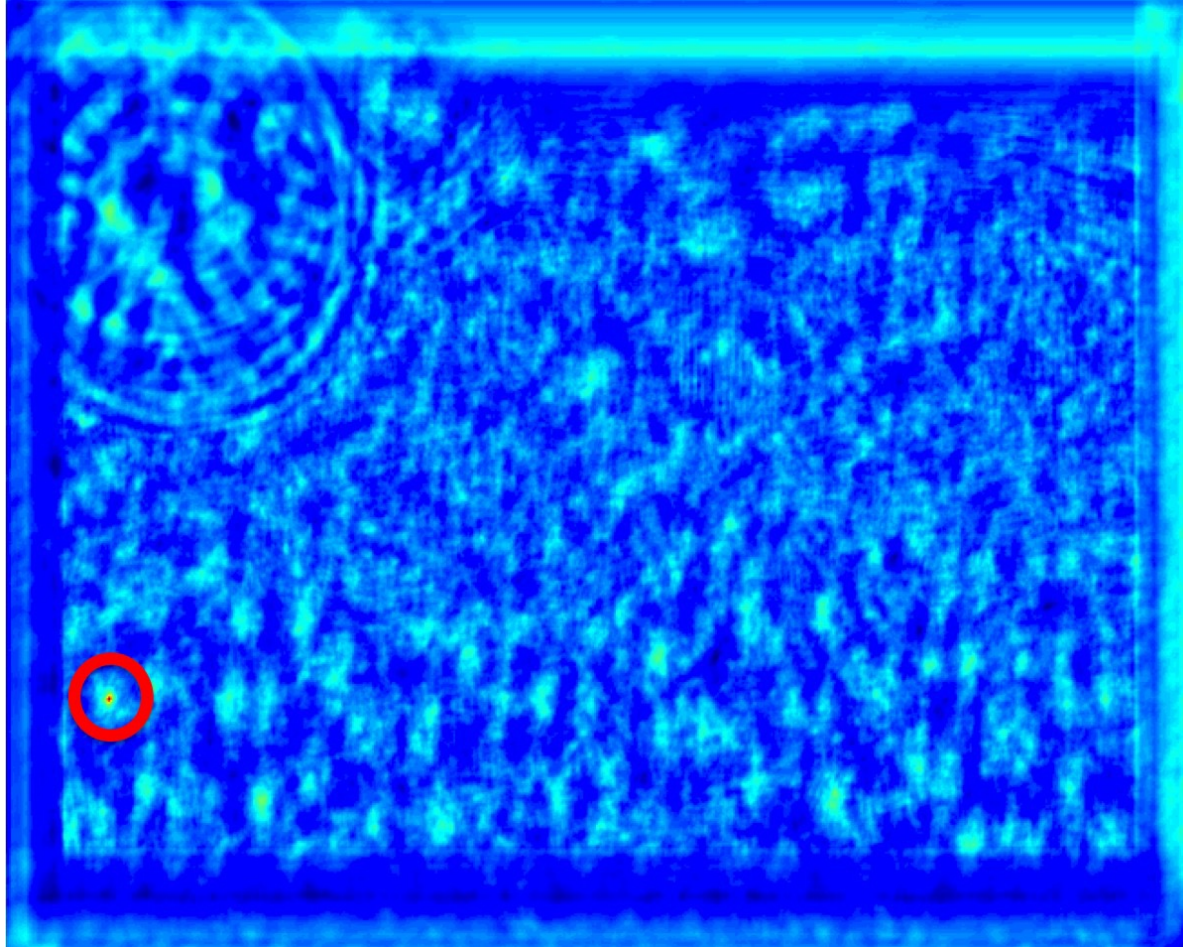
Result of normalized cross-correlation

Back to Finding Waldo



Find the highest peak

Back to Finding Waldo



Find the highest peak

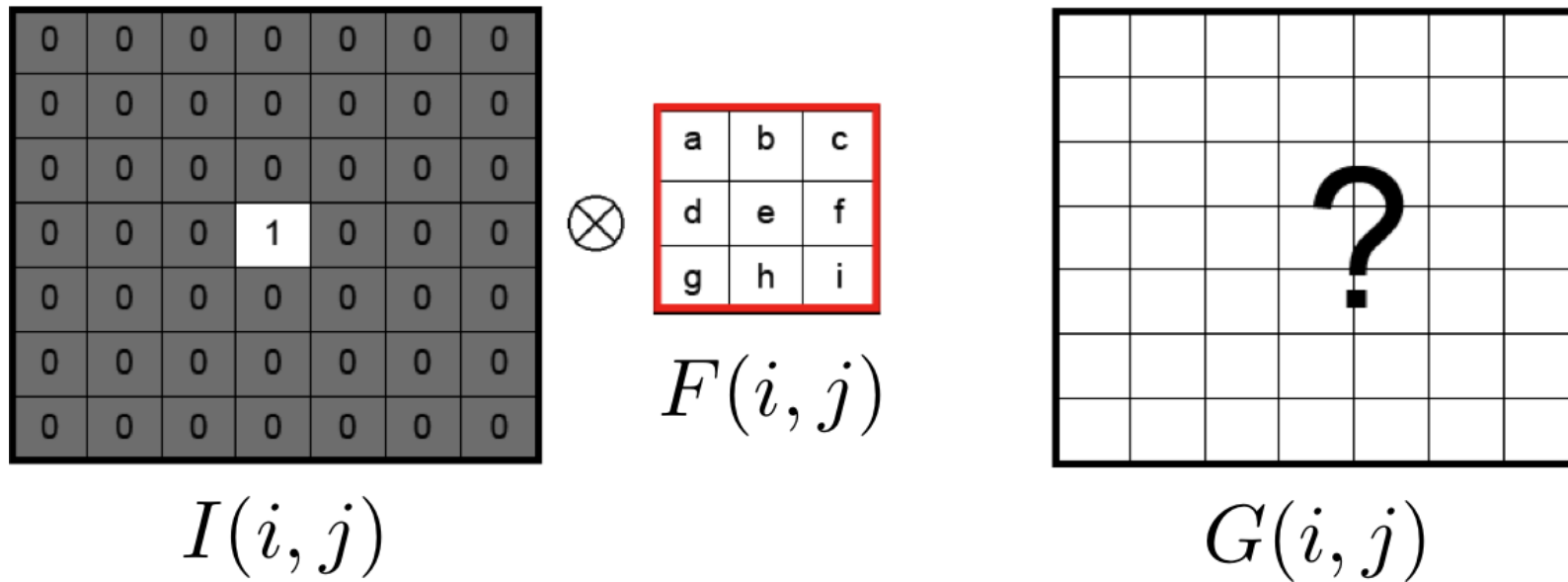
Back to Finding Waldo



With a bounding box (rectangle the size of the template) at the point...

