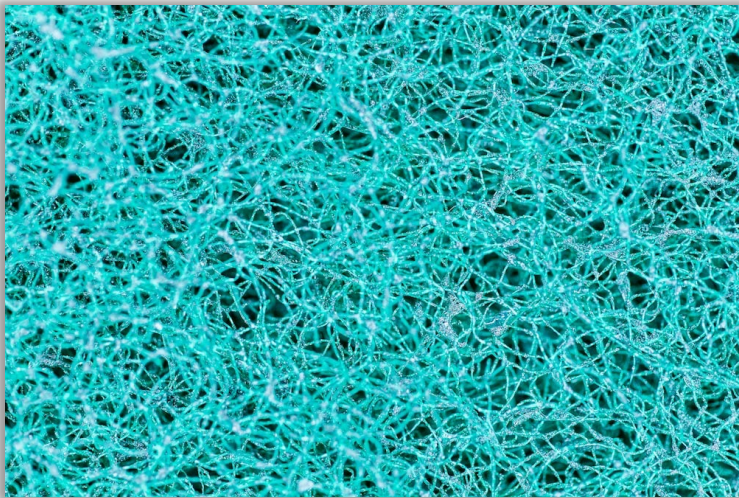


Introduction to Neural Networks

MLPs, CNNs, Backpropagation, Learned Image Processing



CSC2529

David Lindell

University of Toronto

cs.toronto.edu/~lindell/teaching/2529

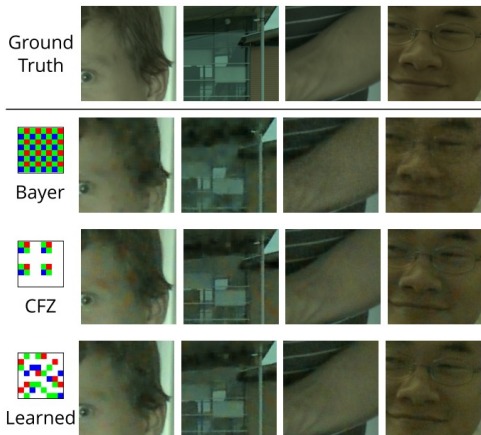
*slides adapted from CS231n at Stanford

Announcements

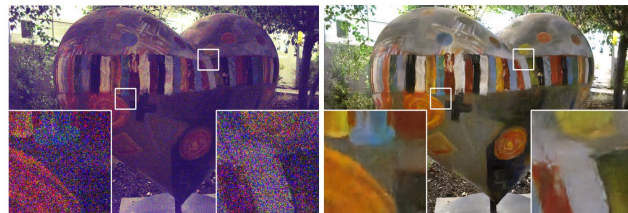
- HW4 due Wednesday 25/10
- HW5 is out
- Problem session for HW5 tomorrow

Neural Networks in Computational Imaging

- Now: learned pipelines for computational imaging



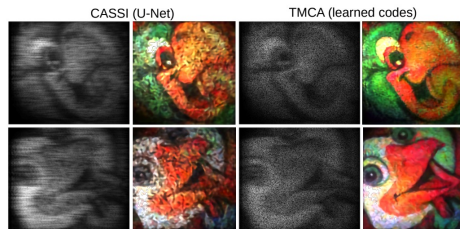
Learning CFAs



(b) Raw data via traditional pipeline

(c) Our result

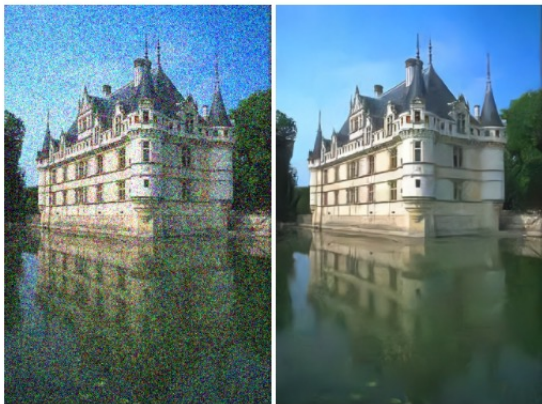
Learning ISPs



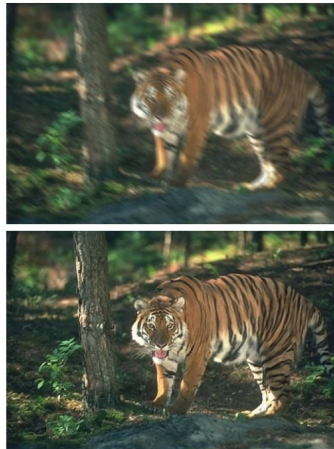
Learning coded apertures

Neural Networks in Computational Imaging

- Now: learned pipelines for computational imaging



Learned denoising



Learned deblurring



HDR Imaging

Today

- What is a neural network?
- Training/optimizing neural nets
- Why “neural”?
- Convolutional neural networks
- Applications & inverse problems

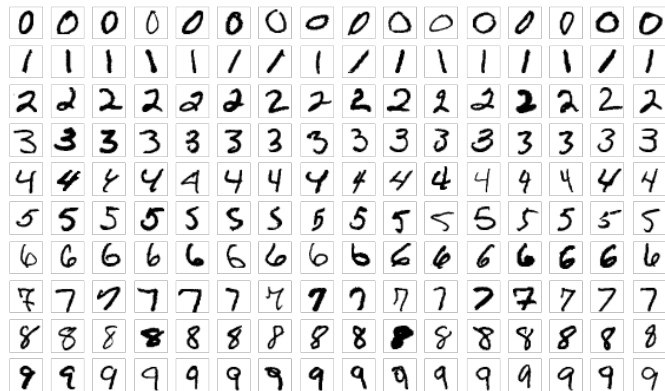
What is a neural network?

- Image classification example

Image Classification

- Image classification example

Images

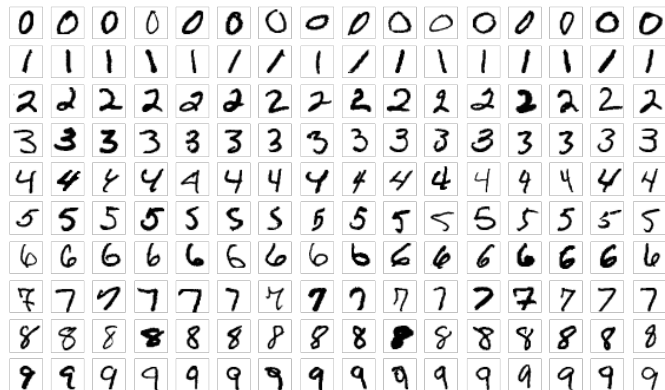


MNIST Dataset

Image Classification

- Image classification example

Images



Class

“zero”

“one”

...

