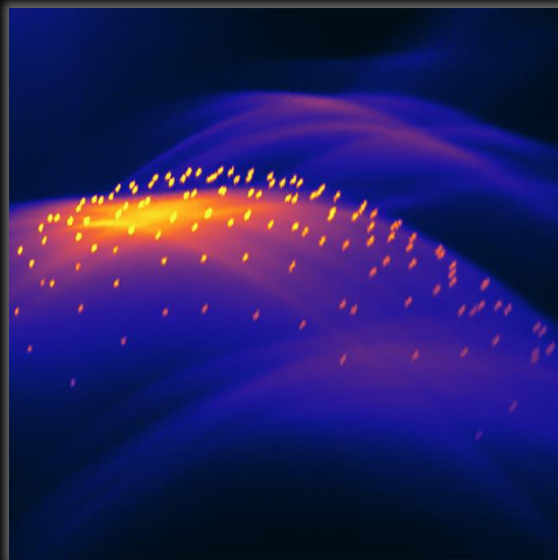


# Time-of-Flight Imaging & Single-Photon Imaging

lidar, non-line-of-sight imaging, ultrafast imaging



**CSC2529**

David Lindell

University of Toronto

[cs.toronto.edu/~lindell/teaching/2529](https://cs.toronto.edu/~lindell/teaching/2529)

\*slides adapted from Matt O'Toole,  
Gordon Wetzstein, Yannis Gkioulekas

# Poster Session

- Tuesday Nov 26 2-4 pm
  - Bahen Atrium
  - you MUST attend in person or make alternative arrangements with me (only in extenuating circumstances)
- **You are responsible for making sure that your poster is printed on time!**
  - use the “same-day” or “next-day” ordering option from <https://utposter.com/>
  - use the offer code on Quercus (“pages”) so you don’t have to pay
  - Pick up poster from 339 Bloor St W
  - bring it to the poster session and put it up before poster session begin
- You have from now until Nov 25 (the day before the poster session) to print your poster using this service.

**\$3.99 / sqft**  
**Next Day Service**

Mon. to Sat. 9am-6pm,  
Order by 2pm.  
Approve eproof by 3pm  
Ready no later than 5pm  
on next day  
**Free** deliver to U of T area.

Product Code: Next day service

Price: \$3.99

### Available Options

\* Shorter Side ( 24-60 inch ):

24

\* Longer Side ( 36-200 inch ):

36

\* Materials:

High Quality Glossy Photo Paper



Tube ( Mandatory for Paper Poster Shipping):

☐ Yes (+\$3.99)

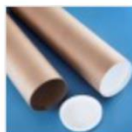
How Many Letter Size Handout Copies ( Optional ):

--- Please Select ---



\* Pick Up /Shipping Options:

Pick Up @ store Mon-Sat 9am-6pm. Exclude Long Weekend Sat & Mon



Use the offer code on Quercus

# Poster Session

- All proposals should have received feedback!
  - messaged you on Piazza
- Check Quercus->Assignments->Final Project for rubric for the poster session, code, report





Scan for Zoom link

## Mohammad Norouzi

Ideogram

“Diffusion Models and The Future of Creative Expression”



Wednesday, Nov 6th



2:00pm–3:00pm



BA5187

*Reception to follow*



## Holographic displays

Manu Gopakumar

Ph.D. Candidate, Stanford

Tuesday Nov 12, 2-4 PM (Zoom)  
(Lecture 11)

## Single photon imaging



Sotiris Nousias

Postdoc, University of Toronto  
Incoming Assistant Professor,  
Purdue University

Tuesday Nov 19, 2-4 PM, here  
(Lecture 11)

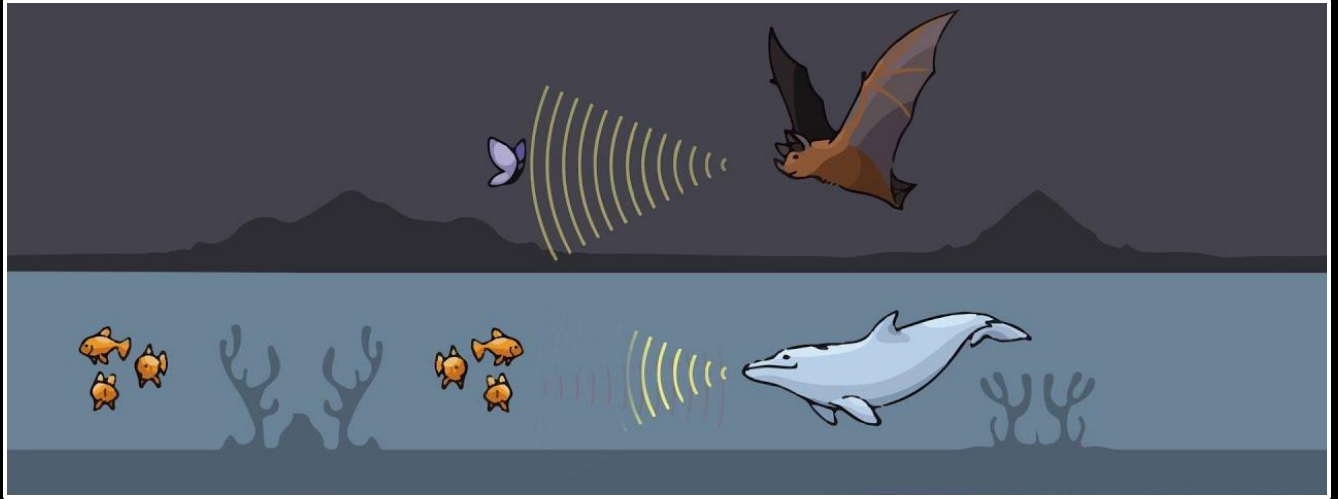
## transient imaging (a.k.a. femtophotography)



## overview

- Time-resolved imaging
- Single-photon avalanche diodes (SPADs)
- Single-photon lidar
- Non-line-of-sight imaging
- Neural rendering for propagating light

# echolocation



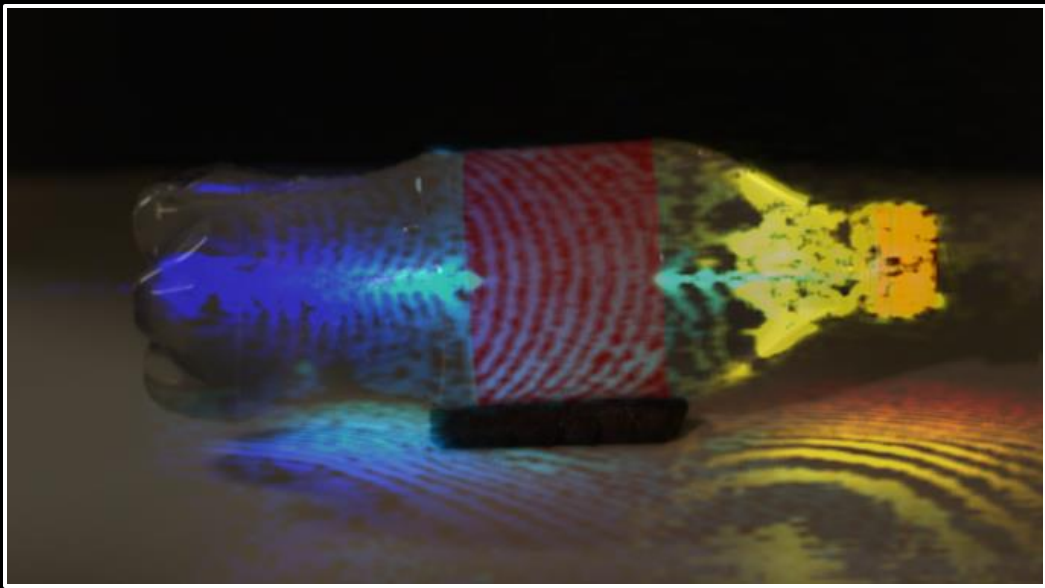
speed of sound in air: 343 meters / sec  
in water: 1480 meters / sec

Light takes 1.255 seconds to travel from the earth to the moon



speed of light in a vacuum: **299,792,458** meters / sec  
(Light travels approximation **1 MILLION** times faster than sound!)

## transient imaging



speed of light in a vacuum: **299,792,458** meters / sec

(Light travels approximation **1 MILLION** times faster than sound!)



# direct and indirect time-of-flight sensors for transient imaging



Direct time-of-flight sensor



Indirect time-of-flight sensor

## direct and indirect time-of-flight sensing



Direct time-of-flight sensor



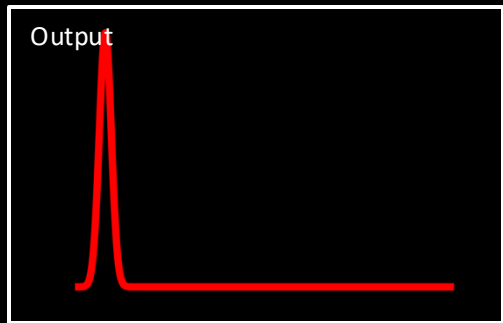
Indirect time-of-flight sensor

## direct and indirect time-of-flight sensing

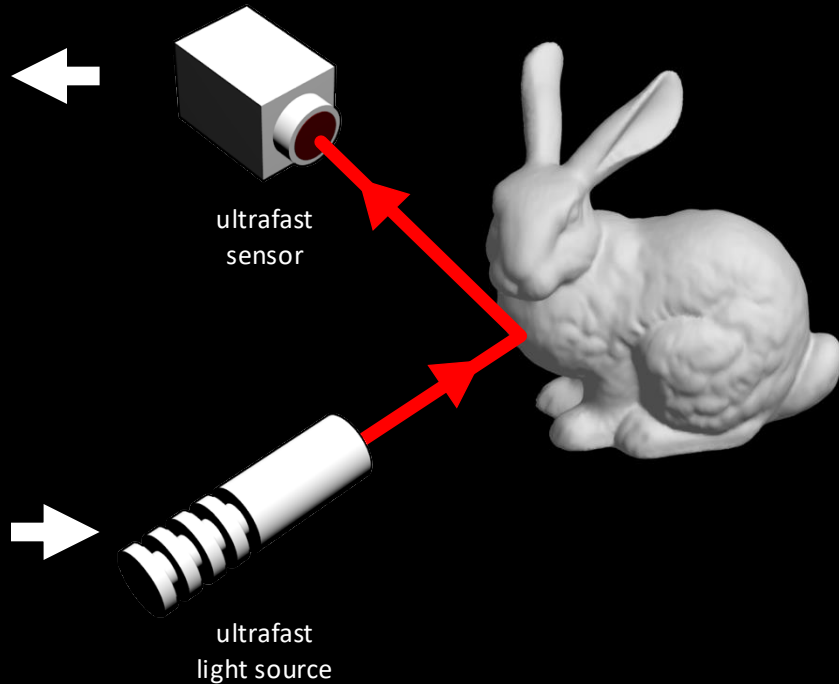
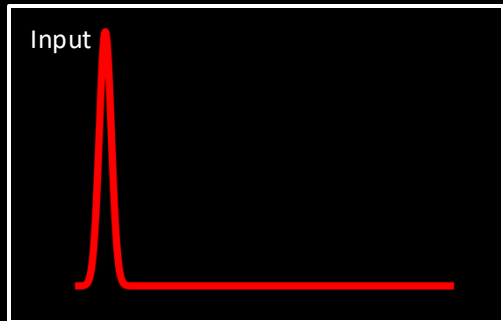


Direct time-of-flight sensor

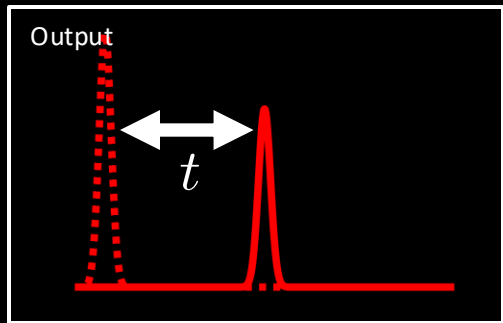
# direct time-of-flight principle



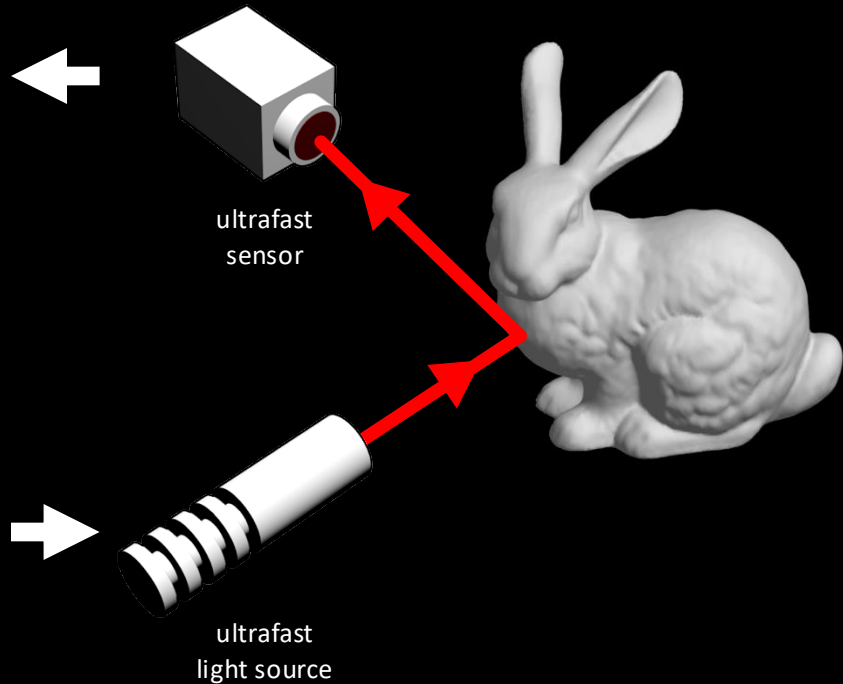
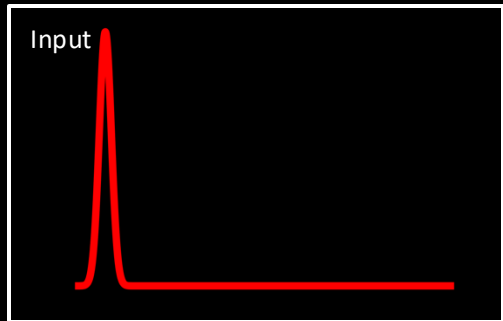
time →



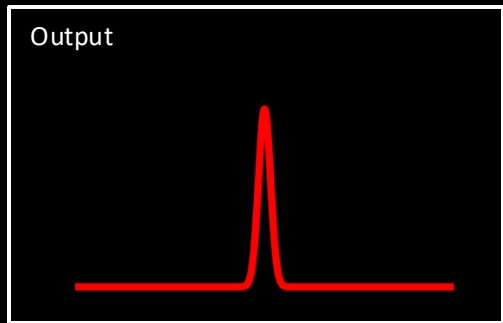
# direct time-of-flight principle



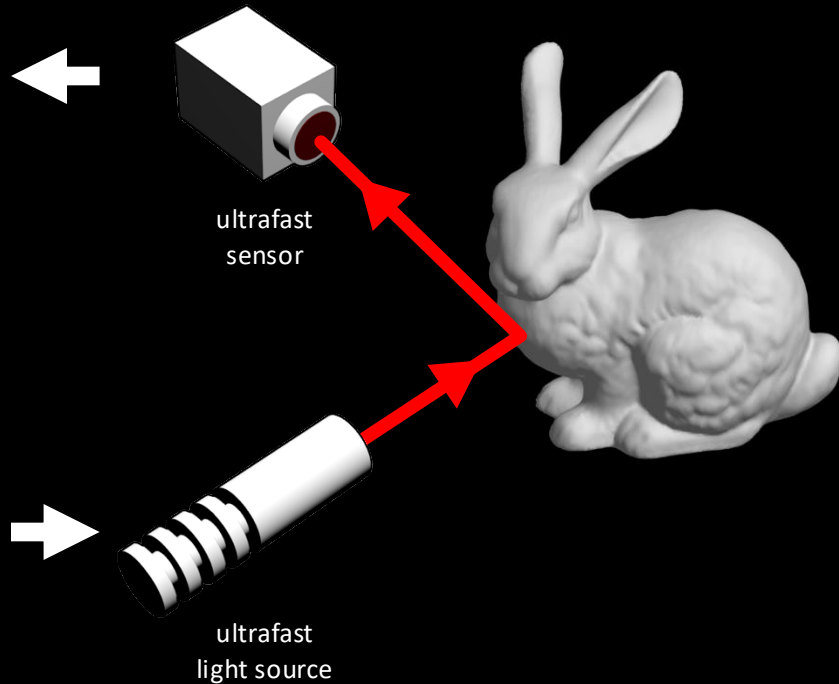
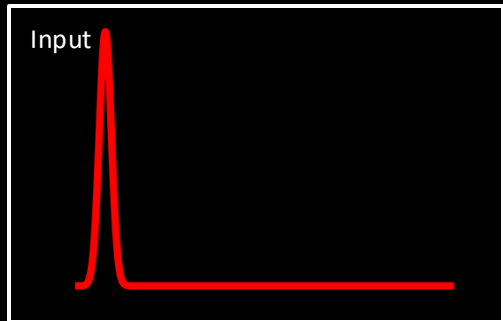
time →



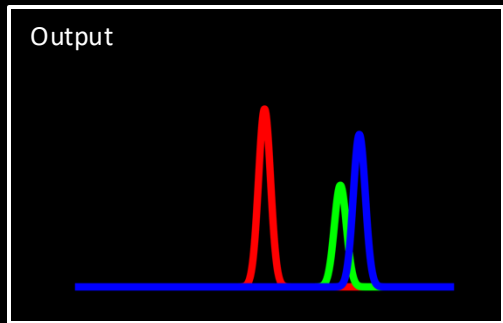
# direct time-of-flight principle



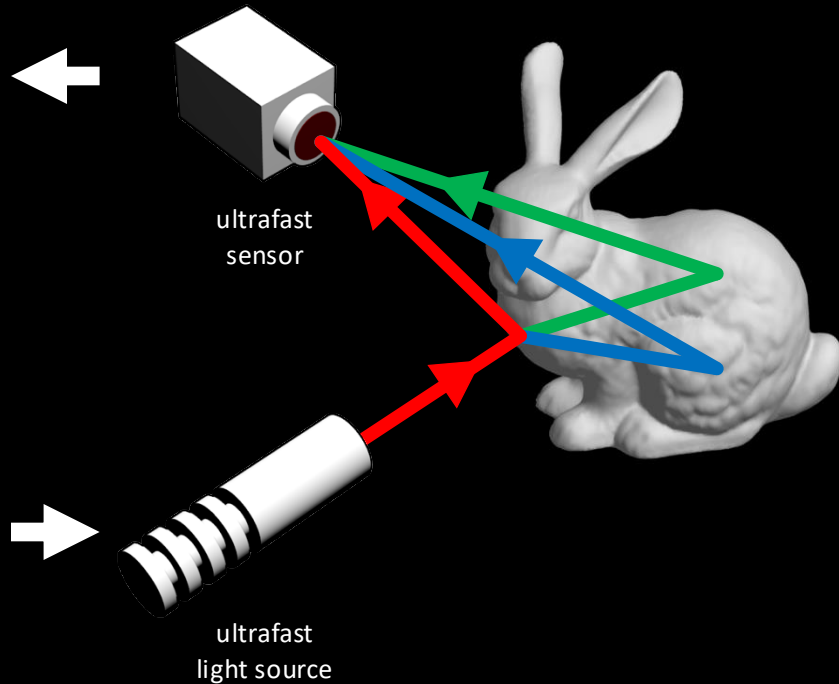
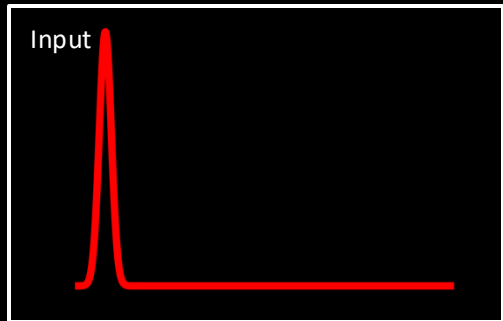
time →



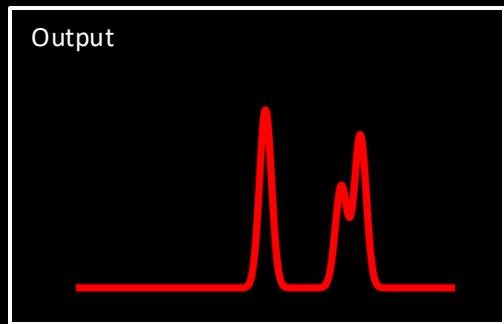
# direct time-of-flight principle



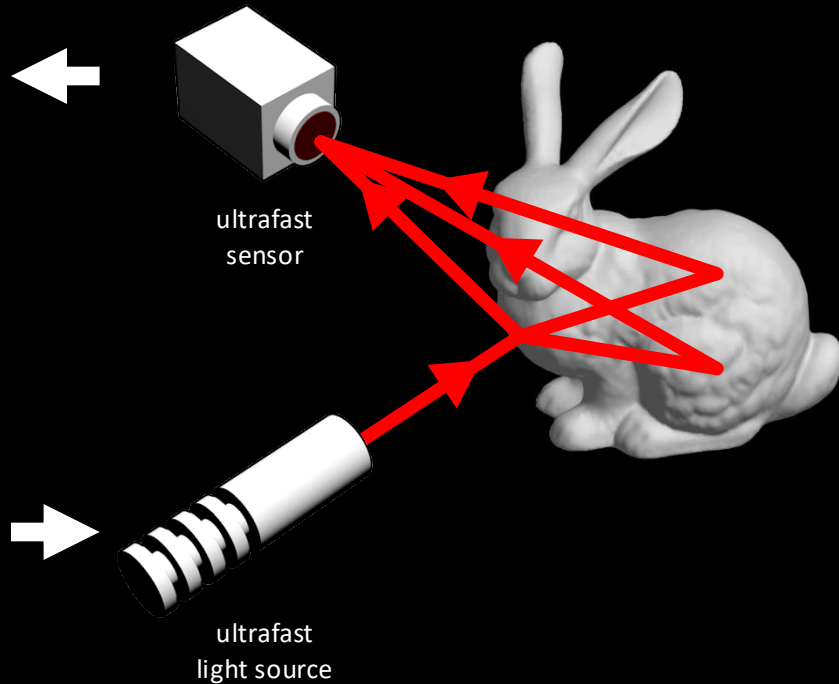
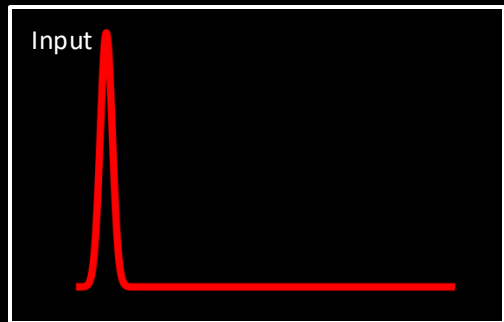
time →



# direct time-of-flight principle



time →





# direct time-of-flight principle



time →

transient measurement

## direct and indirect time-of-flight sensing

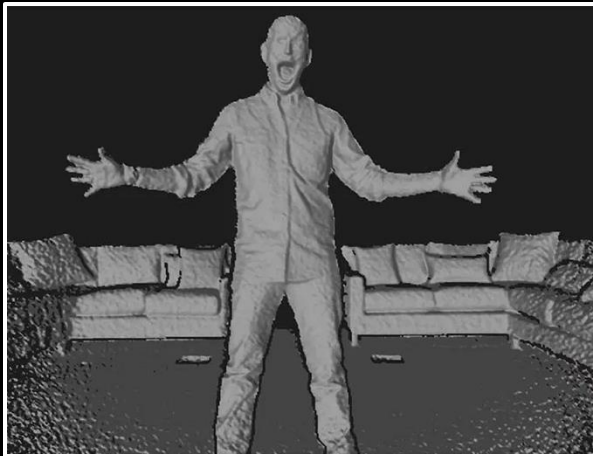


Direct time-of-flight sensor



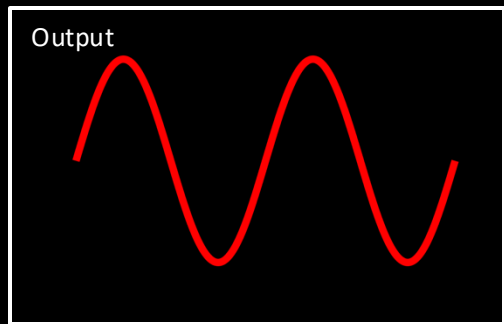
Indirect time-of-flight sensor

## direct and indirect time-of-flight sensing

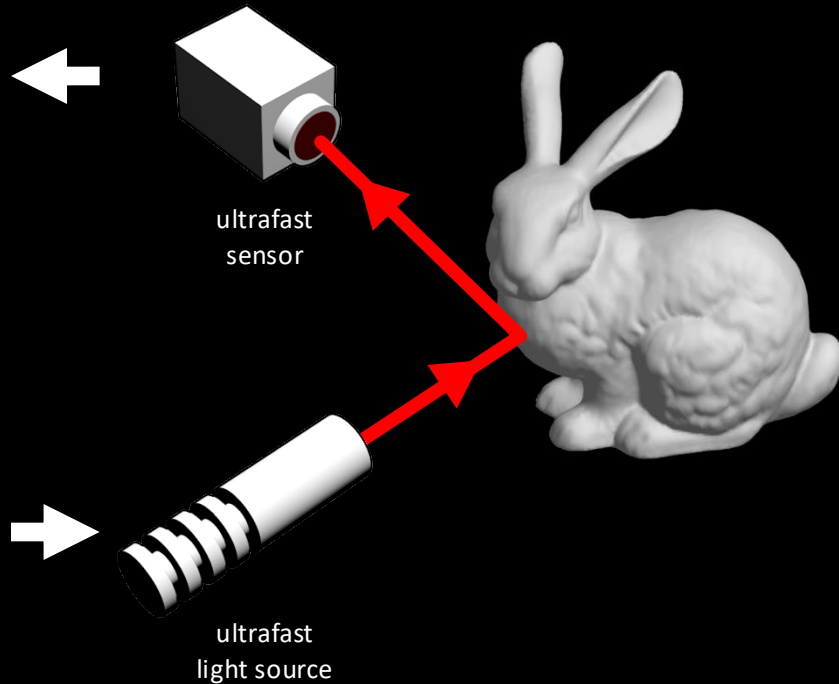
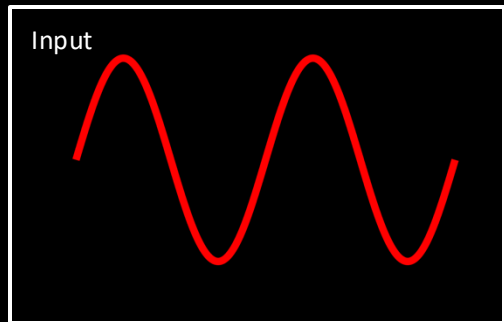


Indirect time-of-flight sensor

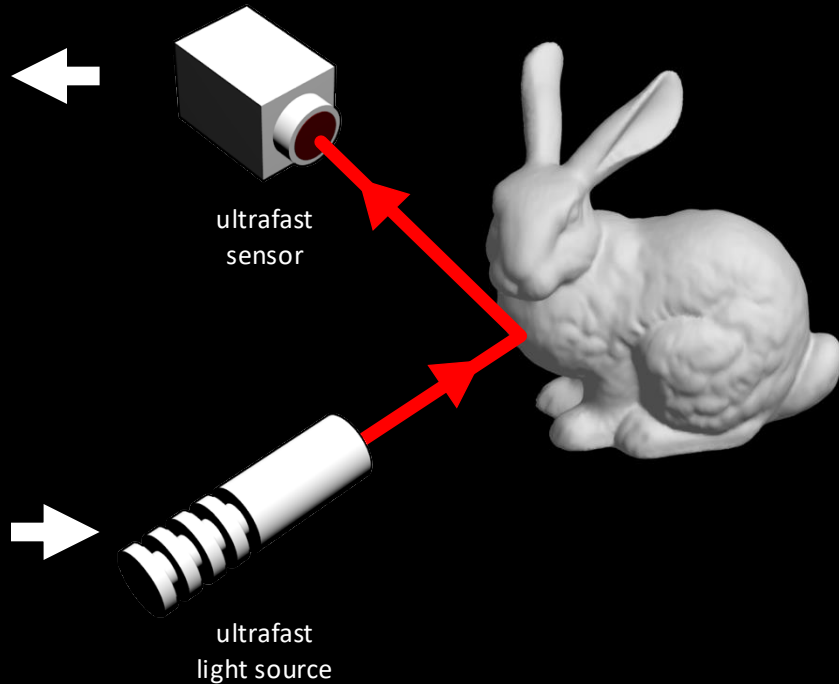
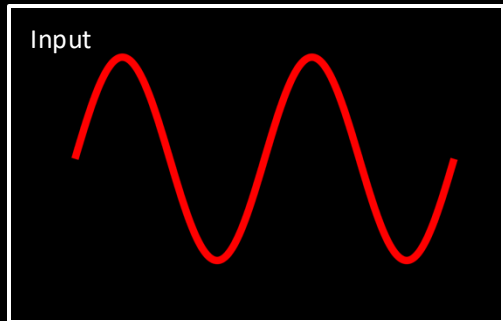
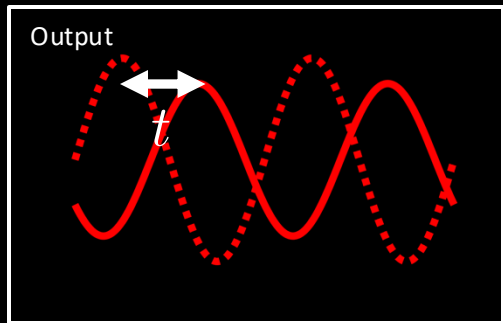
# indirect time-of-flight principle



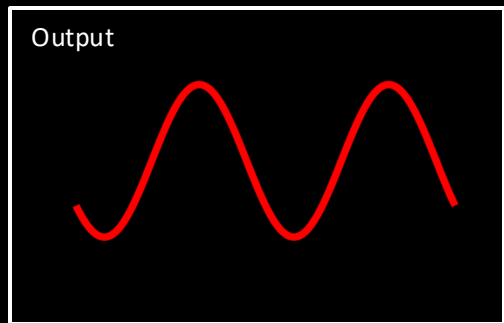
time →



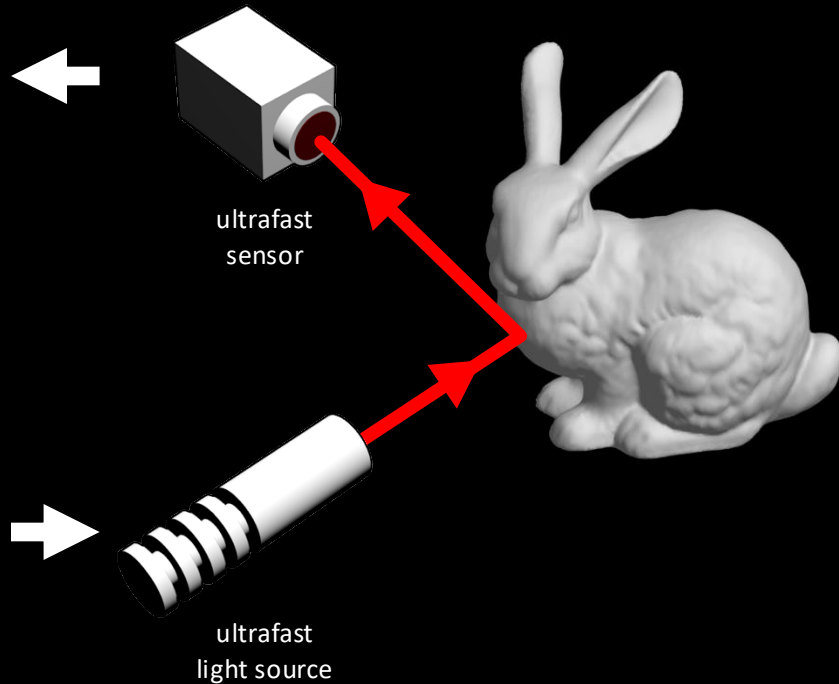
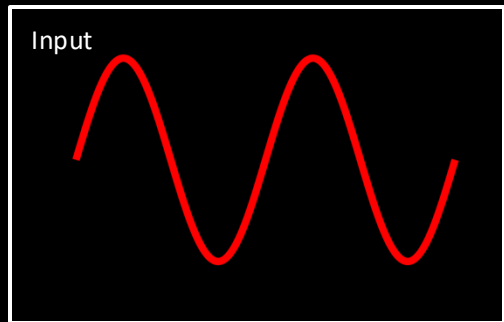
# indirect time-of-flight principle



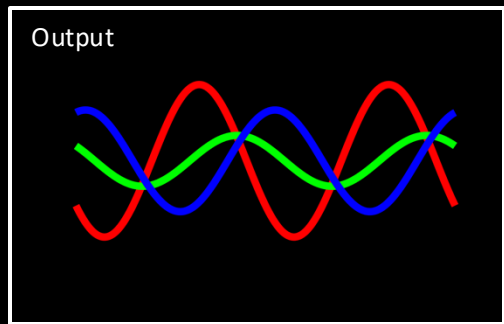
# indirect time-of-flight principle



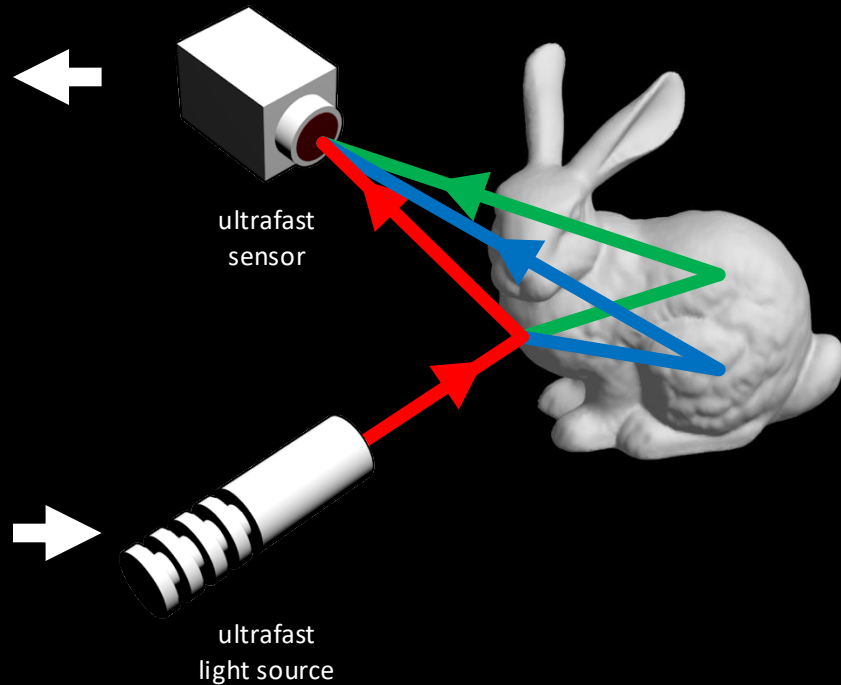
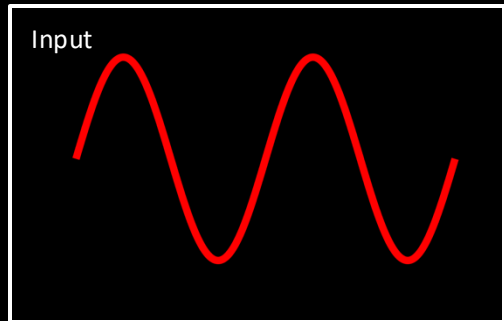
time →



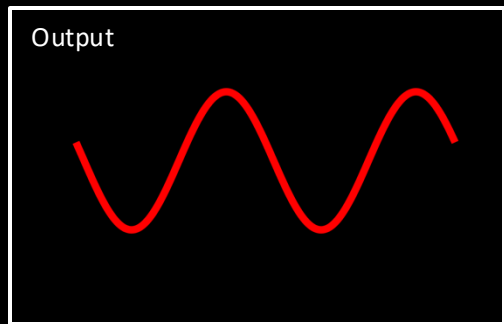
# indirect time-of-flight principle



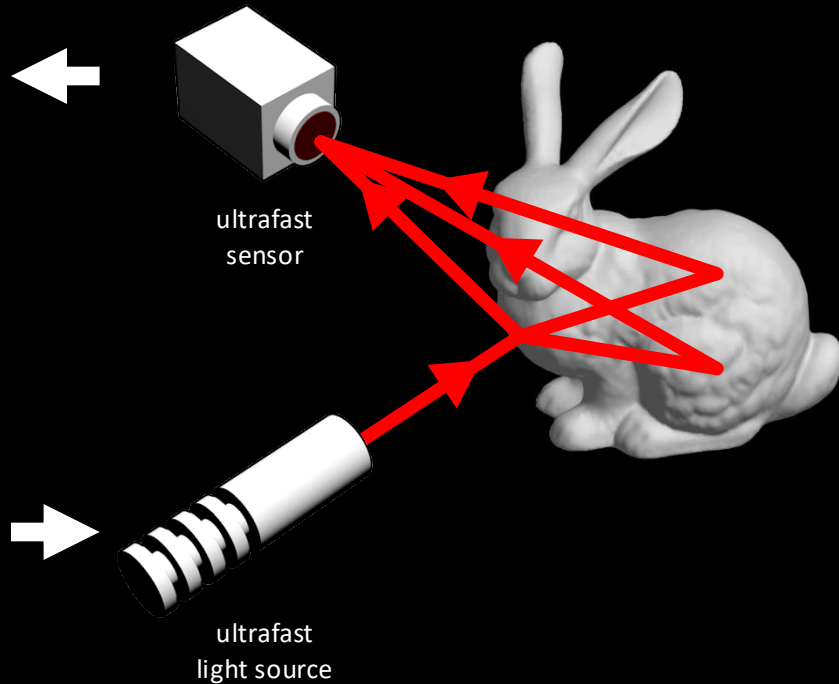
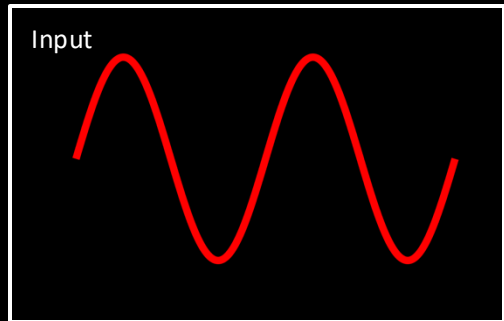
time →



