Time-of-Flight Imaging & Single-Photon Imaging
lidar, non-line-of-sight imaging, ultrafast imaging

CSC2529
David Lindell
University of Toronto
cs.toronto.edu/~lindell/teaching/2529

*slides adapted from Matt O’Toole, Gordon Wetzstein, Yannis Gkioulekas
Poster Session

- **Thursday Dec 7th 2-4pm**
  - Bahen Atrium
  - you MUST attend in person or make alternative arrangements with me (only in extenuating circumstances)
- **You are responsible for making sure that your poster is printed on time!**
  - use the “same-day” or ”next-day” ordering option from https://utposter.com/
  - use the offer code on Quercus (“pages”) so you don’t have to pay
  - Pick up poster from 339 Bloor St W
  - bring it to the poster session and put it up before poster session begin
- You have from November 18 until December 6 (the day before the poster session) to print your poster using this service.
Poster Session

- feedback on proposals out by end of today
  - we will message you on Piazza

- Check Quercus->Assignments->Final Project for rubric for the poster session, code, report
Use the offer code on Quercus
Machine learning for determining protein structure and dynamics from cryo-EM images

Ellen Zhong
Assistant Professor, Princeton University

Wednesday Nov 22, 3-4 PM
DGP Seminar Room (Bahen 5187)

5 bonus points if you attend and upload selfie to the Quercus assignment
Co-Designing Optics and Algorithms for High-Dimensional Visual Computing

Vishwanath Saragadam
Incoming Assistant Professor,
UC Riverside

Monday Nov 27, 2-4 PM (lecture 10)
Next Generation Black-Hole Imaging

Aviad Levis
Incoming Assistant Professor,
University of Toronto

Monday Dec 4, 2-4 PM (lecture 11)
transient imaging (a.k.a. femtophotography)
• Time-resolved imaging
• Single-photon avalanche diodes (SPADs)
• Single-photon lidar
• Non-line-of-sight imaging
• passive ultra-wideband sensing
speed of sound in air: 343 meters / sec
in water: 1480 meters / sec
Light takes 1.255 seconds to travel from the earth to the moon

speed of light in a vacuum: 299,792,458 meters / sec

(Light travels approximation 1 MILLION times faster than sound!)
transient imaging

speed of light in a vacuum: \(299,792,458\) meters / sec

(Light travels approximately 1 MILLION times faster than sound!)
**direct** and **indirect** time-of-flight sensors for transient imaging

- **Direct time-of-flight sensor**
  - Velodyne VLS-128

- **Indirect time-of-flight sensor**
  - Microsoft Kinect v2
direct and indirect time-of-flight sensing

Direct time-of-flight sensor

Velodyne VLS-128

Indirect time-of-flight sensor

Microsoft Kinect v2
direct and indirect time-of-flight sensing

Velodyne VLS-128

Direct time-of-flight sensor
direct time-of-flight principle

ultrafast sensor

ultrafast light source

Output

Input

time ➔
ultrafast light source
ultrafast sensor

direct time-of-flight principle
direct time-of-flight principle

ultrafast light source

ultrafast sensor

Input

Output

time ➔

e

direct time-of-flight principle
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ultrafast light source

ultrafast sensor
direct time-of-flight principle

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ultrafast sensor

Input

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direct time-of-flight principle

Output

time ➔

 transient measurement
direct and indirect time-of-flight sensing

Direct time-of-flight sensor

Indirect time-of-flight sensor
direct and indirect time-of-flight sensing
indirect time-of-flight principle
indirect time-of-flight principle
indirect time-of-flight principle

Output

Input

time ➔

ultrafast light source

ultrafast sensor
indirect time-of-flight principle

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ultrafast light source

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indirect time-of-flight principle

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Output

transient measurement
## Transient Sensing Technologies

<table>
<thead>
<tr>
<th>Temporal Resolution</th>
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**optical coherence tomography**

**streak camera**

**single-photon avalanche diodes**

**time-of-flight cameras**

**avalanche photodiode**

**frame rate**

**transient sensing technologies**
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Velten et al. [2012]
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Micro Photon Devices (MPD) specializes in transient sensing technologies.
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transient sensing technologies