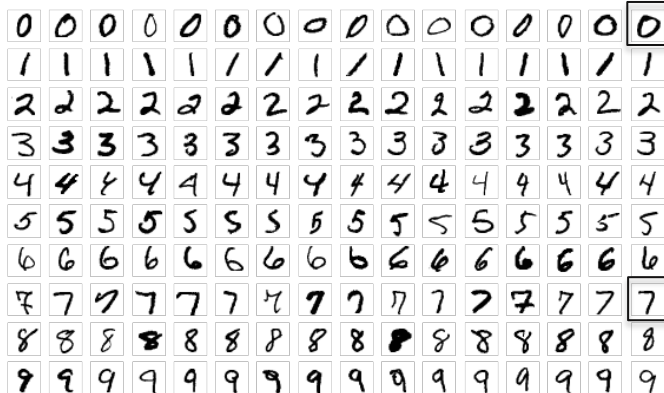
“nine”

Image Classification

- Image classification example

What the computer “sees”

Images



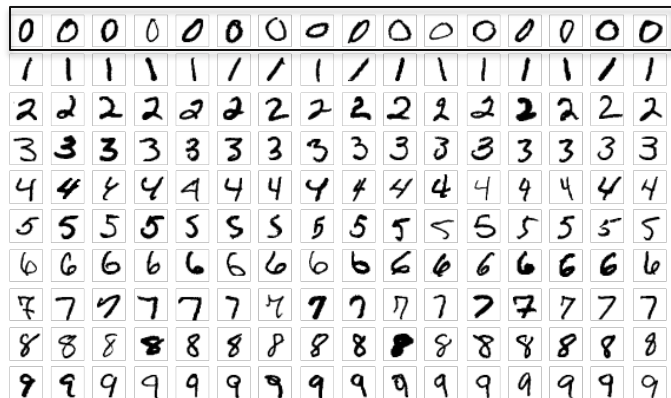
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000 000 000 000 000 000 000 000 000 000 019 146 146 018 000 000 000
000 000 000 000 000 000 000 000 000 023 188 248 236 115 003 000 000
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000 000 000 000 000 017 214 243 145 223 064 180 113 000 000 000
000 000 000 000 014 161 232 137 006 014 000 156 209 003 000 000
000 000 000 000 108 230 042 009 000 000 000 155 227 004 000 000
000 000 000 020 223 138 000 000 000 000 000 156 220 003 000 000
000 000 000 054 237 023 000 000 000 000 024 206 111 000 000 000
000 000 000 055 210 003 000 000 001 057 216 088 006 000 000 000
000 000 000 055 242 085 022 089 160 219 108 001 000 000 000 000
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000 000 000 000 046 234 076 002 000 000 000 000 000 000 000 000
000 000 000 000 010 087 005 000 000 000 000 000 000 000 000 000
```

Image Classification

- Image classification example

Images



Challenges

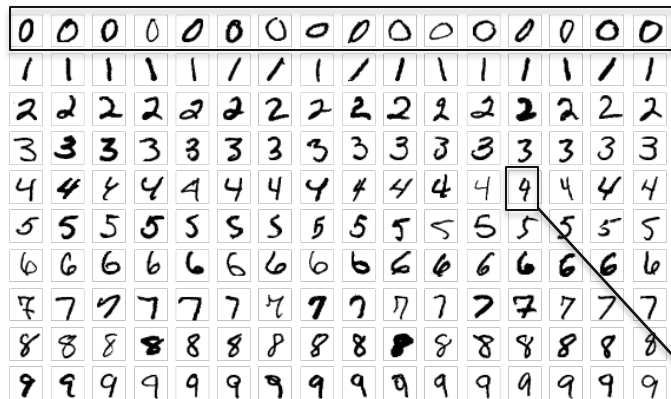
Intra-class variation

- stroke widths
- alignment
- writing styles

Image Classification

- Image classification example

Images



Challenges

Intra-class variation

- stroke widths
- alignment
- writing styles

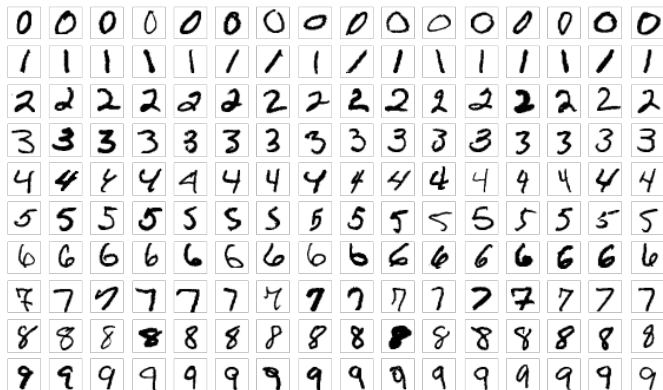
Inter-class similarities

- “four” or “nine”?

Image Classification

- Image classification example

Images



Implementation?

```
def classify_digit(image):  
    # ???  
    return image_class
```

Can't hardcode solution!

Image Classification

- Data-driven approach
 - Collect training images and labels
- Train a classifier using machine learning
- Evaluate the classifier on unseen images

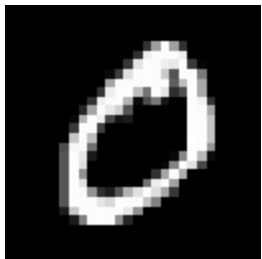
Implementation?