# indirect time-of-flight principle

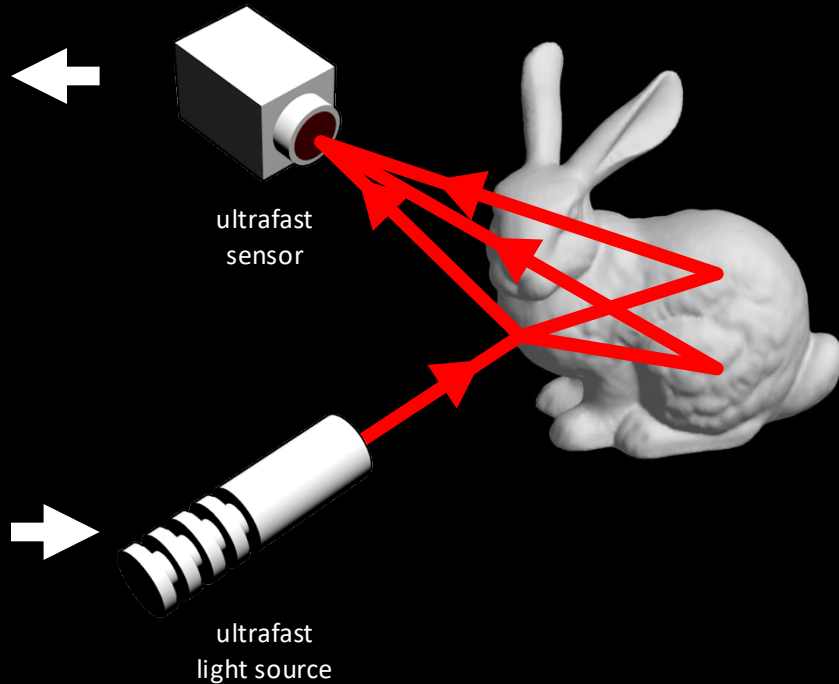
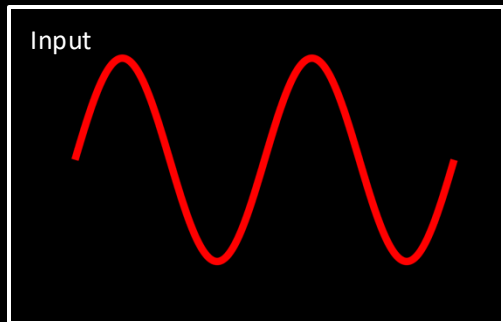
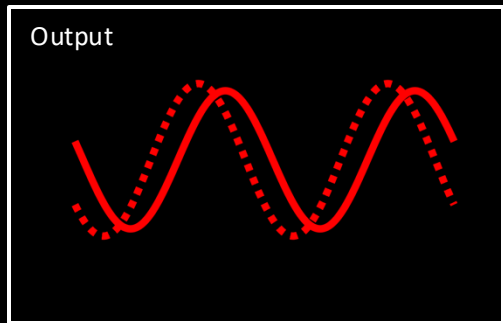


time →

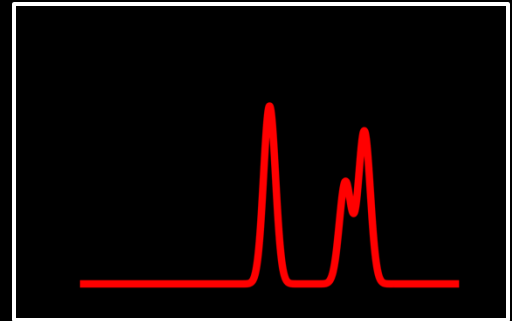
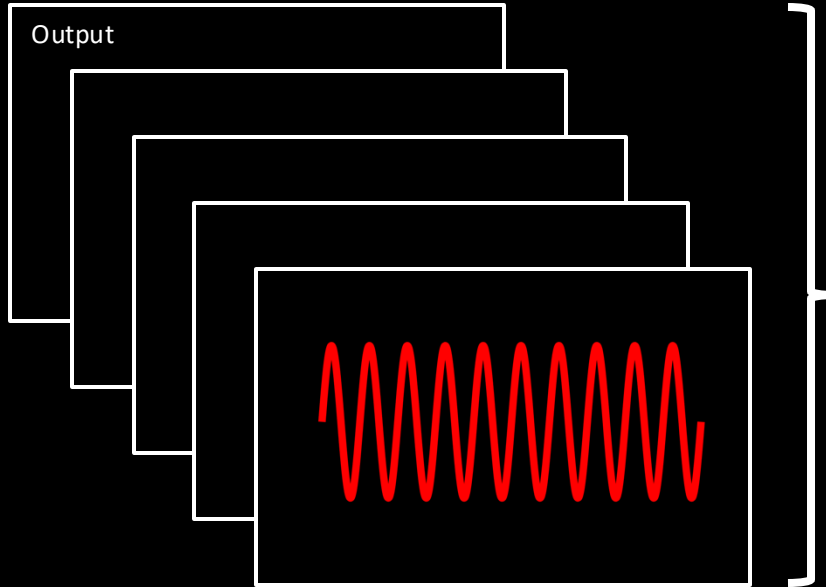




# indirect time-of-flight principle



# indirect time-of-flight principle



time →  
transient measurement

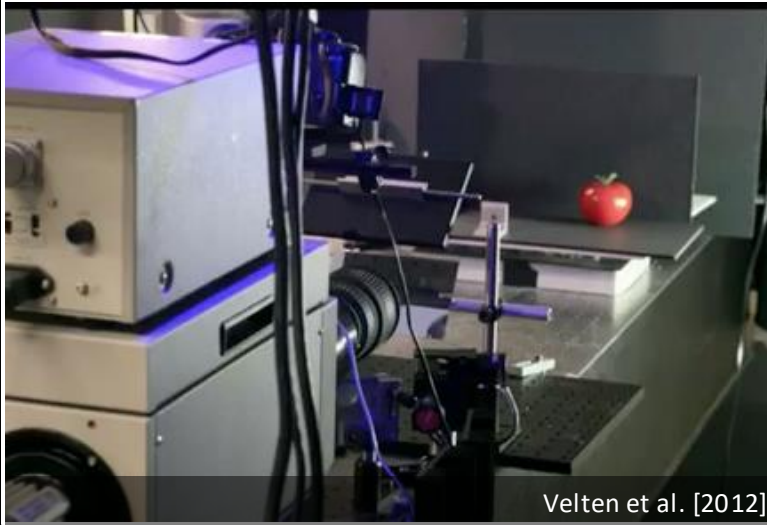
# transient sensing technologies

	optical coherence tomography	streak camera	single-photon avalanche diodes	time-of-flight cameras	avalanche photodiode
temporal resolution	1 femtosecond ( $10^{-15}$ secs)	1 picosecond ( $10^{-12}$ secs)	100 picosecond ( $10^{-10}$ secs)	1 nanosecond ( $10^{-9}$ secs)	10 nanoseconds ( $10^{-8}$ secs)
frame rate	quadrillion fps	trillion fps	10 billion fps	billion fps	100 million fps
distance travelled	1 micron ( $10^{-6}$ meters)	1 millimeter ( $10^{-3}$ meters)	10 centimeters ( $10^{-1}$ meters)	1 meter ( $10^0$ meters)	10 meters ( $10^1$ meters)

# transient sensing technologies

	optical coherence tomography	streak camera	single-photon avalanche diodes	time-of-flight cameras	avalanche photodiode
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# transient sensing technologies

	optical coherence tomography	streak camera	single-photon	time-of-flight	avalanche
temporal resolution	1 femtosecond ( $10^{-15}$ secs)	1 picosecond ( $10^{-12}$ secs)	 <p>Velten et al. [2012]</p>		
frame rate	quadrillion fps	trillion fps			
distance travelled	1 micron ( $10^{-6}$ meters)	1 millimeter ( $10^{-3}$ meters)			
			10 centimeters ( $10^{-1}$ meters)	1 meter ( $10^0$ meters)	10 meters ( $10^1$ meters)

# transient sensing technologies

	optical coherence tomography	streak camera	single-photon avalanche diodes	time-of-flight cameras	avalanche photodiode
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# transient sensing technologies

	optical coherence tomography	streak camera	single-photon	time-of-flight	avalanche photodiode
temporal resolution	1 femtosecond ( $10^{-15}$ secs)	1 picosecond ( $10^{-12}$ secs)	 <p>Velodyne VLS-128</p>		10 nanoseconds ( $10^{-8}$ secs)
frame rate	quadrillion fps	trillion fps			100 million fps
distance travelled	1 micron ( $10^{-6}$ meters)	1 millimeter ( $10^{-3}$ meters)			10 meters ( $10^1$ meters)

# transient sensing technologies

	optical coherence tomography	streak camera	single-photon avalanche diodes	time-of-flight cameras	avalanche photodiode
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# transient sensing technologies

temporal  
resolution

optical coherence  
tomography

1 femtosecond  
( $10^{-15}$  secs)

frame rate

quadrillion fps

distance  
travelled

1 micron  
( $10^{-6}$  meters)

streak camera

single-photon

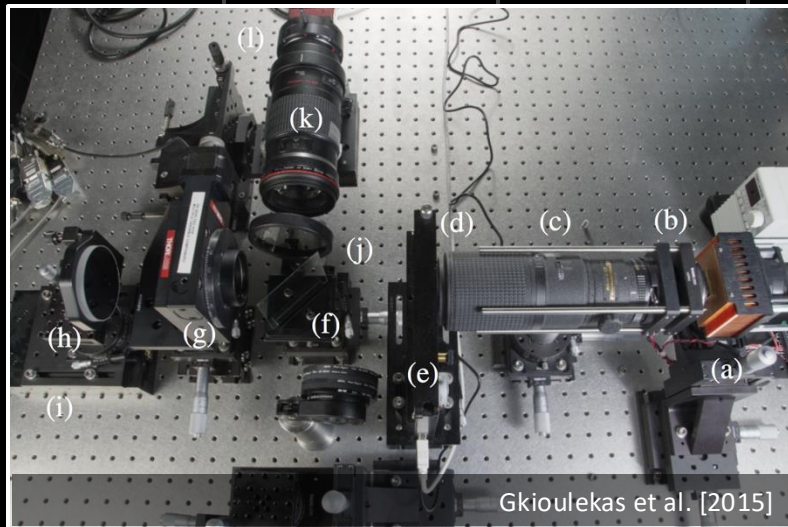
time-of-flight

avalanche  
photodiode

10 nanoseconds  
( $10^{-8}$  secs)

100 million fps

10 meters  
( $10^1$  meters)



Gkioulekas et al. [2015]

1 millimeter  
( $10^{-3}$  meters)

10 centimeters  
( $10^{-1}$  meters)

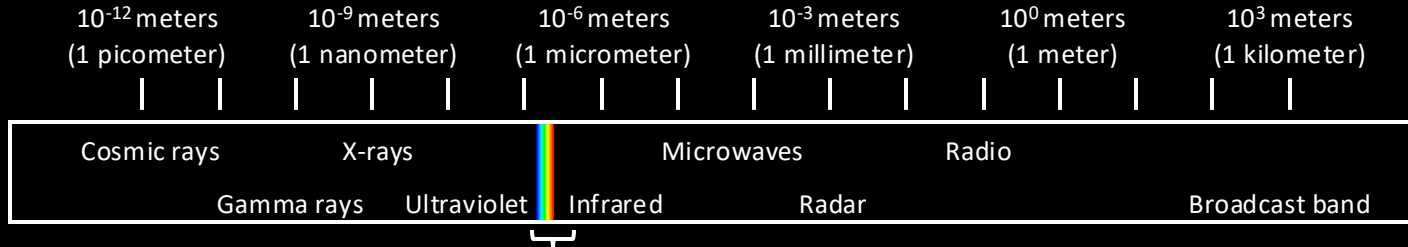
1 meter  
( $10^0$  meters)

# transient sensing technologies

	optical coherence tomography	streak camera	single-photon avalanche diodes	time-of-flight cameras	avalanche photodiode
temporal resolution				1 nanosecond ( $10^{-9}$ secs)	10 nanoseconds ( $10^{-8}$ secs)
frame rate				billion fps	100 million fps
distance travelled	1 micron ( $10^{-6}$ meters)	1 millimeter ( $10^{-3}$ meters)	10 centimeters ( $10^{-1}$ meters)	1 meter ( $10^0$ meters)	10 meters ( $10^1$ meters)

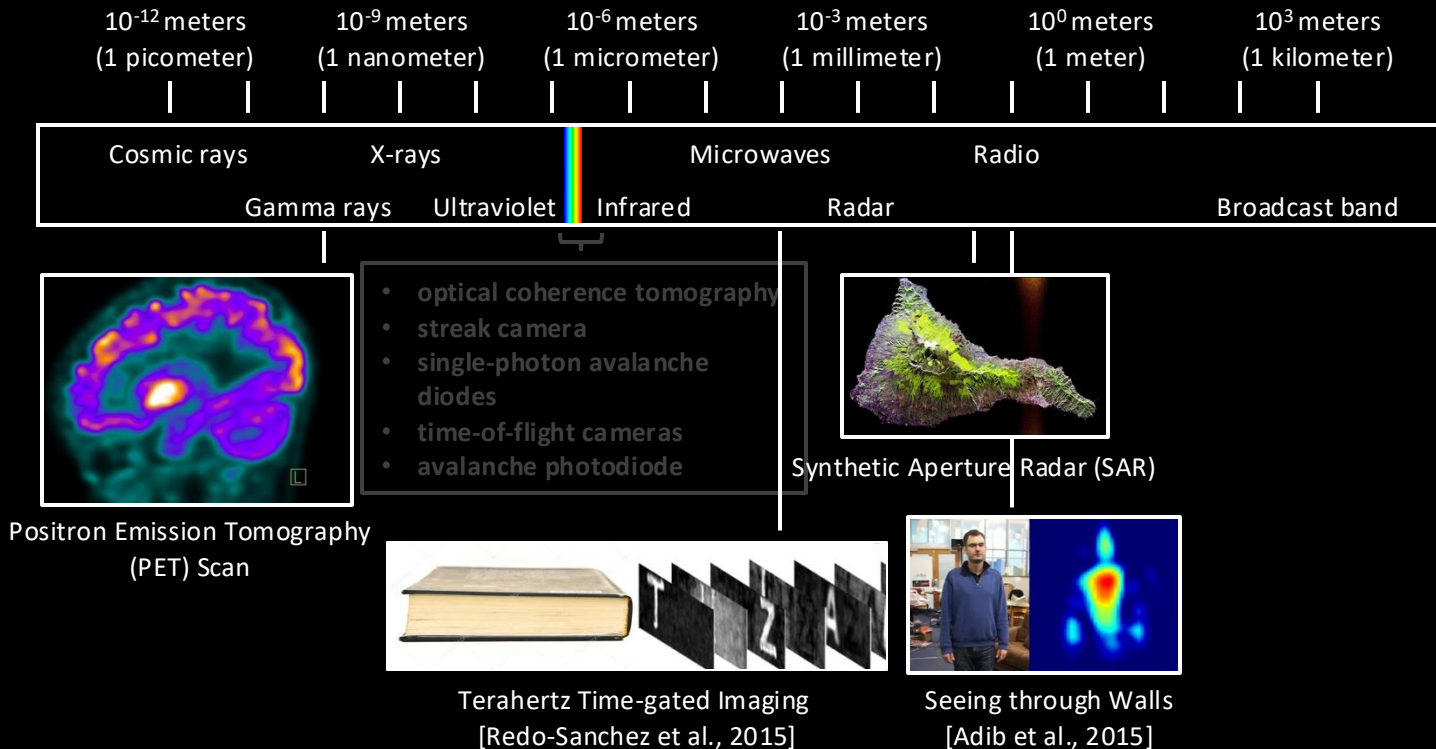
Heide et al. [2013]

# spectrum of transient sensing technologies



- optical coherence tomography
- streak camera
- single-photon avalanche diodes
- time-of-flight cameras
- avalanche photodiode

# spectrum of transient sensing technologies



# spectrum of transient sensing technologies

$10^{-12}$  meters  
(1 picometer)

$10^{-9}$  meters  
(1 nanometer)

$10^{-6}$  meters  
(1 micrometer)

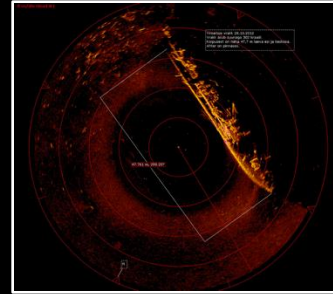
$10^{-3}$  meters  
(1 millimeter)

$10^0$  meters  
(1 meter)

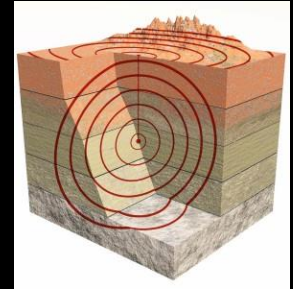
$10^3$  meters  
(1 kilometer)



Ultrasound Imaging



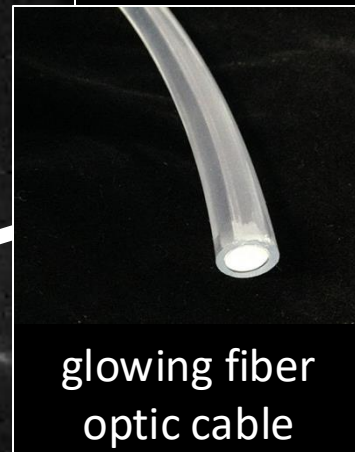
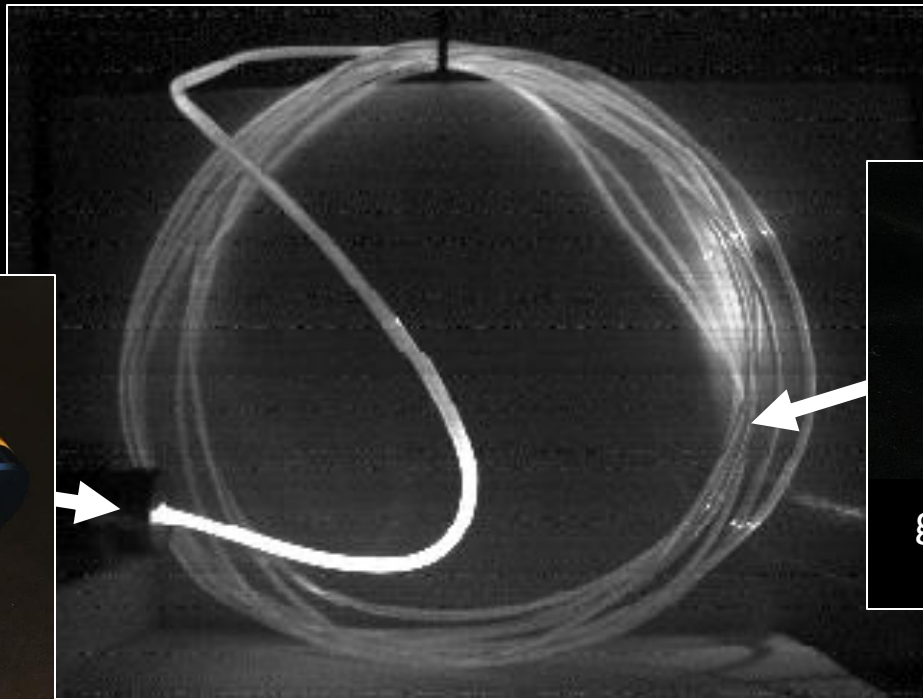
SONAR  
(Sound Navigation and Ranging)



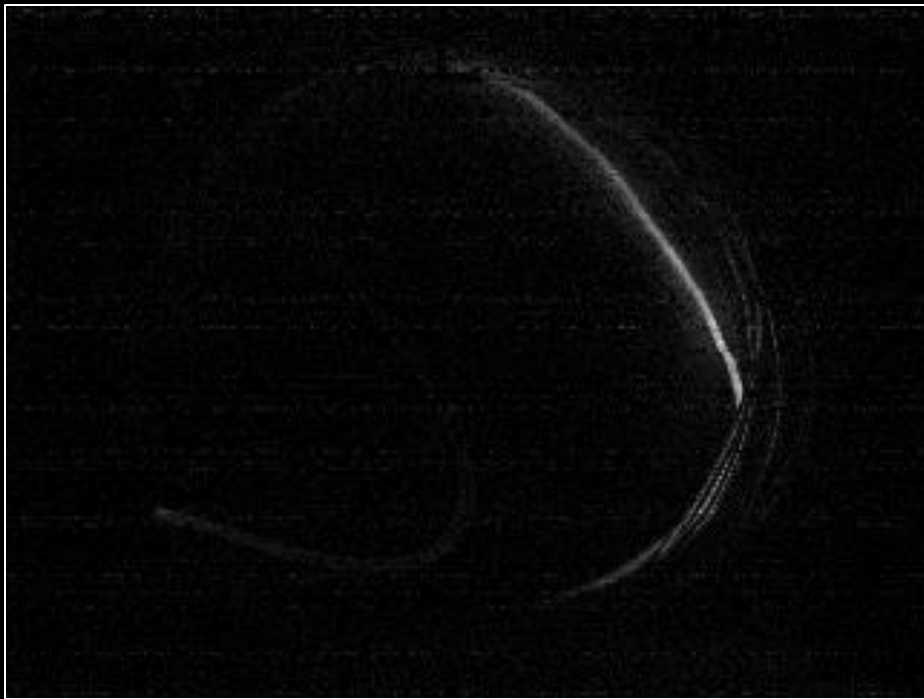
Seismic Imaging

## overview

- Time-resolved imaging
- Single-photon avalanche diodes (SPADs)
- Single-photon lidar
- Non-line-of-sight imaging
- neural rendering for propagating light



regular image



transient image

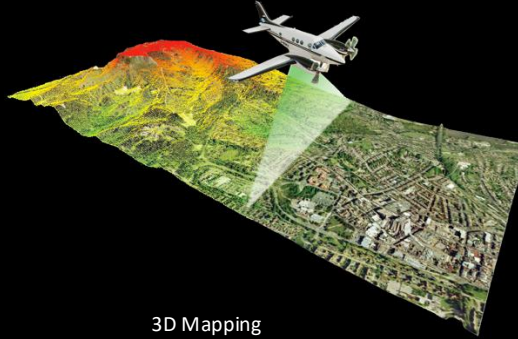


# Applications



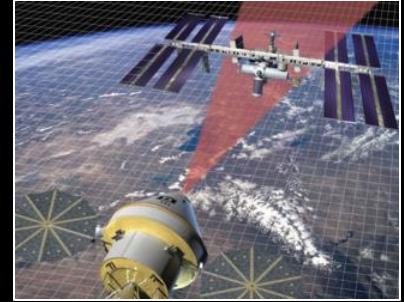
Autonomous Navigation

Image by Wikimedia Commons



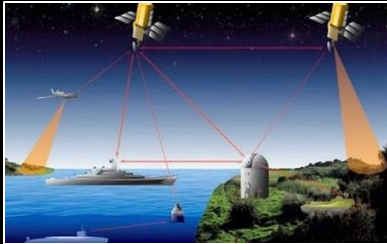
3D Mapping

Image by UDAR-America



Space Station Docking

Image by NASA



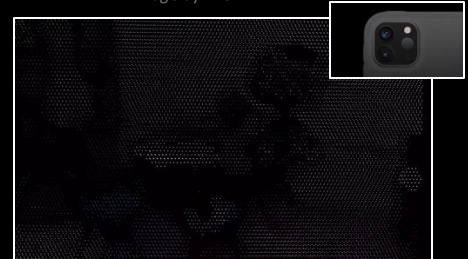
Optical Communications

Image by Siasat Daily



Biomedical Imaging

Image by Washington University



Consumer Electronics (2020 iPad Pro)

Video by Tim Fields

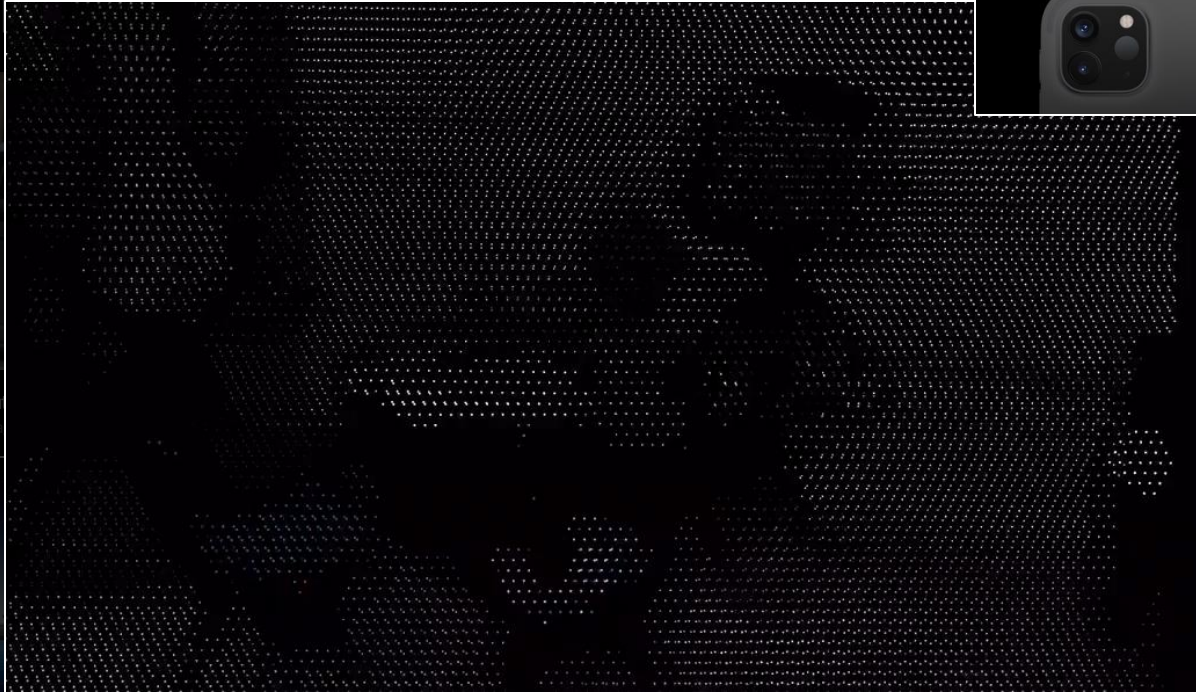
# Applications



Autonomous Imaging

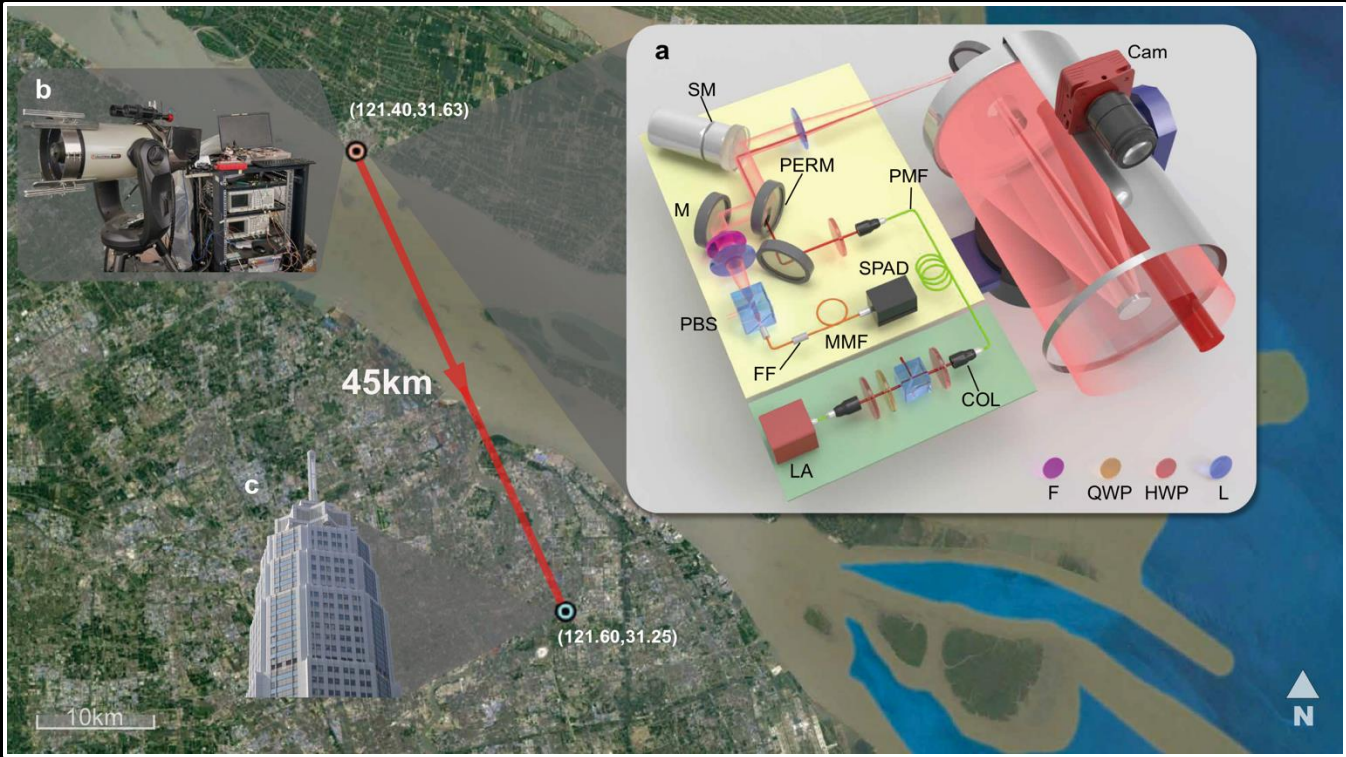


Optical Communications  
Image by Siasat Data



Biomedical Imaging  
Image by Washington University

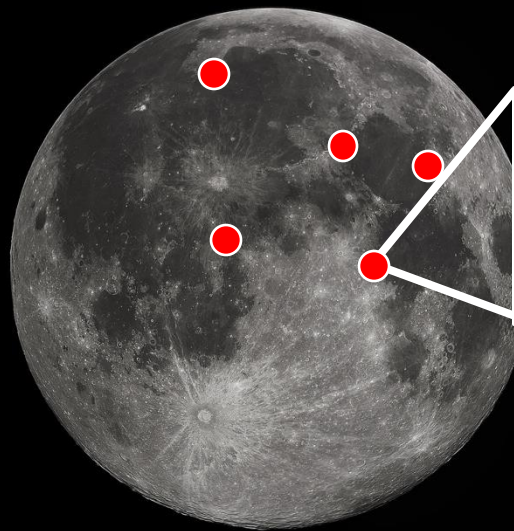
Consumer Electronics (2020 iPad Pro)  
Video by Tim Fields





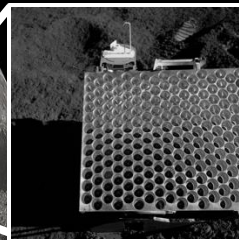


International Laser Ranging Service (ILRS)



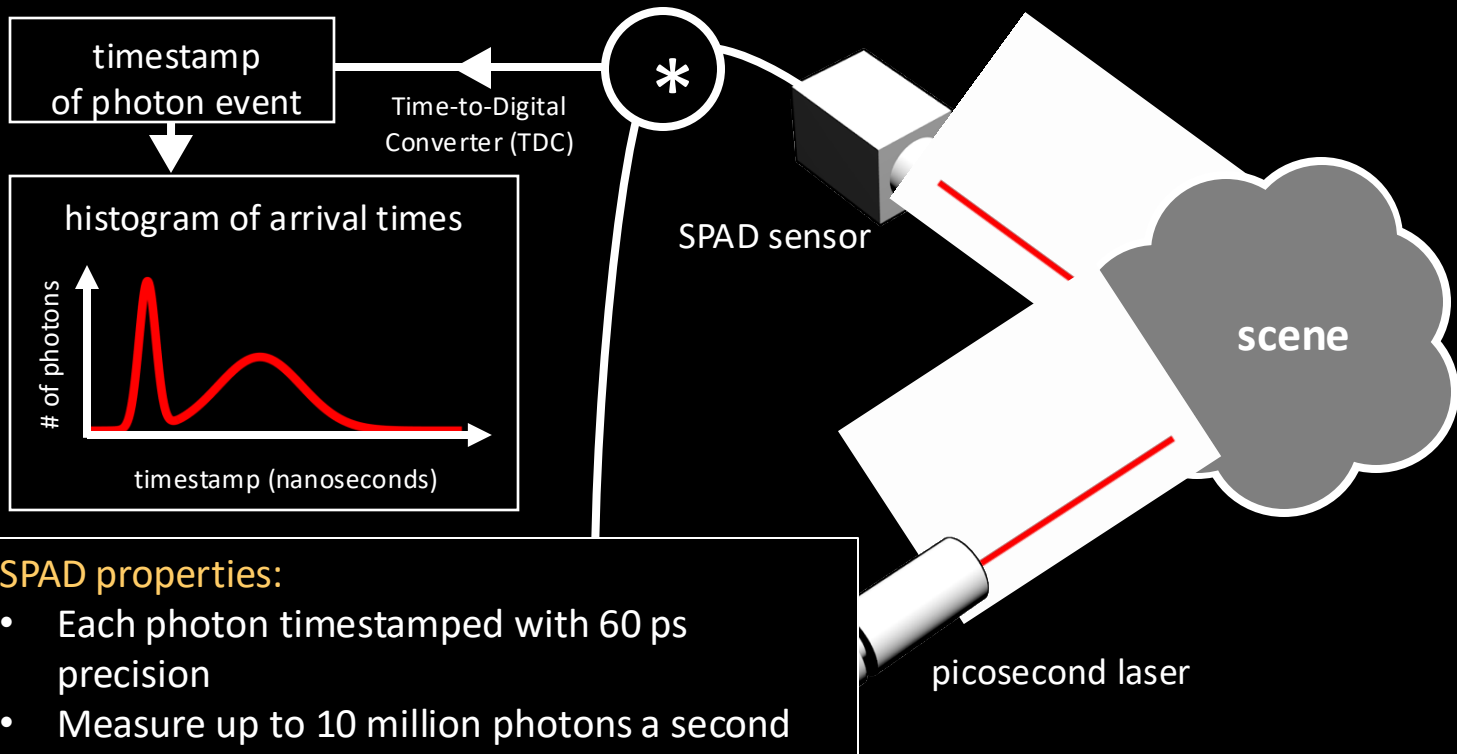
Lunar Laser Ranging (LLR)

● - Location of Lunar Retroreflector

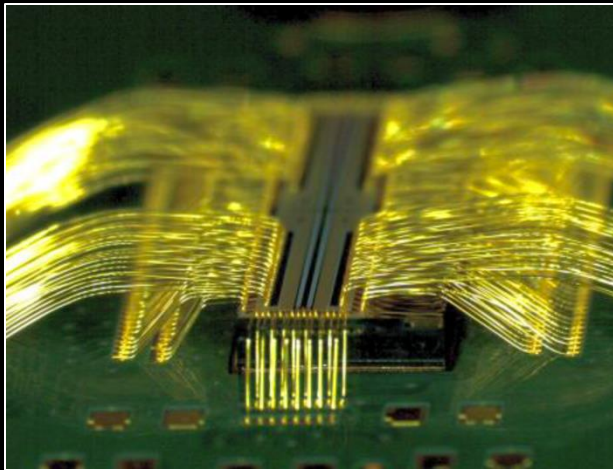


Retroreflector

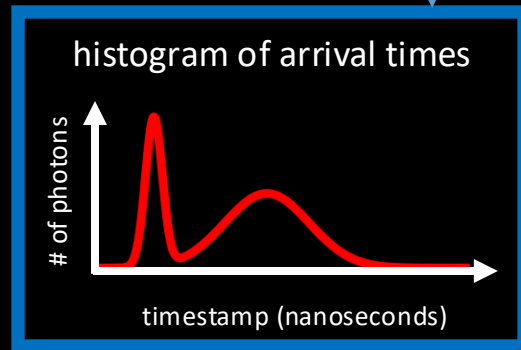
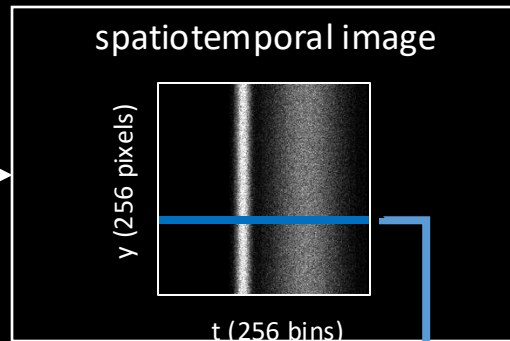
# single-photon avalanche diode (SPAD)