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- **streak camera**
- **single-photon**
- **time-of-flight**

- Gkioulekas et al. [2015]

- Avalanche photodiode
- Avalanche photodiode
- Optical coherence tomography
- Streak camera
- Time-of-flight camera
- Single-photon detector

- 10 nanoseconds (10^{-8} secs)
- 100 million fps
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spectrum of transient sensing technologies

- Cosmic rays
- Gamma rays
- X-rays
- Ultraviolet
- Microwaves
- Infrared
- Radar
- Radio
- Broadcast band

- optical coherence tomography
- streak camera
- single-photon avalanche diodes
- time-of-flight cameras
- avalanche photodiode
spectrum of transient sensing technologies

- Cosmic rays
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- Optical coherence tomography
- Streak camera
- Single-photon avalanche diodes
- Time-of-flight cameras
- Avalanche photodiode
- Synthetic Aperture Radar (SAR)

- Positron Emission Tomography (PET) Scan
- Terahertz Time-gated Imaging [Redo-Sanchez et al., 2015]
- Seeing through Walls [Adib et al., 2015]
spectrum of transient sensing technologies

$10^{-12}$ meters (1 picometer) | $10^{-9}$ meters (1 nanometer) | $10^{-6}$ meters (1 micrometer) | $10^{-3}$ meters (1 millimeter) | $10^{0}$ meters (1 meter) | $10^{3}$ meters (1 kilometer)

Ultrasound Imaging

SONAR (Sound Navigation and Ranging)

Seismic Imaging
• Time-resolved imaging
• Single-photon avalanche diodes (SPADs)
• Single-photon lidar
• Non-line-of-sight imaging
• passive ultra-wideband sensing
regular image

picosecond laser

glowing fiber optic cable
transient image
Applications

Autonomous Navigation
Image by Wikimedia Commons

3D Mapping
Image by LIDAR-America

Space Station Docking
Image by NASA

Optical Communications
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Biomedical Imaging
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Consumer Electronics (2020 iPad Pro)
Video by Tim Fields
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International Laser Ranging Service (ILRS)

Lunar Laser Ranging (LLR)

- Location of Lunar Retroreflector
histogram of arrival times

timestamp of photon event
Time-to-Digital Converter (TDC)

SPAD sensor

scene

SPAD properties:
• Each photon timestamped with 60 ps precision
• Measure up to 10 million photons a second
linear array of SPADs

LinoSPAD from FastTree 3D
scanning procedure

SPAD output

regular image
scanning procedure

transient image

regular image
scanning procedure

t (256 bins)

x (320 pixels)

y (256 pixels)

transient image

regular image
• Time-resolved imaging
• Single-photon avalanche diodes (SPADs)
• Single-photon lidar
• Non-line-of-sight imaging
• passive ultra-wideband sensing
LiDAR

distance

photon count

time delay

time
LiDAR

distance

photon count

photon detections

time
LiDAR

distance

photon count

noise

time

laser return
Challenges

1. Light efficiency / photon sensitivity (determines range)
2. High-speed time stamping (determines accuracy)
3. Computational algorithms (determines range and accuracy)
Challenges

1. Light efficiency / photon sensitivity (determines range)
   - intensity of returned light falls off with $1/d^2$, i.e. very quickly!
   - emit as much light as possible - *fundamentally limited by eye safety* (in most applications)
   - detect as much light as possible, ideally individual photons
Challenges

2. High-speed time stamping (determines accuracy)
   
   • speed of light is \(~300,000,000 \text{ m/s}\)
   
   • \(1 \text{ m} = 3.3 \text{ ns}; 1 \text{ cm} = 33 \text{ ps}; 1 \text{ mm} = 3.3 \text{ ps}\)
   
   • need picosecond-accurate time-stamping \(\rightarrow\) usually high-end electronics, but also done with ASICs, FPGAs
Challenges

3. Computational algorithms (determines range and accuracy)

• robust depth estimation from single photon per pixel!

