Low-Dose CT Image Reconstruction with State-Space Model Denoising Prior

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Motivation

Low-dose CT Imaging

- Computed Tomography (CT) imaging is a medical imaging technique used to take cross-sectional images or slices of internal structures.
- Due to the harmful effects of x-ray exposure, low-dose CT has become a critical area of research. However, the lower radiation dose introduces significant noise and artifacts, which can compromise the quality and reliability of diagnostic images.

Noisy Image Reconstruction

- The problem of noisy image reconstruction is typically expressed as

$$\mathbf{b}=\mathbf{A}\mathbf{x}+\mathbf{n}$$

where:

$\mathbf{A}$: The forward operator modeling the image system (in the case of CT is the Radon Transform)
$\mathbf{x}$: The unknown "true" image to be reconstructed
$\mathbf{b}$: The measured data (in the case of CT is a sinogram)
$\mathbf{n}$: Measurement noise, typically modelled as additive Gaussian noise.

- Solving for x is typically non-trivial as simply inverting A will lead to amplification of the noise.

Related Work

Regularization with Denoising (RED) [1]

- Directly embed a denoising engine into the optimization process
- Unlike implicit priors like Plug-and-Play, RED defines a regularization term based on the denoising function **f**
- The reconstruction is guided by the denoising objective that aims to minimize noise while preserving the image structure

$$E(\mathbf{x}) = \frac{1}{2\sigma^2}\|\mathbf{H}\mathbf{x}-\mathbf{y}\|_2^2 + \frac{\lambda}{2}\mathbf{x}^T(\mathbf{x}-f(\mathbf{x}))$$

Solving Linear Inverse Problems with ADMM [2]

- Simplifies complex optimization problem by splitting them into smaller subproblems subject to a constraint
- Alternate between solving for each variable while enforcing the constraint with a dual variable update

$$L_\rho(x,z,y) = f(x) + g(z) + y^T(Ax+Bz-c) + (\rho/2)\|Ax+Bz-c\|_2^2.$$

Solving Linear Inverse Problems with Fixed Point [1]

- Does not split the problem into subproblems
- Directly updates the solution by solving the gradient condition of the entire objective function

$$\nabla_{\mathbf{x}}\ell(\mathbf{y},\mathbf{x}) + \lambda(\mathbf{x}-f(\mathbf{x})) = 0$$

References

[1] Romano et al., "The little engine that could: Regularization by denoising (RED)," SIAM Journal on Imaging Sciences, 2017.

[2] Boyd et al., "Distributed optimization and statistical learning via the alternating direction method of multipliers," Foundations and Trends® in Machine learning, 2011.

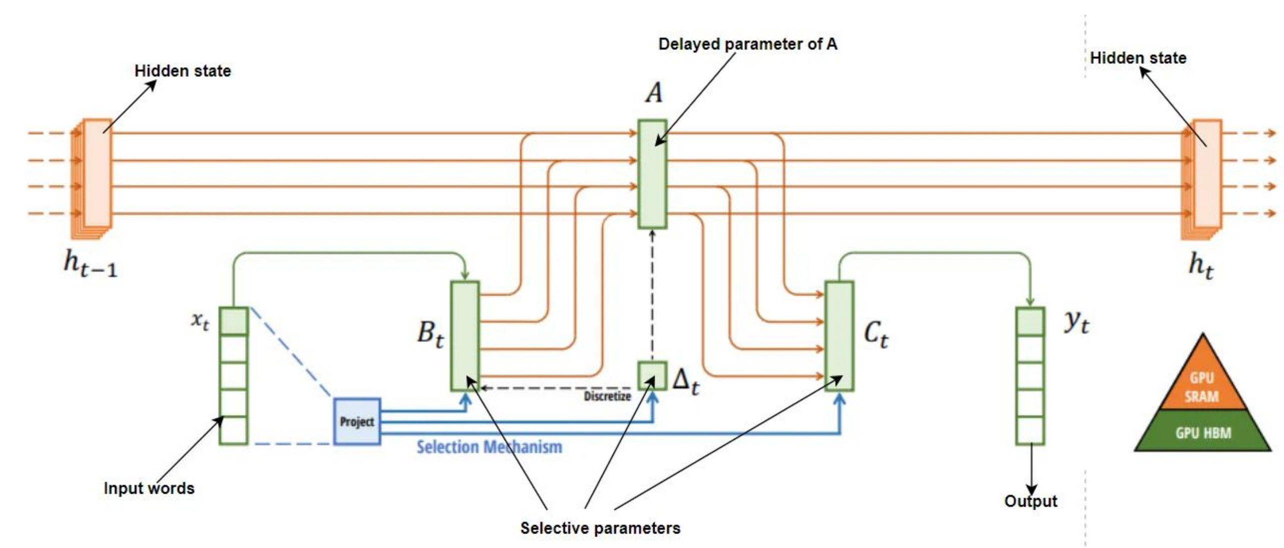
[3] Öztürk et al., "DenoMamba: A fused state-space model for low-dose CT denoising," arXiv, 2024.