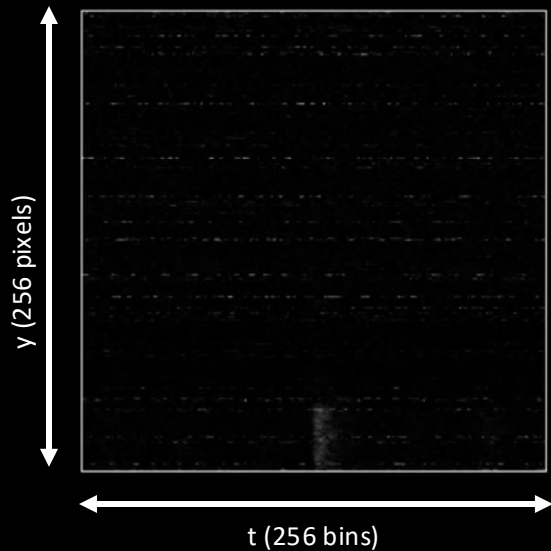
# linear array of SPADs



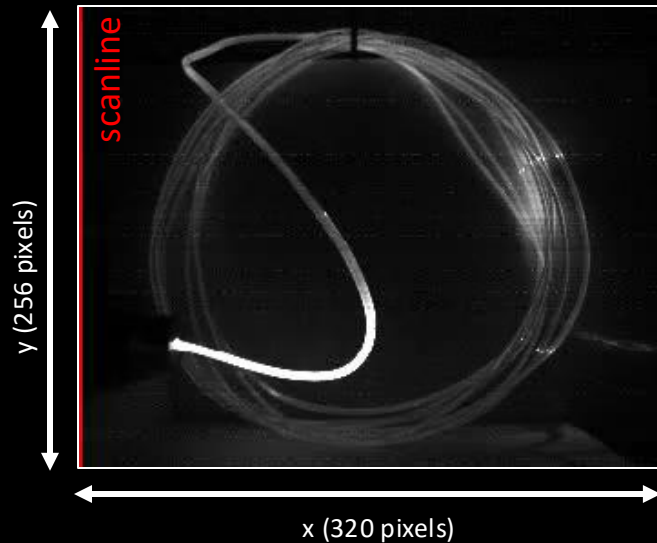
LinoSPAD from FastTree 3D



## scanning procedure

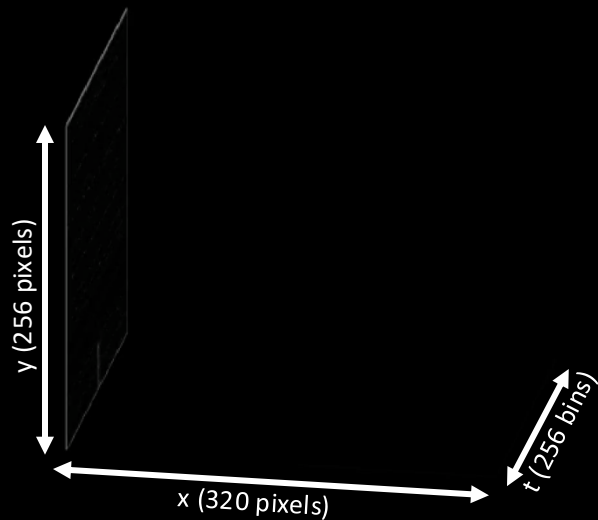


SPAD output

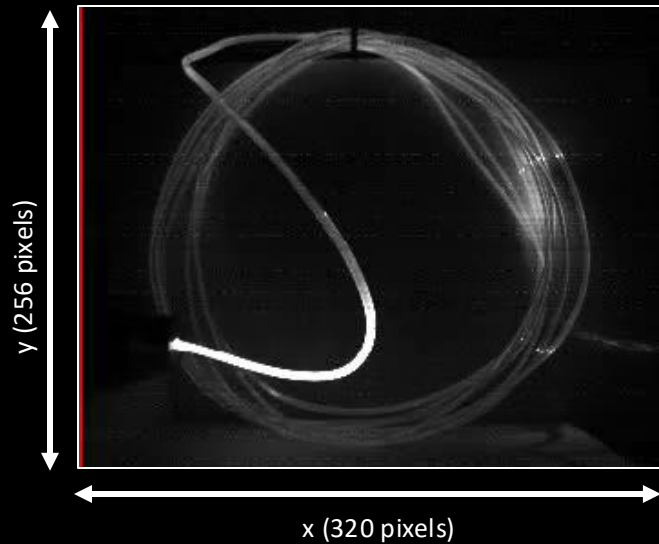


regular image

## scanning procedure



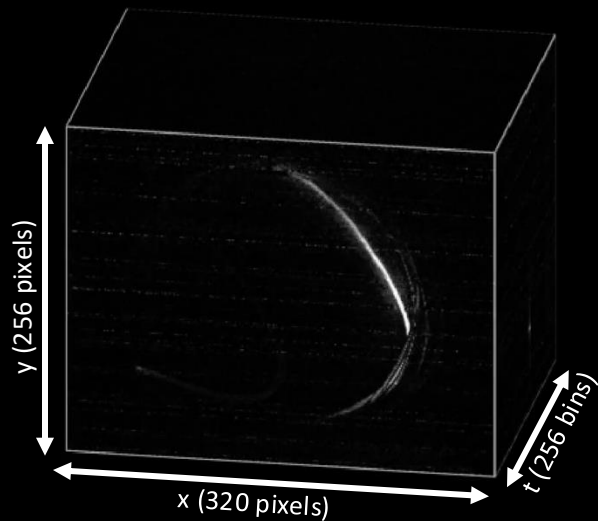
transient image



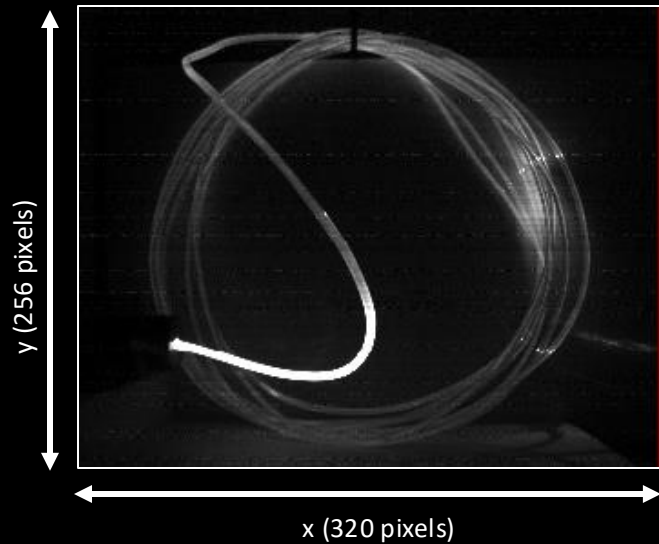
regular image



## scanning procedure



transient image



regular image

## overview

- Time-resolved imaging
- Single-photon avalanche diodes (SPADs)
- Single-photon lidar
- Non-line-of-sight imaging
- passive ultra-wideband sensing

# LiDAR

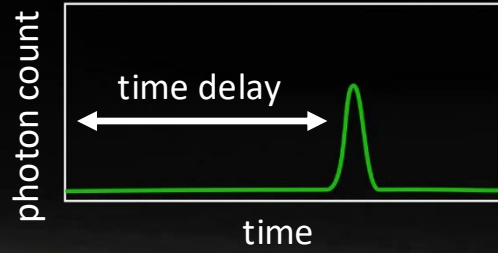
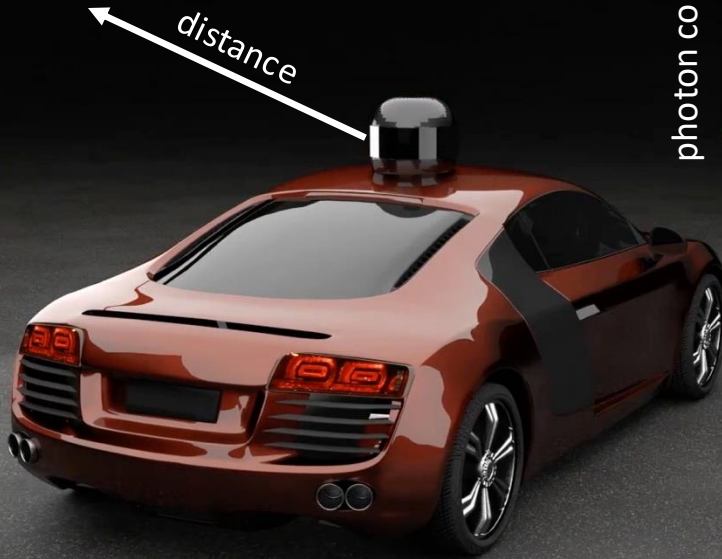


photon count

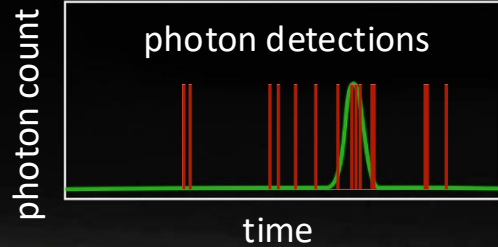
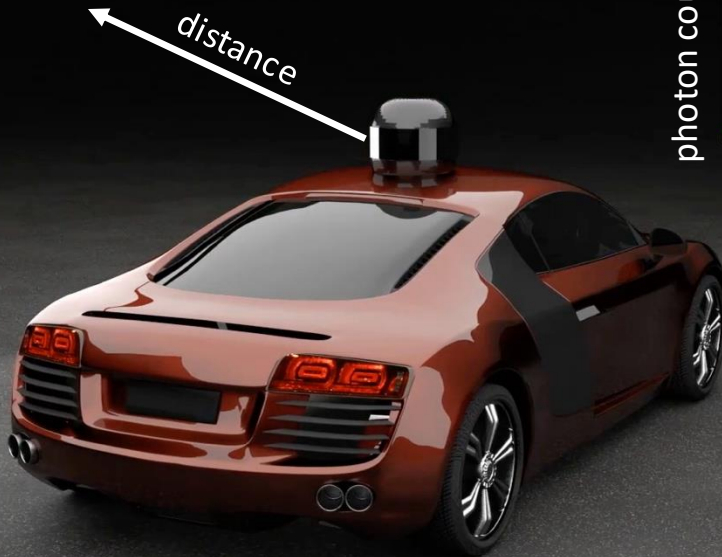


time

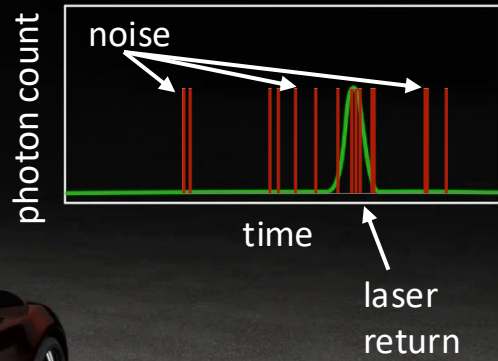
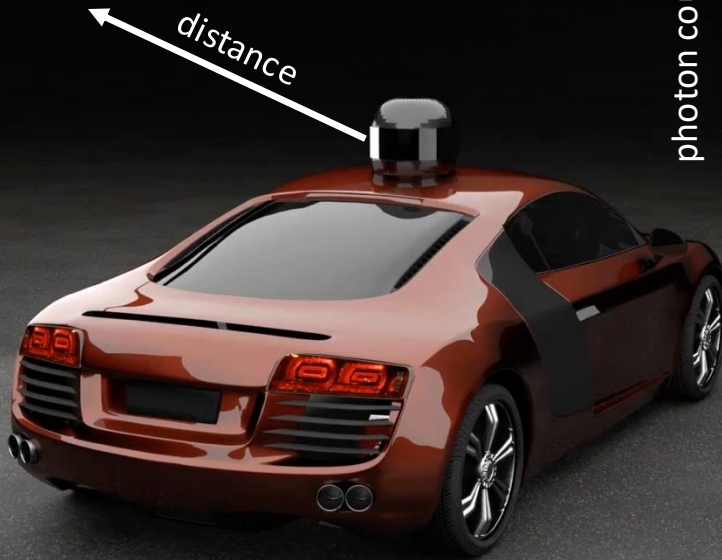
# LiDAR



# LiDAR

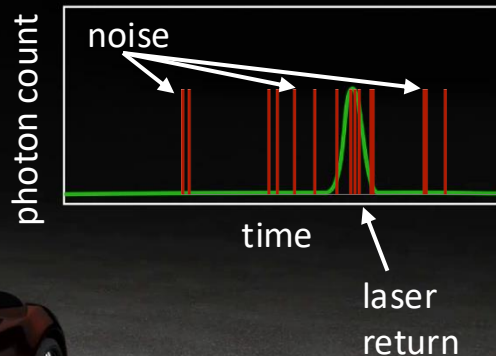
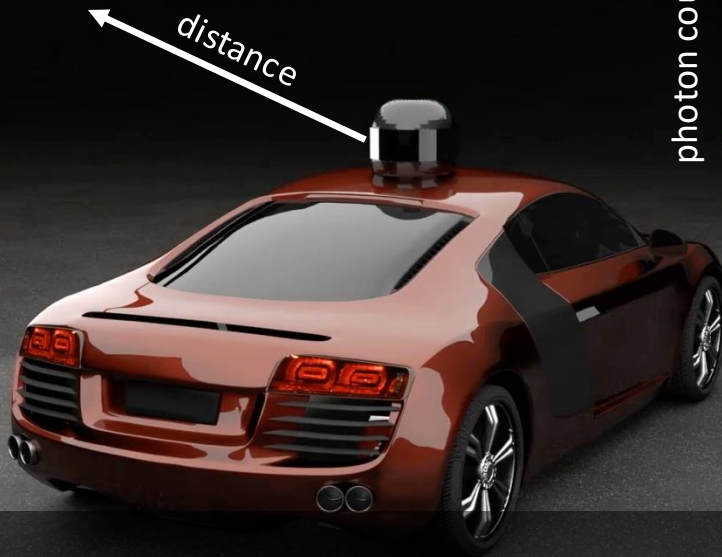


# LiDAR





# LiDAR



## Challenges

1. Light efficiency / photon sensitivity (determines range)
2. High-speed time stamping (determines accuracy)
3. Computational algorithms (determines range and accuracy)

# Challenges

1. Light efficiency / photon sensitivity (determines range)
  - intensity of returned light falls off with  $1/d^2$ , i.e. very quickly!
  - emit as much light as possible - *fundamentally limited by eye safety* (in most applications)
  - detect as much light as possible, ideally individual photons



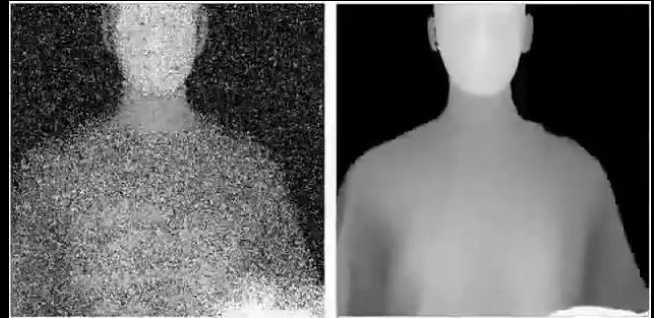
# Challenges

2. High-speed time stamping (determines accuracy)
  - speed of light is  $\sim 300,000,000$  m/s
  - $1\text{ m} = 3.3\text{ ns}$ ;  $1\text{ cm} = 33\text{ ps}$ ;  $1\text{ mm} = 3.3\text{ ps}$
  - need picosecond-accurate time-stamping  $\rightarrow$  usually high-end electronics, but also done with ASICs, FPGAs

# Challenges

3. Computational algorithms (determines range and accuracy)
  - robust depth estimation from single photon per pixel!

Kirmani et al. "First-photon Imaging", Science 2014



conventional method

first-photon imaging

# (Single-photon) Avalanche Photodiodes

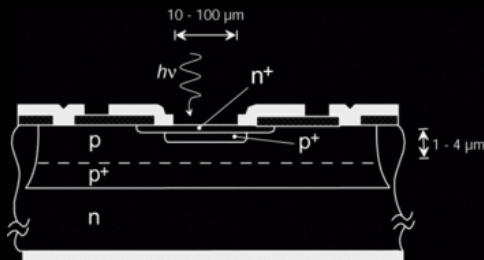
Linear mode (i.e., avalanche photodiode or *APD*):

acts like a conventional photodiode with extremely high gain or amplification  
time resolution  $> 300 \text{ ps} - 10 \text{ ns}$

Geiger mode (i.e., single-photon avalanche photodiode *SPAD*):

500x more sensitive, i.e. single-photon sensitive

time resolution  $\sim 50 \text{ ps}$



Semiconductor devices

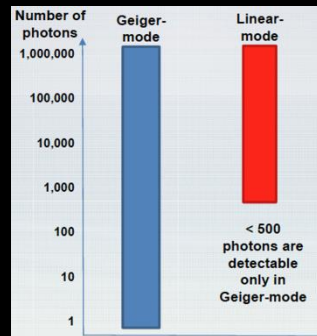
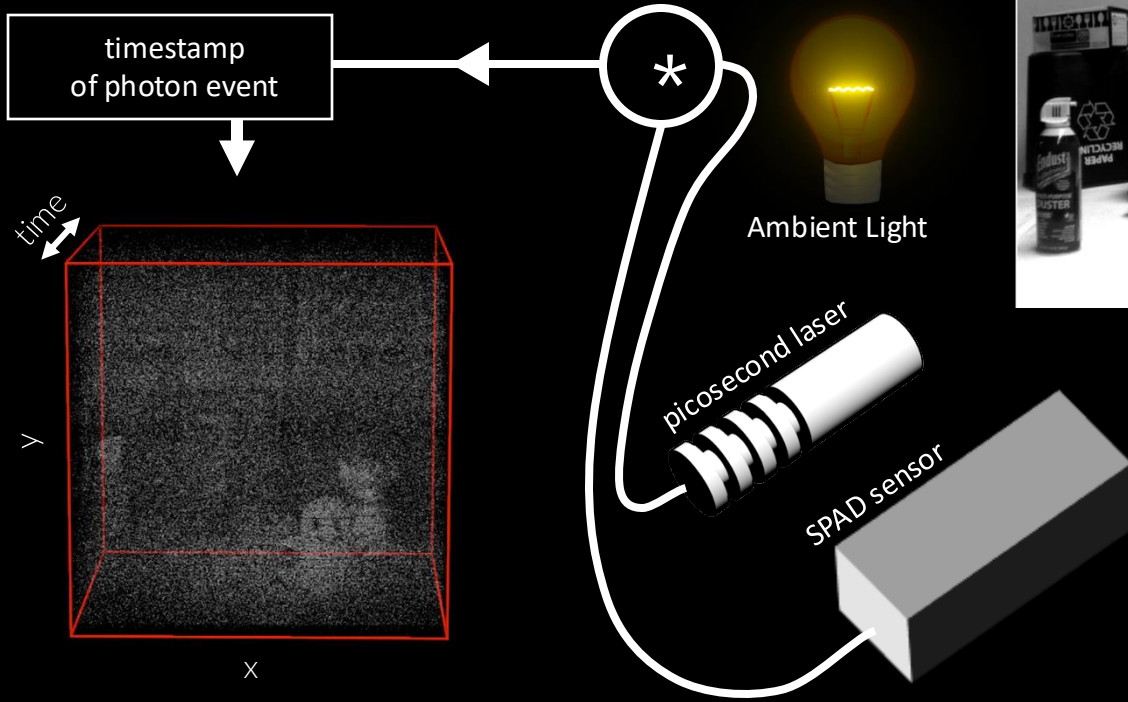
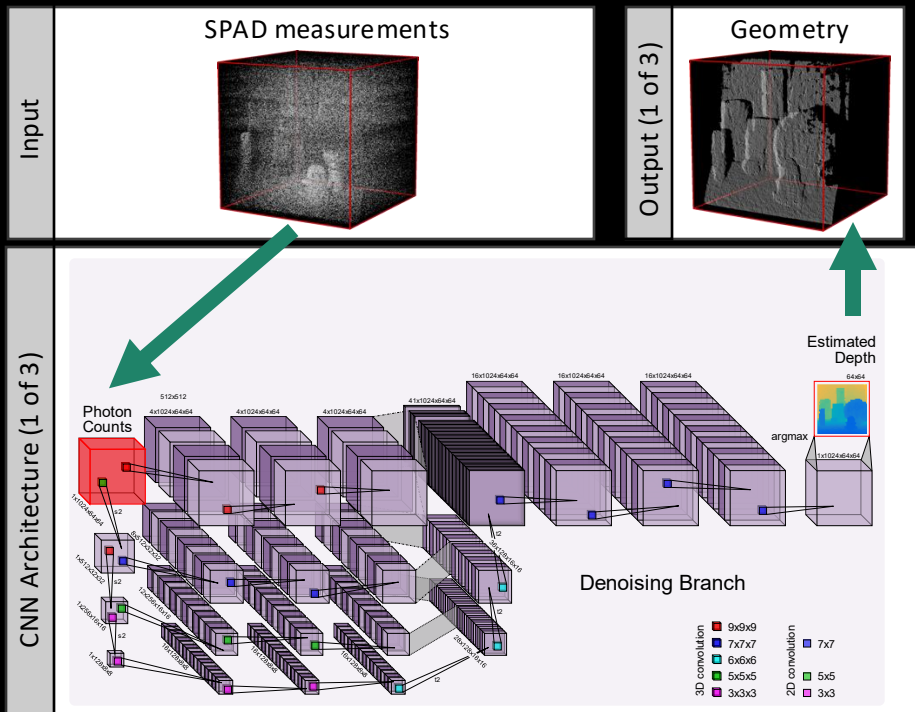


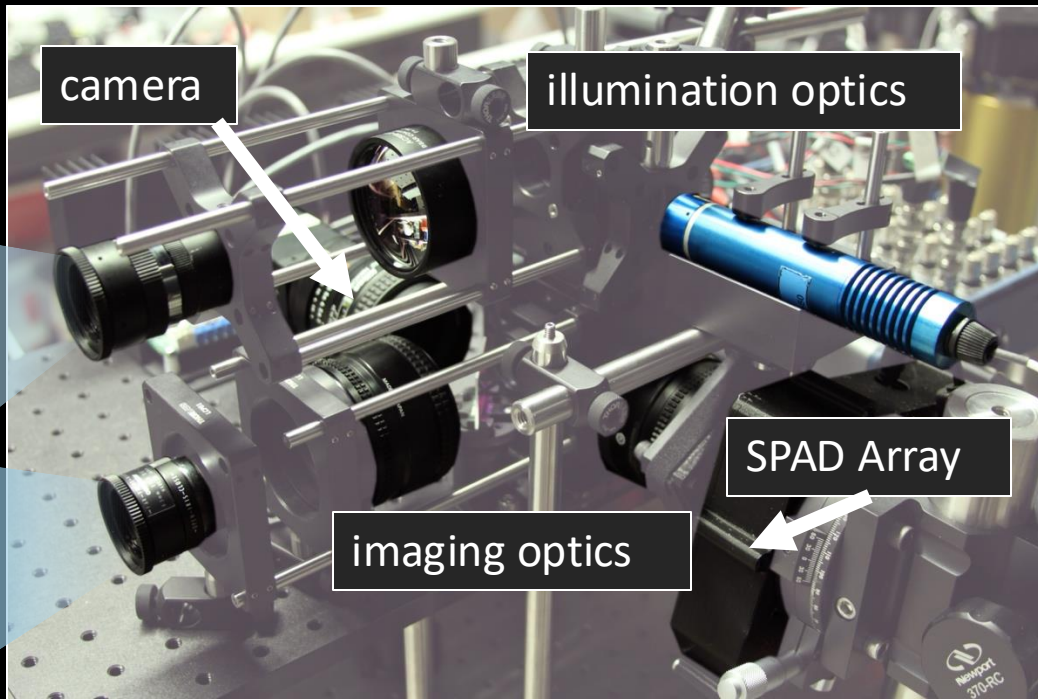
image by Princeton Lightwave

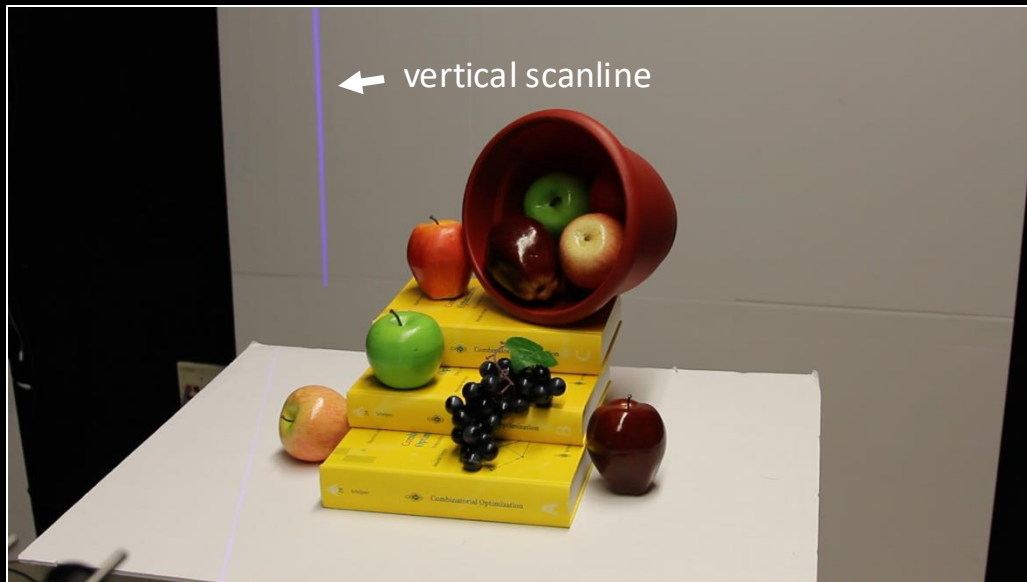
# Single-Photon Avalanche Diodes



# CNN Architecture for Depth Estimation



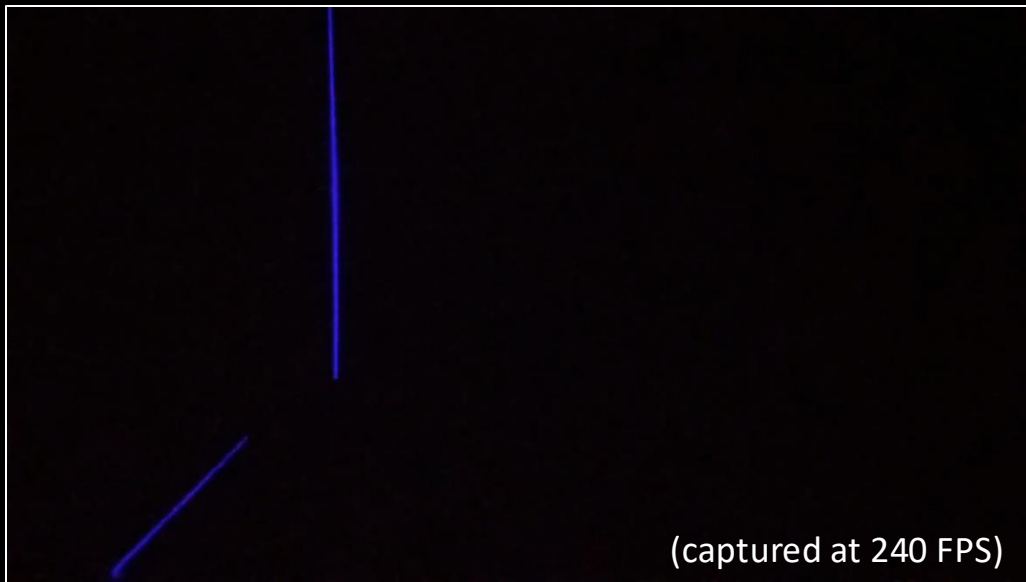




scan rate: 20 Hz

lights on

(note: laser illumination is too weak to observe visually while scanning under ambient light)



scan rate: 20 Hz

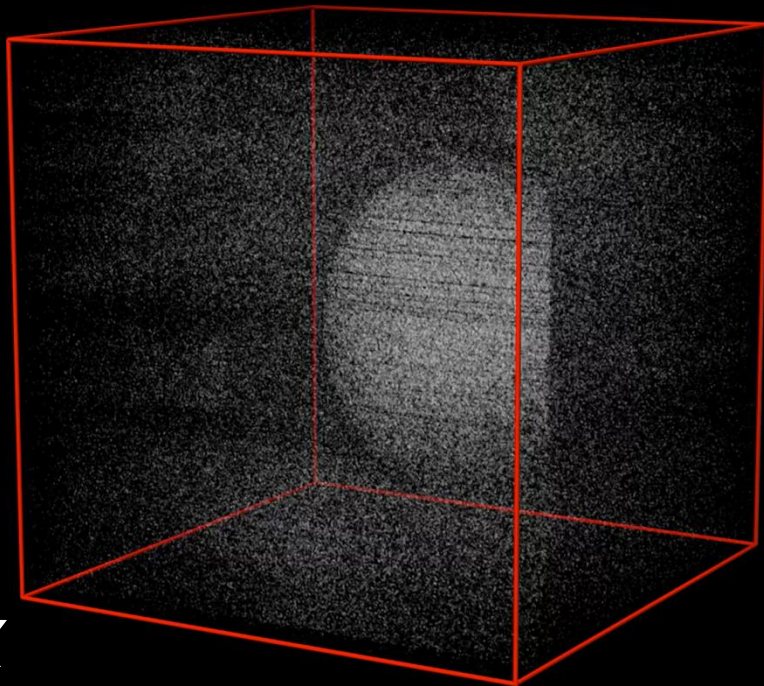
(captured at 240 FPS)

lights **off**





Intensity image

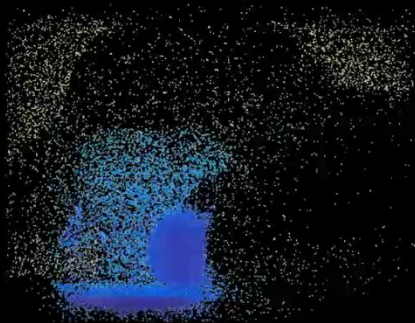


SPAD measurements (20 Hz)

Average per spatial position  
0.64 Signal Detections  
0.87 Background Detections



Intensity image



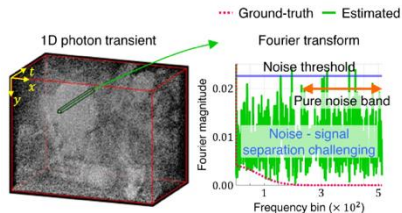
Log-matched filter



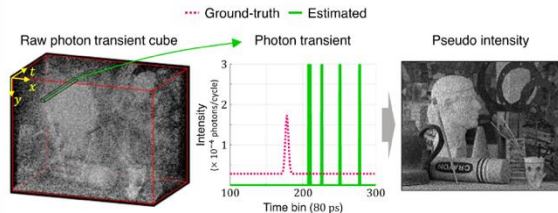
[Rapp and Goyal 2017]

# CASPI: collaborative photon processing for active single-photon imaging

a Photon correlations and noise estimation

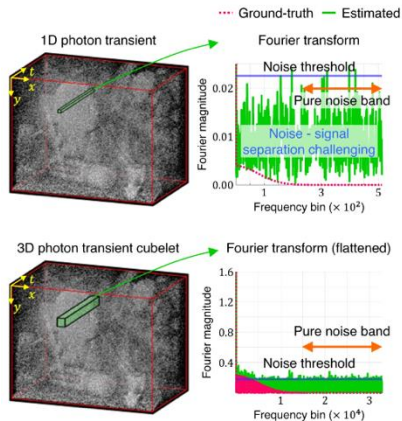


b Hierarchical photon processing

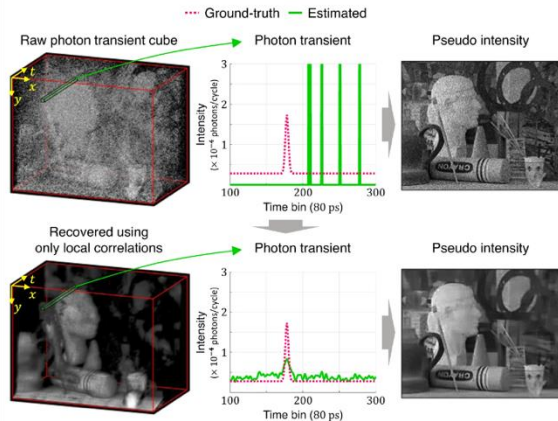


# CASPI: collaborative photon processing for active single-photon imaging

a Photon correlations and noise estimation

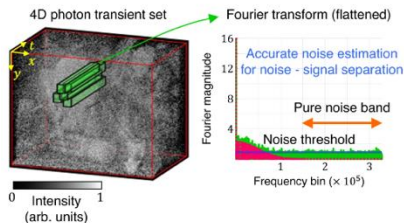
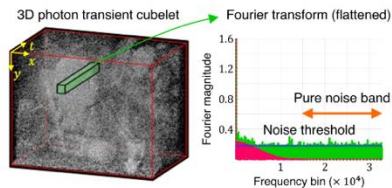
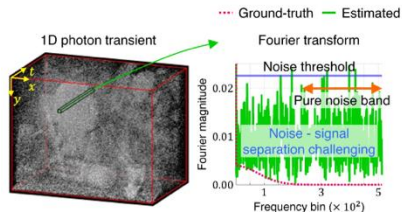


b Hierarchical photon processing

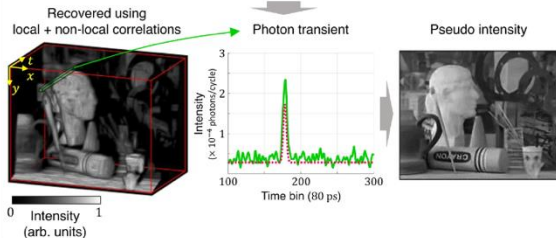
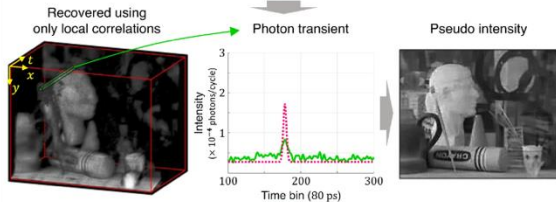
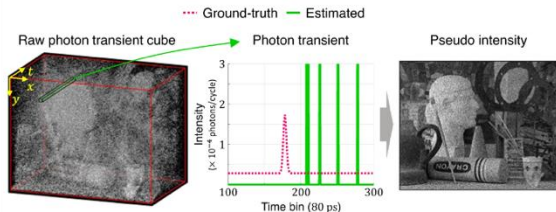


# CASPI: collaborative photon processing for active single-photon imaging

**a Photon correlations and noise estimation**



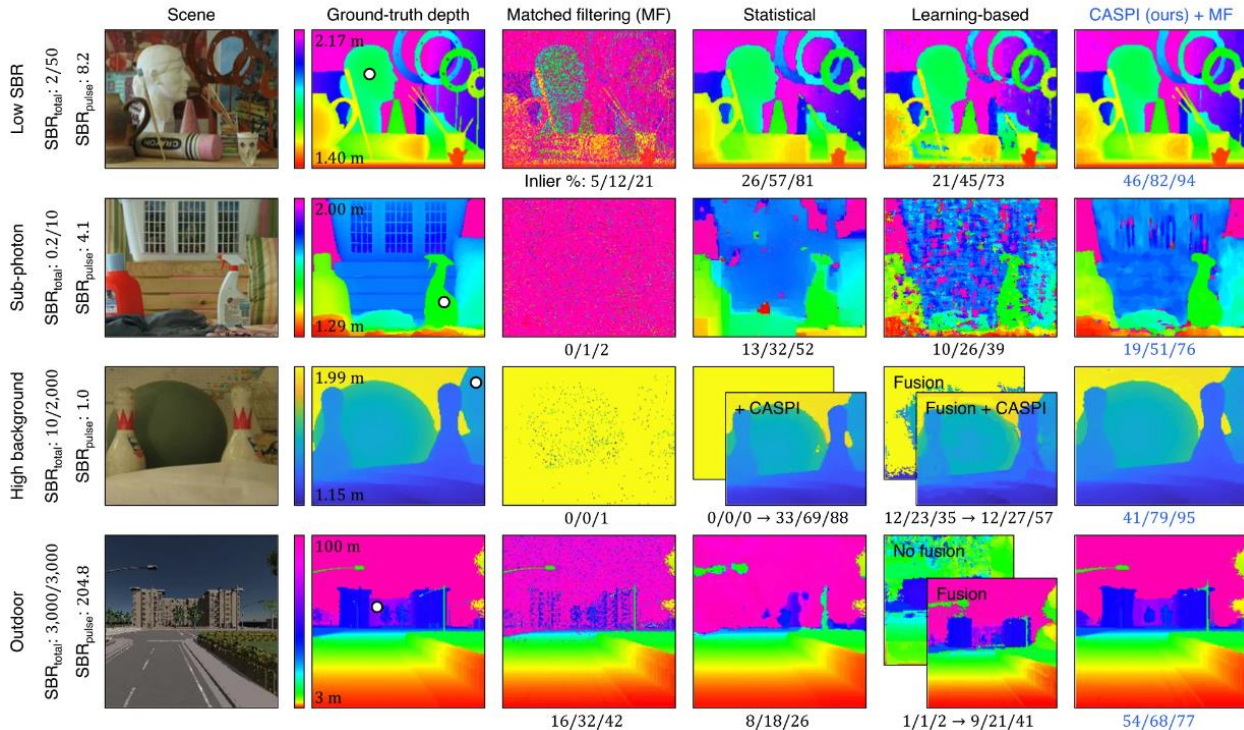
**b Hierarchical photon processing**



# CASPI: collaborative photon processing for active single-photon imaging

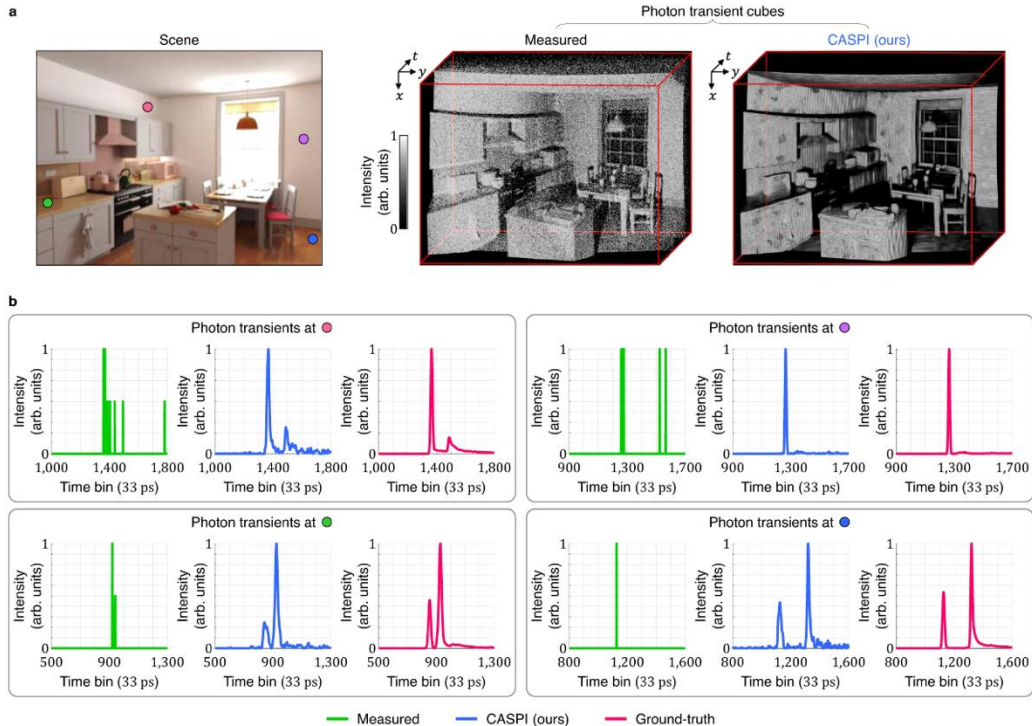
a

Comparisons with direct estimation approaches





# CASPI: collaborative photon processing for active single-photon imaging



## overview

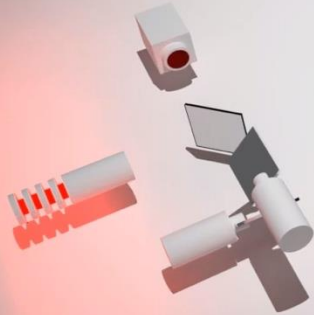
- Time-resolved imaging
- Single-photon avalanche diodes (SPADs)
- Single-photon lidar
- Non-line-of-sight imaging (part 1)
- neural rendering for propagating light