(Single-photon) Avalanche Photodiodes

**Linear mode** (i.e., avalanche photodiode or APD): acts like a conventional photodiode with extremely high gain or amplification
time resolution >300 ps – 10 ns

**Geiger mode** (i.e., single-photon avalanche photodiode SPAD): 500x more sensitive, i.e. single-photon sensitive
time resolution ~50 ps
Single-Photon Avalanche Diodes

- Picosecond laser
- SPAD sensor
- Ambient Light

Timestamp of photon event
Measurements

SPAD measurements
(256 x 256 x 1536)

Intensity image
(1024 x 1024)

Noisy <1 photon per pix., low spatial resolution
clean, high spatial resolution

How to fuse information from both?
CNN Architecture for Depth Estimation

SPAD measurements

Geometry

Input

Output (1 of 3)

CNN Architecture (1 of 3)

Photon Counts

Denoising Branch

Estimation Depth

argmax
vertical scanline

camera

illumination optics

SPAD Array

imaging optics

Lindell et al., SIGGRAPH 2018
scan rate: 20 Hz  lights on

(note: laser illumination is too weak to observe visually while scanning under ambient light)
scan rate: 20 Hz  
lights off  
(captured at 240 FPS)
Intensity image

SPAD measurements (20 Hz)

Average per spatial position:
0.64 Signal Detections
0.87 Background Detections

Lindell et al., SIGGRAPH 2018
Intensity image
Log-matched filter
[Rapp and Goyal 2017]
CASPI: collaborative photon processing for active single-photon imaging
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Comparisons with direct estimation approaches:

Low SBR
- SBR_{plain}: 2.50
- SBR_{max}: 8.2
- Ground-truth depth: 1.40 m
- Scene: 2.17 m
- Inlier %: 5/12/21
- 26/57/81
- 21/45/73
- 46/92/94

Sub-photon
- SBR_{max}: 0.21/0
- SBR_{plain}: 4.1
- Ground-truth depth: 1.29 m
- Scene: 2.00 m
- 0/1/2
- 13/32/52
- 10/26/39
- 19/51/76

High background
- SBR_{max}: 10/2,000
- SBR_{plain}: 1.0
- Ground-truth depth: 1.15 m
- Scene: 1.99 m
- 0/0/1
- 0/0/0 → 33/69/88
- 12/23/35 → 12/27/57
- 41/79/95

Outdoor
- SBR_{max}: 3.000/7,000
- SBR_{plain}: 204.8
- Ground-truth depth: 8 m
- Scene: 100 m
- 16/32/42
- 8/18/26
- 1/1/2 → 9/21/41
- 54/68/77
CASPI: collaborative photon processing for active single-photon imaging
overview

- Time-resolved imaging
- Single-photon avalanche diodes (SPADs)
- Single-photon lidar
- Non-line-of-sight imaging (part 1)
- Passive ultra-wideband sensing
04.800 ns
1st bounce: 2.7 ns
3rd bounce: 4.3 ns
04.800 ns
1st bounce: 2.7 ns
3rd bounce: 4.3 ns
RAW histogram (10 FPS)

Detector samples one location on wall

detector position

EXIT

object
scanning mirrors
laser
detector
scene photo

measurements
scene photo

reconstruction

Dimensions: 2 m x 2 m x 1.5 m
non-confocal sampling
laser and detector focus on this point

laser and detectors illuminate and image same points

confocal sampling
confocal sampling

same path to the object and back
confocal sampling

- simplified NLOS mathematical model
- enables efficient NLOS reconstruction

equivalent to one-way propagation at half-speed
\[ \tau(x', y', t) = \iiint_{\Omega} \frac{1}{r'_1 r_2} \delta (r'_1 + r - tc) \cdot \rho(x, y, z) \, dx \, dy \, dz \]

3D measurements

radiometric term

geometric term

hidden 3D volume

3D measurements

\( (x_0, y_0, t) = \iiint_{\Omega} \cdots \)

\( (r'_1 + r - tc) \cdot \rho(x, y, z) \, dx \, dy \, dz \)
$\tau(x', y', t) = \iiint_{\Omega} \frac{1}{r_1^2 r_2^2} \delta (r_1 + r - tc) \cdot \rho(x, y, z) \, dx \, dy \, dz$