```
1 def train(images, labels):  
2     # machine learning model  
3     return image_class  
4  
5 def evaluate(model, test_images):  
6     # machine learning model  
7     return test_labels  
8
```

Image Classification

- Linear Model

$$f(x, W) = Wx$$



vectorize



x

Image Classification

- Linear Model

$$f(x, W) = Wx$$



vectorize

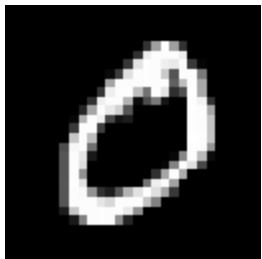


x

Image Classification

- Linear Model

$$f(x, W) = Wx$$



vectorize

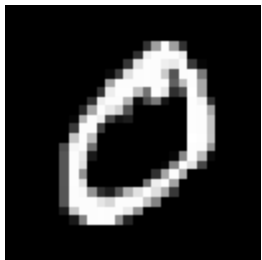


x

Image Classification

- Linear Model

$$f(x, W) = Wx$$



vectorize



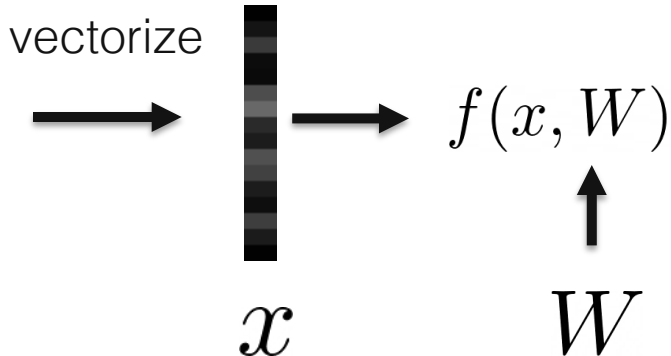
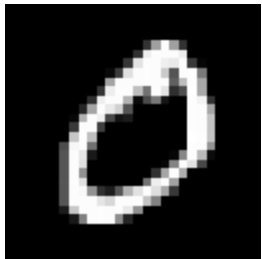
x

Length of this vector is the “dimensionality” of our problem!

Image Classification

- Linear Model

$$f(x, W) = Wx$$



In general: $Wx + b$

Image Classification

- Linear Model

$$f(x, W) = Wx$$

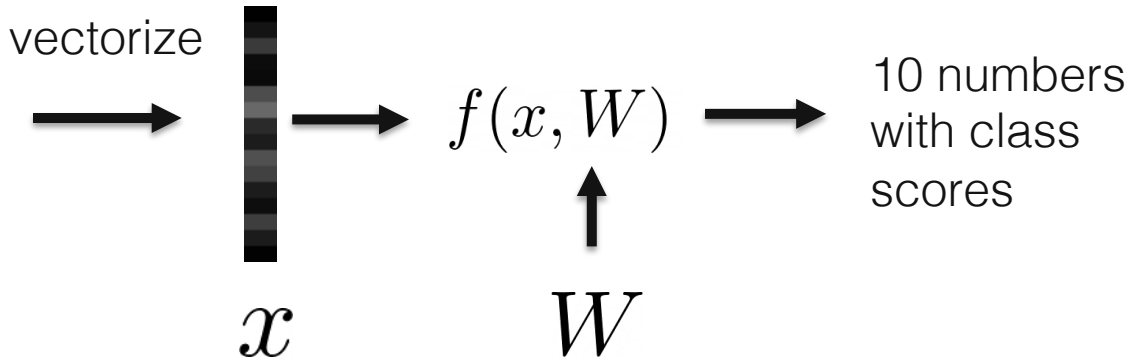
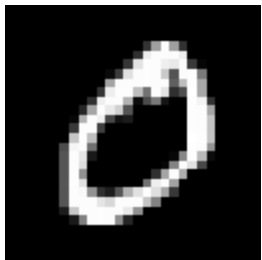
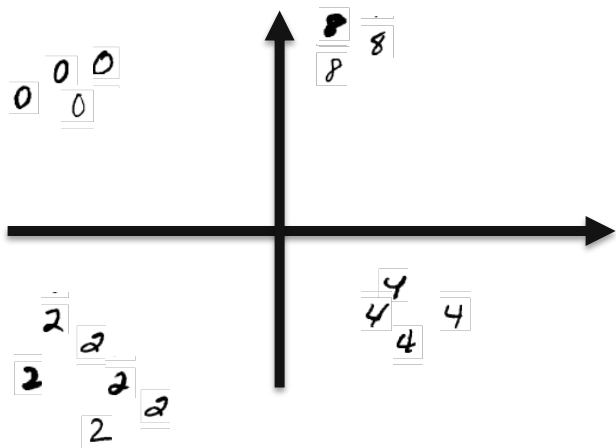


Image Classification

- Linear model: geometric interpretation

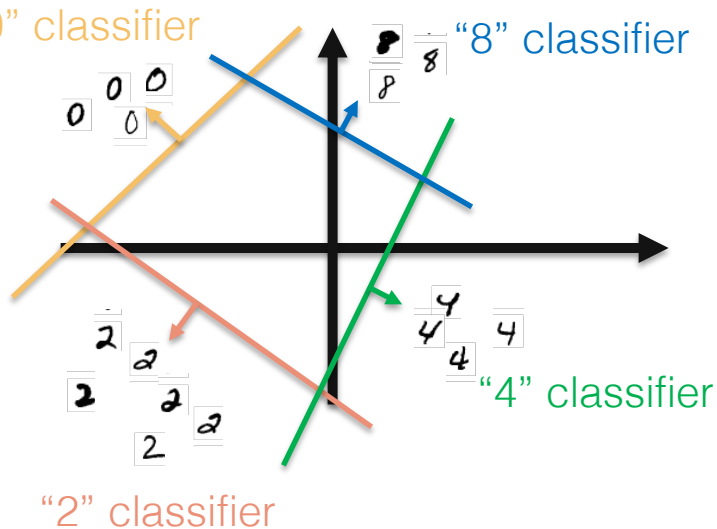


Each image is a point in an N-dimensional space

- N is the number of pixels

Image Classification

- Linear model: geometric interpretation



$$f(x, W) = Wx$$

Computes inner product
between rows of W and x !

- Each row of W is a hyperplane
- Sign of inner product tells you which side of the hyperplane
- “separates” the digits

Image Classification

- Linear model (visual interpretation)

Learned filters (rows of W)

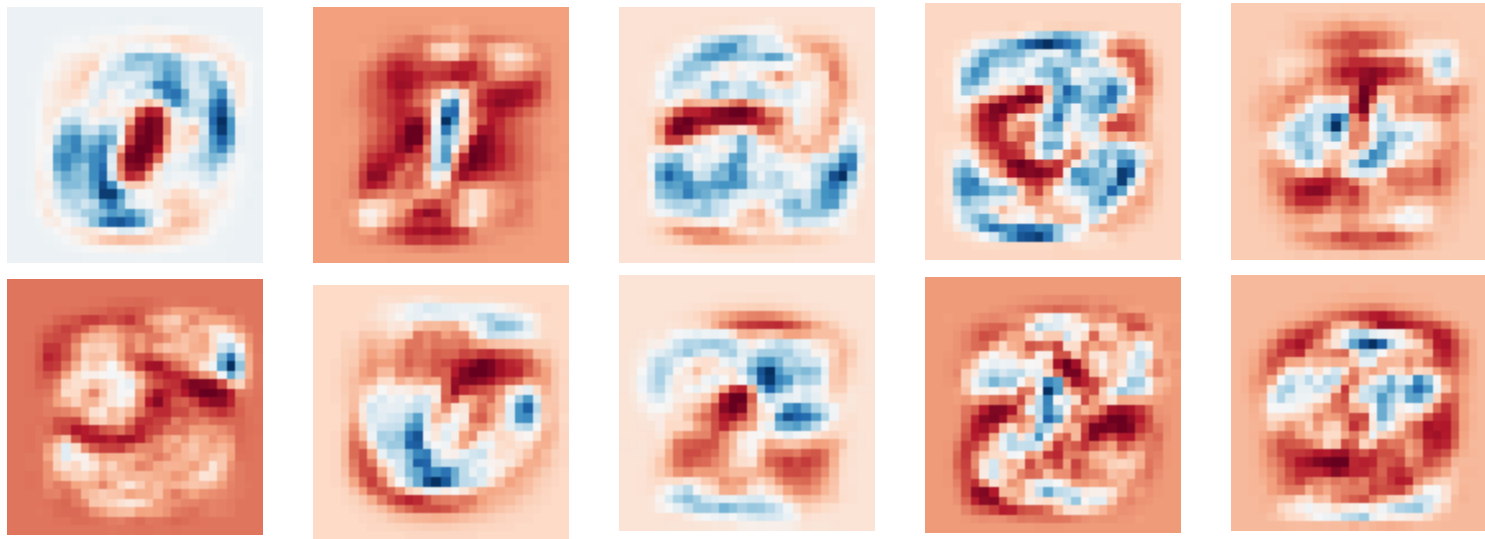
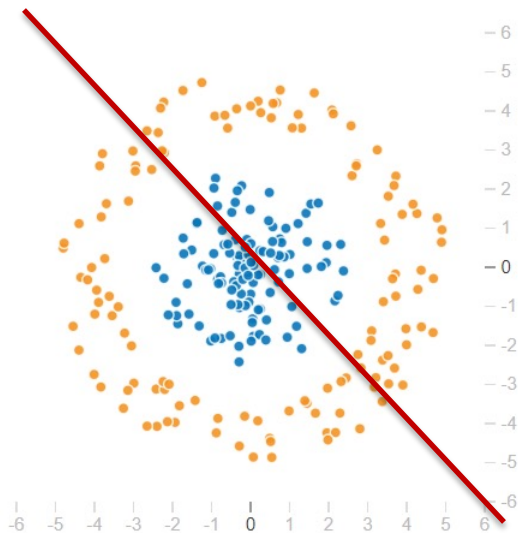


Image Classification

- Limits of linear classifiers

Linear classifiers learn linear decision planes

What if dataset is not linearly separable?



Multilayer Perceptrons (MLPs)

- Linear Model $f = Wx$
- 2-layer MLP $f = W_2 \max(0, W_1 x)$

Multilayer Perceptrons (MLPs)

- Linear Model $f = Wx$
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- 3-layer MLP $f = W_3 \max(0, W_2 \max(0, W_1 x))$