**04.800 ns**

1st bounce: 2.7 ns

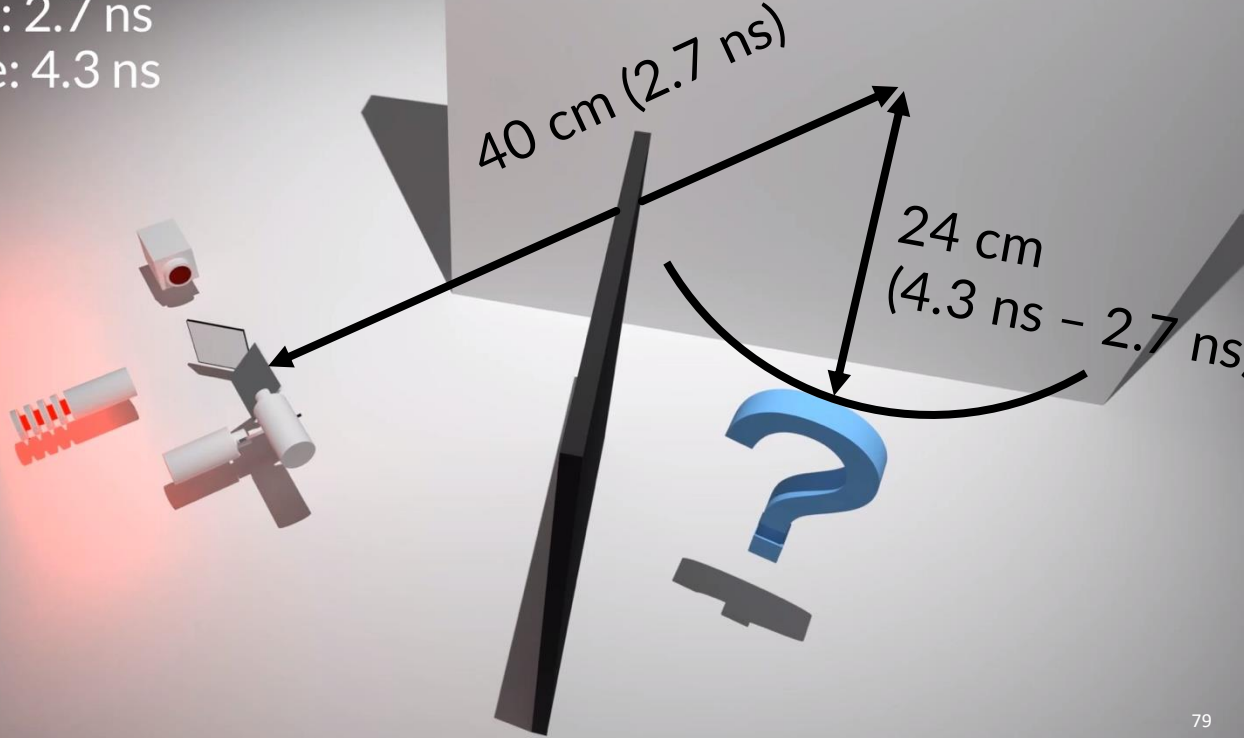
3rd bounce: 4.3 ns



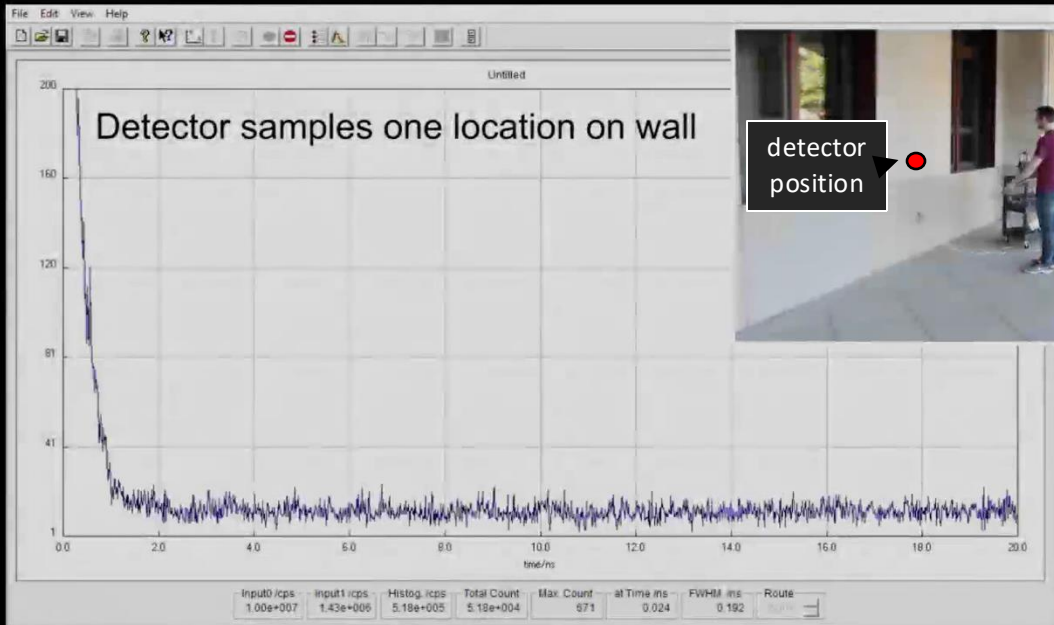
**04.800 ns**

1st bounce: 2.7 ns

3rd bounce: 4.3 ns



# RAW histogram (10 FPS)



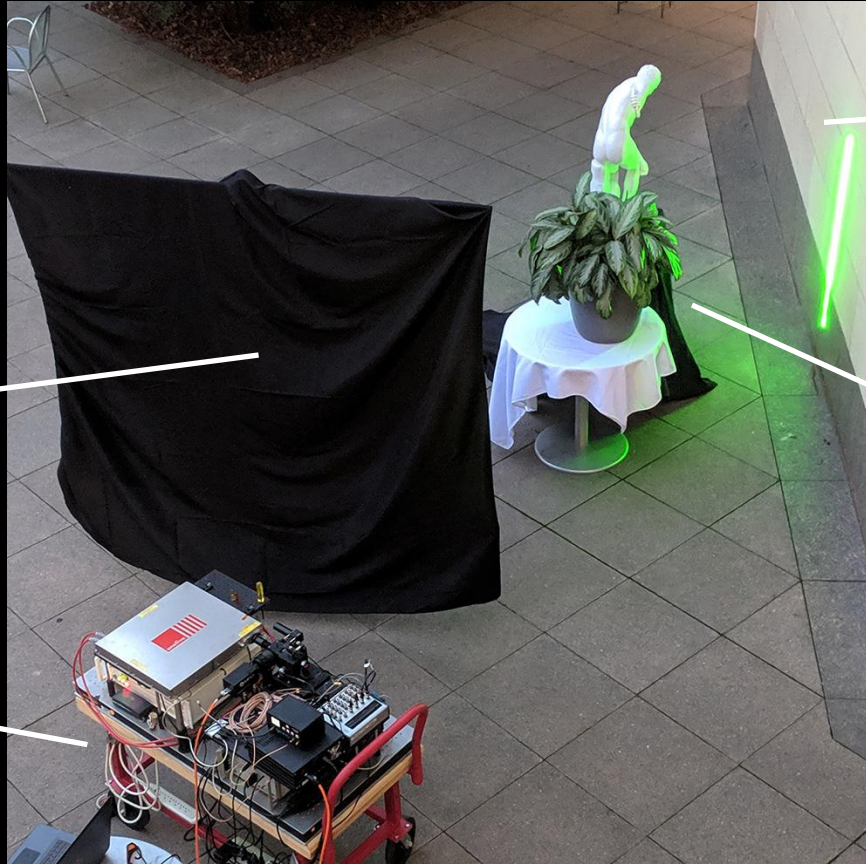
object

occluder

NLOS  
imaging  
system

wall

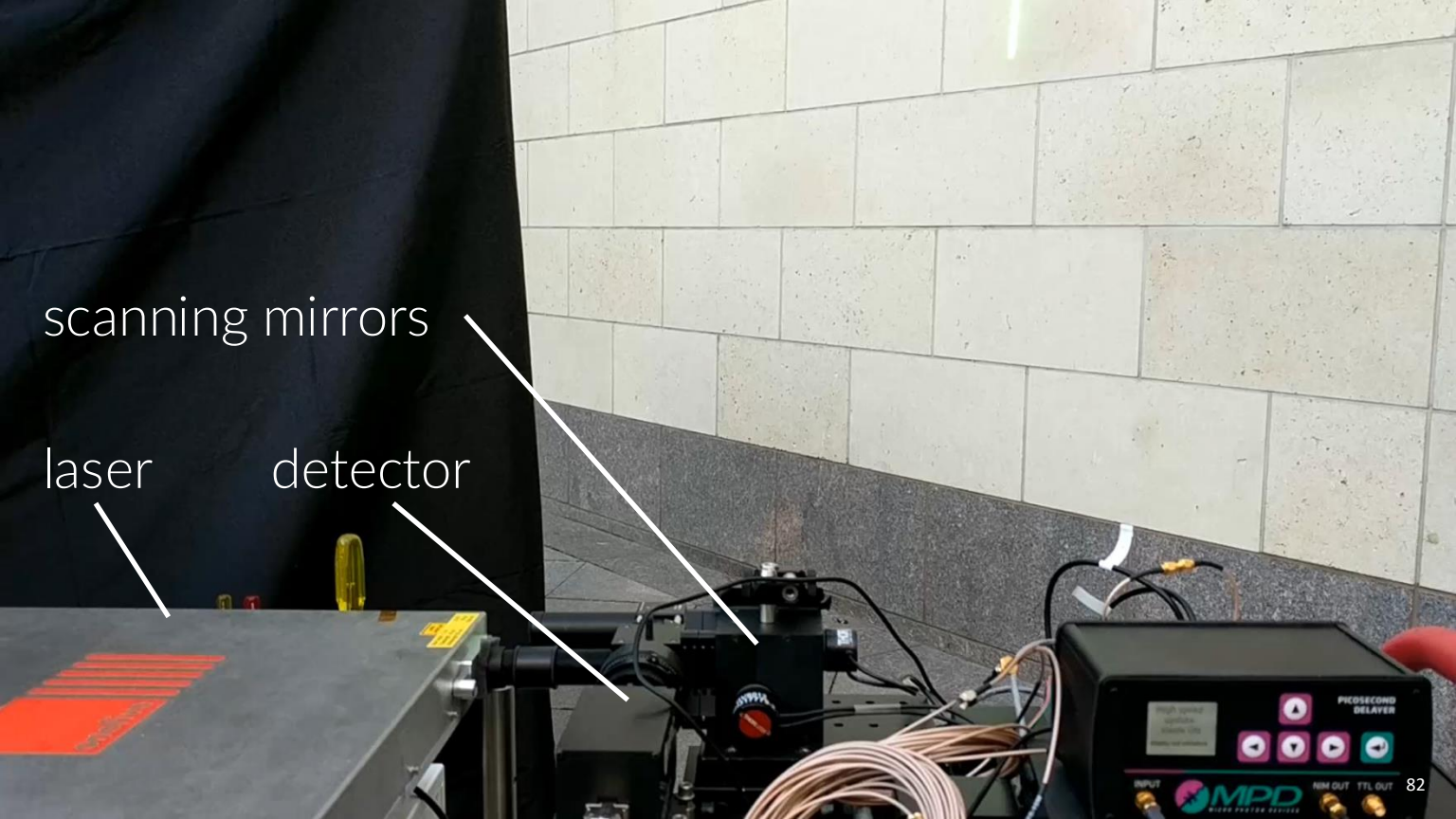
hidden  
scene



scanning mirrors

laser

detector



resolution: 128 x 128  
area: 2 m × 2 m

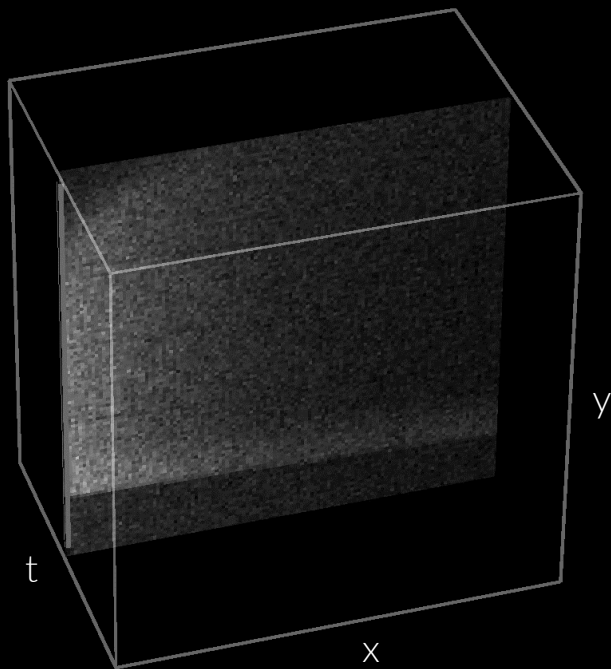




scene photo



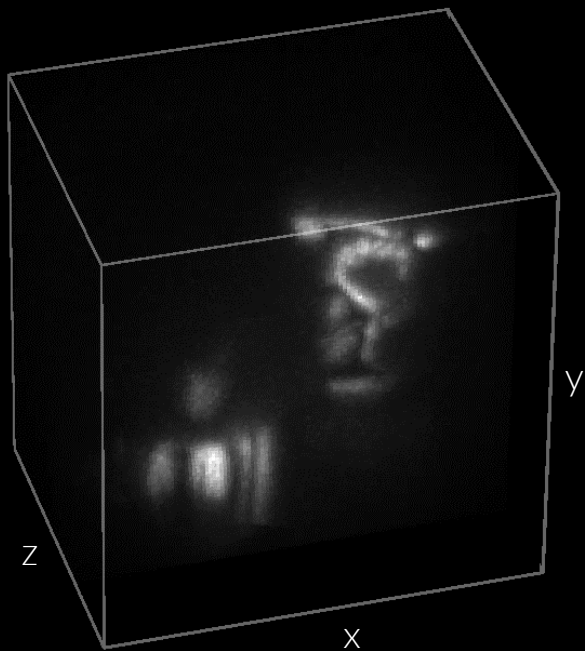
measurements



scene photo

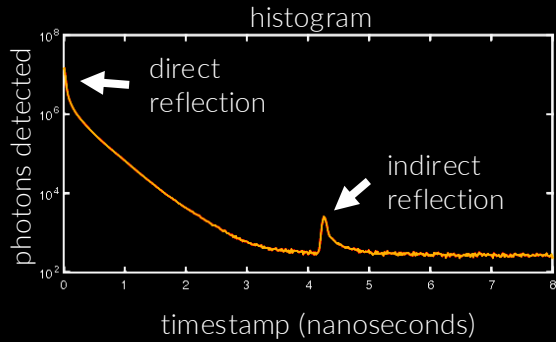


reconstruction

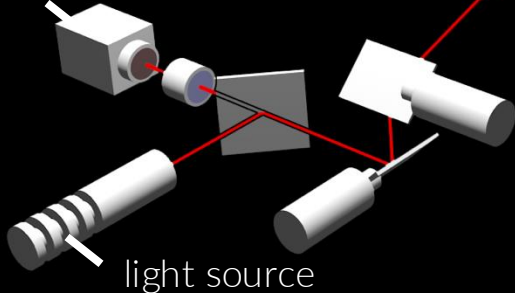


Dimensions: 2 m x 2 m x 1.5 m

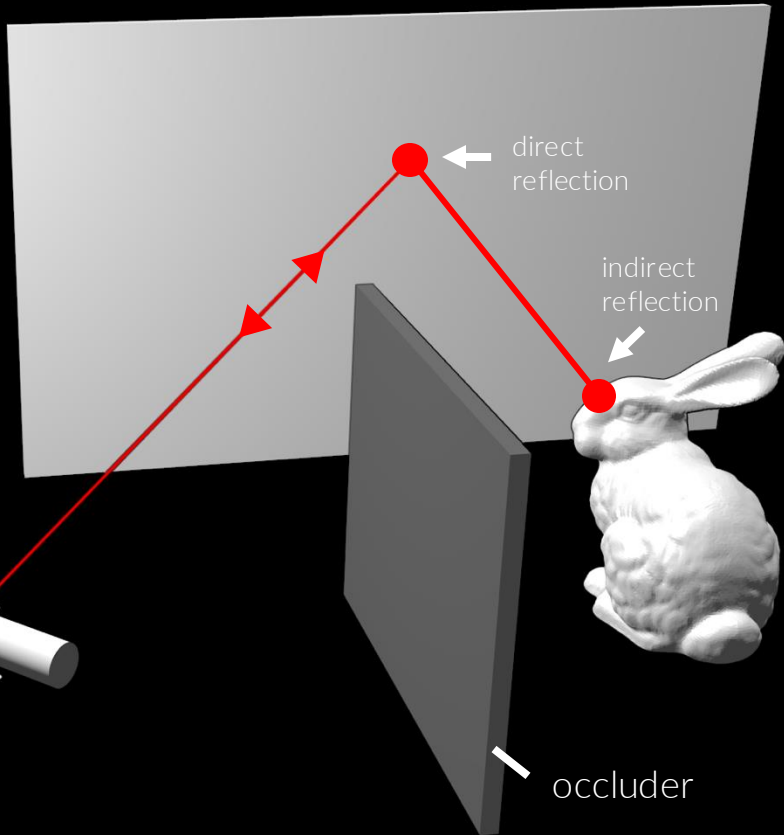


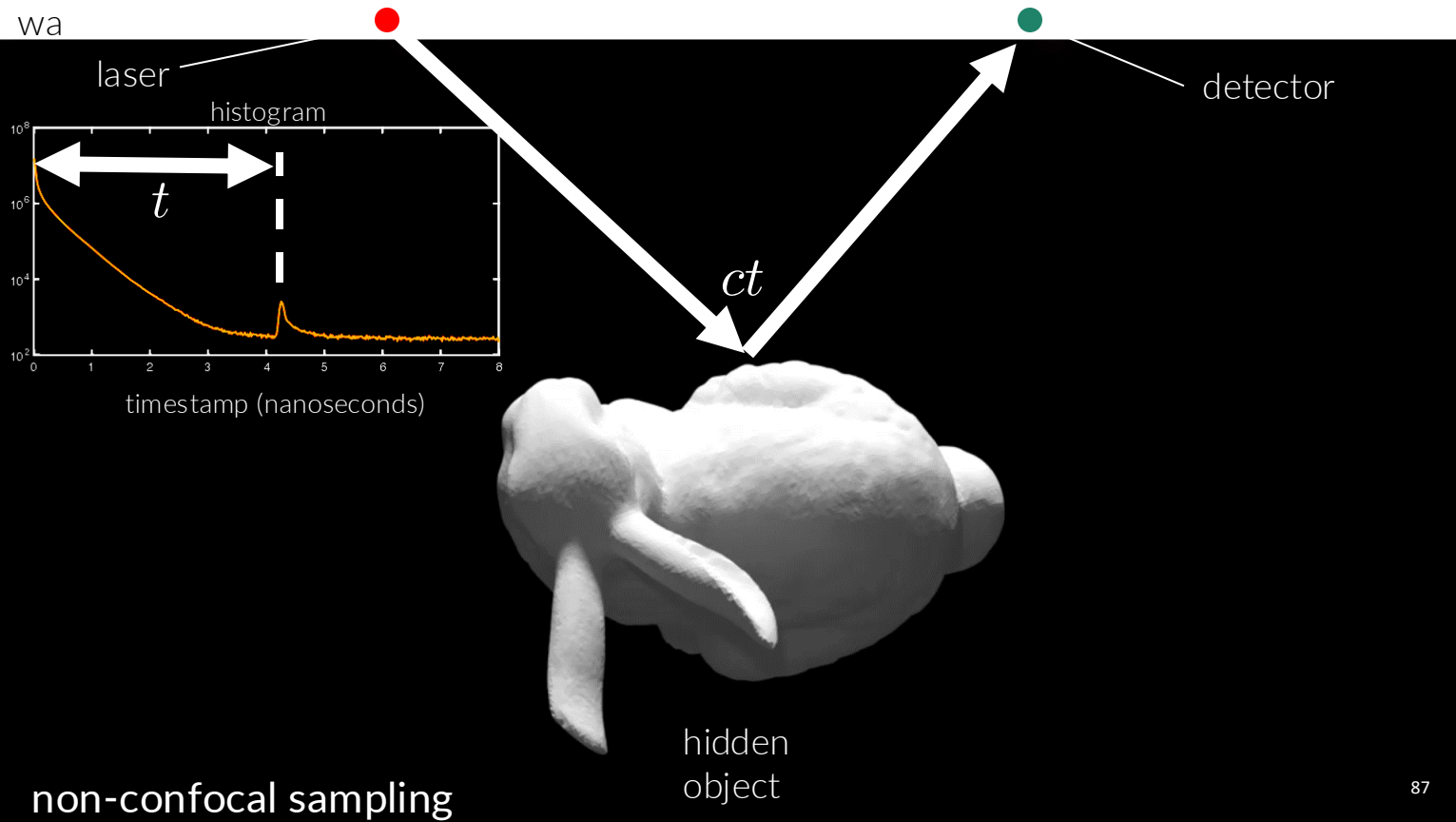


sensor

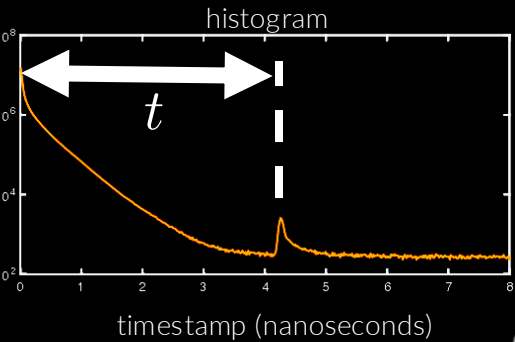


light source





wa



laser and detector  
focus on this point

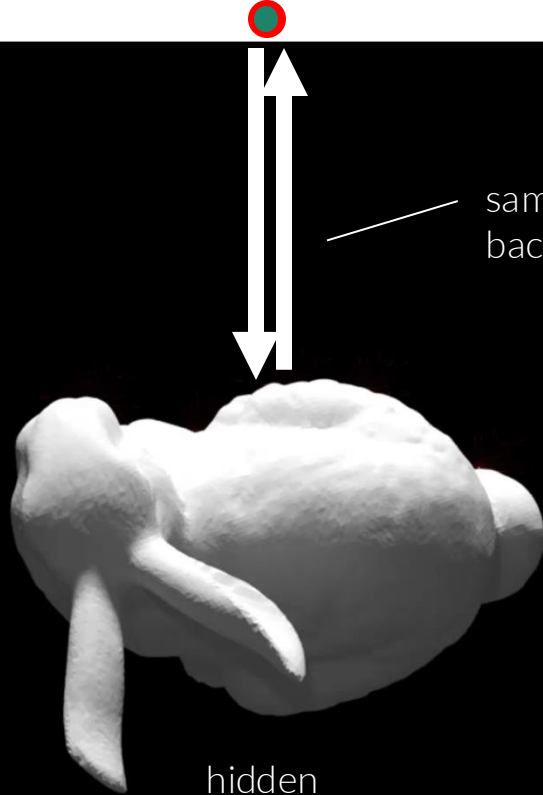


hidden  
object



lasers and detectors  
illuminate and image  
same points

confocal sampling



same path to the object and  
back

hidden  
object

confocal sampling

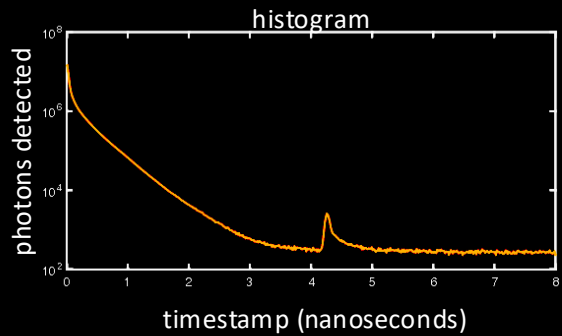
- simplified NLOS mathematical model
- enables efficient NLOS reconstruction



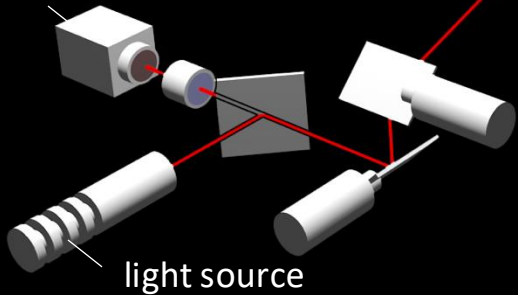
equivalent to one-way  
propagation at half-speed

confocal sampling

hidden  
object

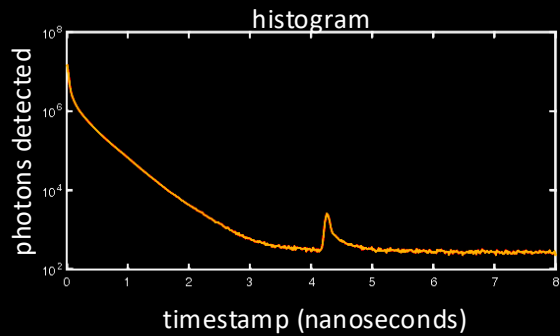


sensor



light source

occluder

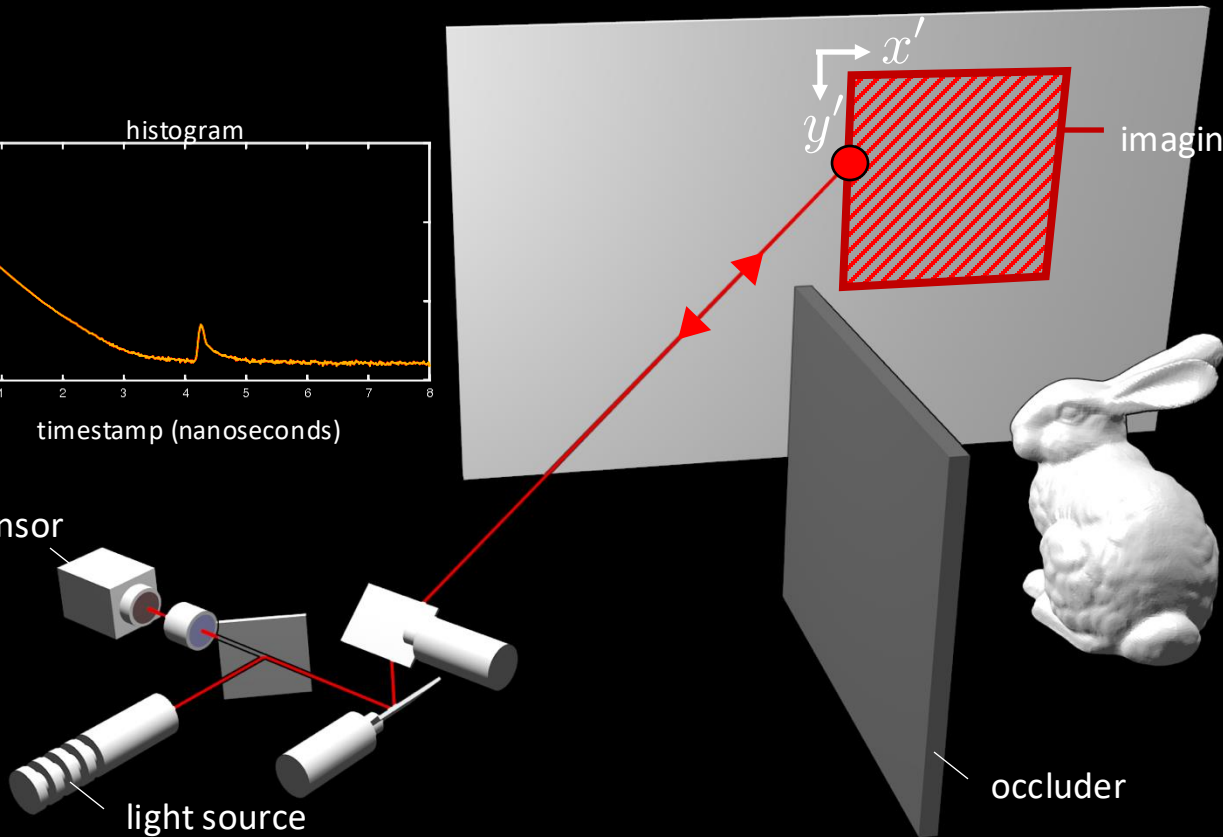


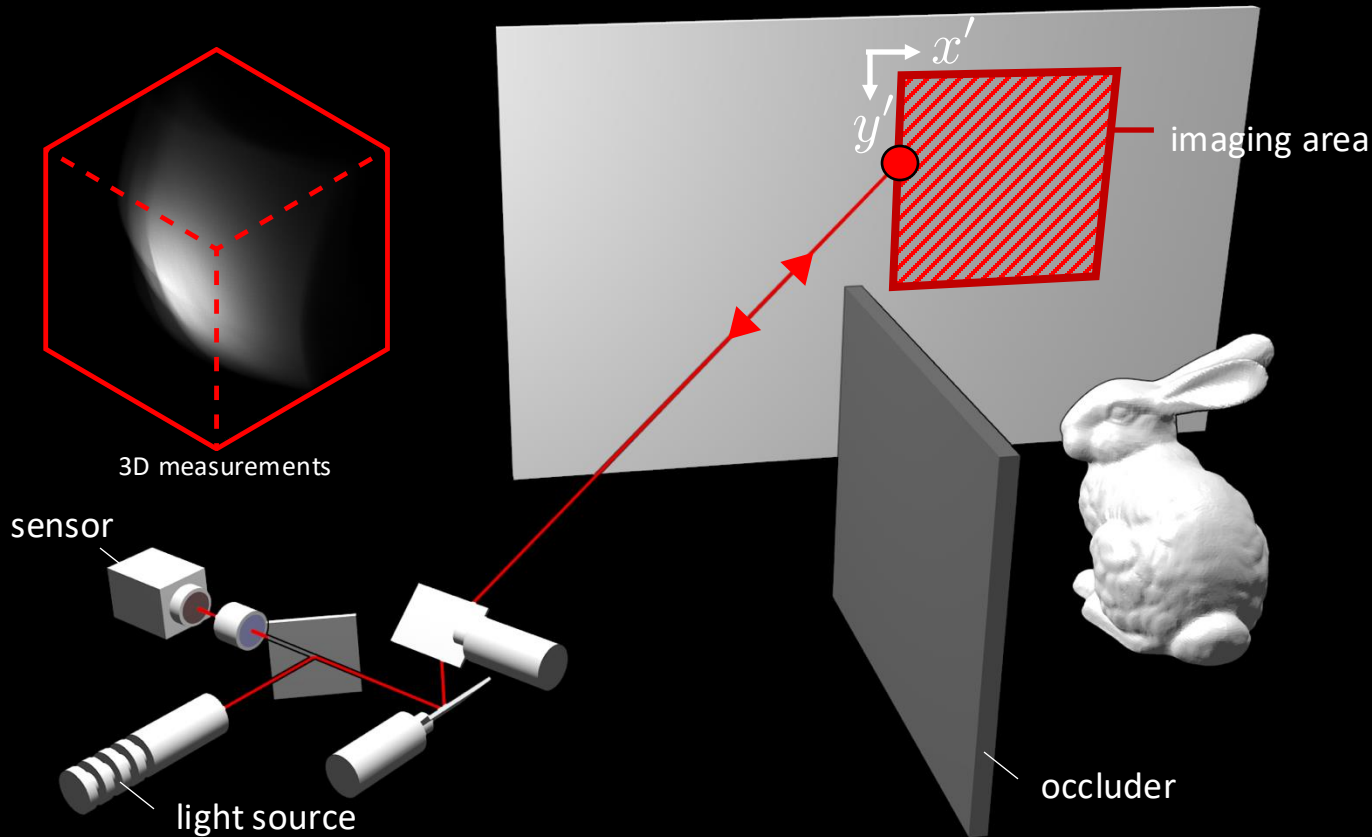
sensor

light source

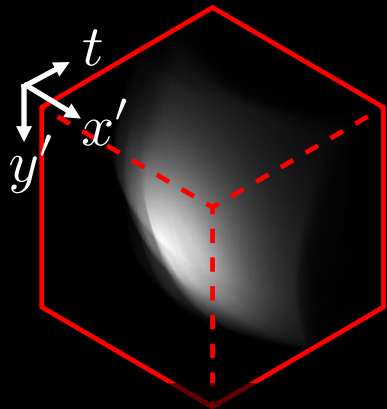
$x'$   
 $y'$   
imaging area

occluder

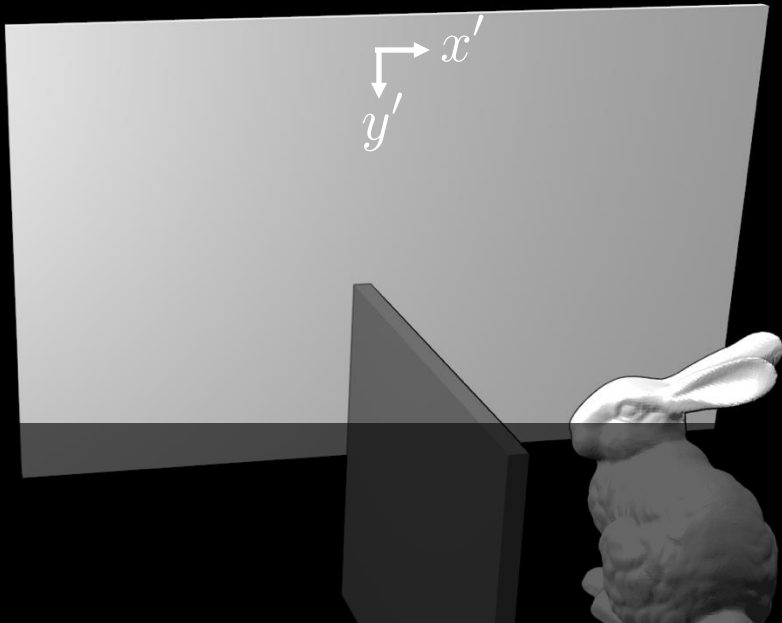




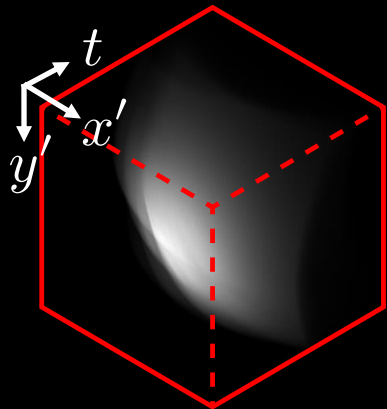




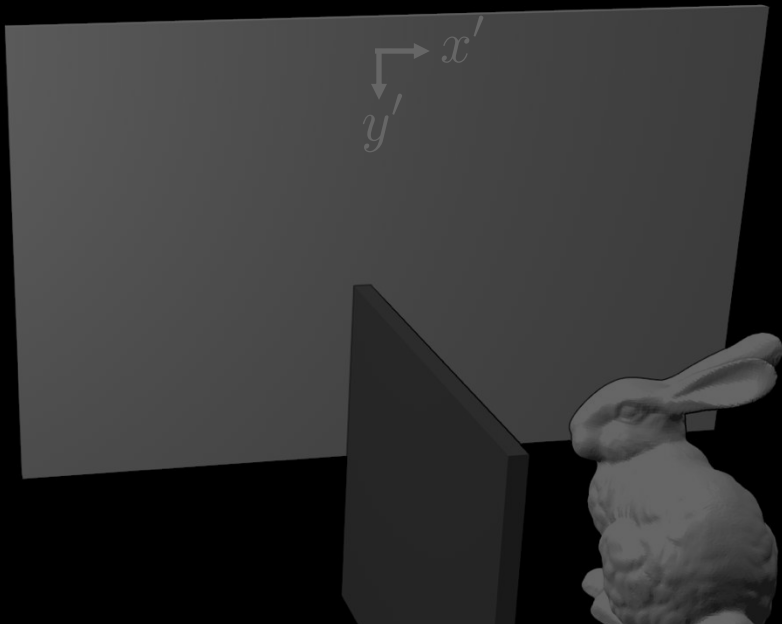
3D measurements



$$\underbrace{\tau(x', y', t)}_{\text{3D measurements}} = \underbrace{\iiint_{\Omega}}_{\text{radiometric term}} \underbrace{\frac{1}{r_l^2 r^2} \delta(r_l + r - tc)}_{\text{geometric term}} \cdot \underbrace{\rho(x, y, z) dx dy dz}_{\text{hidden 3D volume}}$$