Correlation example

What is the result of filtering the impulse signal (image) I with an arbitrary filter F ?



Convolution

Convolution operator

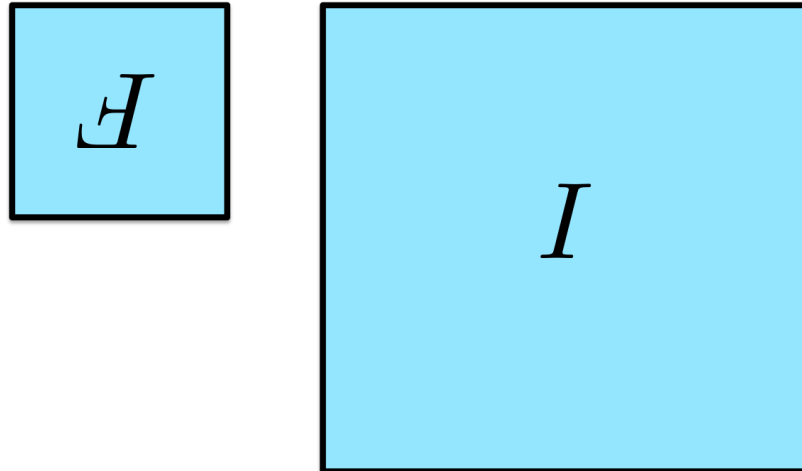
$$G(i, j) = \sum_{u=-k}^k \sum_{v=-k}^k F(u, v) \cdot I(i - u, j - v)$$