- **3D measurements**
- **Radiometric term**
- **Geometric term**
- **Hidden 3D volume**
\[ \tau(x', y', t) = \iiint_{\Omega} \frac{1}{r^2 l r^2} \delta(r_l + r - t c) \cdot \rho(x, y, z) \, dx \, dy \, dz \]
3D measurements

\[ \tau(x', y', t) = \int_\Omega \int_\Omega \int_\Omega \frac{1}{r_l^2 r^2} \delta(r_l + r - tc) \cdot \rho(x, y, z) \, dx \, dy \, dz \]

3D measurements

radiometric term geometric term hidden 3D volume
3D measurements

\[ \tau(x', y', t) = \iiint_{\Omega} \frac{1}{r_l^2 r^2} \delta(r_l + r - tc) \cdot \rho(x, y, z) \, dx \, dy \, dz \]

Assumptions

1. no occlusions in hidden volume
2. light scatters isotropically
3. wall geometry is planar
\[ \tau(x', y', t) = \int \int \int_{\Omega} \frac{1}{r_l^2 r^2} \delta (r_l + r - tc) \cdot \rho(x, y, z) \, dx \, dy \, dz \]

\[ \tau = A \times \rho \]

Assumptions
1. no occlusions in hidden volume
2. light scatters isotropically
3. wall geometry is planar
NLOS image formation mode:
\[ \tau = A \rho \]

- Measurements: \( n^3 \times 1 \)
- Transport matrix: \( n^3 \times n^3 \)
- Unknown volume: \( n^3 \times 1 \)

**PROBLEM:** \( A \) extremely large in practice (e.g., for \( n = 100 \), \( A \) has 1 trillion elements)

**Backpropagation** [Velten 12, Buttafava 15]
- Flops: \( O(n^5) \)
- Memory: \( O(n^3) \)
- Runtime: Approx. 10 min.

**Iterative Inversion** [Gupta 12, Heide 13]
- Flops: \( O(n^5) \) per iter.
- Memory: \( O(n^5) \)
- Runtime: > 1 hour
Our approach

express image formation model as a 3D convolution, by:

1. confocalizing measurements
2. performing a change of variables
(set \( z = \sqrt{u} \), \( t = 2\sqrt{v/c} \))

\[
\tau(x', y', t) = \iiint_{\Omega} \frac{1}{r_l^2 r^2} \delta(r_l + r - tc) \cdot \rho(x, y, z) \, dx \, dy \, dz
\]
Our approach

express image formation model as a 3D convolution, by:

1. confocalizing measurements
2. performing a change of variables (set $z = \sqrt{u}$, $t = 2\sqrt{v/c}$)

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express image formation model as a 3D convolution, by:

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(set $z = \sqrt{u}$, $t = 2\sqrt{v/c}$)

$$v^{3/2} \tau(x', y', \frac{2}{c} \sqrt{v}) = \iiint_{\Omega} \frac{1}{2\sqrt{u}} \delta \left( (x' - x)^2 + (y' - y)^2 + u - v \right) \cdot \rho(x, y, \sqrt{u}) dx dy du$$
**Our approach**

express image formation model as a 3D convolution, by:

1. confocalizing measurements
2. performing a change of variables
   (set $z = \sqrt{u}$, $t = 2\sqrt{v}/c$)

$$v^{3/2} \tau(x', y', \frac{2}{c} \sqrt{v}) = \int\int\int_{\Omega} \frac{1}{2\sqrt{u}} \delta \left( (x' - x)^2 + (y' - y)^2 + u - v \right) \cdot \rho(x, y, \sqrt{u}) dx dy du$$

$$\tau = a * \rho$$
Confocal NLOS image formation mode:

\[ \tau = a \ast \rho \]

NLOS image formation mode:

\[ \tau = A \rho \]

**Backpropagation** [Velten 12, Buttafava 15]

- Flops: \( O(n^5) \)
- Memory: \( O(n^3) \)
- Runtime: Approx. 10 min.