3D measurements



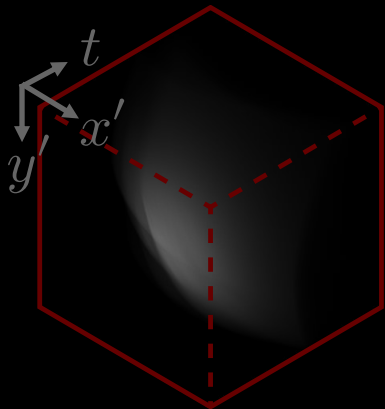
$$\tau(x', y', t) = \iiint_{\Omega} \underbrace{\frac{1}{r_l^2 r^2}}_{\text{radiometric term}} \underbrace{\delta(r_l + r - tc)}_{\text{geometric term}} \cdot \underbrace{\rho(x, y, z)}_{\text{hidden 3D volume}} dx dy dz$$

3D measurements

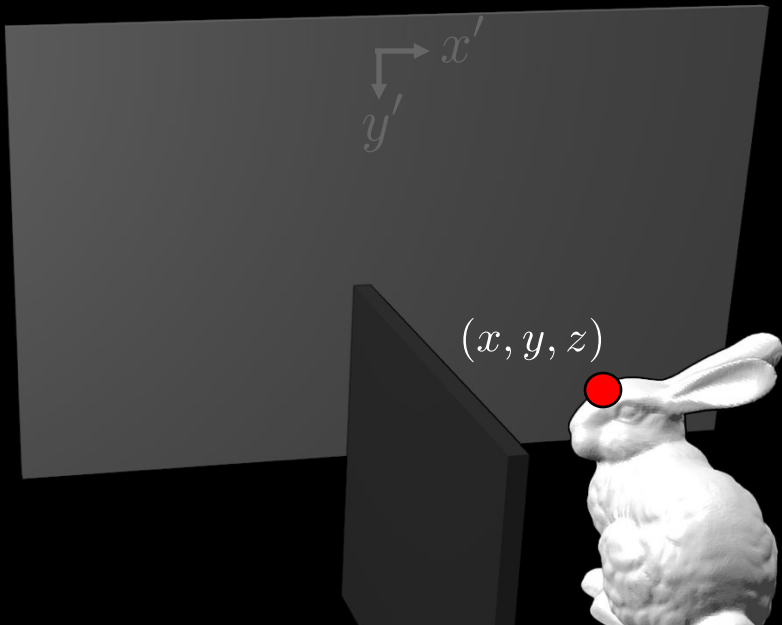
radiometric term

geometric term

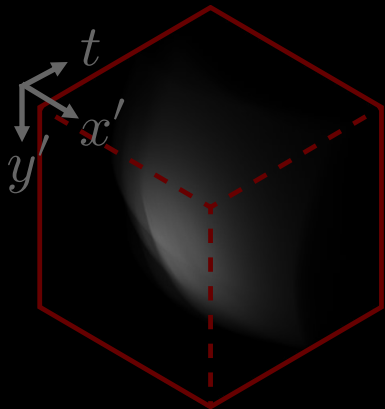
hidden 3D volume



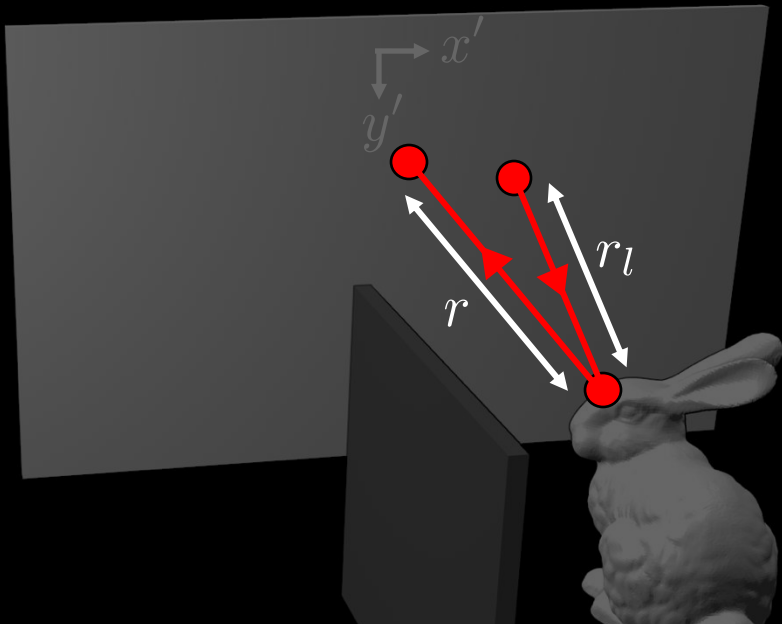
3D measurements



$$\underbrace{\tau(x', y', t)}_{\text{3D measurements}} = \underbrace{\iiint_{\Omega} \frac{1}{r_l^2 r^2} \delta(r_l + r - tc)}_{\text{radiometric term} \quad \text{geometric term}} \cdot \underbrace{\rho(x, y, z) \, dx \, dy \, dz}_{\text{hidden 3D volume}}$$



3D measurements



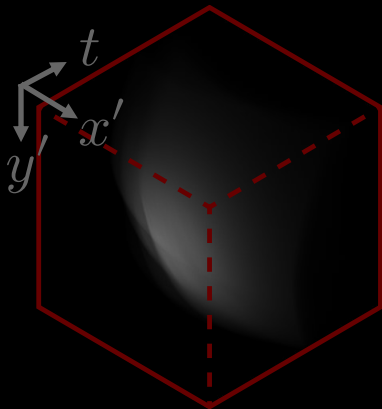
$$\tau(x', y', t) = \iiint_{\Omega} \underbrace{\frac{1}{r_l^2 r^2}}_{\text{radiometric term}} \underbrace{\delta(r_l + r - tc)}_{\text{geometric term}} \cdot \underbrace{\rho(x, y, z)}_{\text{hidden 3D volume}} dx dy dz$$

3D measurements

radiometric term

geometric term

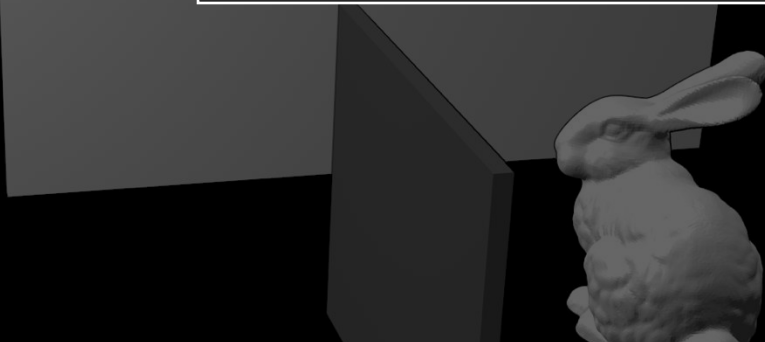
hidden 3D volume



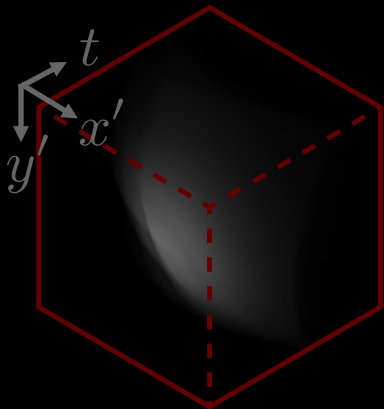
3D measurements

### Assumptions

1. no occlusions in hidden volume
2. light scatters isotropically
3. wall geometry is planar



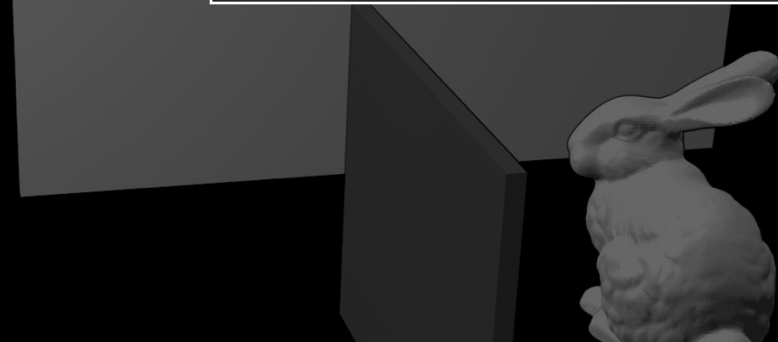
$$\underbrace{\tau(x', y', t)}_{\text{3D measurements}} = \underbrace{\int \int \int_{\Omega}}_{\text{radiometric term}} \underbrace{\frac{1}{r_l^2 r^2} \delta(r_l + r - tc)}_{\text{geometric term}} \cdot \underbrace{\rho(x, y, z) \, dx \, dy \, dz}_{\text{hidden 3D volume}}$$



3D measurements

### Assumptions

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$$\underbrace{\tau(x', y', t)}_{\tau} = \underbrace{\int \int \int_{\Omega} \frac{1}{r_l^2 r^2} \delta(r_l + r - tc)}_{\mathbf{A}} \times \underbrace{\rho(x, y, z) dx dy dz}_{\rho}$$

## NLOS image formation mode:

$$\tau = A\rho$$

measurements  $n^3 \times 1$       transport matrix  $n^3 \times n^3$       unknown volume  $n^3 \times 1$

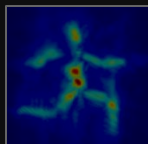
**PROBLEM:**  $A$  extremely large in practice  
(e.g., for  $n = 100$ ,  $A$  has 1 trillion elements)

### Backpropagation [Velten 12, Buttafava 15]

Flops:  $O(n^5)$

Memory:  $O(n^3)$

Runtime: Approx. 10 min.



### Iterative Inversion [Gupta 12, Heide 13]

Flops:  $O(n^5)$  per iter.

Memory:  $O(n^5)$

Runtime: > 1 hour



## Our approach

express image formation model as a 3D convolution, by:

1. confocalizing measurements
2. performing a change of variables  
(set  $z = \sqrt{u}$  ,  $t = 2\sqrt{v}/c$  )

3D measurements

$$\tau(x', y', t) = \iiint_{\Omega} \frac{1}{r_l^2 r^2} \delta(r_l + r - tc) \cdot \rho(x, y, z) \, dx \, dy \, dz$$

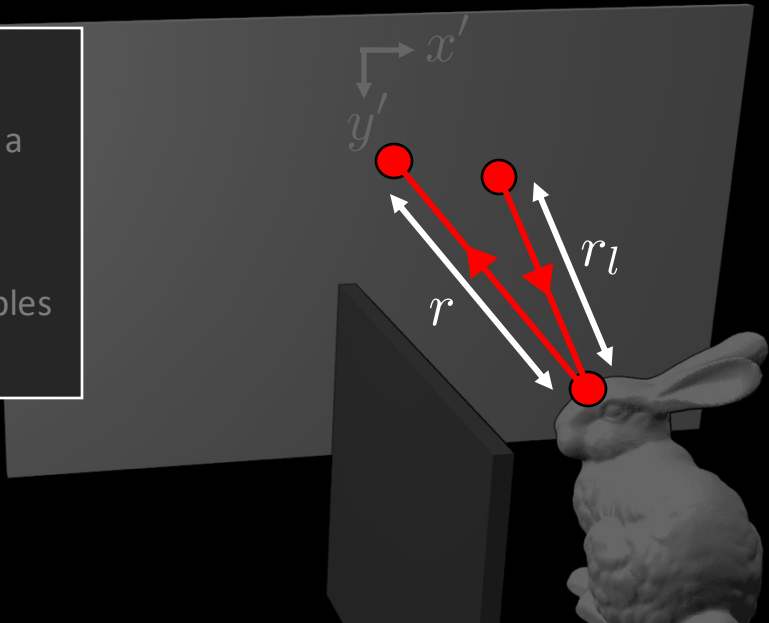


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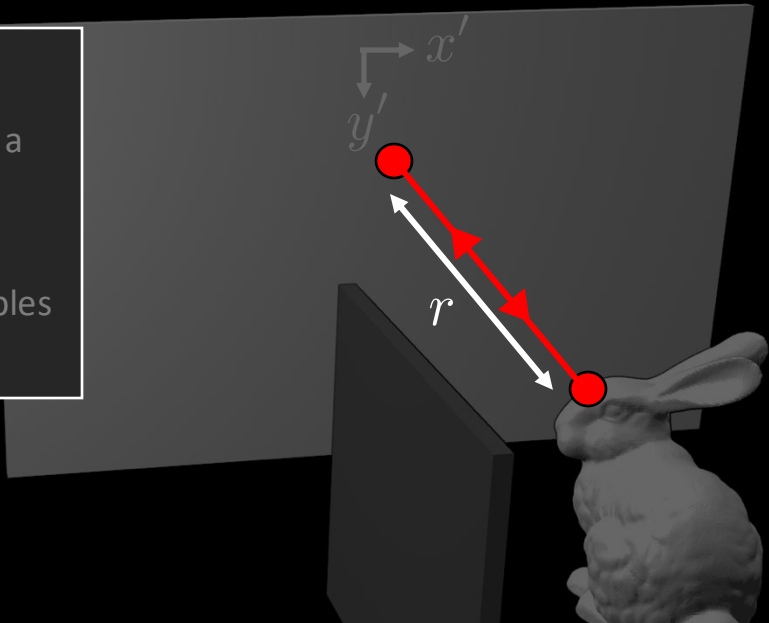
$$\tau(x', y', t) = \iiint_{\Omega} \frac{1}{r_l^2 r^2} \delta(r_l + r - tc) \cdot \rho(x, y, z) dx dy dz$$

## Our approach

express image formation model as a 3D convolution, by:

1. confocalizing measurements
2. performing a change of variables  
(set  $z = \sqrt{u}$  ,  $t = 2\sqrt{v}/c$  )

3D measurements



$$\tau(x', y', t) = \iiint_{\Omega} \frac{1}{r^4} \delta(2r - tc) \cdot \rho(x, y, z) \, dx \, dy \, dz$$

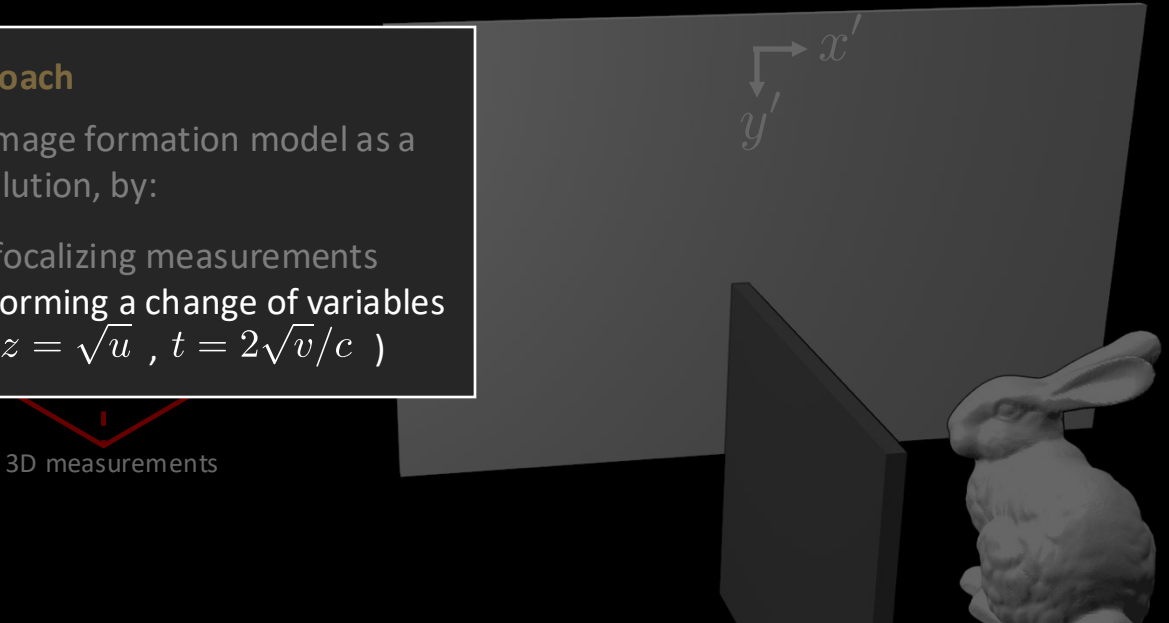
## Our approach

express image formation model as a 3D convolution, by:

1. confocalizing measurements
2. performing a change of variables  
(set  $z = \sqrt{u}$  ,  $t = 2\sqrt{v}/c$  )

3D measurements

$$\tau(x', y', t) = \iiint_{\Omega} \frac{1}{r^4} \delta(2r - tc) \cdot \rho(x, y, z) \, dx \, dy \, dz$$



$x'$   
 $y'$

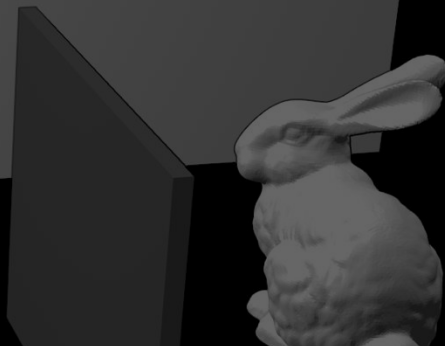
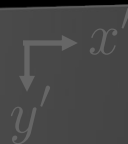
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express image formation model as a 3D convolution, by:

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(set  $z = \sqrt{u}$  ,  $t = 2\sqrt{v}/c$  )

3D measurements

$$v^{3/2}\tau(x', y', \frac{2}{c}\sqrt{v}) = \iiint_{\Omega} \frac{1}{2\sqrt{u}} \delta((x' - x)^2 + (y' - y)^2 + u - v) \cdot \rho(x, y, \sqrt{u}) dx dy du$$

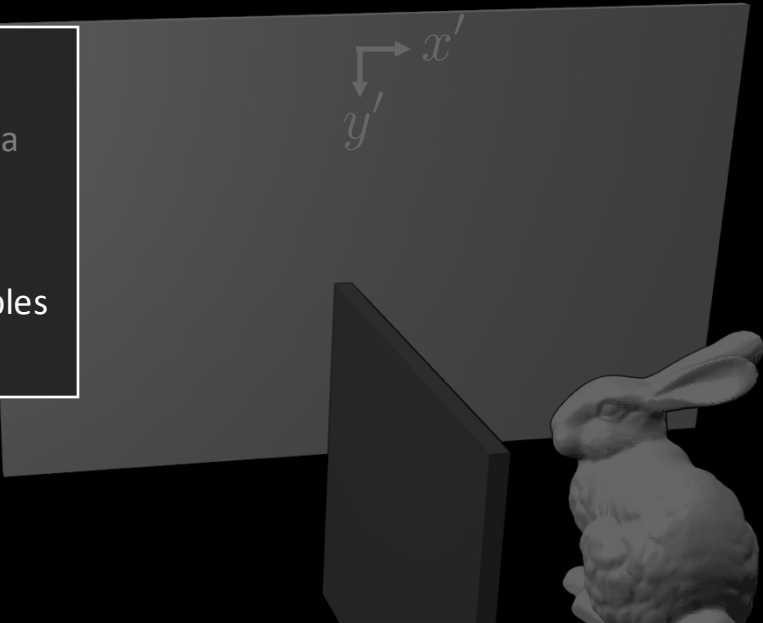


## Our approach

express image formation model as a 3D convolution, by:

1. confocalizing measurements
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3D measurements


$$\underbrace{v^{3/2}\tau(x', y', \frac{2}{c}\sqrt{v})}_{\tau} = \underbrace{\iiint_{\Omega} \frac{1}{2\sqrt{u}} \delta((x' - x)^2 + (y' - y)^2 + u - v)}_{\mathbf{a}} * \underbrace{\rho(x, y, \sqrt{u})}_{\rho} dx dy du$$

## NLOS image formation mode:

$$\tau = A\rho$$

measurements  $n^3 \times 1$       transport matrix  $n^3 \times n^3$       unknown volume  $n^3 \times 1$

## Confocal NLOS image formation mode:

$$\tau = \mathbf{a} * \rho$$

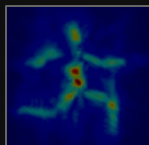
measurements  $n \times n \times n$       blur kernel  $n \times n \times n$       unknown volume  $n \times n \times n$

### Backpropagation [Velten 12, Buttafava 15]

Flops:  $O(n^5)$

Memory:  $O(n^3)$

Runtime: Approx. 10 min.



### Iterative Inversion [Gupta 12, Heide 13]

Flops:  $O(n^5)$  per iter.

Memory:  $O(n^5)$

Runtime: > 1 hour



## NLOS image formation mode:

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measurements  $n^3 \times 1$       transport matrix  $n^3 \times n^3$       unknown volume  $n^3 \times 1$

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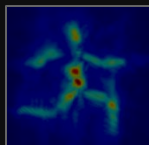
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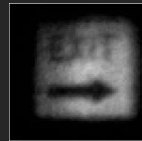
### 3D Deconvolution (with Light Cone Transform)

[O'Toole et al. 2018]

Flops:  $O(n^3 \log(n))$

Memory:  $O(n^3)$

Runtime: < 1 second



### Iterative Inversion [Gupta 12, Heide 13]

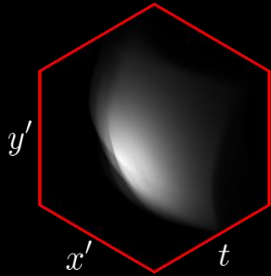
Flops:  $O(n^5)$  per iter.

Memory:  $O(n^5)$

Runtime: > 1 hour

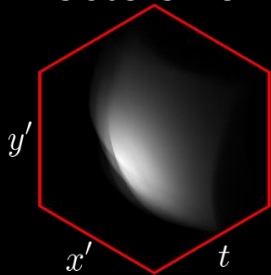


measurements

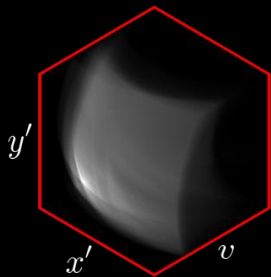




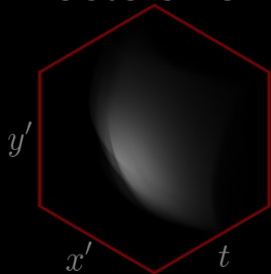
measurements



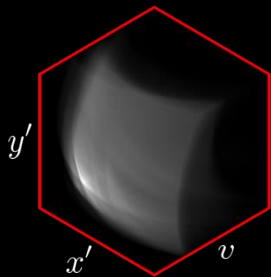
**Step 1:** resample  
and attenuate  
along  $t$  -axis



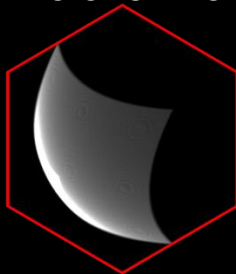
measurements



**Step 1:** resample  
and attenuate  
along  $t$ -axis

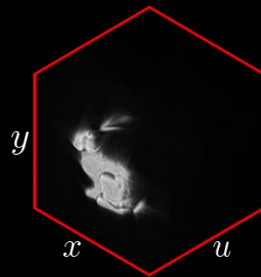
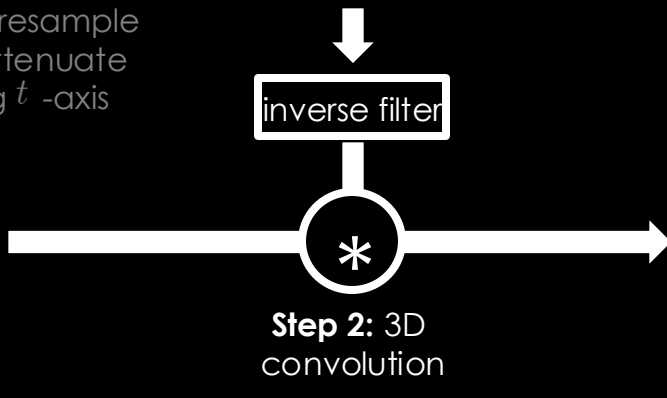


convolution kernel

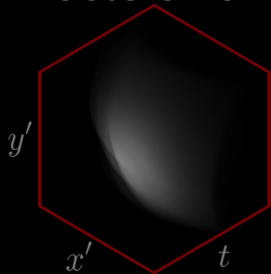


inverse filter

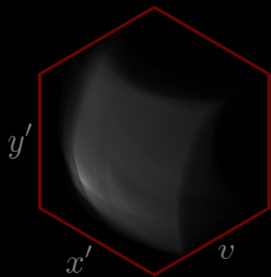
**Step 2:** 3D  
convolution



measurements



**Step 1:** resample  
and attenuate  
along  $t$ -axis



convolution kernel

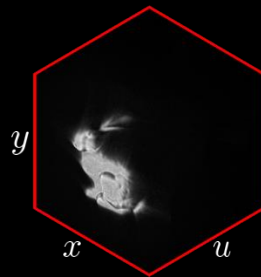
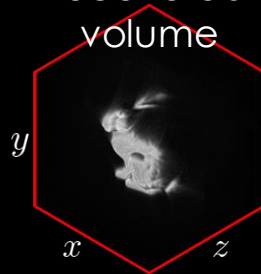


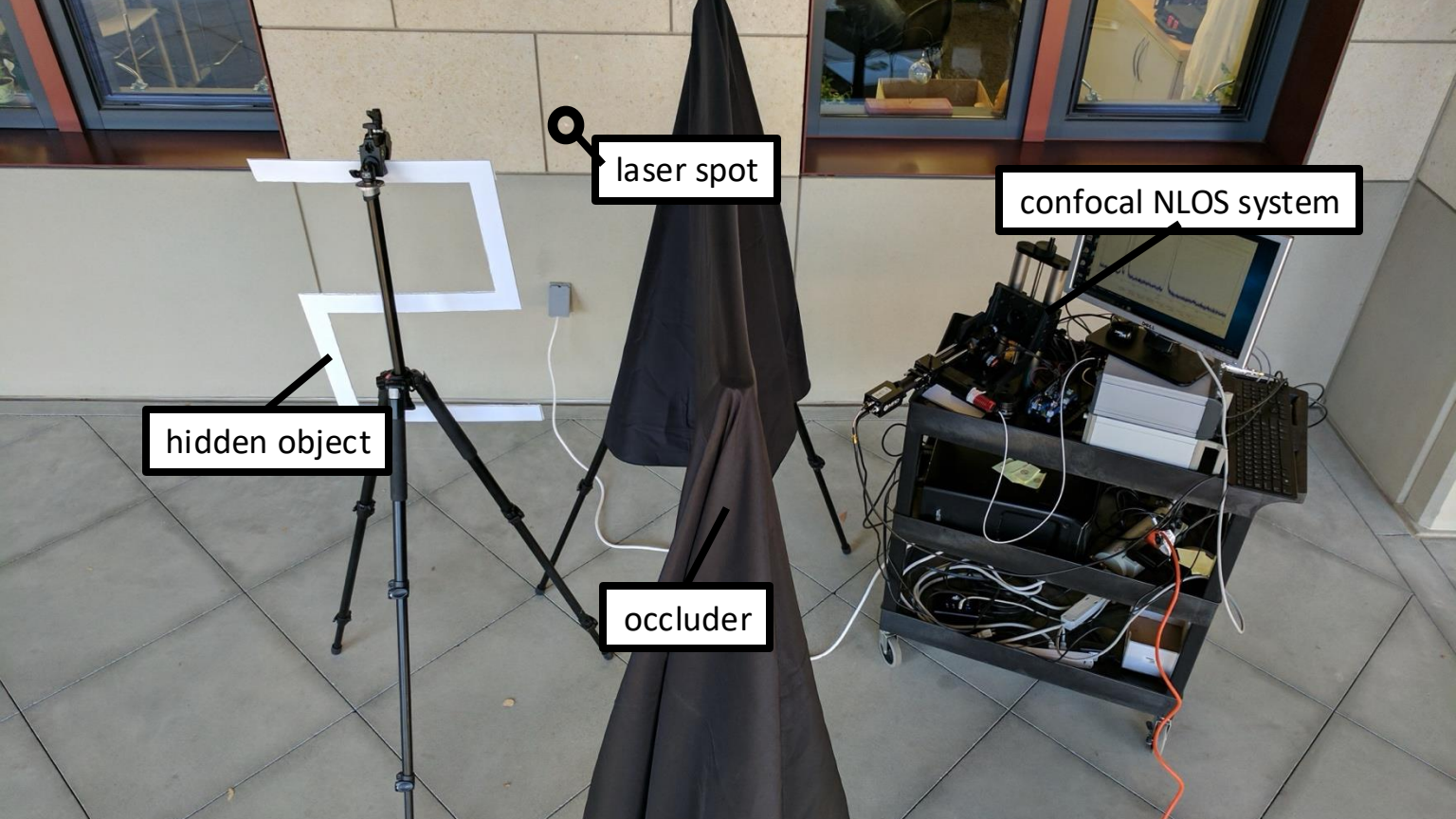
inverse filter

**Step 2:** 3D  
convolution

**Step 3:** resample  
and attenuate  
along  $z$ -axis

recovered  
volume





laser spot

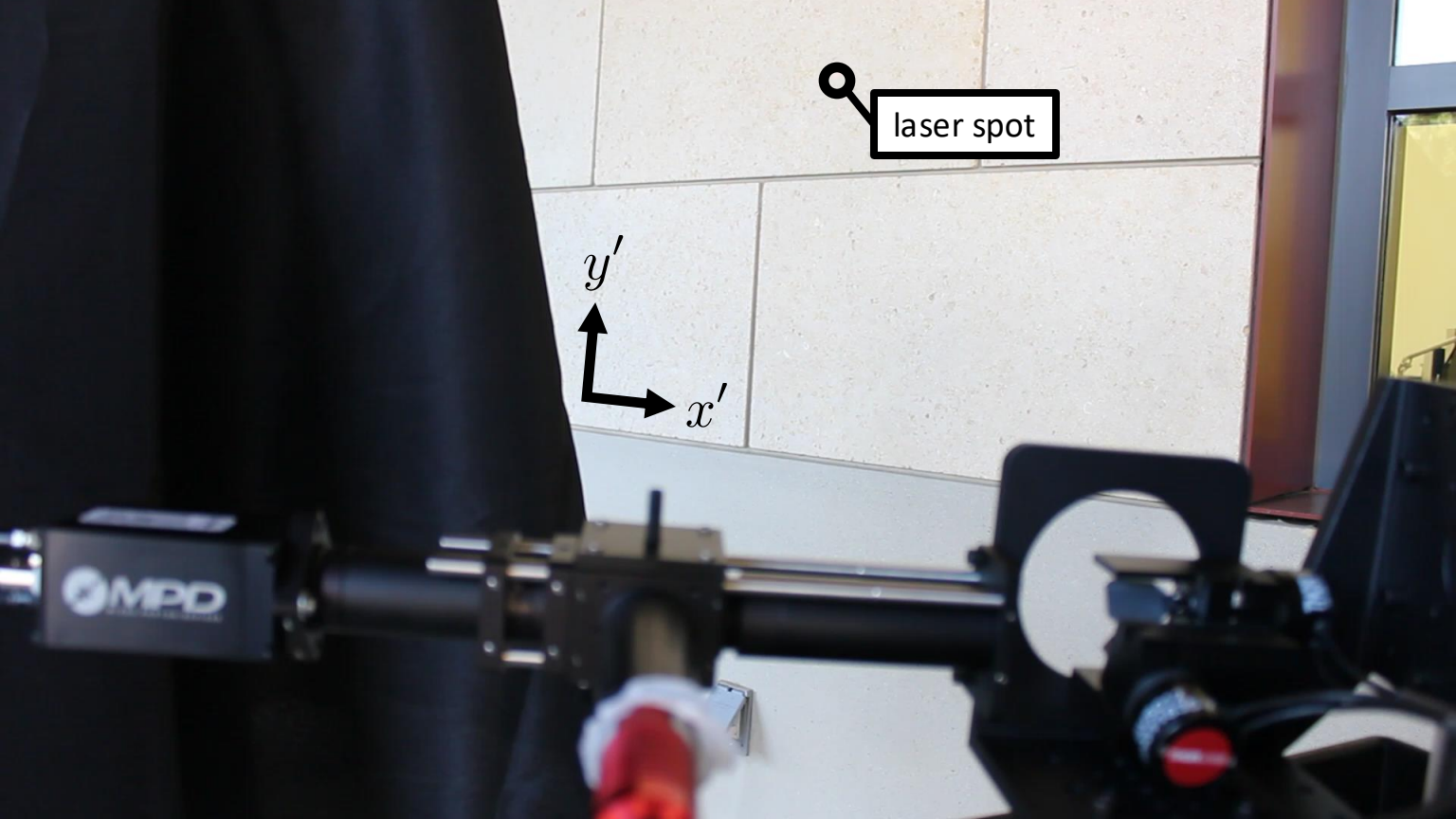
confocal NLOS system

hidden object

occluder



1.25 meters

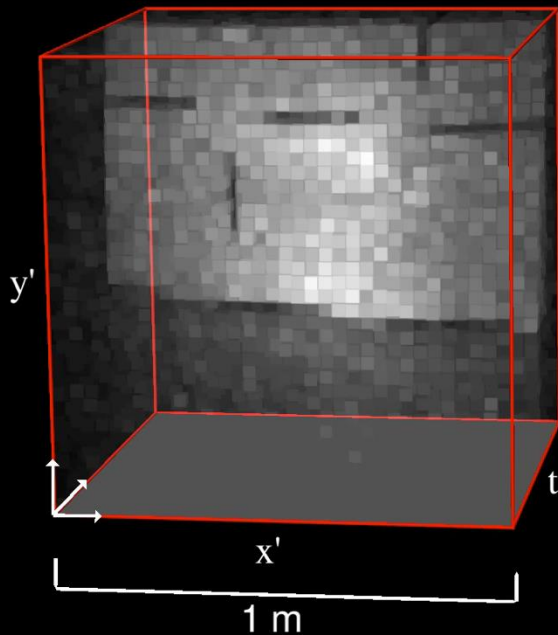


laser spot

$y'$   
 $x'$

MPD

measurements



Maximum Intensity Projection

## NLOS image formation model:

$$\tau = A \rho$$

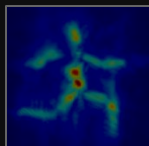
measurements  $n^3 \times 1$       transport matrix  $n^3 \times n^3$       unknown volume  $n^3 \times 1$

### Backprojection [Velten 12, Buttafava 15]

Flops:  $O(n^5)$

Memory:  $O(n^3)$

Runtime: Approx. 10 min.



### Iterative Inversion [Gupta 12, Wu 12, Heide

Flops:  $O(n^5)$  <sup>13</sup> per iter.