Convolution

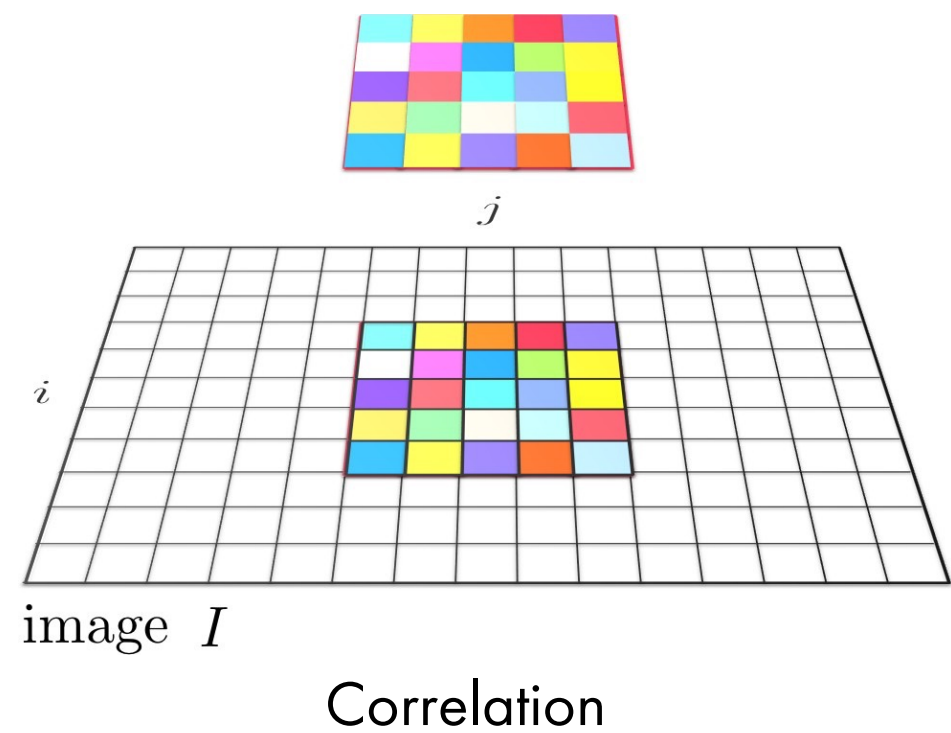
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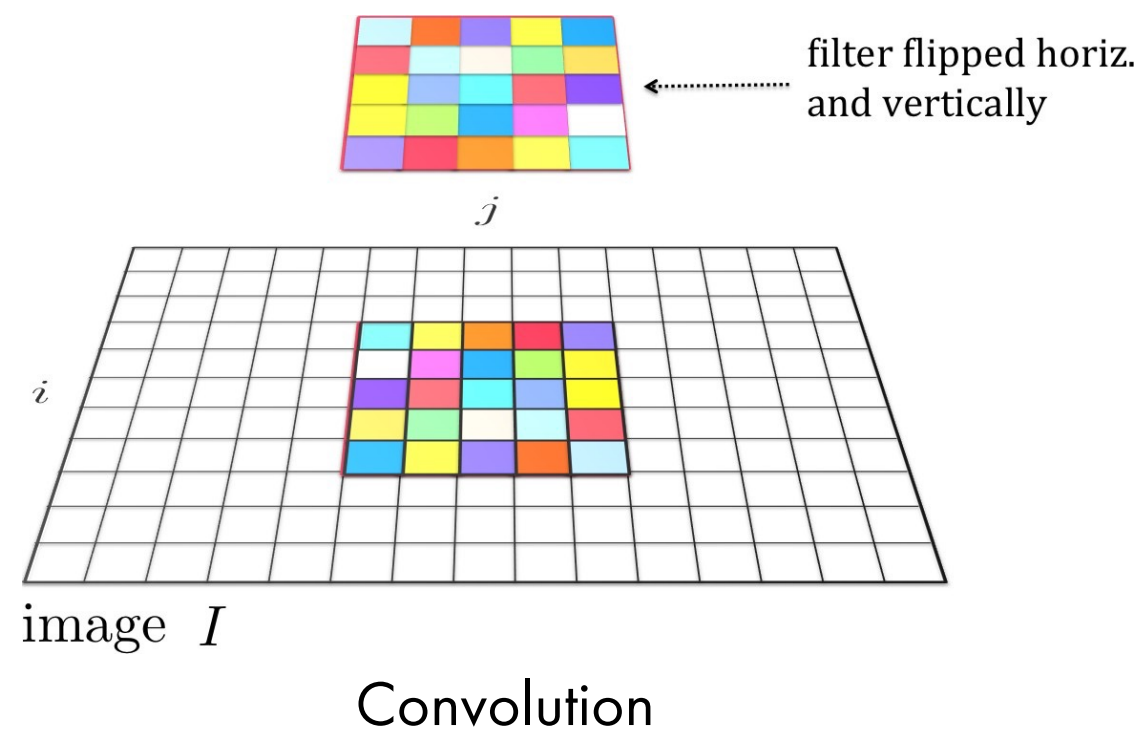
Equivalent to flipping the filter in both dimensions (bottom to top, right to left) and apply correlation.