Multilayer Perceptrons (MLPs)

- Linear Model $f = Wx$
- 2-layer MLP $f = W_2 \max(0, W_1 x)$
- 3-layer MLP $f = W_3 \max(0, W_2 \max(0, W_1 x))$



Non-linearity/activation function between linear layers

Multilayer Perceptrons (MLPs)

- Linear Model $f = Wx$
- 2-layer MLP $f = W_2 \max(0, W_1 x)$
- 3-layer MLP $f = W_3 \max(0, W_2 \max(0, W_1 x))$

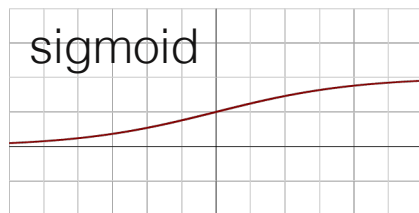
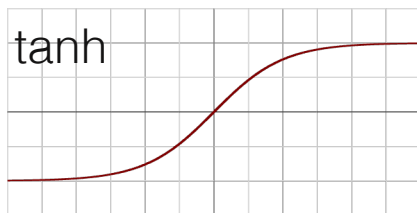
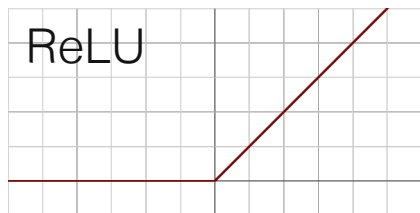
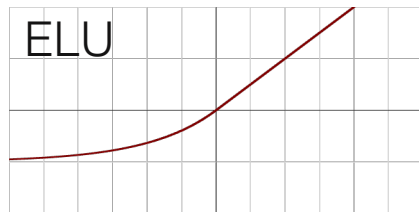
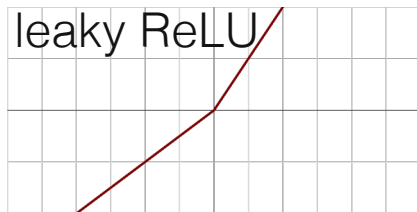
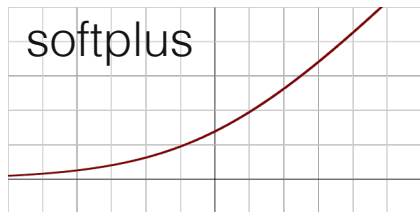


Otherwise we have:

$$f = W_3 W_2 W_1 x$$

Activation Functions

...many to choose from

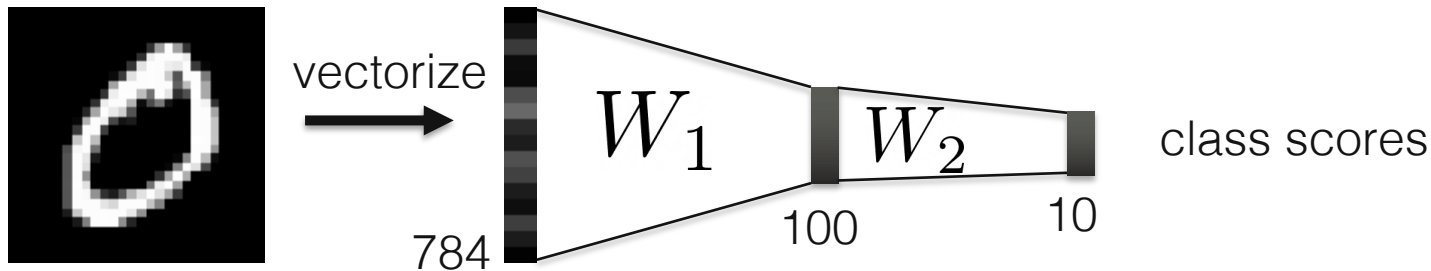


... ReLU is a good general-purpose choice: $\text{ReLU}(x) = \max(0, x)$

Multilayer Perceptrons (MLPs)

- Linear Model $f = Wx$
- 2-layer MLP $f = W_2 \max(0, W_1 x)$

Back to our classification example...

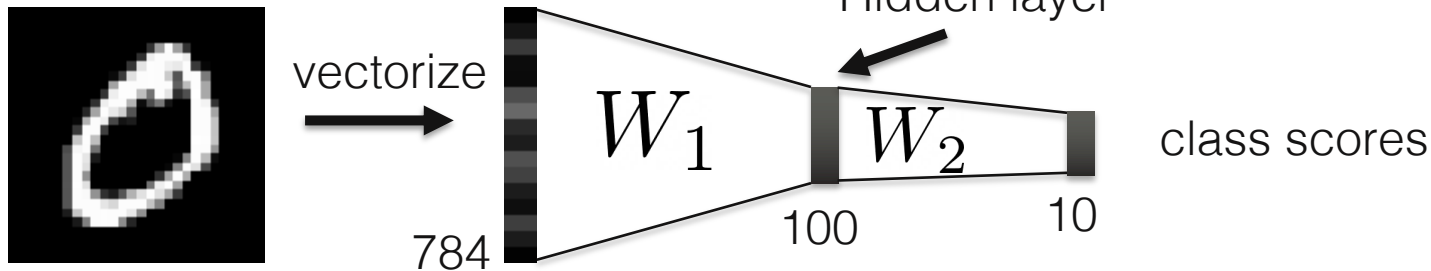


$$x \in \mathbb{R}^D, W_1 \in \mathbb{R}^{H \times D}, W_2 \in \mathbb{R}^{C \times H}$$

Multilayer Perceptrons (MLPs)

- Linear Model $f = Wx$
- 2-layer MLP $f = W_2 \max(0, W_1 x)$

Back to our classification example...

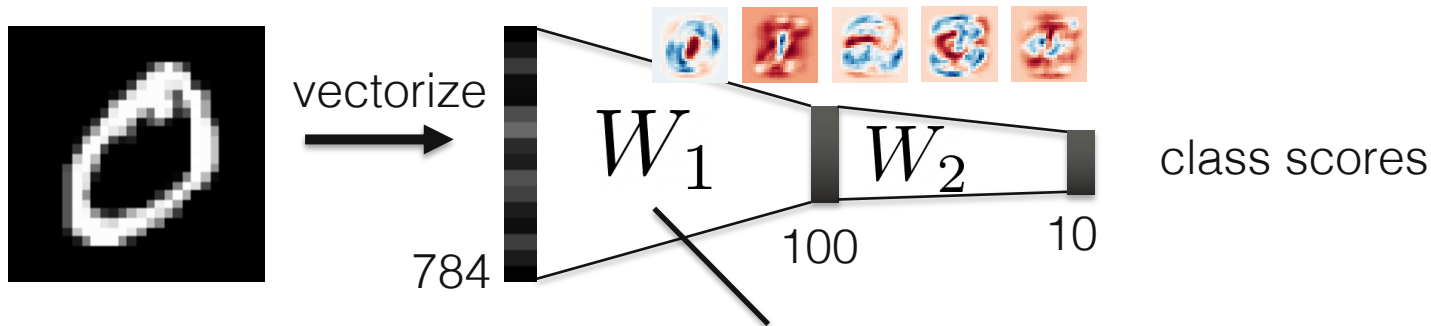


$$x \in \mathbb{R}^D, W_1 \in \mathbb{R}^{H \times D}, W_2 \in \mathbb{R}^{C \times H}$$

Multilayer Perceptrons (MLPs)

- Linear Model $f = Wx$
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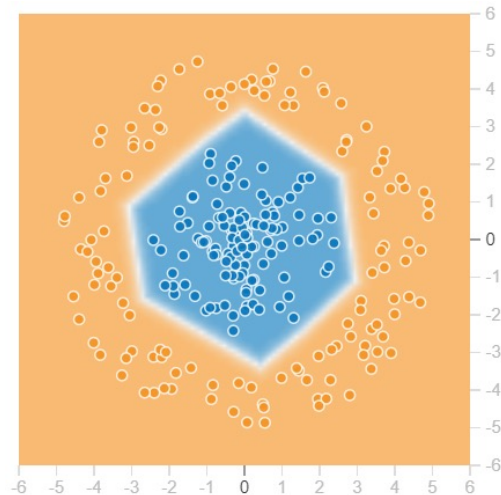
Back to our classification example...



Now we have 100 shape templates, shared between classes

Multilayer Perceptrons (MLPs)

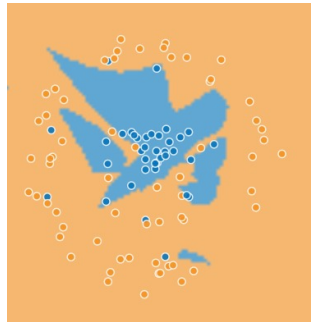
- Overcomes limits of linear classifiers
- Can learn non-linear decision boundaries
- Complexity scales with the number of neurons/hidden layers



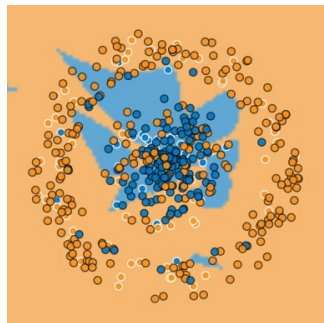
Multilayer Perceptrons (MLPs)

- More parameters is not always better!