**Iterative Inversion** [Gupta 12, Heide 13]

- Flops: \( O(n^5) \) per iter.
- Memory: \( O(n^5) \)
- Runtime: > 1 hour
NLOS image formation mode:

\[ \tau = A \rho \]

- Measurements: \( n^3 \times 1 \)
- Transport matrix: \( n^3 \times n^3 \)
- Unknown volume: \( n^3 \times 1 \)

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Confocal NLOS image formation mode:

\[ \tau = a \ast \rho \]

- Measurements: \( n \times n \times n \)
- Blur kernel: \( n \times n \times n \)
- Unknown volume: \( n \times n \times n \)

**3D Deconvolution** (with Light Cone Transform) [O’Toole et al. 2018]

- Flops: \( O(n^3 \log(n)) \)
- Memory: \( O(n^3) \)
- Runtime: < 1 second
Step 1: resample and attenuate along $t$-axis
Step 1: resample and attenuate along $t$-axis

inverse filter

Step 2: 3D convolution
Step 1: resample and attenuate along $t$-axis

Step 2: 3D convolution

Step 3: resample and attenuate along $z$-axis
laser spot
hidden object
occluder
confocal NLOS system
NLOS image formation model:
\[
\tau = A \rho
\]
\(\begin{align*}
\text{measurements} & \quad n^3 \times 1 \\
\text{transport matrix} & \quad n^3 \times n^3 \\
\text{unknown volume} & \quad n^3 \times 1
\end{align*}\)

Backprojection [Velten 12, Buttafava 15]
Flops: \(O(n^5)\)
Memory: \(O(n^3)\)
Runtime: Approx. 10 min.

Iterative Inversion [Gupta 12, Wu 12, Heide 13]
Flops: \(O(n^5)\) per iter.
Memory: \(O(n^5)\)
Runtime: > 1 hour

Confocal scanning and Light-Cone Transform
\[
\tilde{\tau} = \alpha * \tilde{\rho}
\]
\(\begin{align*}
\text{measurements} & \quad n \times n \times n \\
\text{blur kernel} & \quad n \times n \times n \\
\text{unknown volume} & \quad n \times n \times n
\end{align*}\)

3D Deconvolution (with Light-Cone Transform) [O’Toole et al. 2018]
Flops: \(O(n^3 \log(n))\)
Memory: \(O(n^3)\)
Runtime: < 1 second

Assumption:
- Isotropic scattering (only diffuse or retroreflective objects)
overview

- Time-resolved imaging
- Single-photon avalanche diodes (SPADs)
- Single-photon lidar
- Non-line-of-sight imaging (part 2)
- Passive ultra-wideband sensing
\[ \nabla^2 \psi - \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} = 0 \]
confocal sampling
image formation model

\[ \Psi(x, z, t) \]

wavefield

wall \( (z = 0) \)

\[ \Psi(x, z, t = 0) \]

hidden object

\[ \Psi(x, z = 0, t) \]

confocal measurements
general solution (time reversal)

\[ \Psi(x, z, t = 03.000 \text{ ns}) \]

wavefield

wall \((z = 0)\)

\[ \Psi(x, z = 0, t) \]

confocal measurements

hidden object
1. approximate wave equation with finite differences

$$\frac{\partial^2 \Psi}{\partial t^2} \approx \frac{\Psi_{i}^{n+1} - 2\Psi_{i}^{n} + \Psi_{i}^{n-1}}{(\Delta t)^2}$$

2. solve for previous timestep

$$\Psi_{i}^{n-1} = f(\Psi_{i}^{n}, \Psi_{i}^{n+1})$$

3. repeatedly update $\Psi$ at all grid cells
general solution (time reversal)

finite-difference time-domain method

1. approximate wave equation with finite differences
   \[ \frac{\partial^2 \Psi}{\partial t^2} \approx \frac{\Psi_{i}^{n+1} - 2 \Psi_{i}^{n} + \Psi_{i}^{n-1}}{\Delta t^2} \]

2. solve for previous timestep
   \[ \Psi_{i}^{n-1} = f(\Psi_{i}^{n}, \Psi_{i}^{n+1}) \]