Correlation vs Convolution



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Correlation vs Convolution

For a Gaussian or box filter, how will the outputs $F * I$ and $F \otimes I$ differ?

Correlation vs Convolution

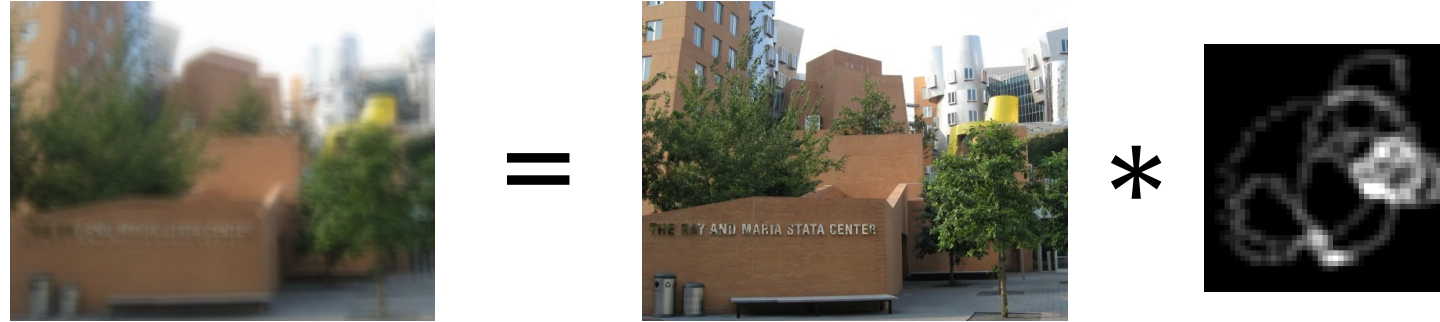
For a Gaussian or box filter, how will the outputs $F * I$ and $F \otimes I$ differ?

How will the outputs differ for:

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

“Optical” Convolution

Camera Shake



[Fergus et al. ,SIGGRAPH 2006]

Blur in out-of-focus regions of an image



Bokeh: <http://lullaby.homepage.dk/diy-camera/bokeh.html>

[Source: N. Snavely]

Properties of Convolution

Commutative: $f * g = g * f$

Associative: $f * (g * h) = (f * g) * h$

Distributive: $f * (g + h) = f * g + f * h$

Assoc. with scalar multiplier: $\lambda \cdot (f * g) = (\lambda \cdot f) * g$

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Why is this good news?

- Hint: Think of complexity of convolution and Fourier Transform
- What if we wanted to undo the result of convolution?

Separable Filters: Speed-up Trick

- The process of performing a convolution requires K^2 operations per pixel, where K is the size (width or height) of the convolution filter

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Separable Filters: Speed-up Trick

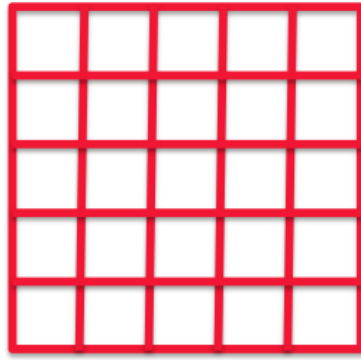
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Separable Filters: Speed-up Trick

- The process of performing a convolution requires K^2 operations per pixel, where K is the size (width or height) of the convolution filter
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- In many cases (**not all!**), this operation can be sped up by first performing a 1D horizontal convolution followed by a 1D vertical convolution, **requiring only $2K$ operations**
- If this is possible, then the convolutional filter is called **separable**
- And it is the outer product of two filters:

$$\mathbf{F} = \mathbf{v}\mathbf{h}^T$$

How it works



filter

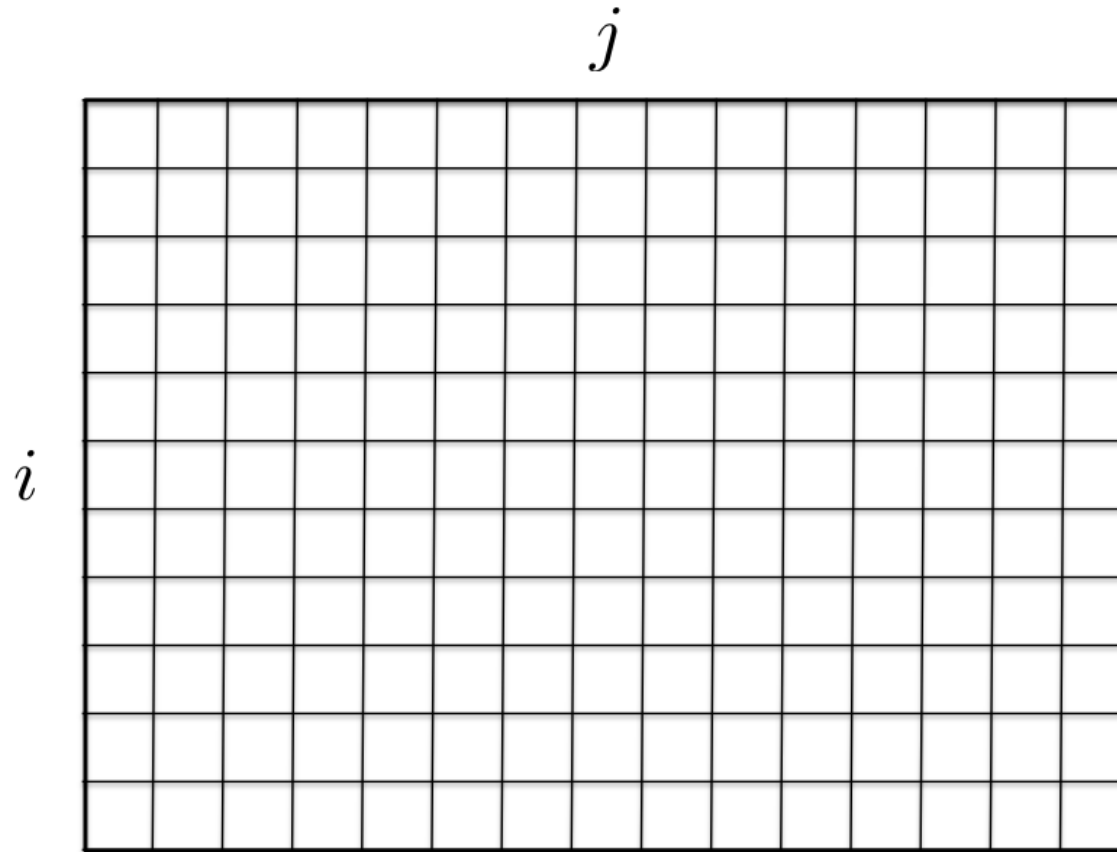
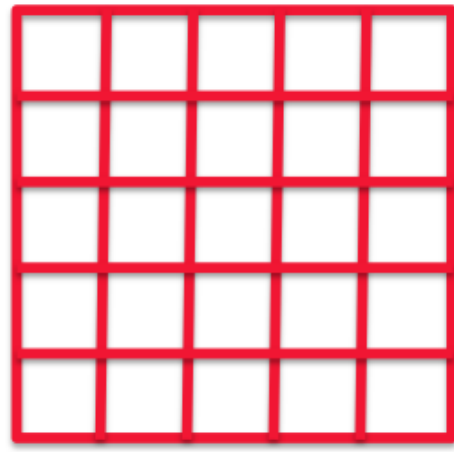