 - Can lead to overfitting the training data
 - Performance on test data is worse

train

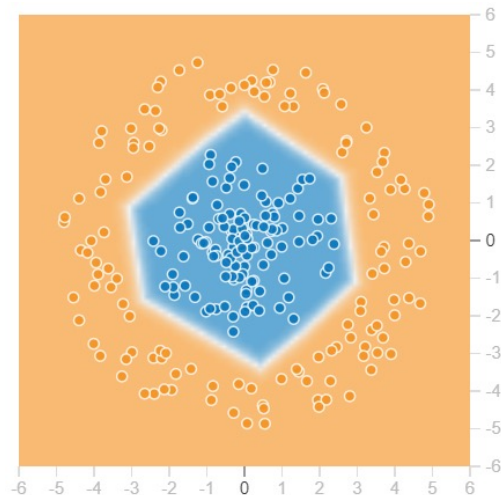


test



Multilayer Perceptrons (MLPs)

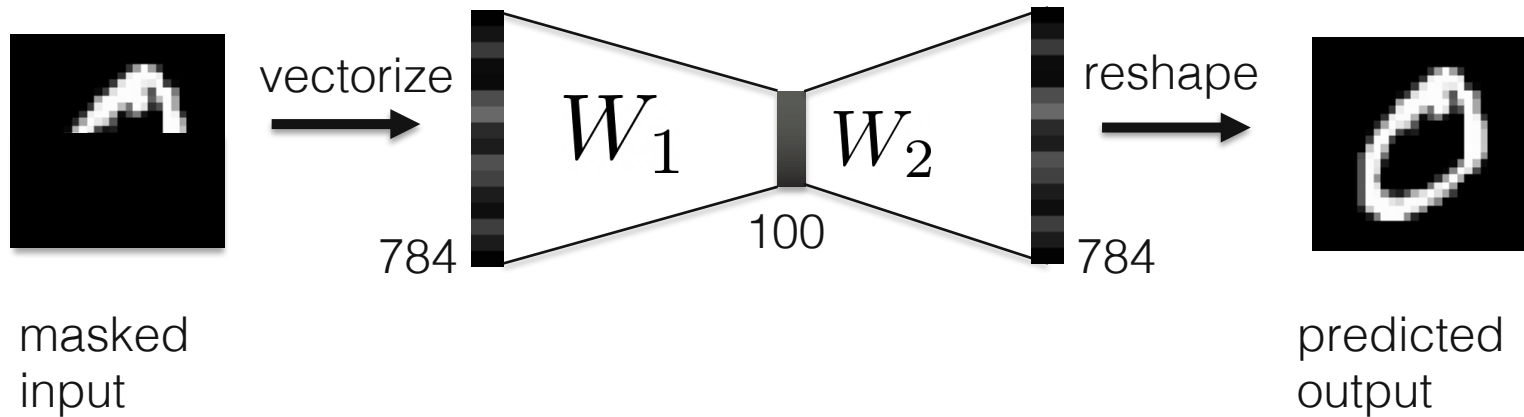
- More on classification...
- <https://cs231n.github.io/linear-classify/>
- <https://csc413-uoft.github.io/>



Today

- What is a neural network?
- Training/optimizing neural nets
- Why “neural”?
- Convolutional neural networks
- Applications & inverse problems

Image Inpainting



Training the MLP

Image inpainting example

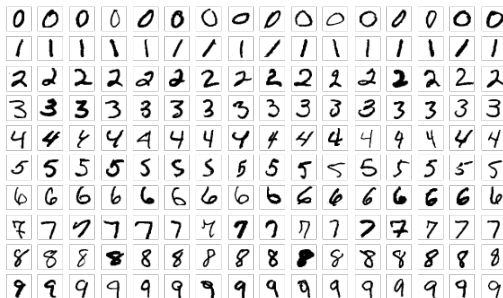
Training dataset:

- masked and complete image pairs
- train network to predict the complete image

masked images



ground truth



Training the MLP

Train the network to minimize the loss function

$$\mathcal{L}_\theta = \frac{1}{2} \|y - \hat{y}\|_2^2$$

network
parameters

$$\theta = \{W_1, W_2\}$$

Training the MLP

Train the network to minimize the loss function

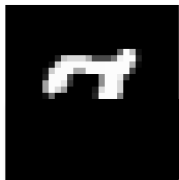
$$\mathcal{L}_{\theta} = \frac{1}{2} \|y - \hat{y}\|_2^2$$

network
parameters

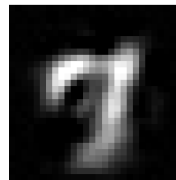
$$\theta = \{W_1, W_2\}$$

ground truth image

network prediction



input



Training the MLP

How do we figure out θ ?


$$\mathcal{L}_{\theta} = \frac{1}{2} \|y - \hat{y}\|_2^2$$

network parameters
 $\theta = \{W_1, W_2\}$

ground truth image

network prediction

input

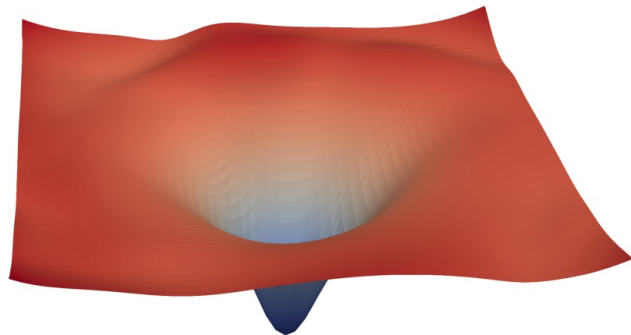


The diagram illustrates the loss function for training an MLP. The equation is $\mathcal{L}_{\theta} = \frac{1}{2} \|y - \hat{y}\|_2^2$. Arrows point from the terms to their corresponding visual representations: θ points to 'network parameters', y points to 'ground truth image', and \hat{y} points to 'network prediction'. Below these are three images: 'input' (a noisy digit 7), 'ground truth image' (a clear digit 7), and 'network prediction' (a blurry digit 7).

Training the MLP

Gradient-based optimization