[4] Gu and Dao, "Mamba: Linear-Time Sequence Modeling with Selective State Spaces," arXiv, 2024.

New Technique

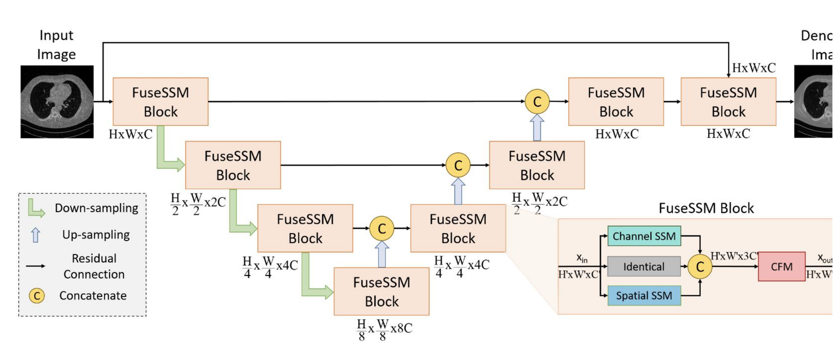
Selective State-Space Model Denoising

- Selective state-space models provide better long-range contextual capture in comparison to CNNs with better efficiency than attention-based architectures.[3]

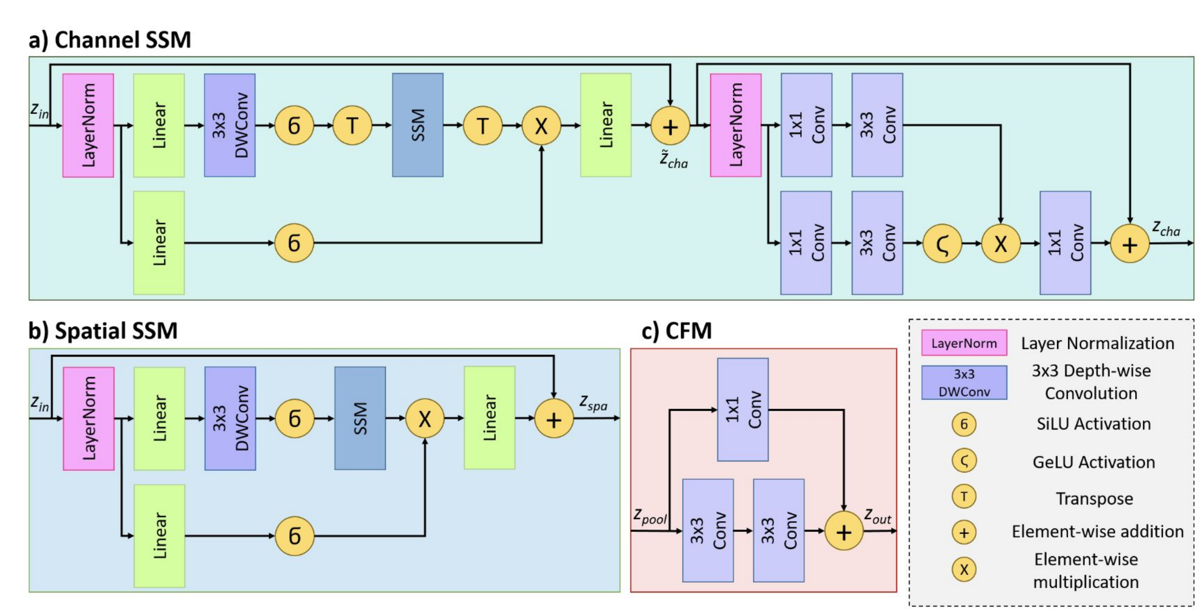


Selective State-Space Model Architecture

- We used Denomamba [4], an SSM based denoising architecture as the denoiser within the RED framework and evaluated against other common CT reconstruction algorithms.



Denomamba Architecture



FuseSSM Block

Datasets

Denoiser Training: 2016 Low Dose CT Grand Challenge

Reconstructions: LoDoPaB-CT Benchmark

Experimental Results