image I

How it works

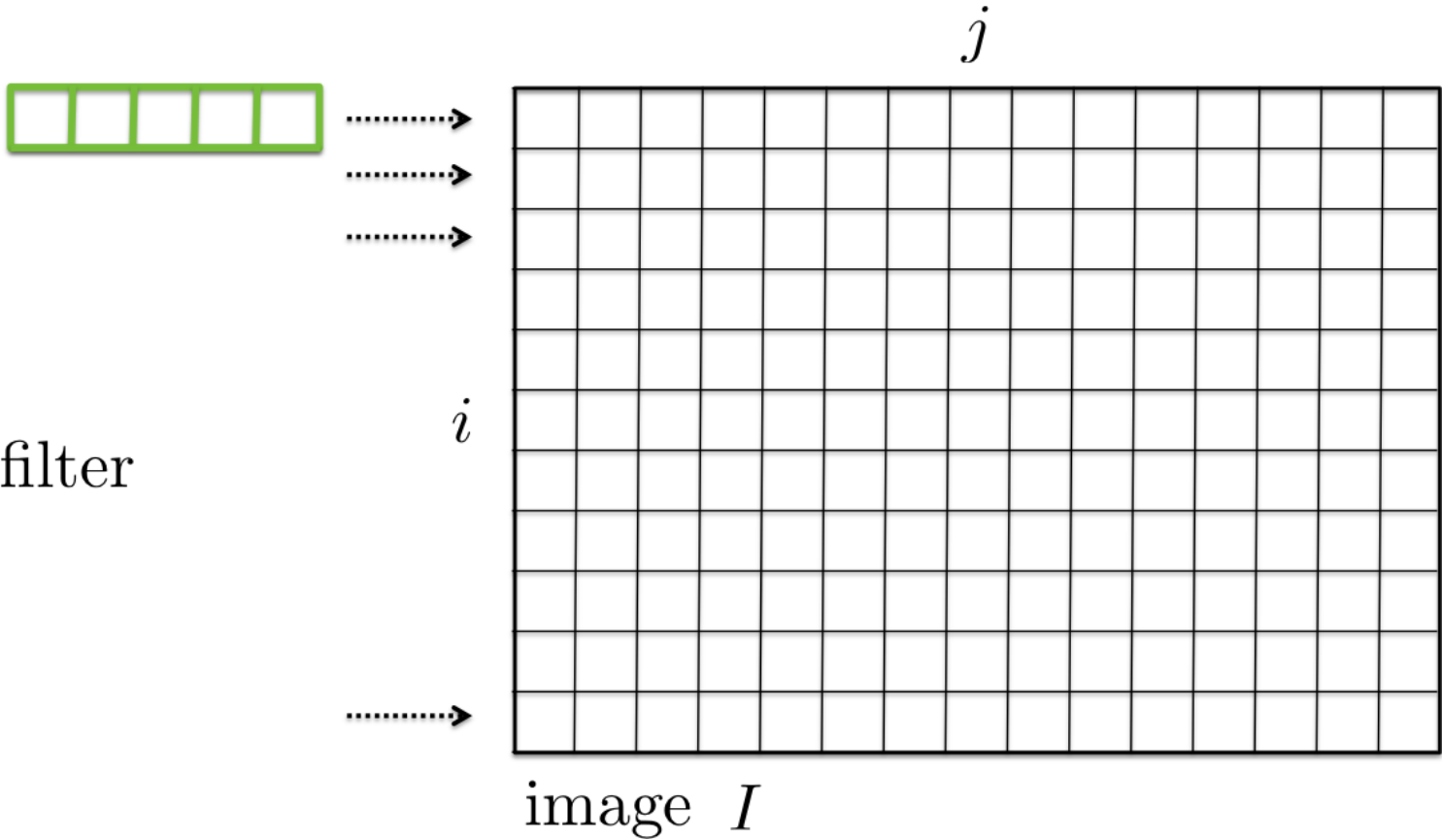


filter

=



How it works

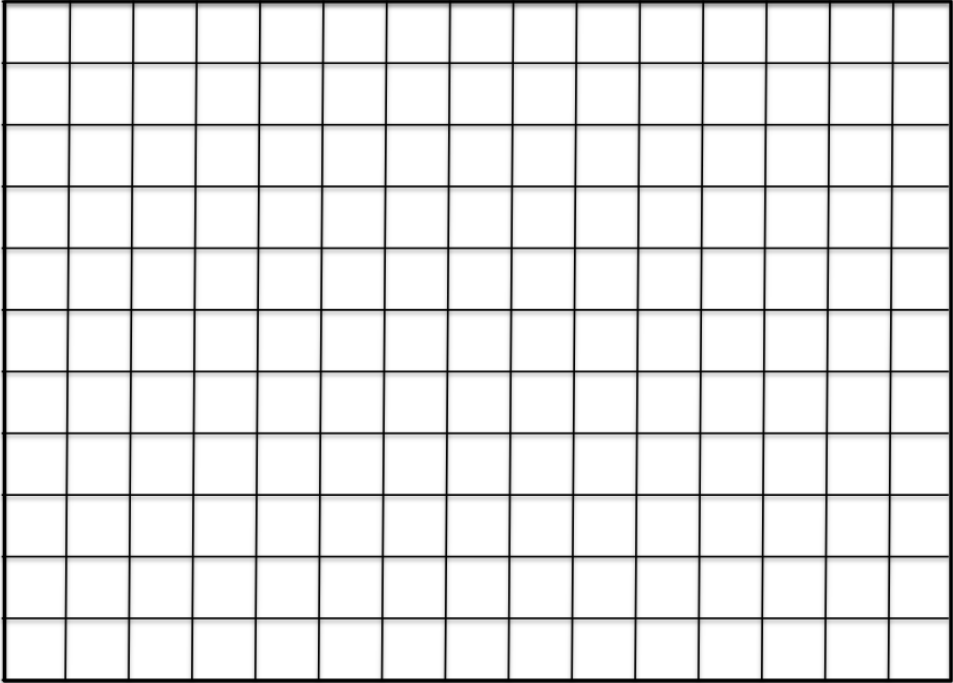


How it works

filter



j



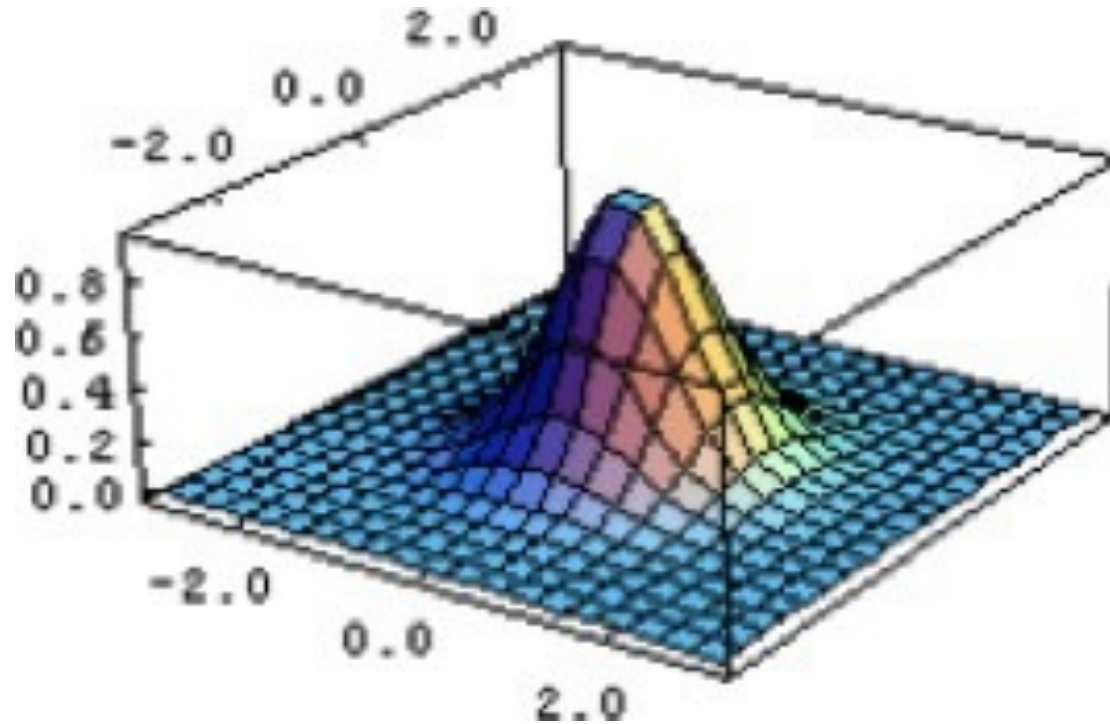
i