$$\nabla_{\theta} \mathcal{L}_{\theta}$$



Loss Landscape

[Li et al. '18]

Training the MLP

$$\frac{\partial}{\partial W_1} \mathcal{L}_\theta = \frac{\partial}{\partial W_1} \frac{1}{2} \|y - \hat{y}\|_2^2$$

$$\frac{\partial}{\partial W_2} \mathcal{L}_\theta = \frac{\partial}{\partial W_2} \frac{1}{2} \|y - \hat{y}\|_2^2$$

Need to calculate the partial derivative with respect to each parameter

Training the MLP

Generally there are 3 options

1. Numerical differentiation
2. Symbolic differentiation
3. “Automatic” differentiation

Numerical Differentiation

$$\frac{\partial f(x)}{\partial x} \approx \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Not very accurate, computationally expensive

Easy to implement! Can be used to check your analytical answers..

Symbolic Differentiation

$$\begin{aligned}\frac{\partial \mathcal{L}_\theta}{\partial W_1} &= \frac{\partial}{\partial W_1} \frac{1}{2} \|y - \hat{y}\|_2^2 \\ &= \frac{\partial}{\partial W_1} \frac{1}{2} (W_2 \sigma(W_1 x))^T (W_2 \sigma(W_1 x)) \\ &= \frac{\partial}{\partial W_1} \frac{1}{2} \sigma(W_1 x)^T W_2^T W_2 \sigma(W_1 x) \\ &= \dots \quad \text{chain rule, product rule...}\end{aligned}$$

Accurate, but must be manually calculated for each term
Tedious!

Automatic Differentiation

Think about the problem as a “computational graph”

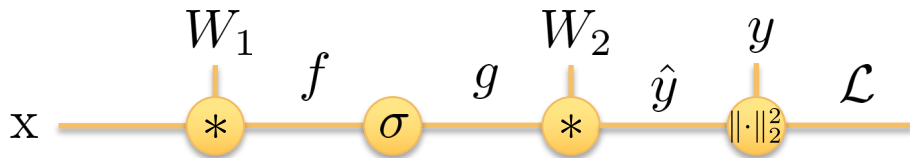
Divide and conquer using the chain rule

Enables “backpropagation” – an efficient way to take derivatives of all parameters in a computational graph

Automatic Differentiation

Think about the problem as a “computational graph”

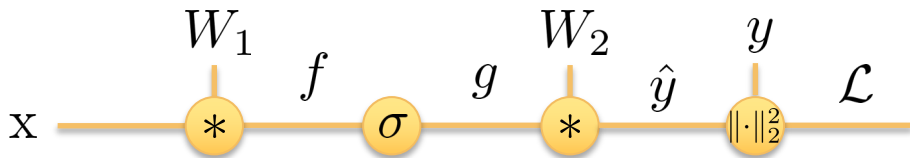
Divide and conquer using the chain rule



Automatic Differentiation

Think about the problem as a “computational graph”

Divide and conquer using the chain rule

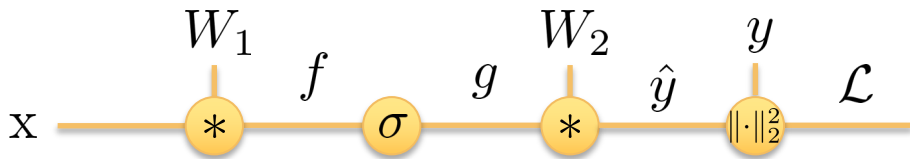


$$\frac{\partial \mathcal{L}}{\partial W_2} = \frac{\partial \hat{y}}{\partial W_2} \frac{\partial \mathcal{L}}{\partial \hat{y}}$$