3. repeatedly update \( \Psi \) at all grid cells

Slow to get \( t=0 \) at high-resolution!
frequency–wavenumber ($f-k$) Migration

$$\Psi(x, z, t = 0)$$

wavefield

wall ($z = 0$)

$$\Psi(x, z = 0, t)$$

confocal measurements

FLOPS: $O(n^3 \log n)$
hardware prototype
hardware prototype
f-k Migration

\[ \Psi(x, y, t) \]
Measurements (z=0)

\[ \Phi(k_x, k_y, f) \]
Spectrum

\[ \Phi(k_x, k_y, k_z) \]
Interpolated Spectrum

\[ \Psi(x, y, z) \]
Hidden Volume (t=0)

\[ \mathcal{F} \]

\[ \mathcal{F}^{-1} \]
**f-k Migration**

Express wavefield as function of measurement spectrum (plane wave decomposition)

\[
\Psi(x, y, z, t) = \iiint \Phi(k_x, k_y, f) e^{2\pi i (k_x x + k_y y + k_z z - f t)} \, dk_x \, dk_y \, df
\]

Almost an inverse Fourier Transform!

Set \( t=0 \) to get migrated solution

\[
\Psi(x, y, z, t = 0) = \iiint \Phi(k_x, k_y, f) e^{2\pi i (k_x x + k_y y + k_z z)} \, dk_x \, dk_y \, df
\]
f-k Migration

Set \( t=0 \) to get migrated solution

\[
\Psi(x, y, z, t = 0) = \iiint \Phi(k_x, k_y, f) e^{2\pi i (k_x x + k_y y + k_z z)} \, dk_x \, dk_y \, df
\]

Almost an inverse Fourier Transform!

Use dispersion relation\(^1\) to perform substitution of variables

\[
f = v \sqrt{k_x^2 + k_y^2 + k_z^2}
\]

\[
f \Rightarrow k_z
\]

Use dispersion relation\(^1\) to perform substitution of variables

\[
f = v \sqrt{k_x^2 + k_y^2 + k_z^2}
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\[f \Rightarrow k_z\]
Use dispersion relation\(^1\) to perform substitution of variables

\[ f = v \sqrt{k_x^2 + k_y^2 + k_z^2} \]

\[ f \Rightarrow k_z \]
Use dispersion relation\(^1\) to perform substitution of variables

\[ f = v \sqrt{k_x^2 + k_y^2 + k_z^2} \]

\[ f \rightarrow k_z \]

The migrated solution is an inverse Fourier Transform!

\[
\Psi(x, y, z, t = 0) = \int \int \int \Phi(k_x, k_y, k_z) e^{2\pi i (k_x x + k_y y + k_z z)} \, dk_x \, dk_y \, dk_z
\]
Reconstruction Comparison

dimensions: 2 m x 2 m x 1.5 m

Filtered Backprojection
hardware prototype
hardware prototype
real-time scanning

Framerate: 4 Hz
Resolution: 32 x 32
Dimensions: 2 m x 2 m x 2 m
Reconstruction time: ~1 s per frame

Lindell et al., SIGGRAPH 2019
Outlook

Directional Light-Cone Transform

hidden scene  Recovered surface

[Young et al., CVPR 2020]
Outlook

Keyhole NLOS Imaging

[Metzler et al., IEEE TCI 2021]
• Time-resolved imaging
• Single-photon avalanche diodes (SPADs)
• Single-photon lidar
• Non-line-of-sight imaging
• passive ultra-wideband sensing
f qef qncbb gk[edge]e mb[w]
ϕ_kcp_rc8/.1*...Fx
ϕ_k c p rc8/ ....... Fx
streak cameras

interferometry
- dp \_ k c $ ct cl r bsp \_ rgnl

- a_nrs\_pc ct cl rq_\_apm\_pq rdk cqa_jcq qn_l l gj e l /_. mpcpq \_mk _el qsboc
- n_oppj c* l mowl af pm1 gx_rgnl
- jmu qsv
sjrp_dqgk_edge mbw

npcagp qv af pmx gx_rgm

ugf qp ejc bcba_rcb gef qncb jef r
conventional flash bulb (ca. 1940)  

Edgerton’s high-speed flash
Single-shot ultrafast imaging attaining 70 trillion frames per second