Memory:  $O(n^5)$

Runtime: > 1 hour



## Confocal scanning and Light-Cone Transform:

$$\tilde{\tau} = a * \tilde{\rho}$$

measurements  $n \times n \times n$       blur kernel  $n \times n \times n$       unknown volume  $n \times n \times n$

### 3D Deconvolution (with Light-Cone Transform)

[O'Toole et al. 2018]

Flops:  $O(n^3 \log(n))$

Memory:  $O(n^3)$

Runtime: < 1 second



### Assumption:

- Isotropic scattering (only diffuse or retroreflective objects)



## overview

- Time-resolved imaging
- Single-photon avalanche diodes (SPADs)
- Single-photon lidar
- Non-line-of-sight imaging (part 2)
- neural rendering for propagating light

$$\nabla^2 \Psi - \frac{1}{v^2} \frac{\partial^2 \Psi}{\partial t^2} = 0$$

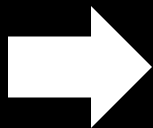


confocal sampling

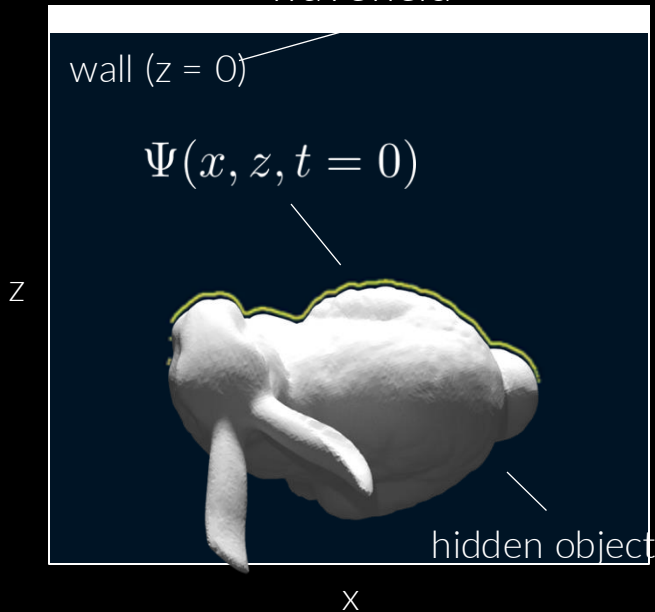
hidden  
object

# image formation model

$\Psi(x, z, t)$   
wavefield



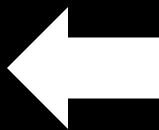
$\Psi(x, z = 0, t)$   
confocal measurements



# general solution (time reversal)

$$\Psi(x, z, t = 03.000 \text{ ns})$$

wavefield



wall ( $z = 0$ )

$z$



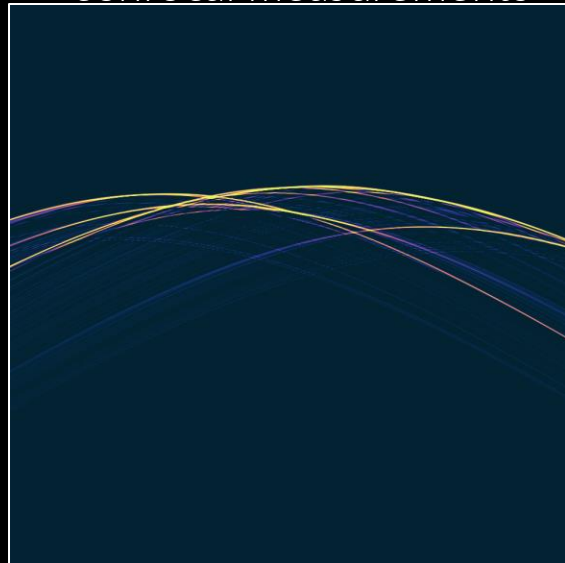
hidden object

$x$

$$\Psi(x, z = 0, t)$$

confocal measurements

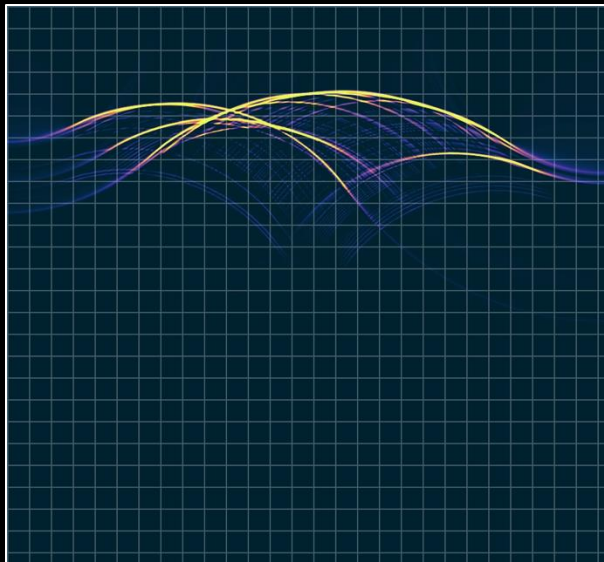
$t$



$x$

# general solution (time reversal)

finite-difference time-  
domain method



1. approximate wave equation with finite differences

$$\frac{\partial^2 \Psi}{\partial t^2} \approx \frac{\Psi_i^{n+1} - 2\Psi_i^n + \Psi_i^{n-1}}{(\Delta t)^2}$$

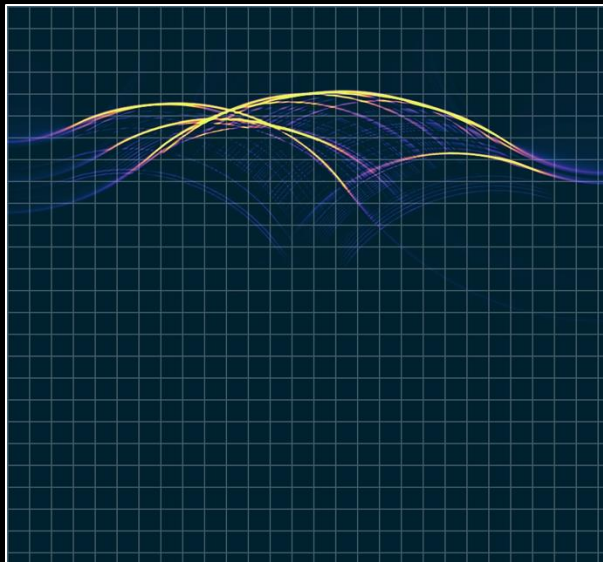
2. solve for previous timestep

$$\Psi_i^{n-1} = f(\Psi^n, \Psi^{n+1})$$

3. repeatedly update  $\Psi$  at all grid cells

# general solution (time reversal)

finite-difference time-domain method



1. approximate wave equation with finite differences

$$\frac{\partial^2 \Psi}{\partial t^2} \approx \frac{\Psi_i^{n+1} - 2\Psi_i^n + \Psi_i^{n-1}}{\Delta t^2}$$

2. solve for  $\Psi_i^n$  at each timestep

$$\Psi_i^{n+1} = f(\Psi_i^n, \Psi_{i-1}^n, \Psi_{i+1}^n)$$

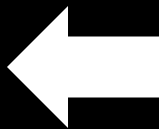
3. repeatedly update  $\Psi$  at all grid cells

**Slow to get t=0 at high-resolution!**

# frequency–wavenumber ( $f$ – $k$ ) Migration

$$\Psi(x, z, t = 0)$$

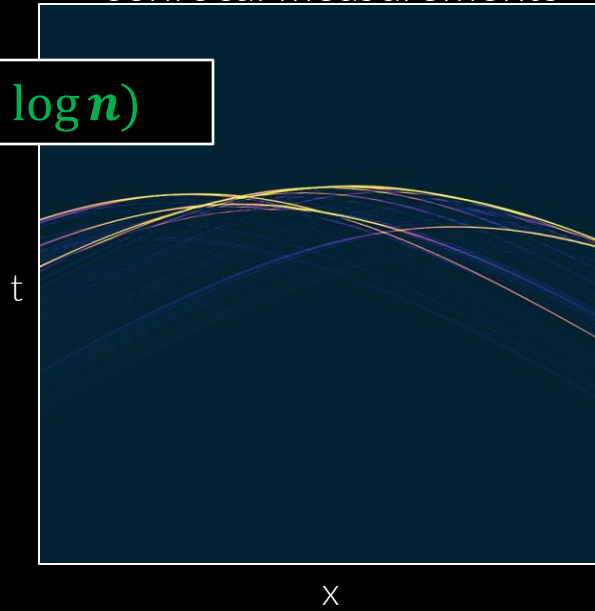
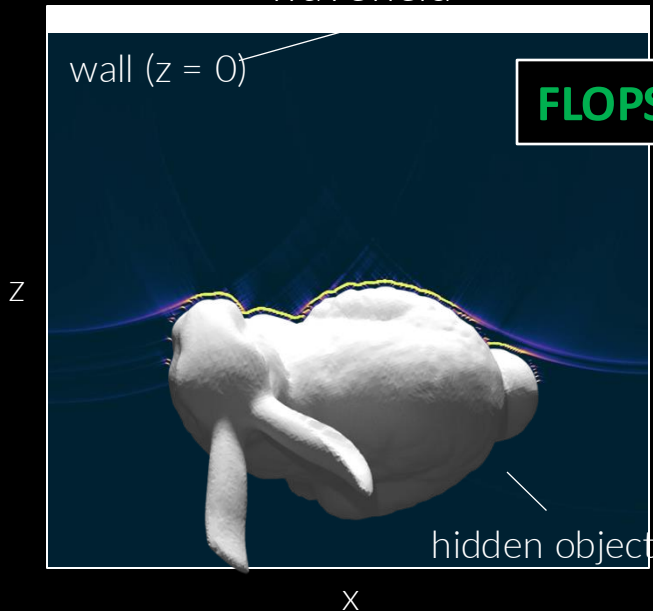
wavefield



$$\Psi(x, z = 0, t)$$

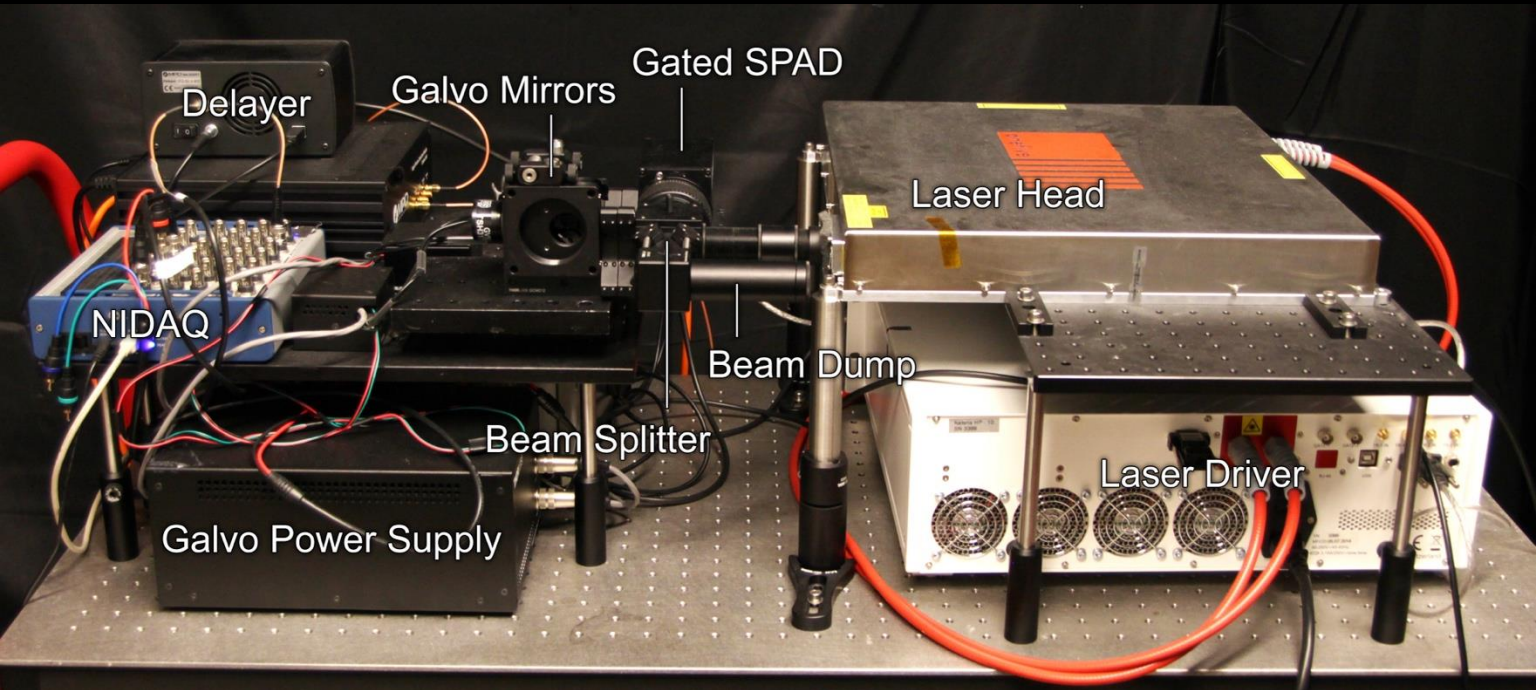
confocal measurements

**FLOPS:  $O(n^3 \log n)$**

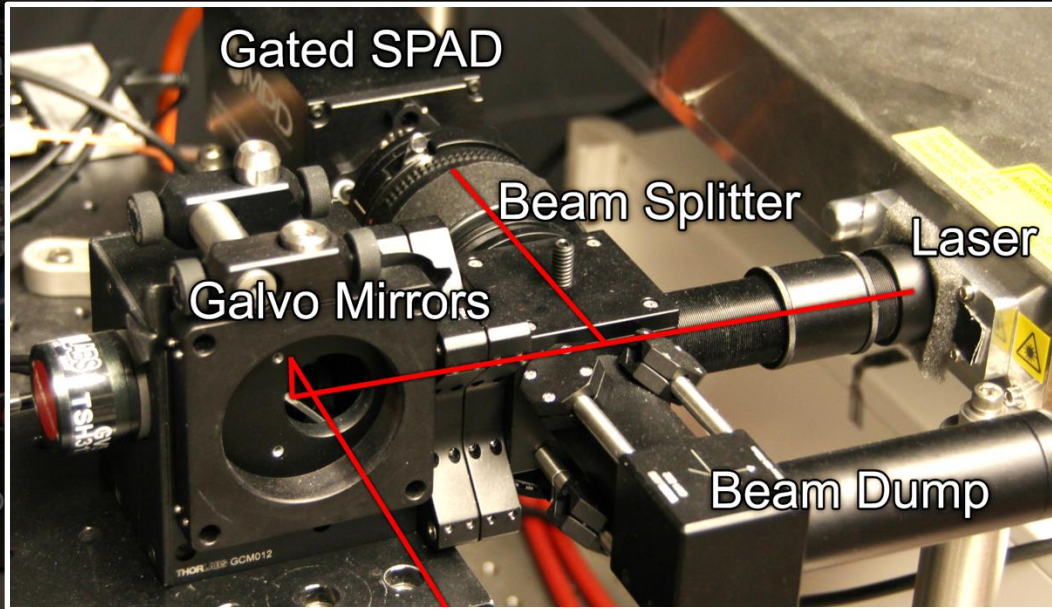




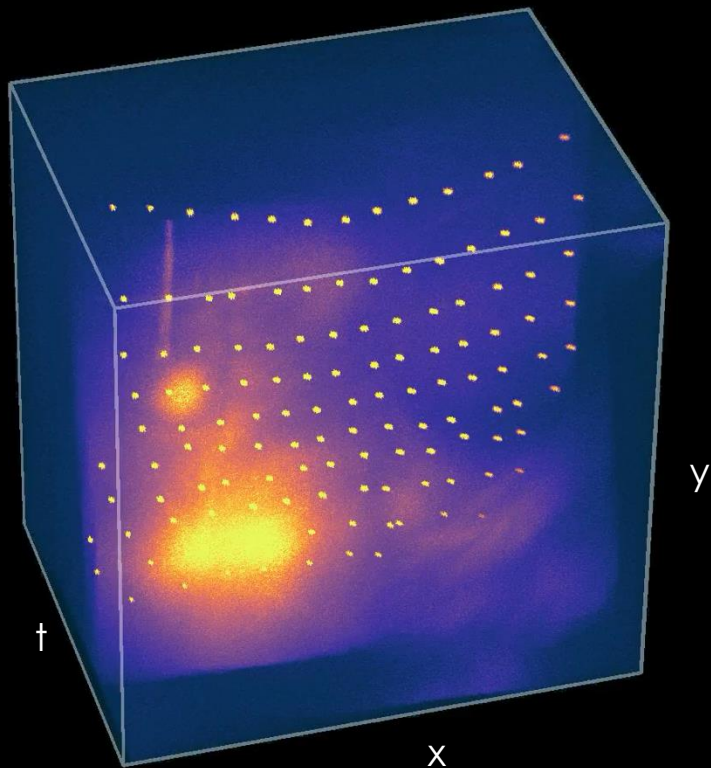
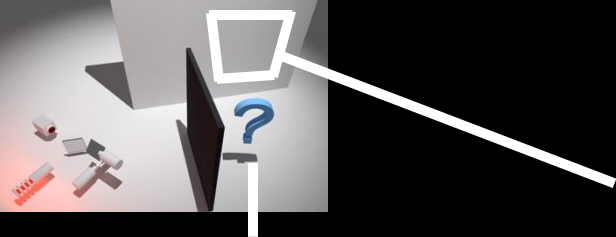
# hardware prototype



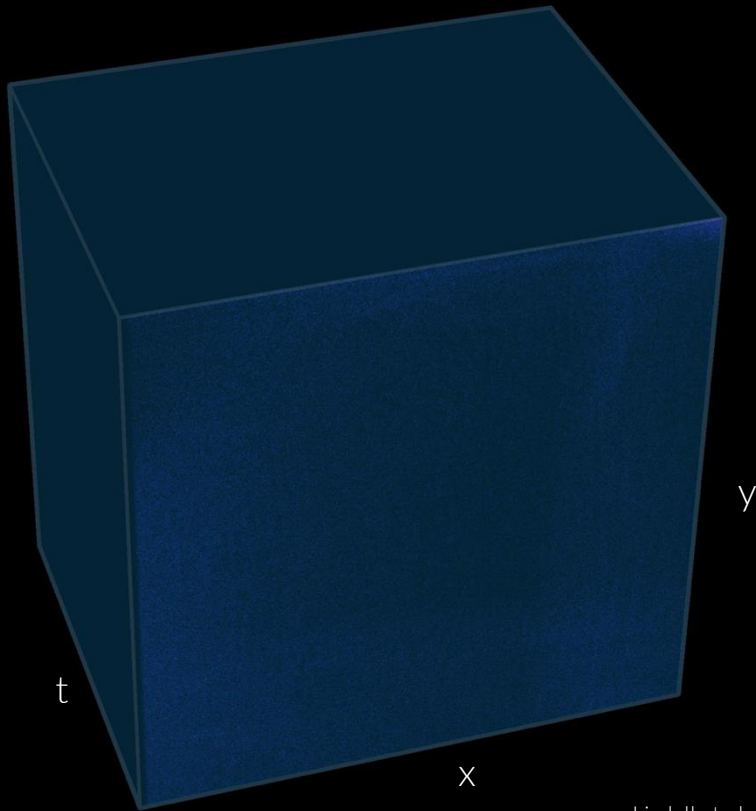
# hardware prototype







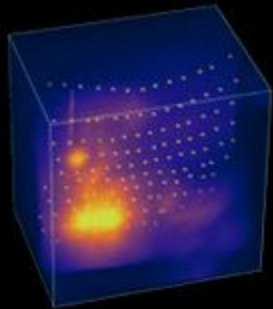
Captured



# $f$ - $k$ Migration

$$\Psi(x, y, t)$$

Measurements ( $z=0$ )



$$\bar{\Phi}(k_x, k_y, f)$$

Spectrum

$$\Phi(k_x, k_y, k_z)$$

Interpolated Spectrum

$$\Psi(x, y, z)$$

Hidden Volume ( $t=0$ )

$\rightarrow$   
 $\mathcal{F}$

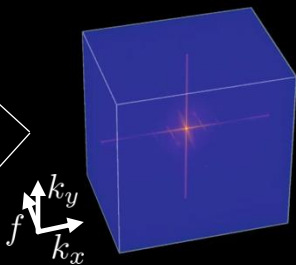
$\rightarrow$   
resample

$\rightarrow$   
 $\mathcal{F}^{-1}$

# $f$ - $k$ Migration

Express wavefield as function of measurement spectrum (plane wave decomposition)

$$\underbrace{\Psi(x, y, z, t)}_{\text{wavefield}} = \underbrace{\iiint \bar{\Phi}(k_x, k_y, f) e^{2\pi i(k_x x + k_y y + k_z z - ft)} dk_x dk_y df}_{\text{Fourier transform of measurements}}$$



Set  $t=0$  to get migrated solution

$$\Psi(x, y, z, t = 0) = \iiint \bar{\Phi}(k_x, k_y, f) e^{2\pi i(k_x x + k_y y + \underbrace{k_z z}_{\text{underbrace}})} dk_x dk_y \underbrace{df}_{\text{underbrace}}$$

*Almost an inverse Fourier Transform!*



# $f$ - $k$ Migration

Set  $t=0$  to get migrated solution

$$\Psi(x, y, z, t = 0) = \iiint \bar{\Phi}(k_x, k_y, f) e^{2\pi i(k_x x + k_y y + \underbrace{k_z z})} dk_x dk_y \underbrace{df}$$

Almost an inverse Fourier Transform!

Use dispersion relation<sup>1</sup> to perform substitution of variables

$$f = v \sqrt{k_x^2 + k_y^2 + k_z^2}$$

$$\boxed{f \Rightarrow k_z}$$

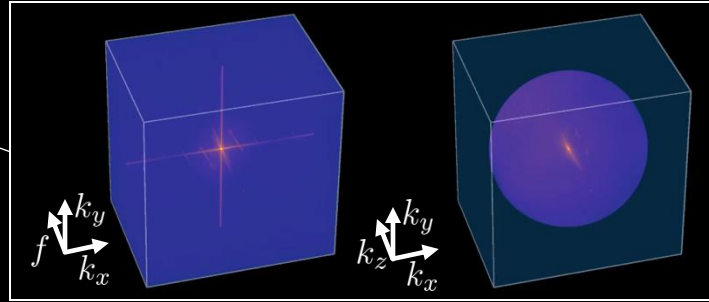
<sup>1</sup>Georgi, Howard. *The physics of waves*. Englewood Cliffs, NJ: Prentice Hall, 1993.



Use dispersion relation<sup>1</sup> to perform substitution of variables

$$f = v \sqrt{k_x^2 + k_y^2 + k_z^2}$$

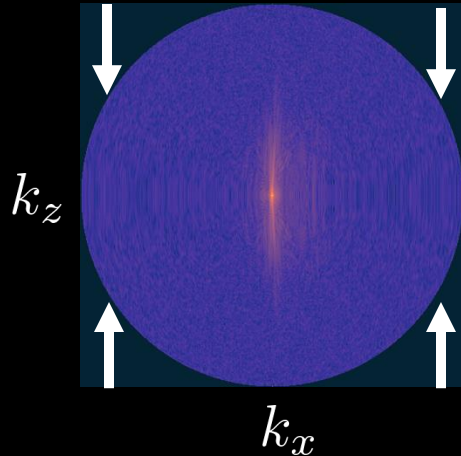
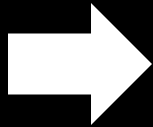
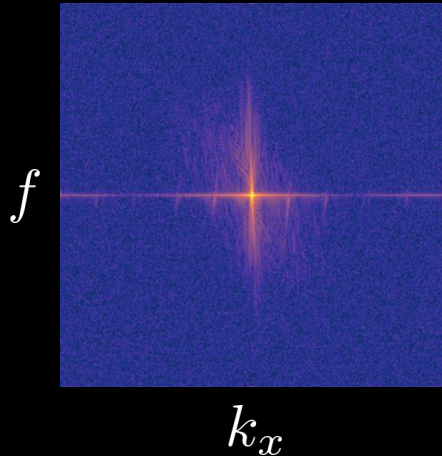
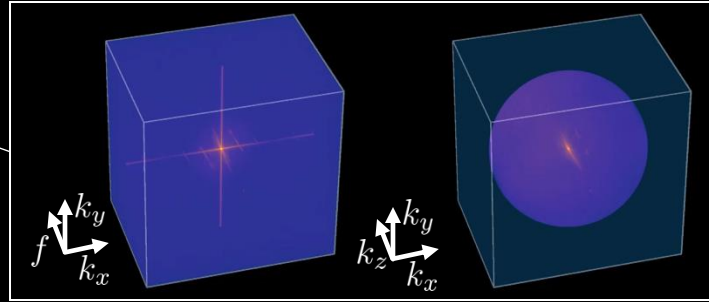
$$f \Rightarrow k_z$$



Use dispersion relation<sup>1</sup> to perform substitution of variables

$$f = v \sqrt{k_x^2 + k_y^2 + k_z^2}$$

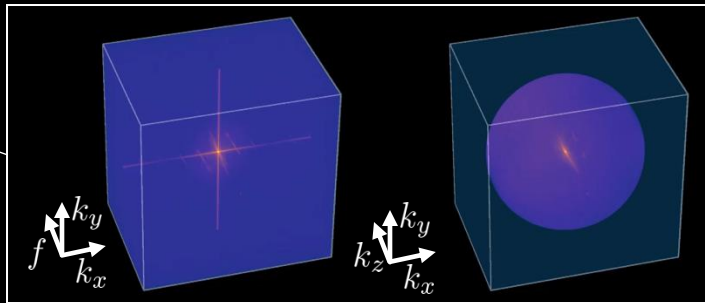
$$f \Rightarrow k_z$$



Use dispersion relation<sup>1</sup> to perform substitution of variables

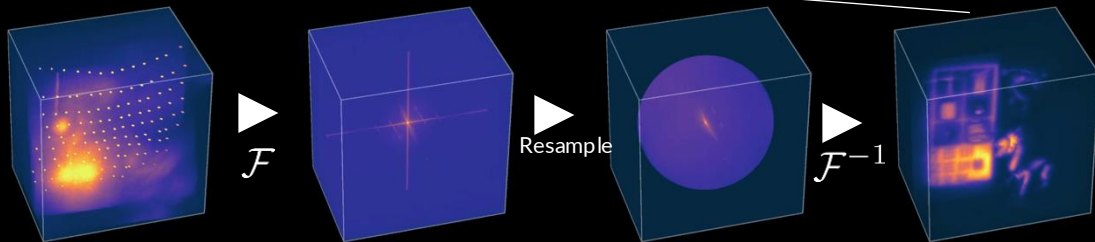
$$f = v\sqrt{k_x^2 + k_y^2 + k_z^2}$$

$$f \Rightarrow k_z$$

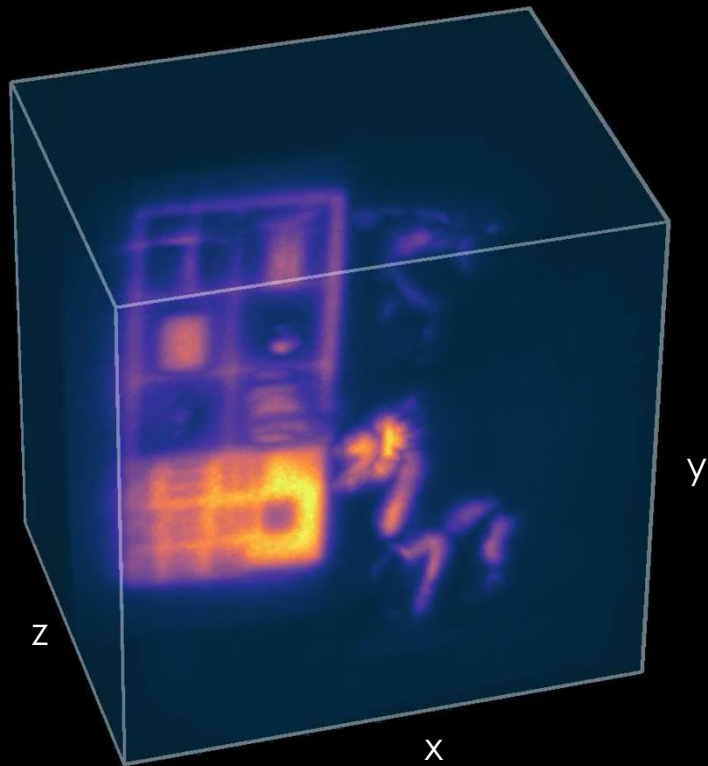


The migrated solution is an inverse Fourier Transform!

$$\Psi(x, y, z, t = 0) = \iiint \Phi(k_x, k_y, k_z) e^{2\pi i(k_x x + k_y y + k_z z)} dk_x dk_y dk_z$$



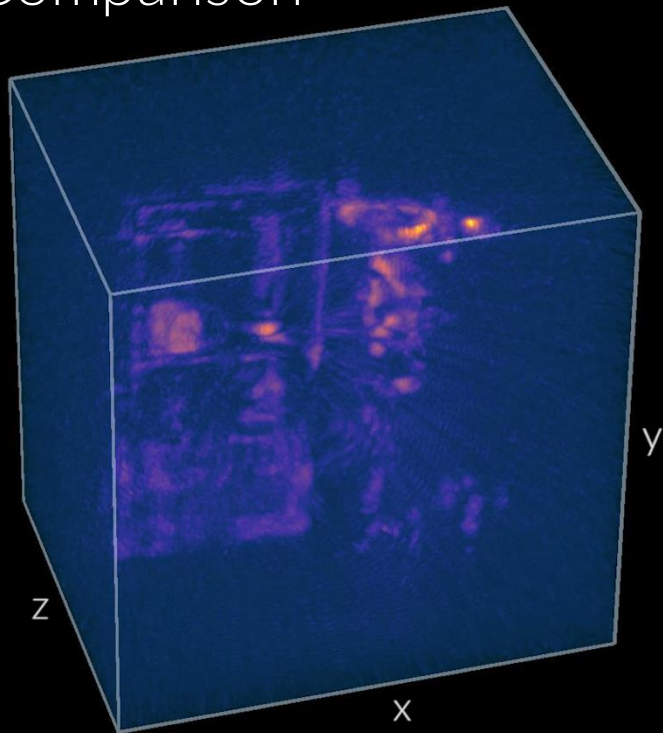
# $f$ - $k$ Migration



Dimensions: 2 x 2 m  
Exposure: 180 min  
Reconstruction time: ~90 sec

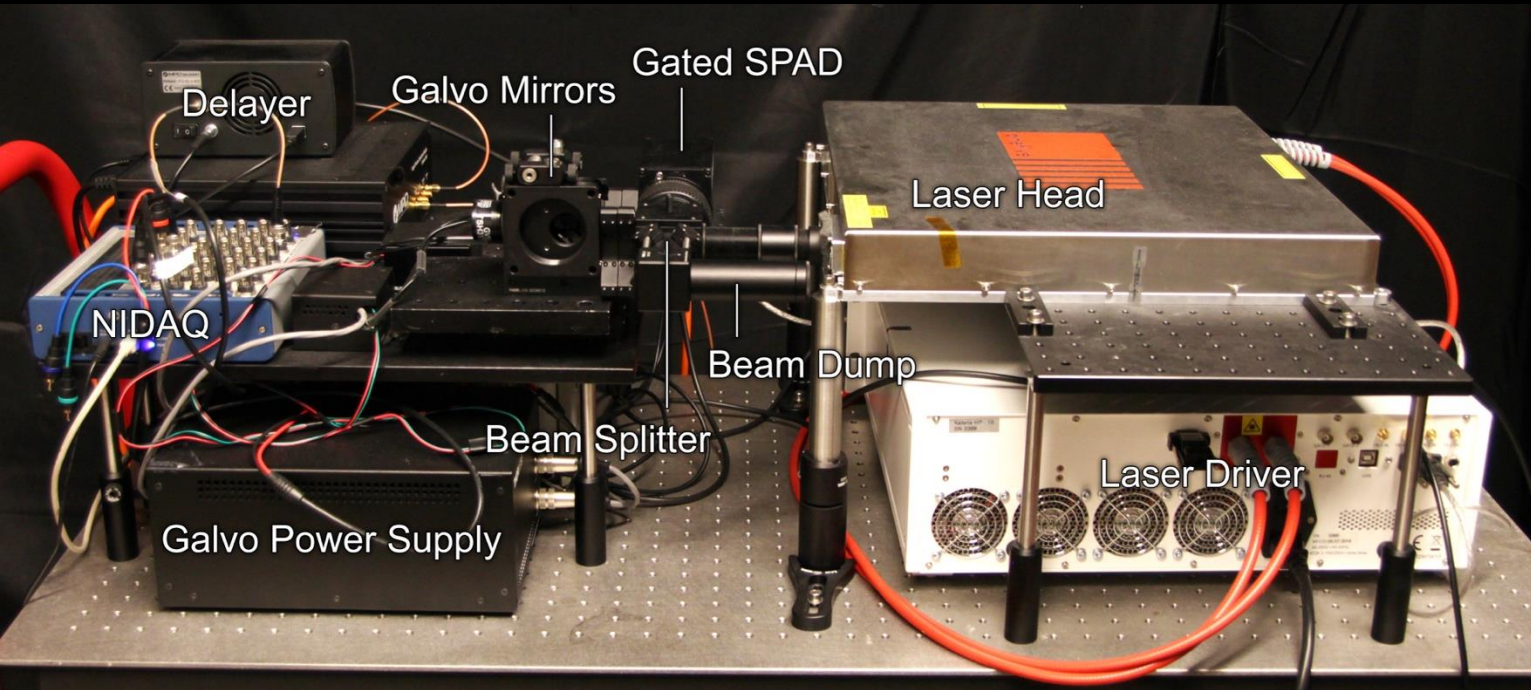
# Reconstruction Comparison

dimensions: 2 m x 2 m x 1.5 m



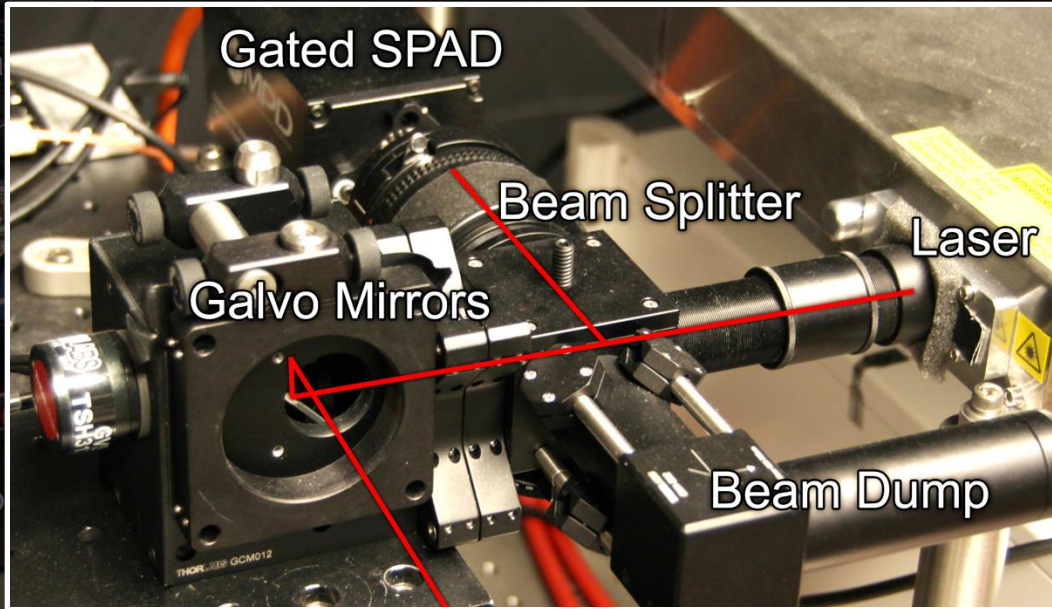
**Filtered  
Backprojection**

# hardware prototype





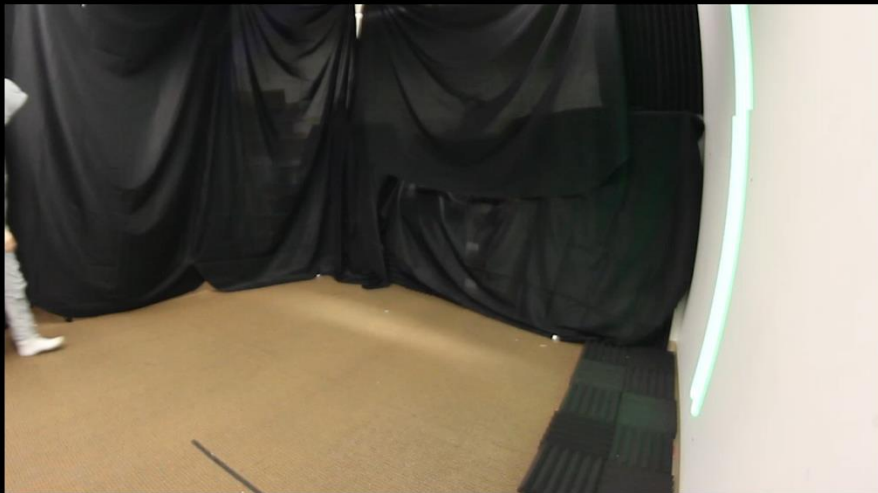
# hardware prototype







# real-time scanning

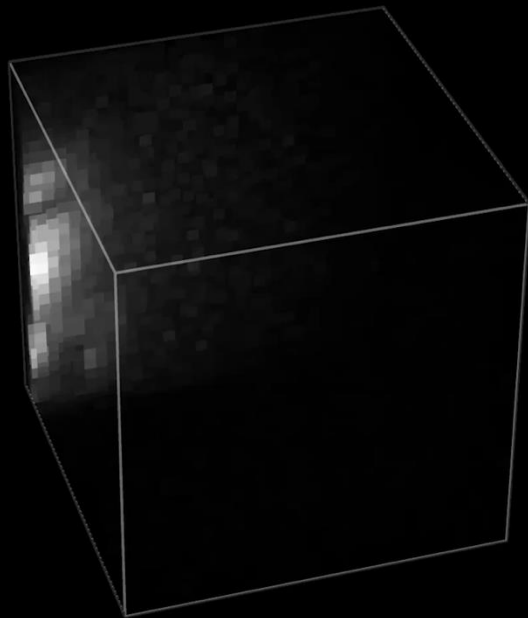


Framerate: 4 Hz

Resolution: 32 x 32

Dimensions: 2 m x 2 m x 2 m

Reconstruction time: ~1 s per frame



# Outlook

## Directional Light-Cone Transform



hidden scene

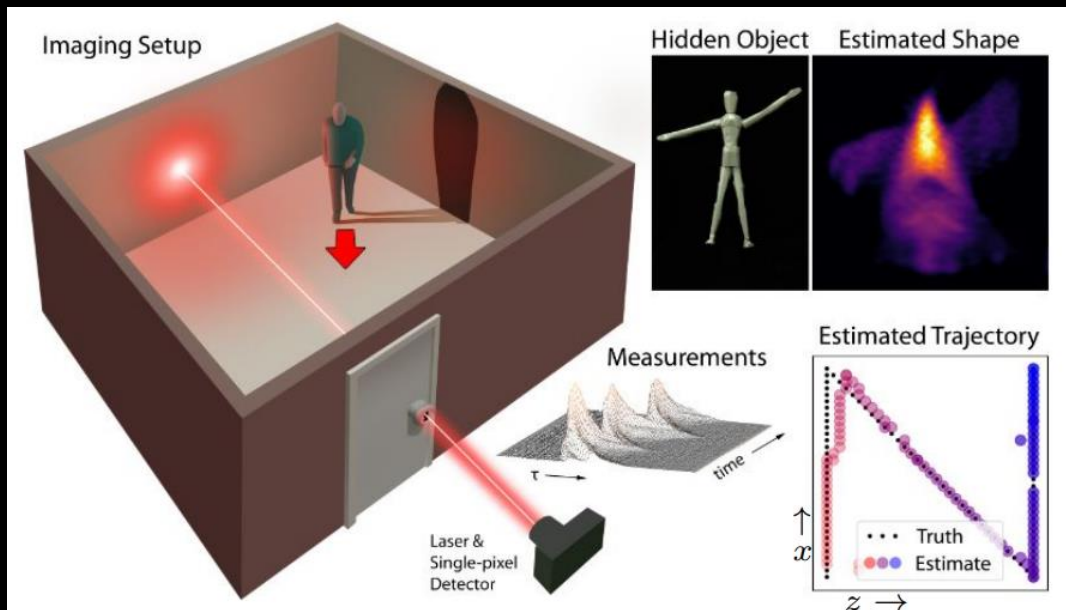


Recovered surface

[Young et al., CVPR 2020]