output of horizontal convolution

How it works

One famous separable filter we already know:

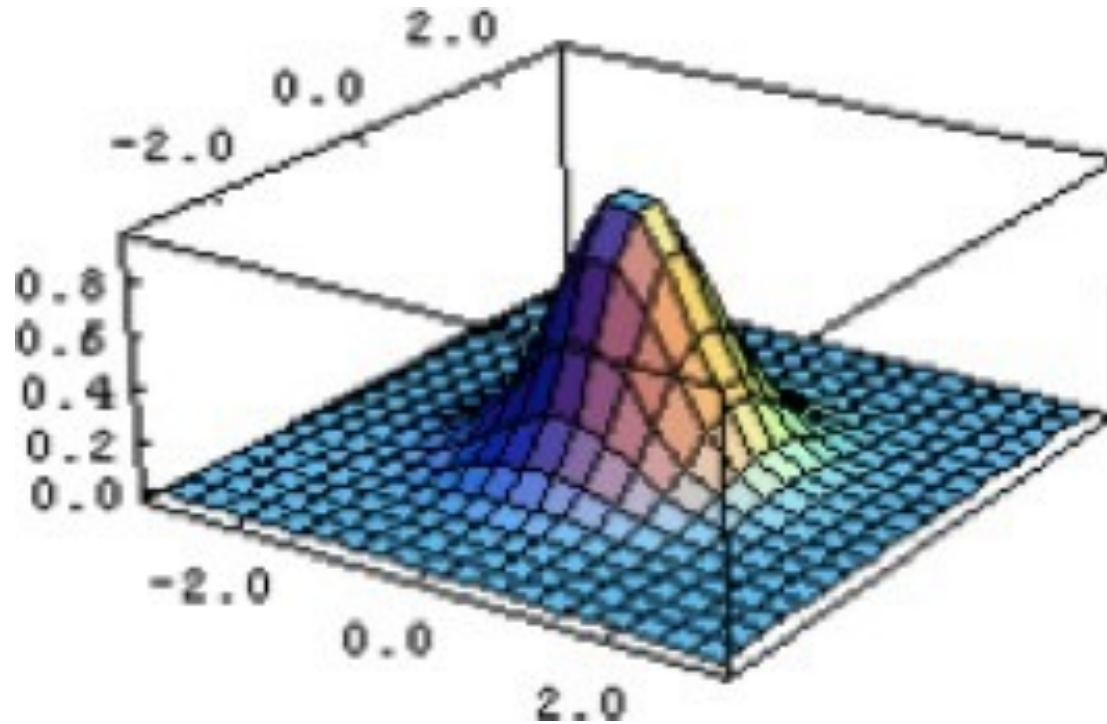
Gaussian: $f(x, y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2+y^2}{\sigma^2}\right)$