Automatic Differentiation

Think about the problem as a “computational graph”

Divide and conquer using the chain rule

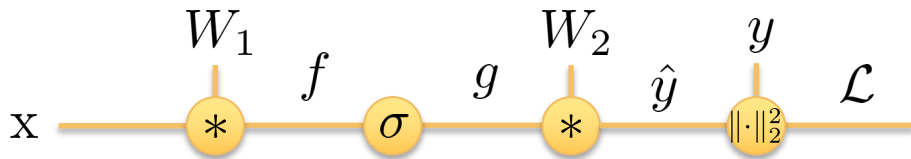


$$\frac{\partial \mathcal{L}}{\partial W_1} = \frac{\partial f}{\partial W_1} \frac{\partial g}{\partial f} \frac{\partial \hat{y}}{\partial g} \frac{\partial \mathcal{L}}{\partial \hat{y}}$$

Automatic Differentiation

Think about the problem as a “computational graph”

Divide and conquer using the chain rule

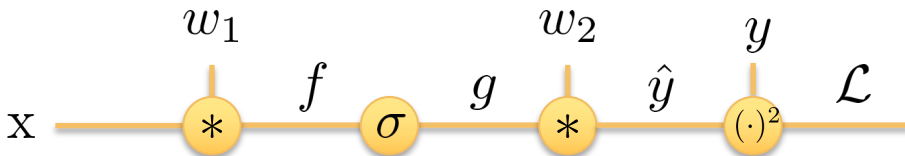


$$\frac{\partial \mathcal{L}}{\partial W_1} = \frac{\partial f}{\partial W_1} \frac{\partial g}{\partial f} \frac{\partial \hat{y}}{\partial g} \frac{\partial \mathcal{L}}{\partial \hat{y}}$$

We can calculate analytical expressions for each of these terms and then plug in our values

Autodiff Example

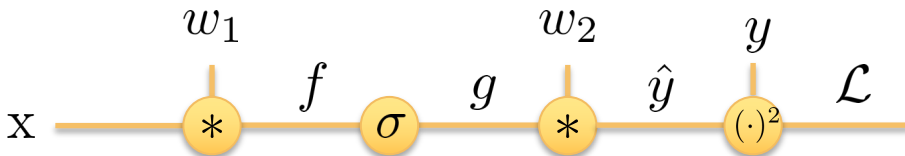
(assume scalar values for now)



$$\frac{\partial \mathcal{L}}{\partial w_1} = \frac{\partial f}{\partial w_1} \frac{\partial g}{\partial f} \frac{\partial \hat{y}}{\partial g} \boxed{\frac{\partial \mathcal{L}}{\partial \hat{y}}}$$

Autodiff Example

(assume scalar values for now)

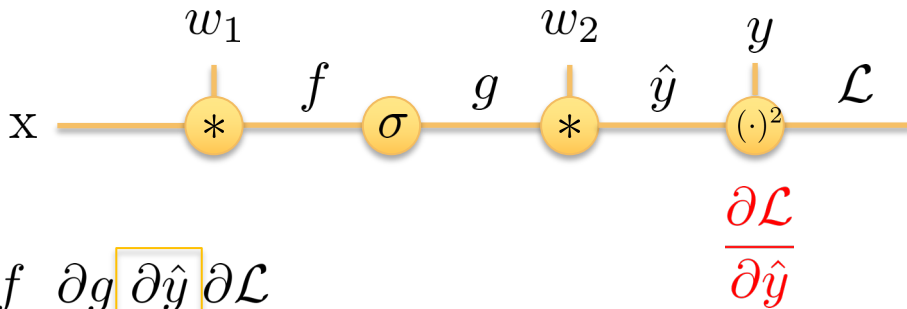


$$\frac{\partial \mathcal{L}}{\partial w_1} = \frac{\partial f}{\partial w_1} \frac{\partial g}{\partial f} \frac{\partial \hat{y}}{\partial g} \boxed{\frac{\partial \mathcal{L}}{\partial \hat{y}}}$$

$$\frac{\partial \mathcal{L}}{\partial \hat{y}} = \frac{\partial}{\partial \hat{y}} \frac{1}{2} (\hat{y} - y)^2 = \hat{y} - y$$

Autodiff Example

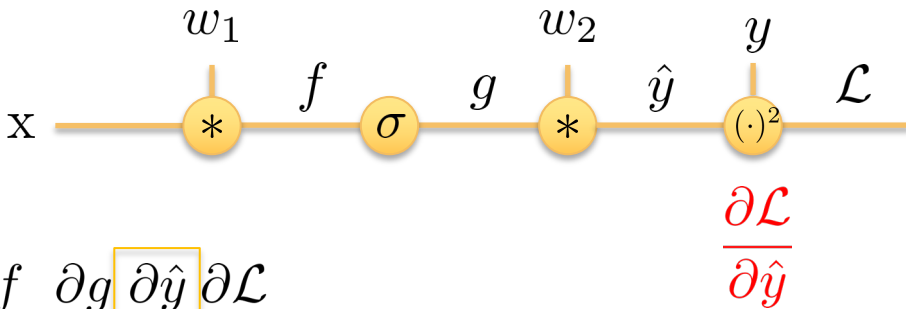
(assume scalar values for now)



$$\frac{\partial \mathcal{L}}{\partial w_1} = \frac{\partial f}{\partial w_1} \frac{\partial g}{\partial f} \boxed{\frac{\partial \hat{y}}{\partial g}} \frac{\partial \mathcal{L}}{\partial \hat{y}}$$

Autodiff Example

(assume scalar values for now)

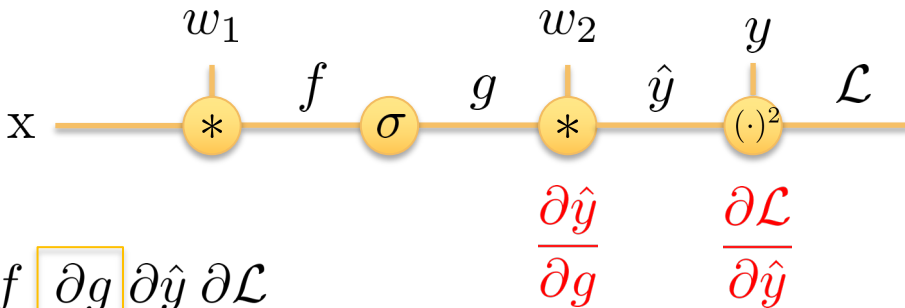