Peng Wang, Jinyang Liang & Lihong V. Wang
Single-shot ultrafast imaging attaining 70 trillion frames per second
\( \phi_k c \$ c t e l r b s p \_ r m n i \)
once "frozen in time" we can keep collecting light as long as needed
rp_lagc_rgk_edge & Tcjrcr cr_j*0. / 1'

φ_k c p_{rc83} . . EFx

( q̃m c φcsccl aw853 K Fx
 lmctcl rglmu cprf _ l . . / 1 l _ mcpaml bqa_l ` c gk_ecb
Elapsed time: 26.1s + 15ms + 169us + 910ns + 848ps
Frame rate: 250.0GHz
Measurement Model

flux function

[Graph showing a horizontal line at 0 photons/sec over the interval 0 to 2 seconds]
Active histogram-based imaging: Role of sync frequency flux function

photon detections

absolute time $t$ (sec)
Active histogram-based imaging: Role of sync frequency

flux function

photon detections

histogram (sync freq = 02.00 Hz = freq of blue signal)
Active histogram-based imaging: Role of sync frequency flux function

photon detections

histogram (sync freq = 32.00 Hz = freq of green signal)

Zoomed in view
Active histogram-based imaging: Role of sync frequency

flux function

photon detections

absolute time $t$ (sec)

histogram (sync freq = 04.02 Hz)
Flux Probing

\[ N(t) = \int_0^t \varphi(u) \, du + M(t) \]

- Counting process
- Flux integral up to time \( t \)
- Martingale noise
\[ p(\mathcal{T}) = \langle p, \varphi \rangle + M_p(t) \]

\[
p(\mathcal{T}) \overset{\text{def}}{=} \sum_{\tau \in \mathcal{T}} p(\tau)
\]
Flux Probing

flux function

photon detections

integral of flux function (solid)
counting process (dashed)

Fourier probing output
Frame rate: 3 KHz
Elapsed: 000 ms
SPAD line of sight
SPAD line of sight

3 MHz strobe (picosecond laser)
SPAD line of sight

40 MHz strobe (picosecond laser)
SPAD line of sight
laser projector
playing a video
Sampling Rate: 1.0 kHz
Timespan: 1.00 s
Sampling Rate: 1.0 kHz
Timespan: 1.00 s
Sampling Rate: 36.2 kHz
Timespan: 28.31 ms

900 Hz light bulb flicker

absolute time (ms)
Sampling Rate: 137.4 kHz
Timespan: 7.45 ms

projector laser beam modulation
Sampling Rate: 39.1 MHz
Timespan: 26.17 µs

pulse from 3 MHz laser

absolute time (µs)
Sampling Rate: 1.0 GHz
Timespan: 1.01 μs

pulse from 3 MHz laser

pulse from 40 MHz laser

absolute time (μs)
non-line-of-sight video reconstruction

(reconstruction with lasers and bulb turned off)

(total of 4,000 photons used in reconstruction per frame)

(total of 3,000 photons used in reconstruction per frame)
\begin{itemize}
\item $\delta l \ b_k \ c \ l \ r_j j_k g q$
\item $c d \ k \ g a \ c l \ a w$
\item $b c r c a m p b c \ b \ r g \ k \ c$
\end{itemize}

\[ f_{\text{max}} \leq \frac{1}{2Q} \]
concluding remarks

- K_\_lw_{nnjga\_rnmqdmprk} c-r\_nd\_g\_efr \_gk_\_edge
  - Jd_p
  - Lm_l_{jgj} c-r\_nd\_g\_efr
  - Rp_l_{apil} r\_gk_\_edge
  - G_{edge} e r\_f_{prosef} qa_{rcp}

- L_{cu a_{n\_\_gqgqrf prosef ank\_\_dj} c_{ckeck {epdep} e_{gpl cpmgupu} g_{f anknr_{rnm}}}
Co-Designing Optics and Algorithms for High-Dimensional Visual Computing

Vishwanath Saragadam
Incoming Assistant Professor,
UC Riverside

Monday Nov 27, 2-4 PM (lecture 10)
References