How it works

One famous separable filter we already know:

Gaussian: $f(x, y) = \left(\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{\sigma^2}} \right) \cdot \left(\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{\sigma^2}} \right)$



How it works

Is this separable? If yes, what's the separable version?

$$\frac{1}{K^2}$$

1	1	...	1
1	1	...	1
\vdots	\vdots	1	\vdots
1	1	...	1

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$$\frac{1}{K}$$

1	1	...	1
---	---	-----	---

What does this filter do?

How it works

Is this separable? If yes, what's the separable version?

$$\frac{1}{16}$$

1	2	1
2	4	2
1	2	1

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---	---	---

What does this filter do?

How it works

Is this separable? If yes, what's the separable version?

$$\frac{1}{8}$$

-1	0	1
-2	0	2
-1	0	1

How it works

Is this separable? If yes, what's the separable version?

$$\frac{1}{8} \begin{array}{|c|c|c|} \hline -1 & 0 & 1 \\ \hline -2 & 0 & 2 \\ \hline -1 & 0 & 1 \\ \hline \end{array}$$
$$\frac{1}{2} \begin{array}{|c|c|c|} \hline -1 & 0 & 1 \\ \hline \end{array}$$

What does this filter do?

How can we tell if a given filter F is indeed separable?

- Inspection... this is what we were doing.

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- Python: `np.linalg.svd`
- $\sqrt{\sigma_1}\mathbf{u}_1$ and $\sqrt{\sigma_1}\mathbf{v}_1$ are the vertical and horizontal filters

Summary – Stuff You Should Know

- **Correlation:** Slide a filter across image and compare (via dot product)
- **Convolution:** Flip the filter to the right and down and do correlation
- **Smooth** image with a Gaussian kernel: bigger σ means more blurring
- **Some** filters (like Gaussian) are separable: you can filter faster. First apply 1D convolution to each row, followed by another 1D conv. to each column

OpenCV:

- `Filter2D` (or `sepFilter2D`): can do both correlation and convolution
- `GaussianBlur`: create a Gaussian kernel
- `medianBlur`, `medianBlur`, `bilateralFilter`

Edges

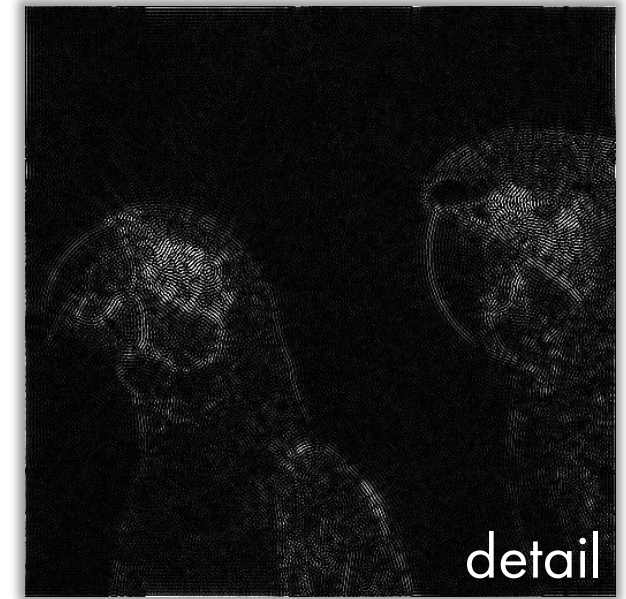
- What does blurring take away?



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Next time:

Edge Detection