# Outlook

## Keyhole NLOS Imaging

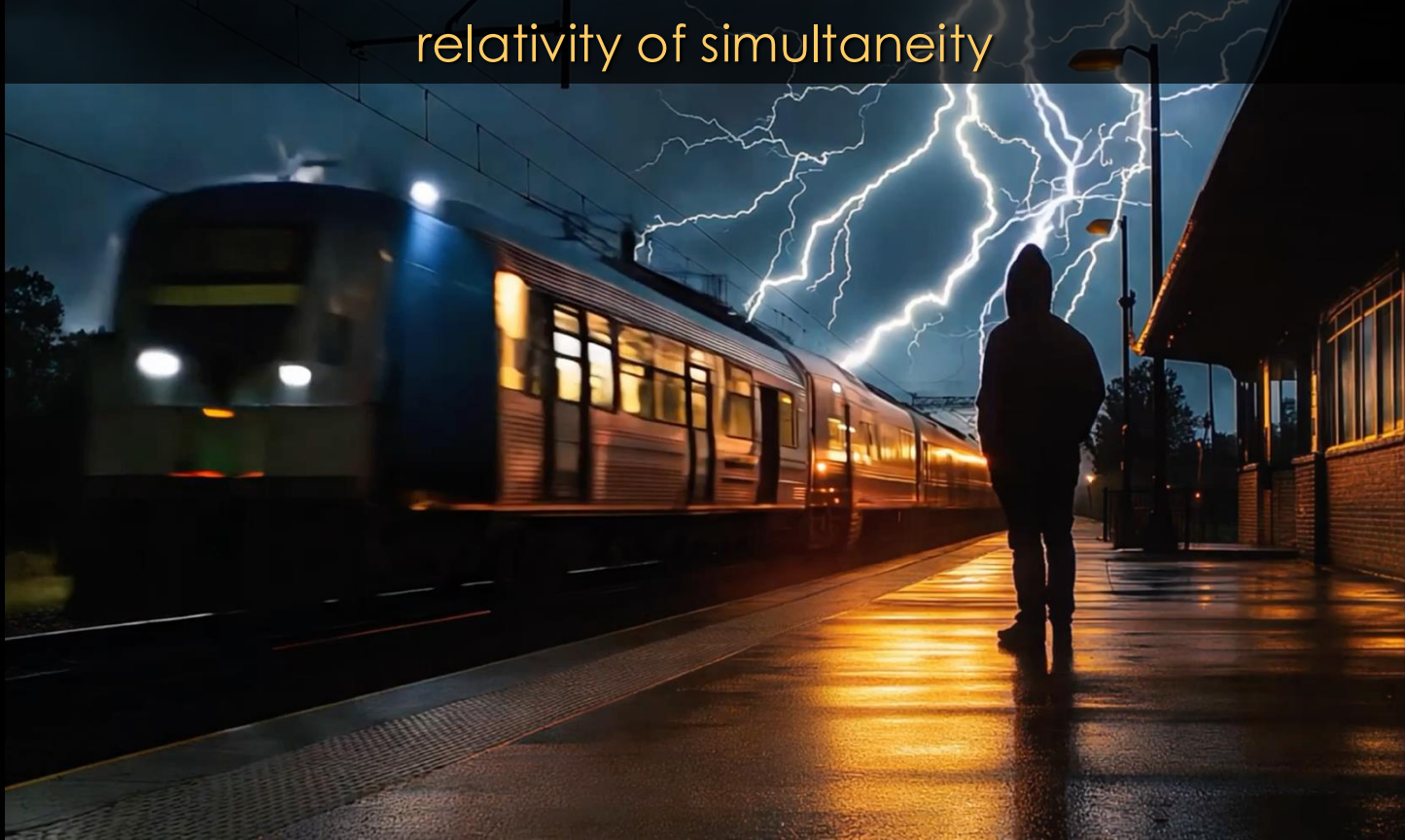


[Metzler et al., IEEE TCI 2021]

## overview

- Time-resolved imaging
- Single-photon avalanche diodes (SPADs)
- Single-photon lidar
- Non-line-of-sight imaging
- neural rendering for propagating light

relativity of simultaneity



relativity of simultaneity

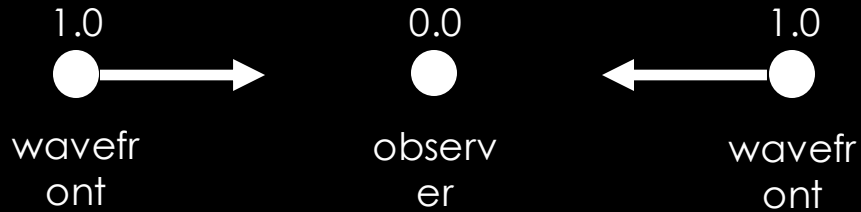




relativity of simultaneity

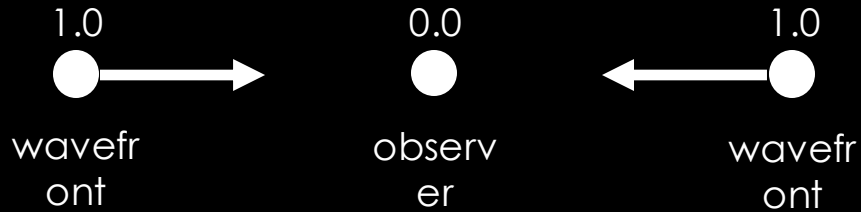


# relativity of simultaneity



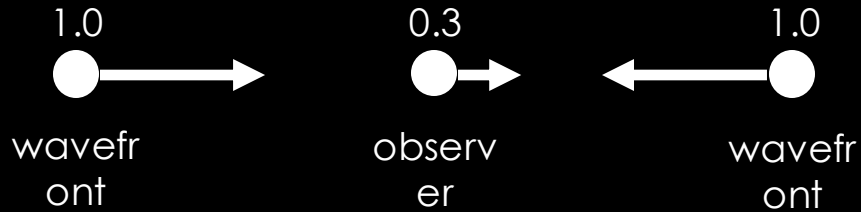


# relativity of simultaneity

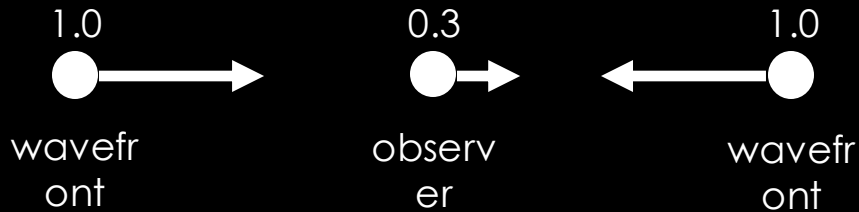


*appear to arrive  
simultaneously*

# relativity of simultaneity



# relativity of simultaneity



***wavefront to the right closes faster—right event happens first!***

## relativity of simultaneity

- Can we capture and visualize light propagation from moving viewpoints?
- Can we observe viewpoint-dependent changes in light propagation as Einstein predicted?

"transient" videography at 250 billion frames per second



"transient" videography at 250 billion frames per second



pulsed  
laser



# "transient" videography at 250 billion frames per second



pulsed  
laser



single-photon avalanche  
diode (SPAD)

[Malik et al. '2

"transient" videography at 250 billion frames per second



or this corner?

will we observe light reach this corner first?



light pulse



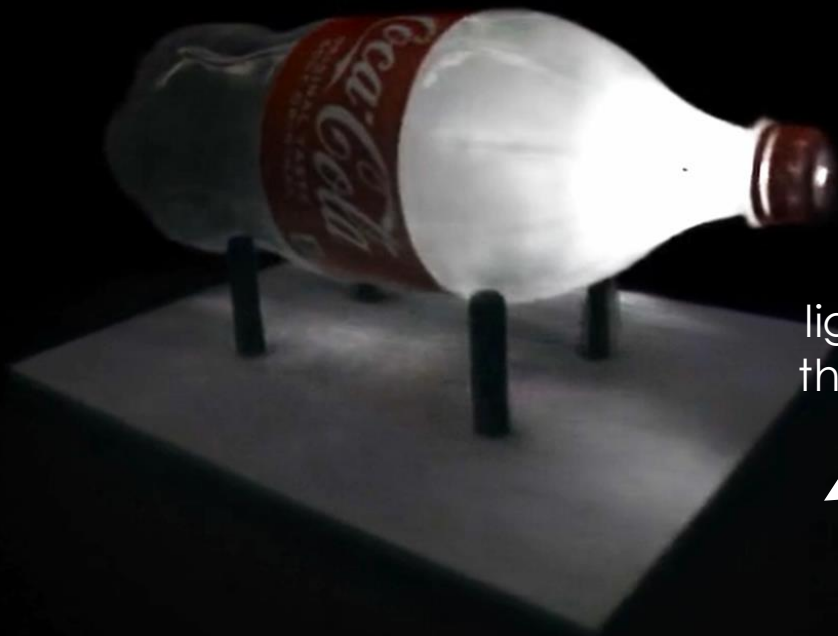
refracted light

0.82ns

light focused at ca



1.43ns



light reaches  
this corner first



1.90ns

← wavefront propagates  
away from camera

**2.33ns**

light reaches the  
back of the bottle



2.77ns





shortest path length  
from bottle to  
camera



## relativity of simultaneity

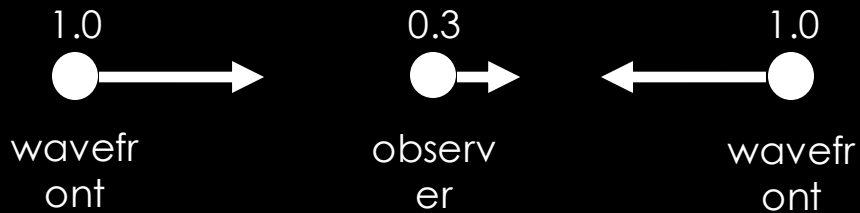
- Can we capture and visualize light propagation from moving viewpoints?
- Can we observe viewpoint-dependent changes in light propagation as Einstein predicted?

## relativity of simultaneity

Yes! (and no)

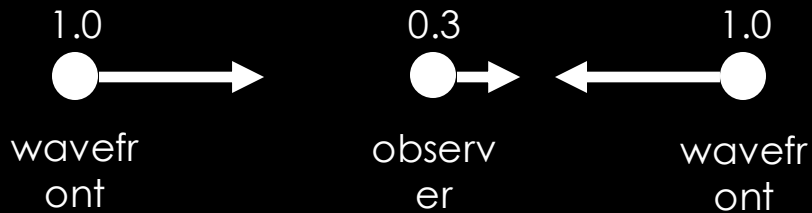
- we can't move at relativistic speeds

# relativity of simultaneity



***does the right wavefront appear to travel  $1.3c$ ?***

# relativity of simultaneity



***No! space contracts so that the speed of light is maintained***

***(Lorentz contraction)***

## relativity of simultaneity

Yes! (and no)

- we can't move at relativistic speeds
- we can capture and visualize propagation of light, including viewpoint-dependent effects

related work

# related work

## transient imaging



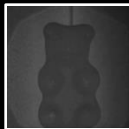
streak  
cameras

[Velten '13, Gao '14,  
...]



time-of-flight cameras

[Heide '13, O'Toole '14, ...]



interferome  
try

[Gkioulekas '15,  
...]



SPADs

[O'Toole '17, Lindell, '18,  
...]

# related work

## transient imaging



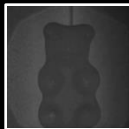
streak  
cameras

[Velten '13, Gao '14,  
...]



time-of-flight cameras

[Heide '13, O'Toole '14, ...]



interferome  
try

[Gkioulekas '15,  
...]



SPADs

[O'Toole '17, Lindell, '18,  
...]

- limited to single-viewpoint  
capture



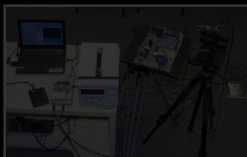
# related work

## transient imaging



streak  
cameras

[Velten '13, Gao '14,  
...]



time-of-flight cameras

[Heide '13, O'Toole '14, ...]



interferome  
try

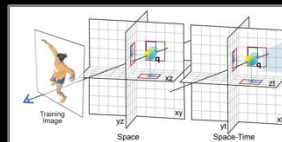
[Gkioulekas '15,  
...]



SPADs

[O'Toole '17, Lindell, '18,  
...]

## NeRFs & video novel view synthesis



[Li '22]

[Cao & Johnson '23]

[Fridovich-Keil '23]

[Wang '23]

- limited to single-viewpoint  
capture

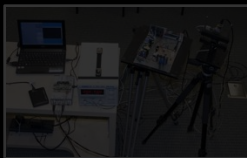
# related work

## transient imaging



streak  
cameras

[Velten '13, Gao '14,  
...]



time-of-flight cameras

[Heide '13, O'Toole '14, ...]



interferome  
try

[Gkioulekas '15,  
...]

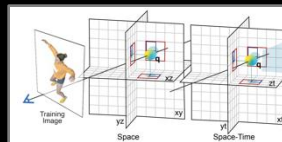


SPADs

[O'Toole '17, Lindell, '18,  
...]

- limited to single-viewpoint  
capture

## NeRFs & video novel view synthesis



[Li '22]

[Cao & Johnson '23]

[Fridovich-Keil '23]

[Wang '23]

- do not account for finite speed  
of light

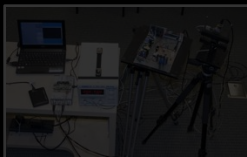
# related work

## transient imaging



streak  
cameras

[Velten '13, Gao '14,  
...]



time-of-flight cameras

[Heide '13, O'Toole '14, ...]



interferome  
try

[Gkioulekas '15,  
...]

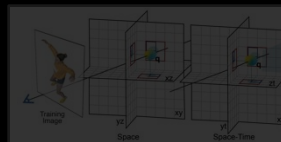


SPADs

[O'Toole '17, Lindell, '18,  
...]

- limited to single-viewpoint  
capture

## NeRFs & video novel view synthesis



[Li '22]

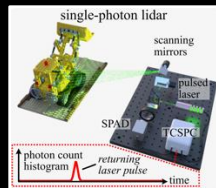
[Cao & Johnson '23]

[Fridovich-Keil '23]

[Wang '23]

- do not account for speed  
of light

## Transient NeRF



[Malik  
'23]

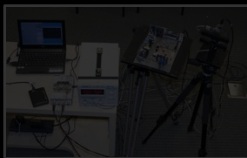
# related work

## transient imaging



streak  
cameras

[Velten '13, Gao '14,  
...]



time-of-flight cameras

[Heide '13, O'Toole '14, ...]



interferome  
try

[Gkioulekas '15,  
...]

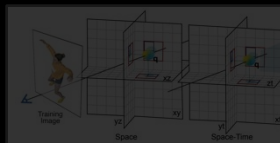


SPADs

[O'Toole '17, Lindell, '18,  
...]

- limited to single-viewpoint  
capture

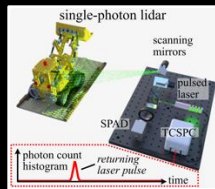
## NeRFs & video novel view synthesis



[Li '22]  
[Cao & Johnson '23]  
[Fridovich-Keil '23]  
[Wang '23]

- do not account for speed  
of light

## Transient NeRF

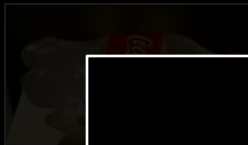


[Malik  
'23]

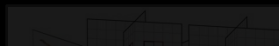
- only models direct  
reflections

# related work

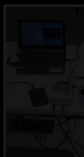
transient imaging



video novel view synthesis



## contributions



[Gkioulekas '15,  
...]

SPADs

[O'Toole '17, Lindell, '18,  
...]



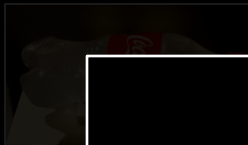
[Malik  
'23]

- limited to single-viewpoint capture

- only models direct reflections

# related work

transient imaging

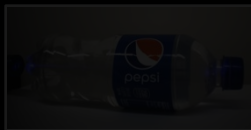


video novel view synthesis



## contributions

- new method for novel view synthesis of light propagation videos



[Gkioulekas '15,  
...]

SPADs

[O'Toole '17, Lindell, '18,  
...]



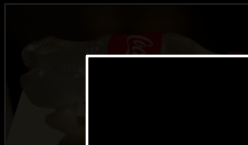
[Malik  
'23]

- limited to single-viewpoint capture

- only models direct reflections

# related work

transient imaging

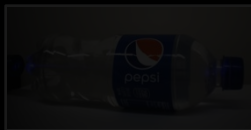


video novel view synthesis



## contributions

- new method for novel view synthesis of light propagation videos
- introduce a new neural field parameterization to make this possible



[Gkioulekas '15,  
...]

SPADs

[O'Toole '17, Lindell, '18,  
...]



[Malik  
'23]

- limited to single-viewpoint

capture

- only models direct reflections

# related work

transient imaging

video novel view synthesis

## contributions

- new method for novel view synthesis of light propagation videos
- introduce a new neural field parameterization to make this possible
- demonstrate on a new multi-viewpoint transient dataset

SPADs

[O'Toole '17, Lindell, '18, ...]



[Malik '23]

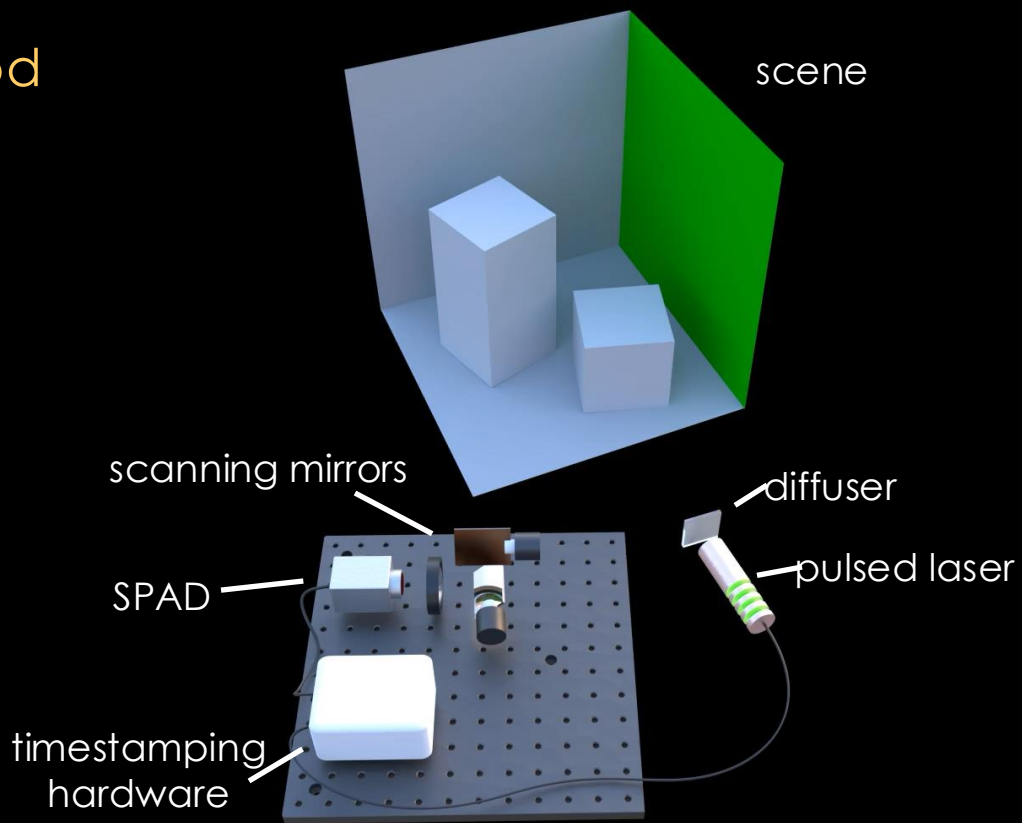
- limited to single-viewpoint capture

- only models direct reflections



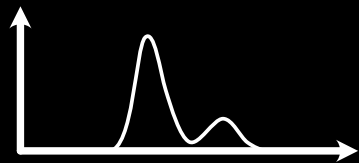
method

method



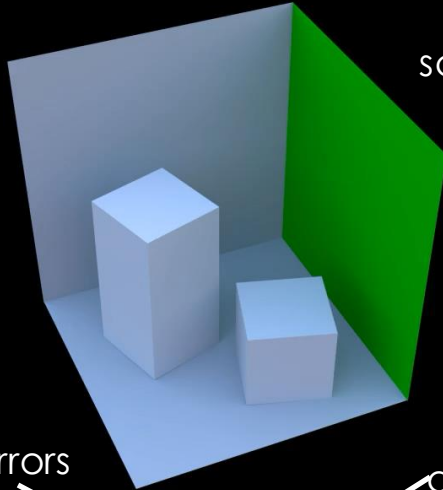
# method

transient (histogram)



time (ps)

scene



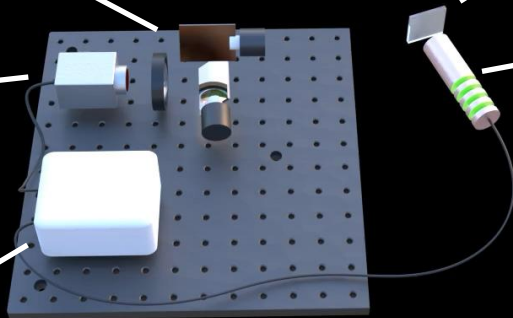
scanning mirrors

SPAD

timestamping  
hardware

diffuser

pulsed laser



method

scene

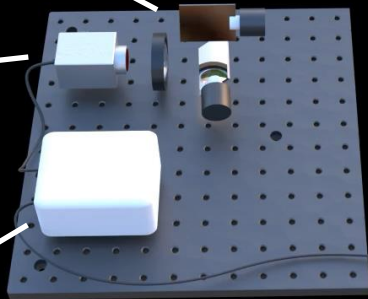
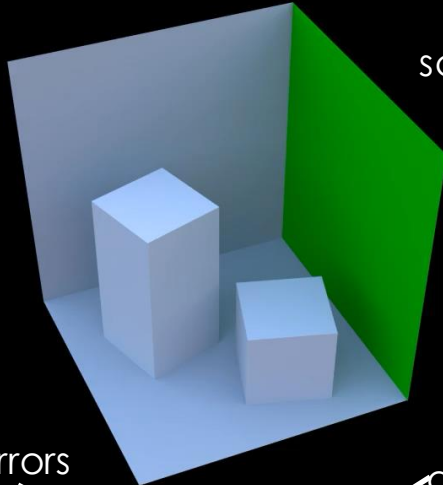
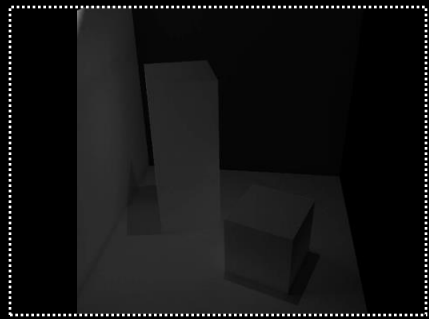
scanning mirrors

SPAD

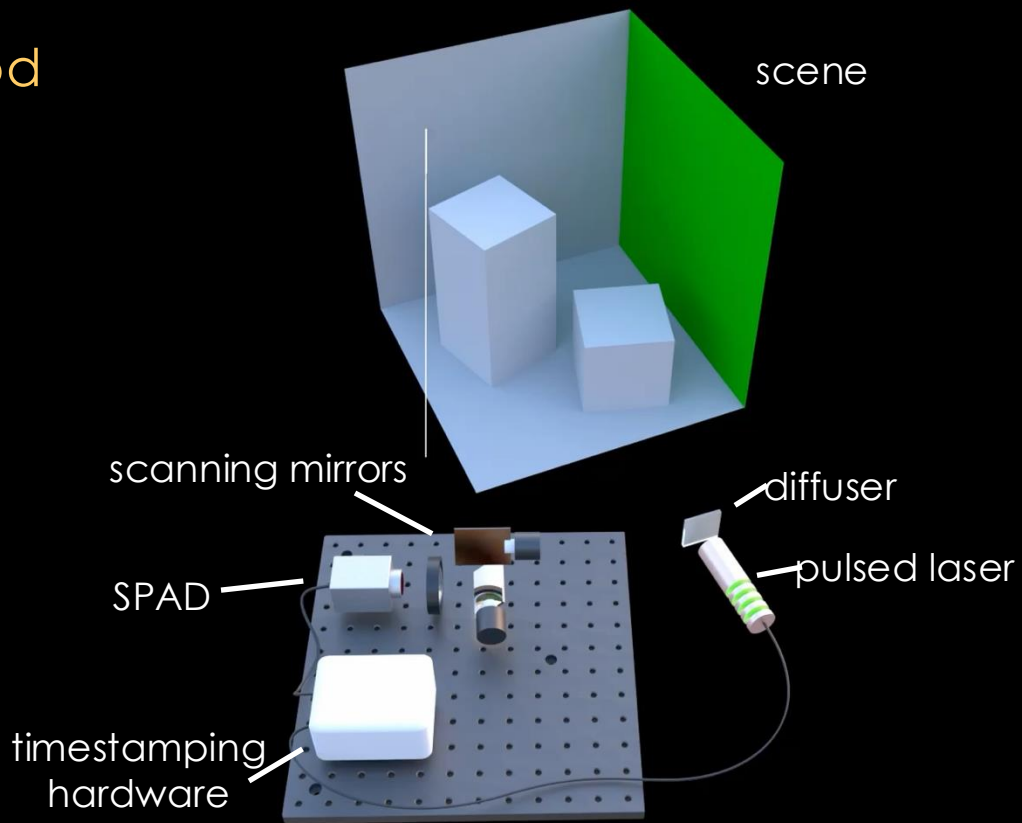
timestamping  
hardware

diffuser

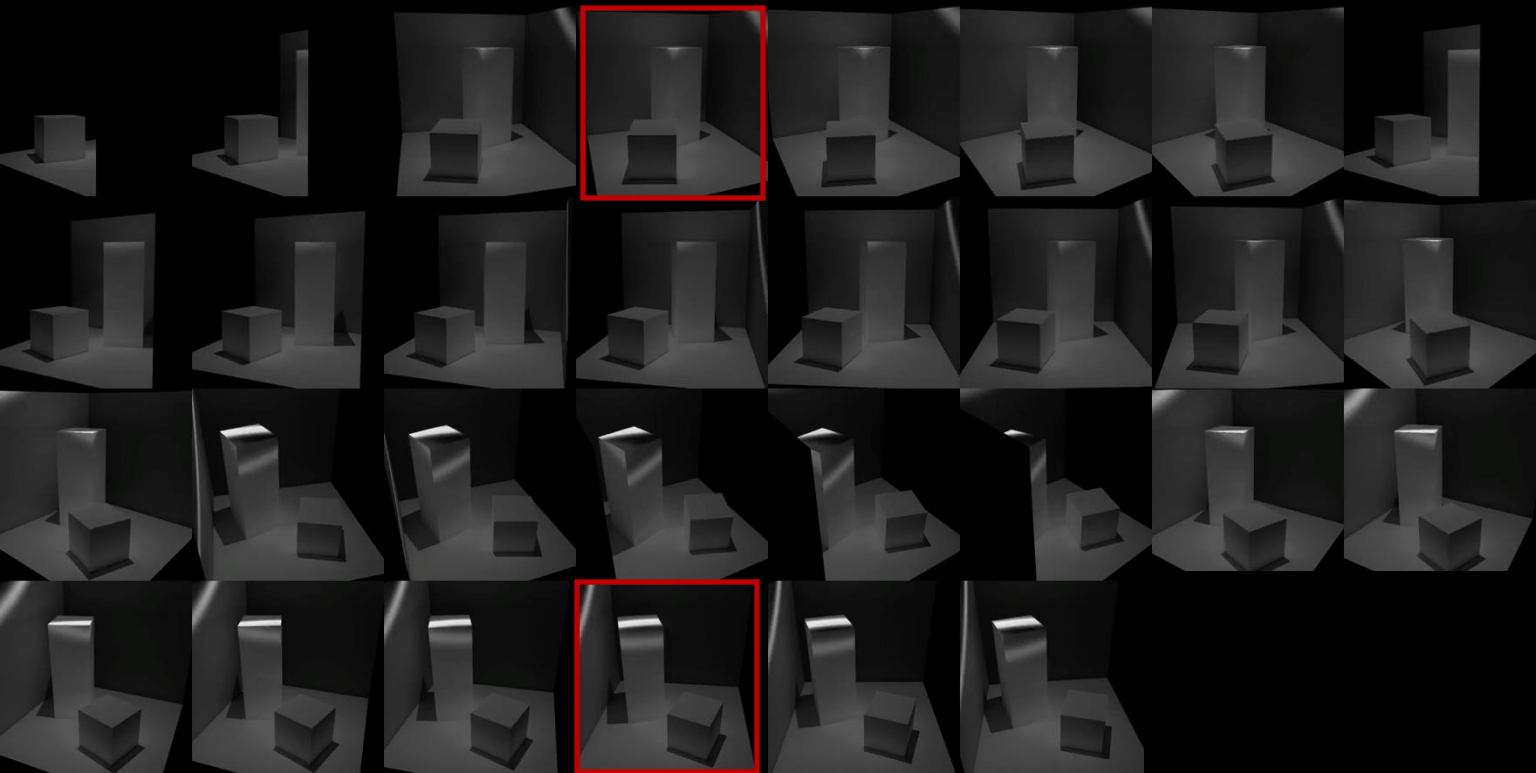
pulsed laser



method

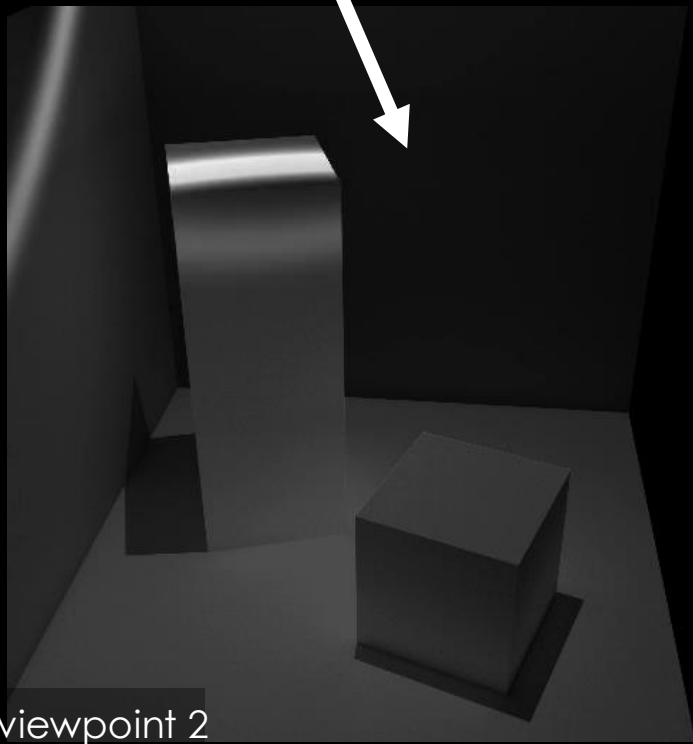
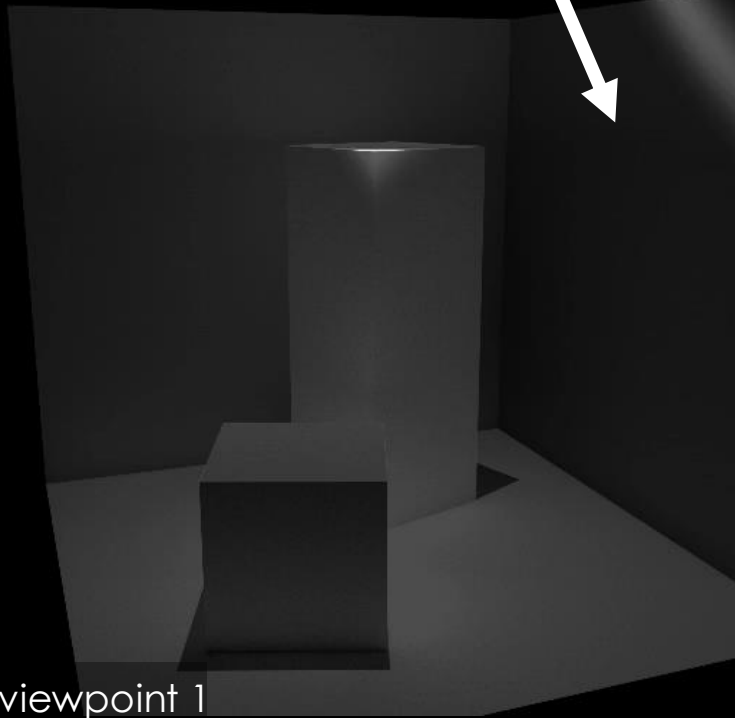


method



method

transient light transport is viewpoint dependent!



# method

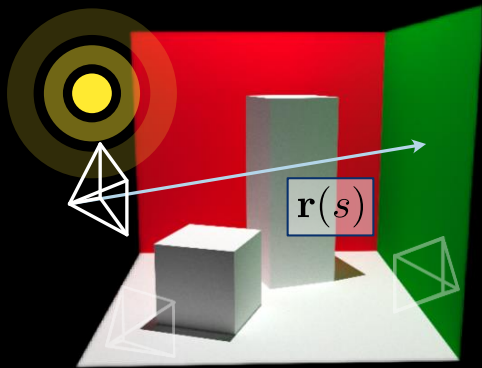
- every camera viewpoint has its own temporal reference frame!

viewpoint 1

viewpoint 2

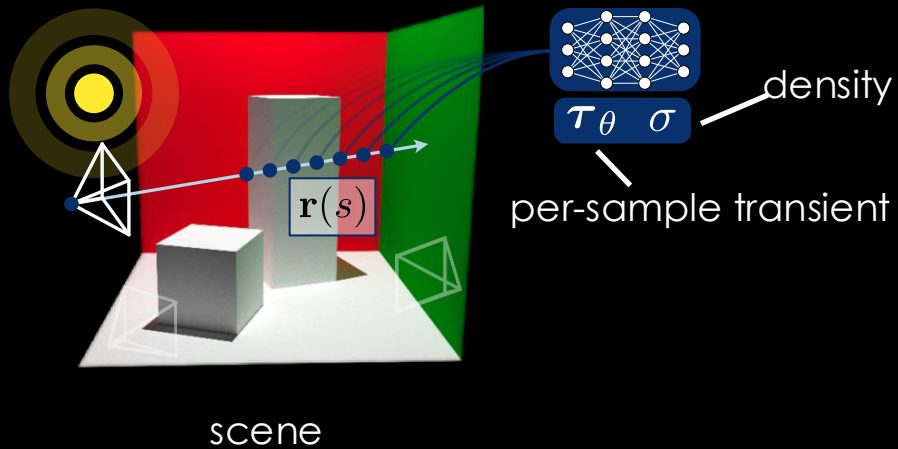


method

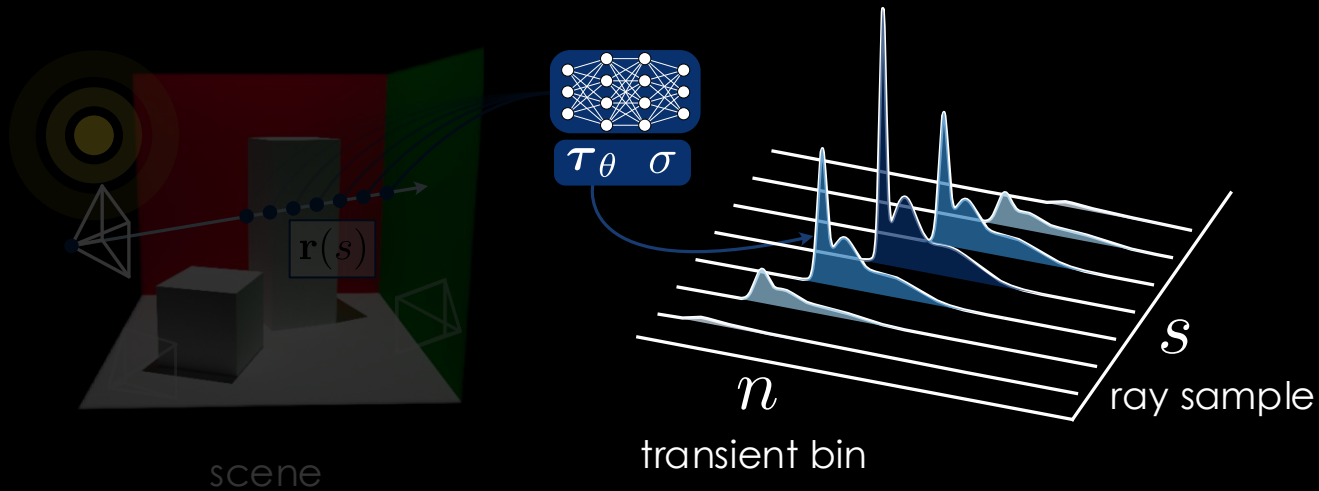


scene

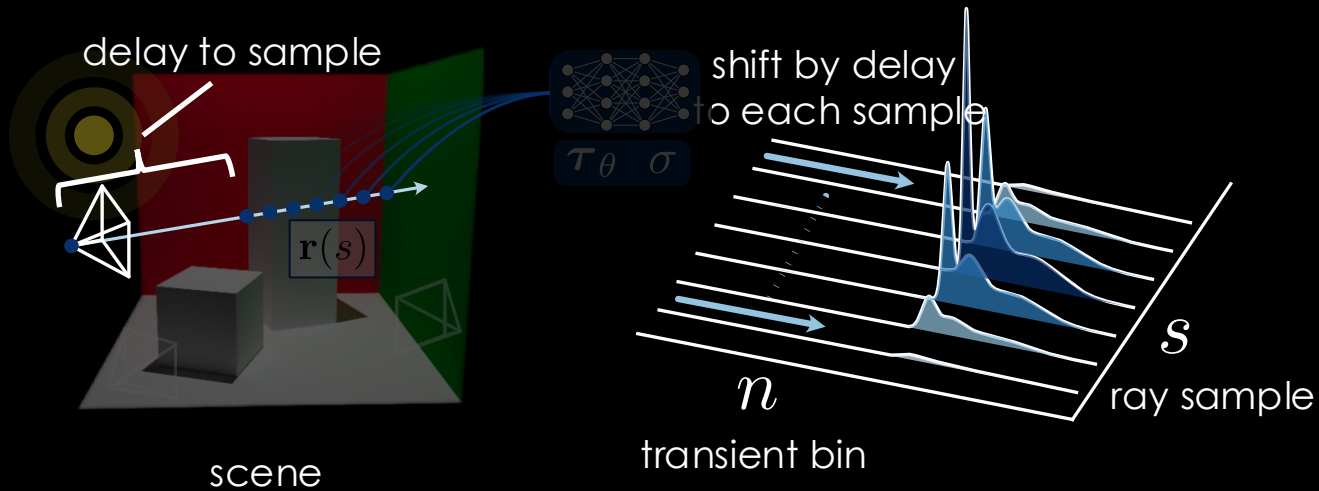
# method



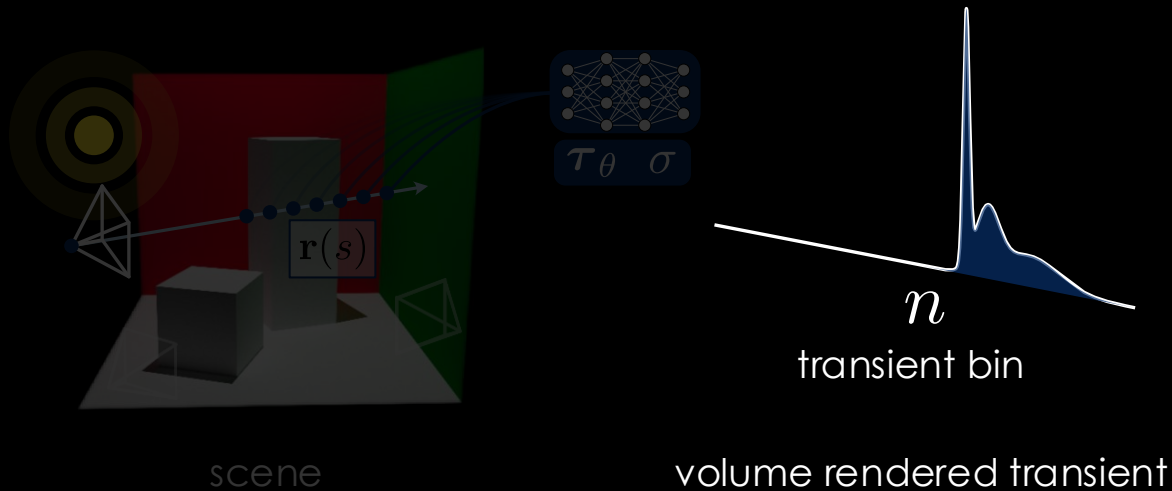
# method



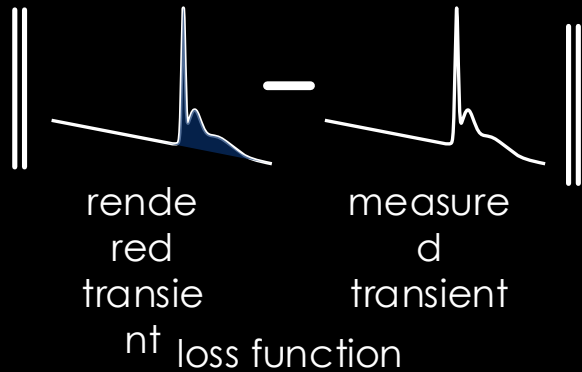
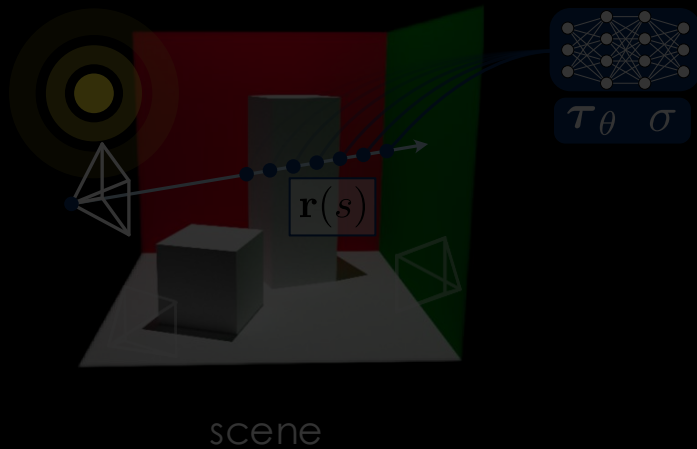
# method



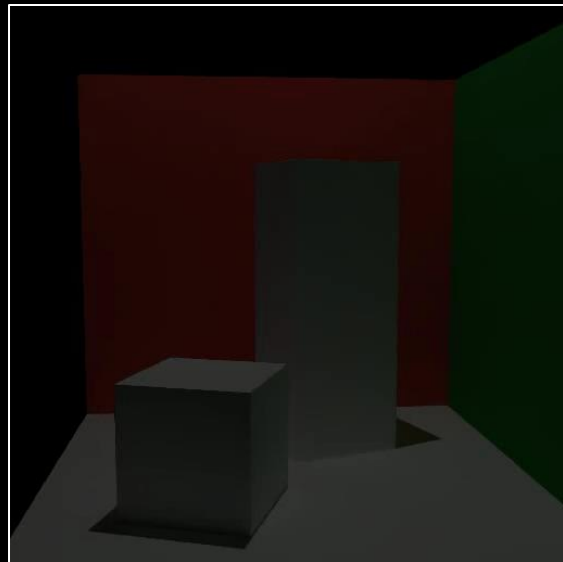
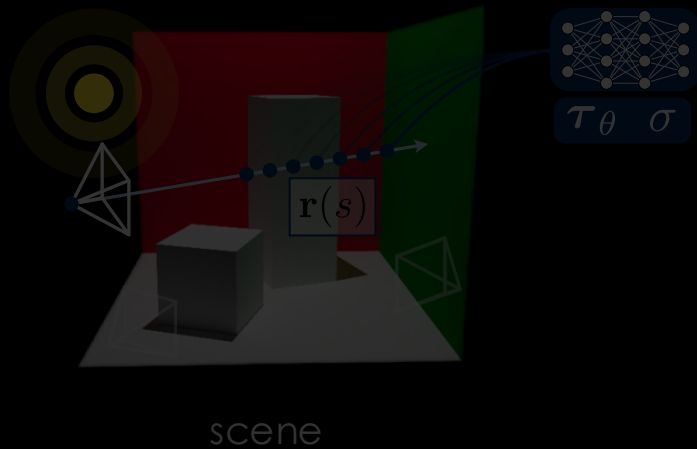
# method



# method



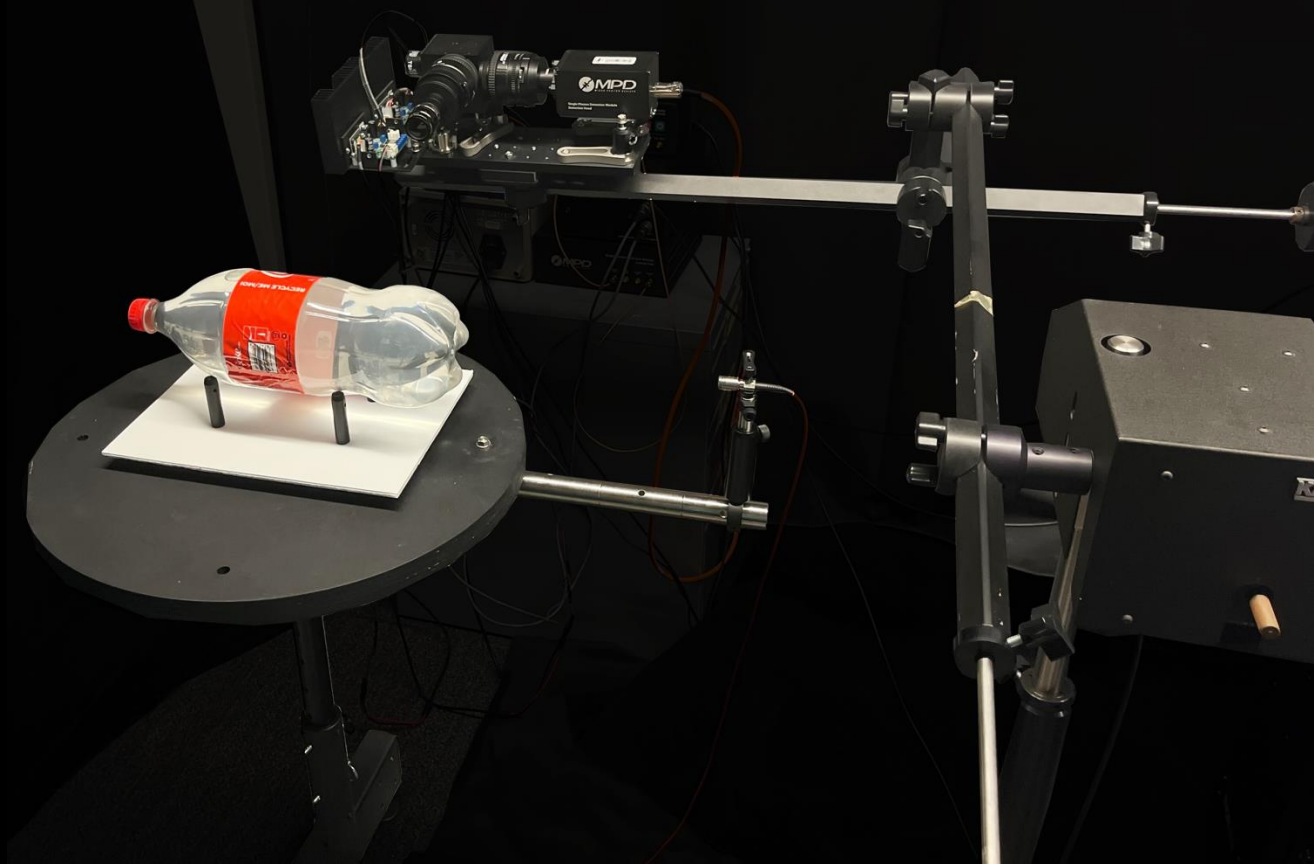
# method



- render from given viewpoint
- linearly increase time

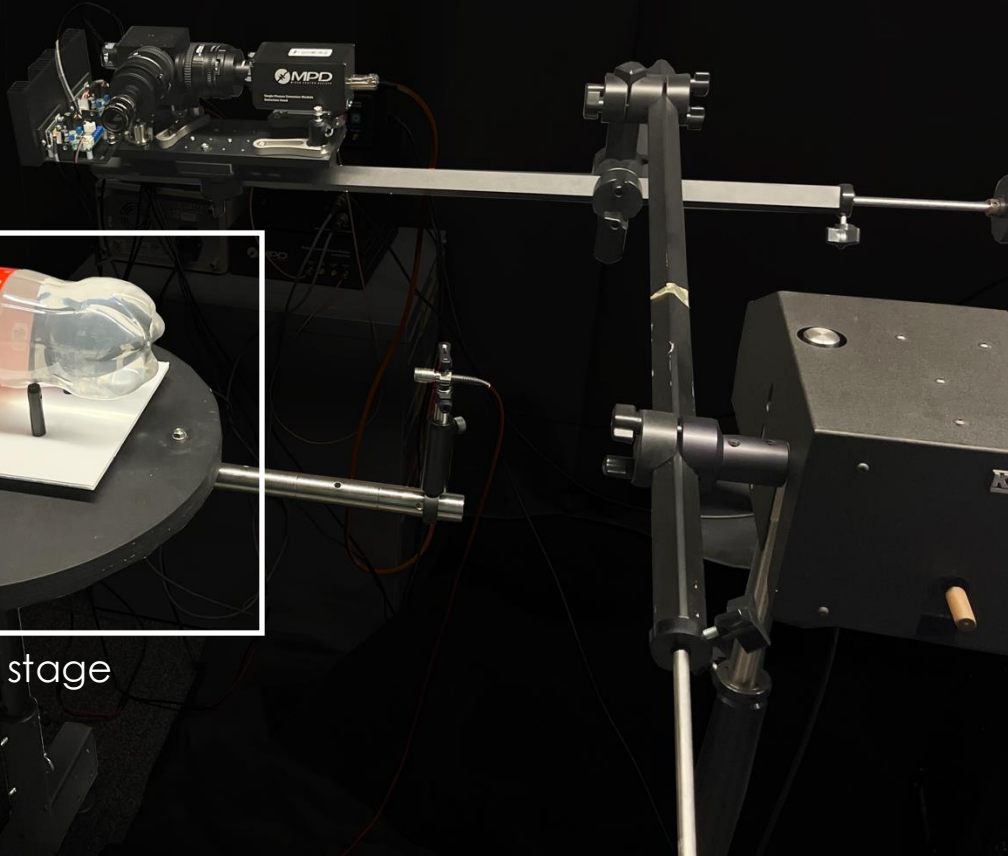
hardware prototype

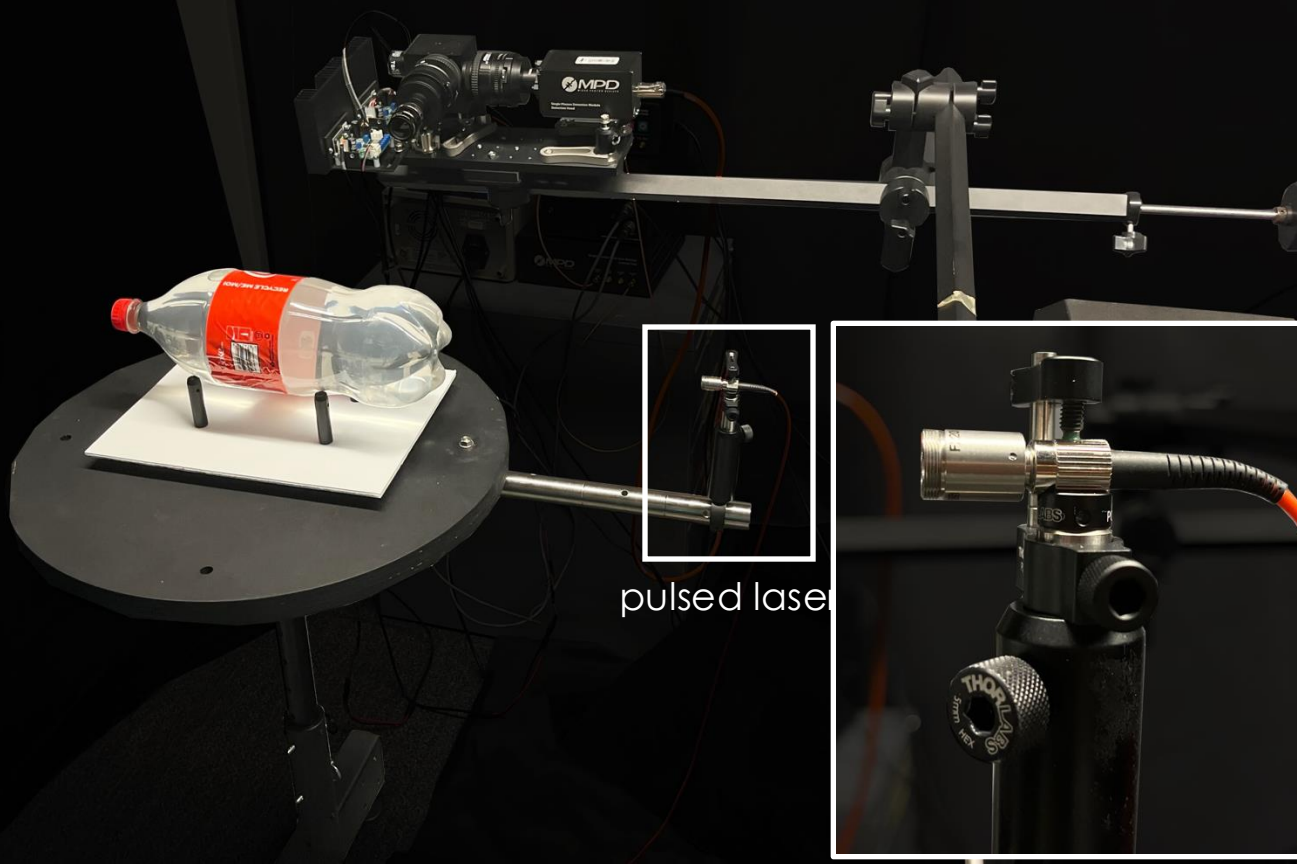






rotation stage



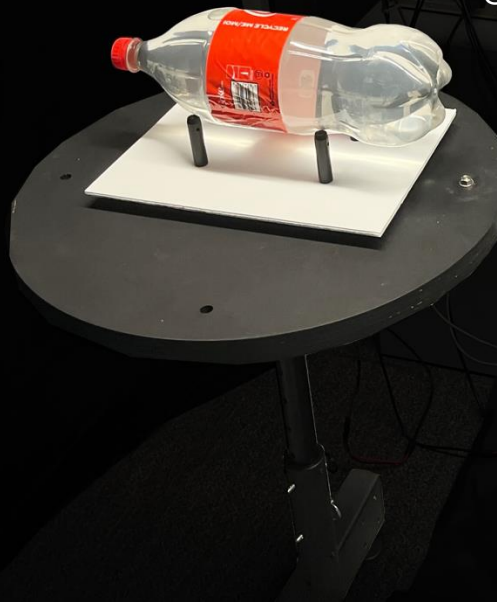


pulsed laser

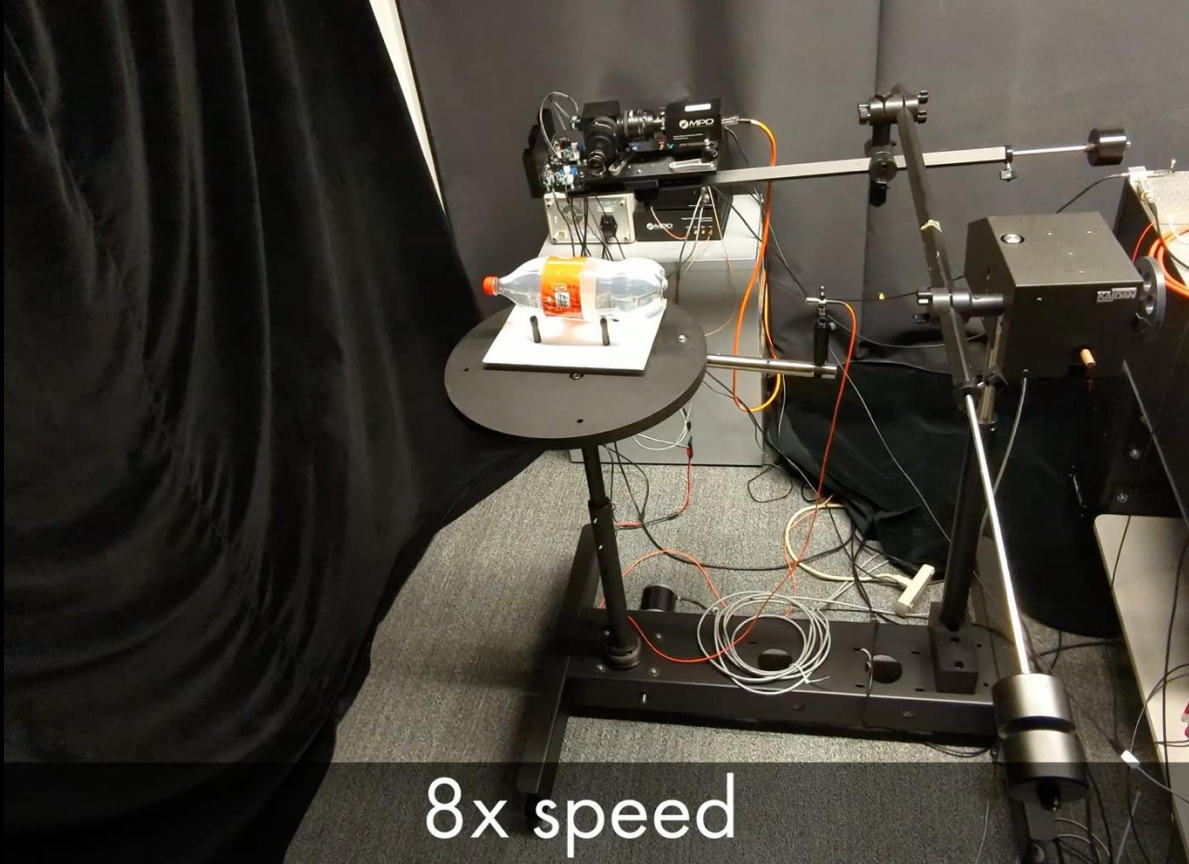




scanning SPAD



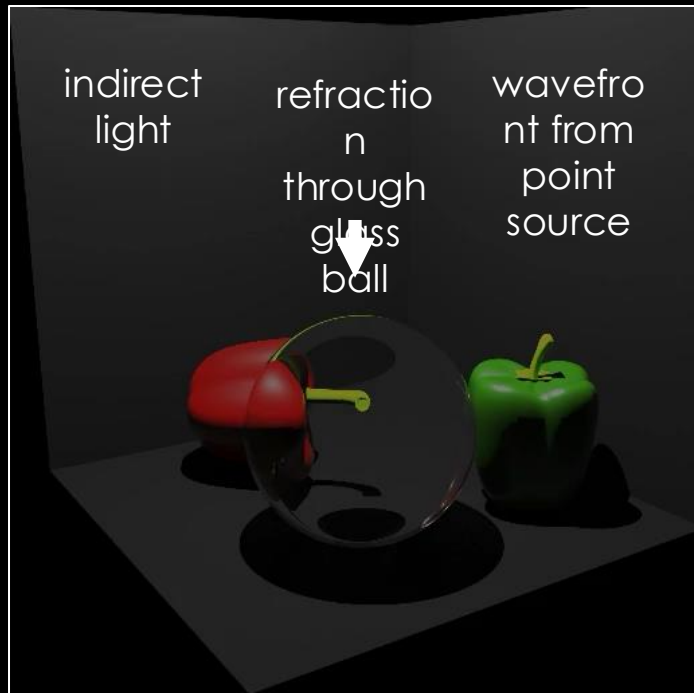
objective lens



8x speed

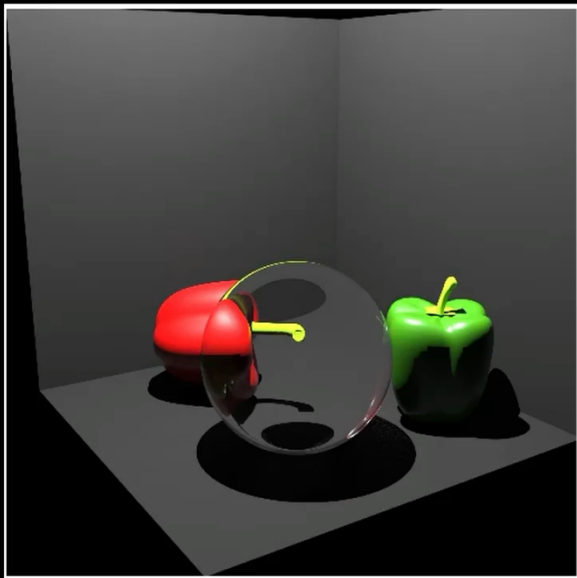
results

# simulated scene (ground truth)

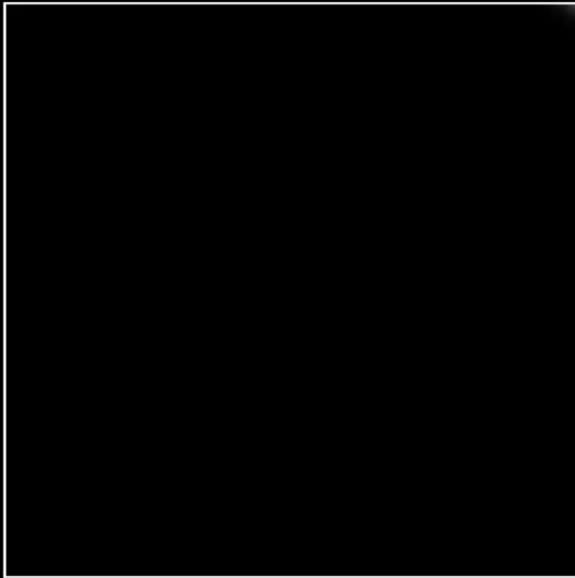


# simulated scene (ground truth)

integrated image



transient





# moving camera (novel viewpoints)

Transient NeRF



K-Planes



proposed

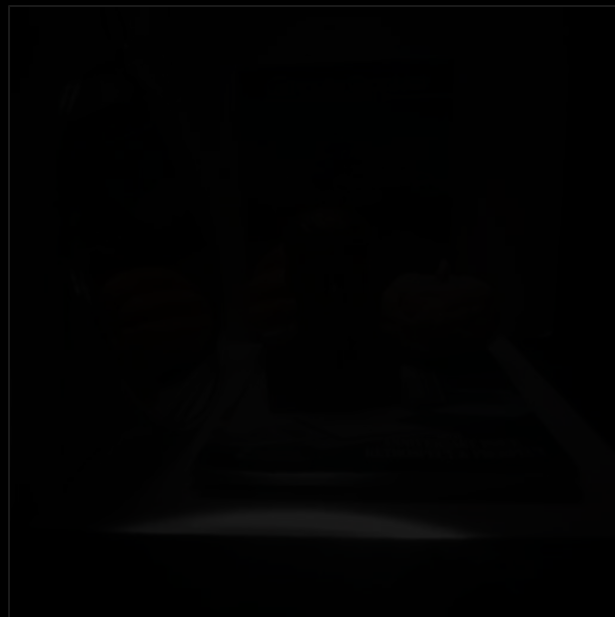


# moving camera (novel viewpoints —captured data)

color image



transient video (proposed)



# moving camera (novel viewpoints — captured data)

color image



transient video (proposed)



# moving camera (novel viewpoints — captured data)

color image



transient video (proposed)



# moving camera (novel viewpoints—captured data)

color image

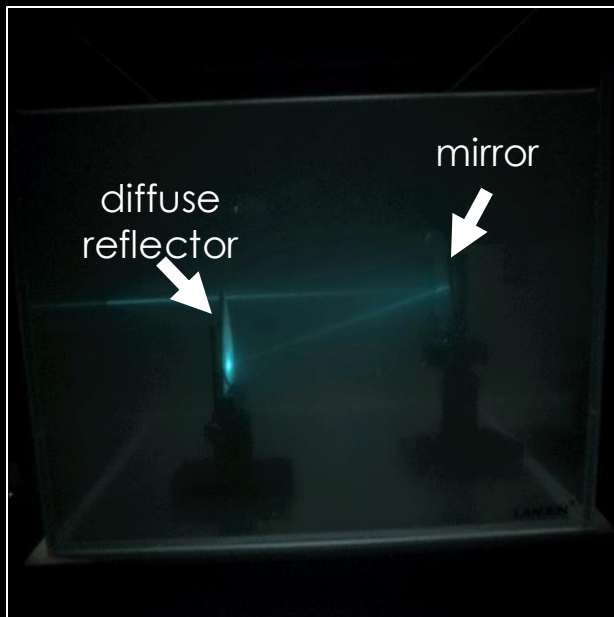


transient video (proposed)



# moving camera (novel viewpoints—captured data)

color image

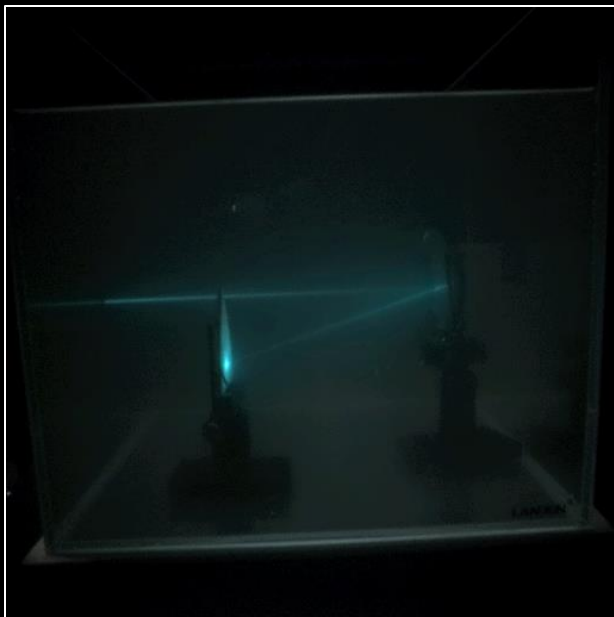


transient video (proposed)



# moving camera (novel viewpoints—captured data)

color image

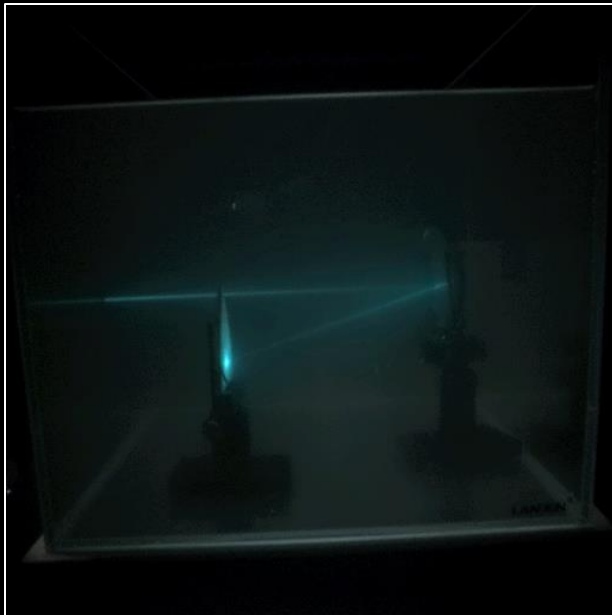


transient video (proposed)

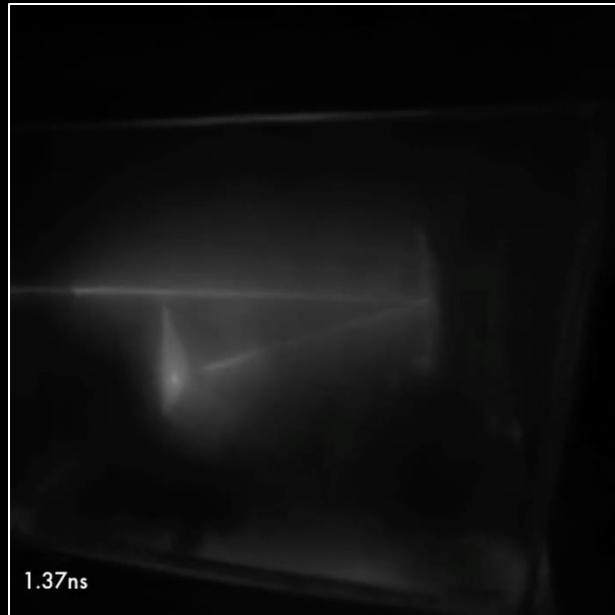


# moving camera (novel viewpoints—captured data)

color image



transient video (proposed)





# moving camera (novel viewpoints—captured data)

color image



transient video (proposed)

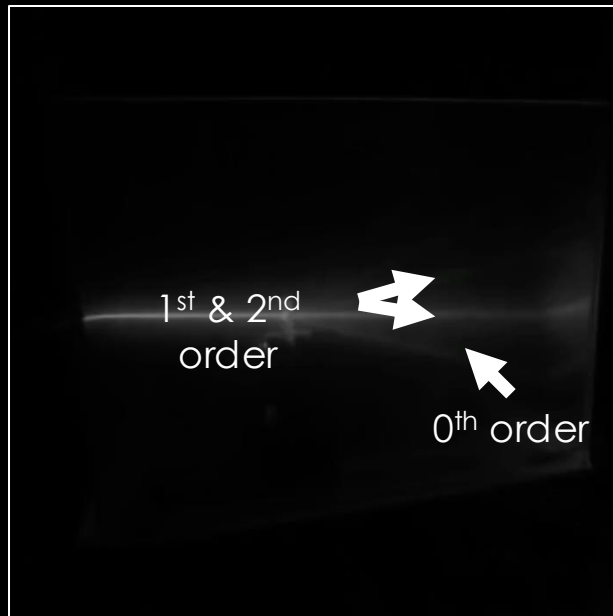


# moving camera (novel viewpoints—captured scene)

color image



transient video (proposed)

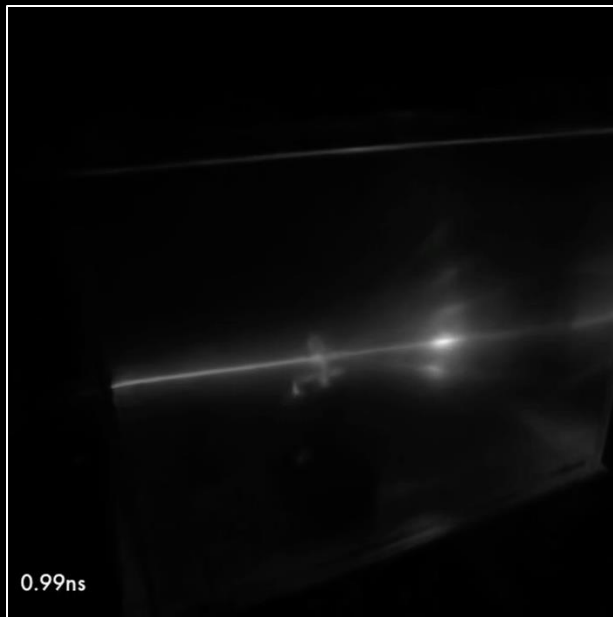


# moving camera (novel viewpoints—captured scene)

color image



transient video (proposed)



# captured scene (novel viewpoints)

color image



transient video



# captured scene (novel viewpoints)

color image



transient video



# captured scene (novel viewpoints)

color image



transient video

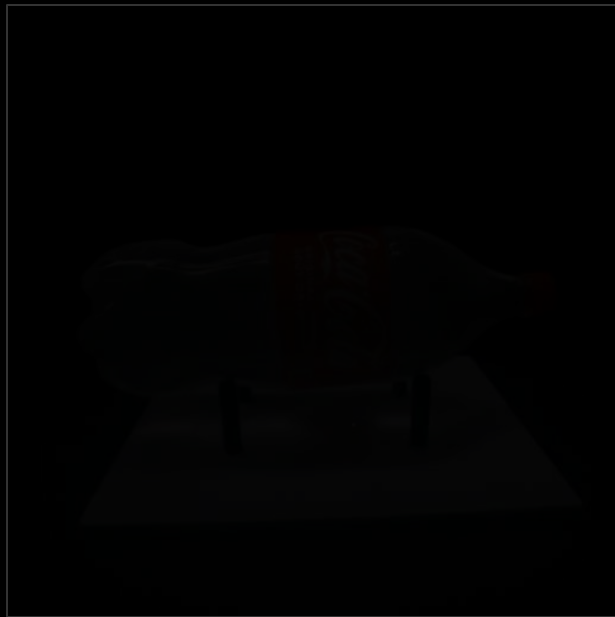


# time-warping (Velten et al. 2012)

no warping



depth-based warping



# time-warping (Velten et al. 2012)

no warping



depth-based warping





discussion

## future work

- Can we render & incorporate relativistic effects?

## future work

- Can we render & incorporate relativistic effects?
- this work focuses primarily on visualization, but...
  - can we use multiply-scattered light to recover 3D geometry?
  - what about material or reflectance properties?
  - imaging through scattering media?

## future work

- 3D visualization of light transport
  - artistic, educational, and scientific visualizations
  - visualize non-linear or quantum optical effects?
  - imaging fast & slow

Frame rate: 10.0Hz

Intensity tone mapped

Elapsed time: 24s + 500ms

[Wei et al. '23]



## concluding remarks

- Many applications for time-of-flight imaging
  - Lidar
  - Non-line-of-sight
  - Transient imaging
  - Imaging through scatter
- New capabilities through combining emerging sensors with computation!

# References

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- Malik, Anagh, et al. "Flying With Photons: Rendering Novel Views of Propagating Light." *arXiv preprint arXiv:2404.06493* (2024).