$$\frac{\partial \mathcal{L}}{\partial w_1} = \frac{\partial f}{\partial w_1} \frac{\partial g}{\partial f} \frac{\partial \hat{y}}{\partial g} \frac{\partial \mathcal{L}}{\partial \hat{y}}$$

$$\frac{\partial \hat{y}}{\partial g} = \frac{\partial}{\partial g} w_2 \cdot g = w_2$$

Autodiff Example

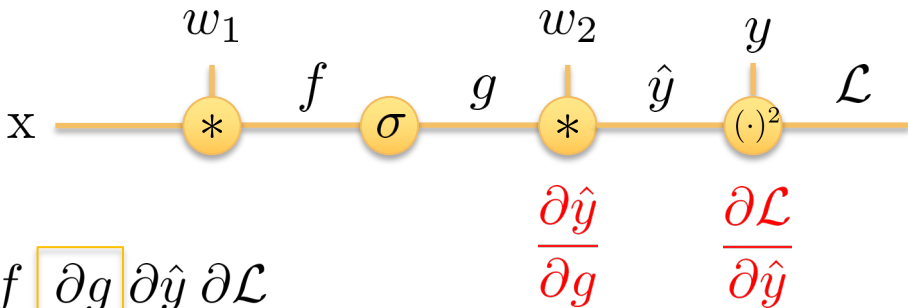
(assume scalar values for now)



$$\frac{\partial \mathcal{L}}{\partial w_1} = \frac{\partial f}{\partial w_1} \boxed{\frac{\partial g}{\partial f}} \frac{\partial \hat{y}}{\partial g} \frac{\partial \mathcal{L}}{\partial \hat{y}}$$

Autodiff Example

(assume scalar values for now)

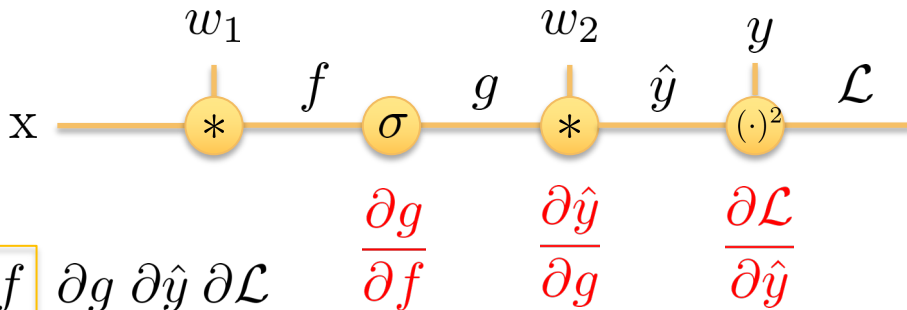


$$\frac{\partial \mathcal{L}}{\partial w_1} = \frac{\partial f}{\partial w_1} \boxed{\frac{\partial g}{\partial f}} \frac{\partial \hat{y}}{\partial g} \frac{\partial \mathcal{L}}{\partial \hat{y}}$$

$$\frac{\partial g}{\partial f} = \frac{\partial}{\partial f} \sigma(f) = \frac{\partial}{\partial f} \max(0, f) = \begin{cases} 0, & f < 0 \\ 1 & \text{else} \end{cases}$$

Autodiff Example

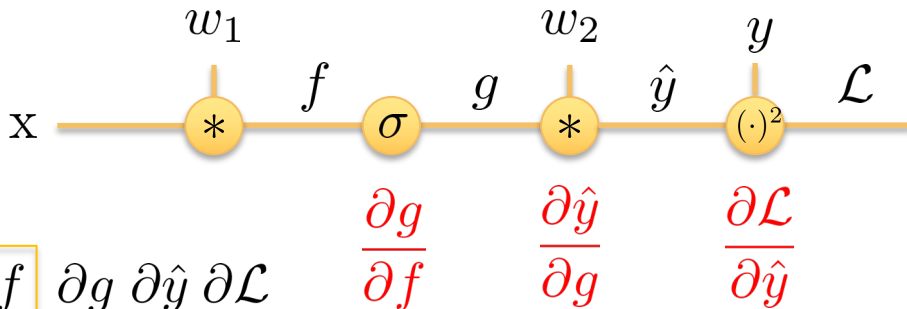
(assume scalar values for now)



$$\frac{\partial \mathcal{L}}{\partial w_1} = \boxed{\frac{\partial f}{\partial w_1}} \frac{\partial g}{\partial f} \frac{\partial \hat{y}}{\partial g} \frac{\partial \mathcal{L}}{\partial \hat{y}}$$

Autodiff Example

(assume scalar values for now)

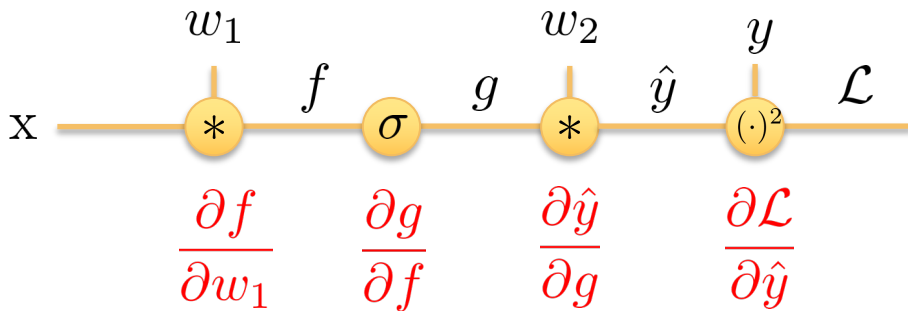


$$\frac{\partial \mathcal{L}}{\partial w_1} = \boxed{\frac{\partial f}{\partial w_1}} \frac{\partial g}{\partial f} \frac{\partial \hat{y}}{\partial g} \frac{\partial \mathcal{L}}{\partial \hat{y}}$$

$$\frac{\partial f}{\partial w_1} = \frac{\partial}{\partial w_1} w_1 \cdot x = x$$

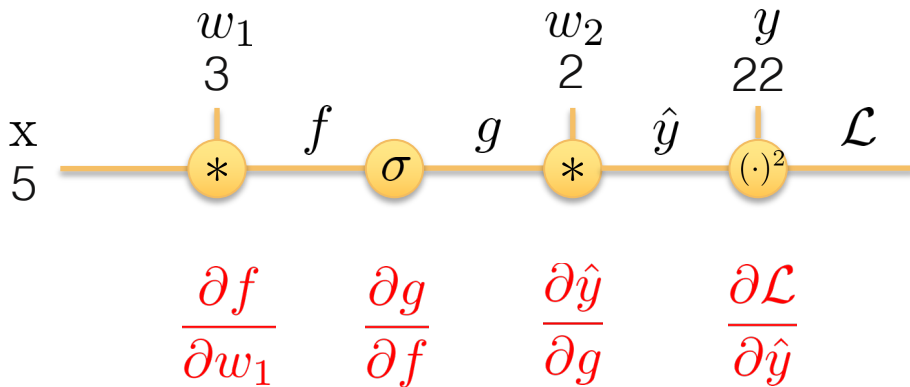
Autodiff Example

(assume scalar values for now)



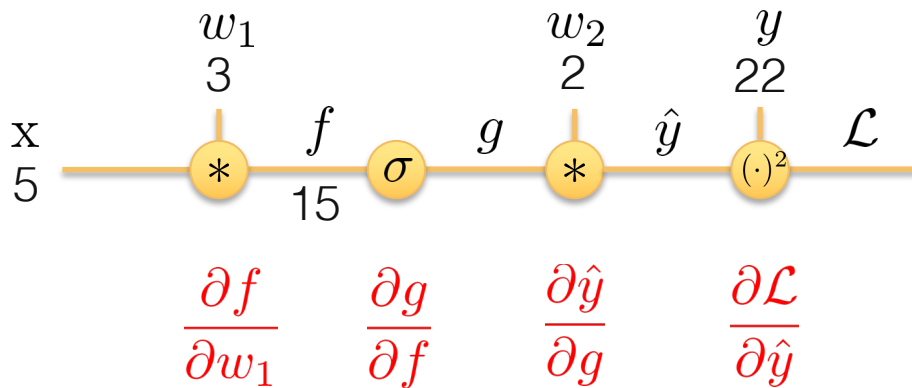
Autodiff Example

Let's plug in the values now...



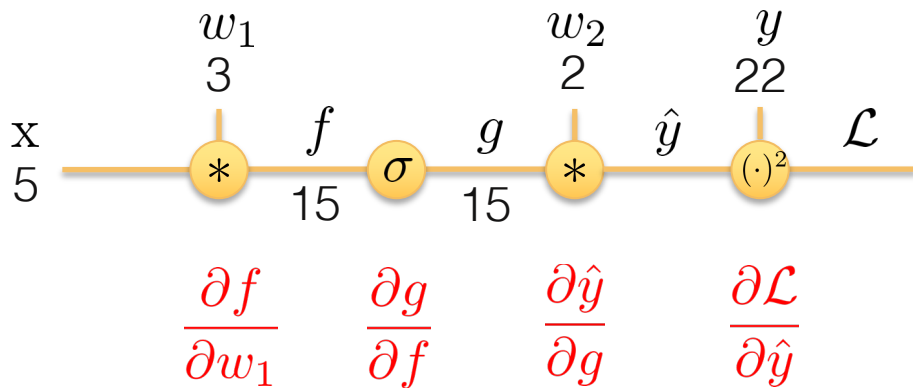
Autodiff Example

Let's plug in the values now...



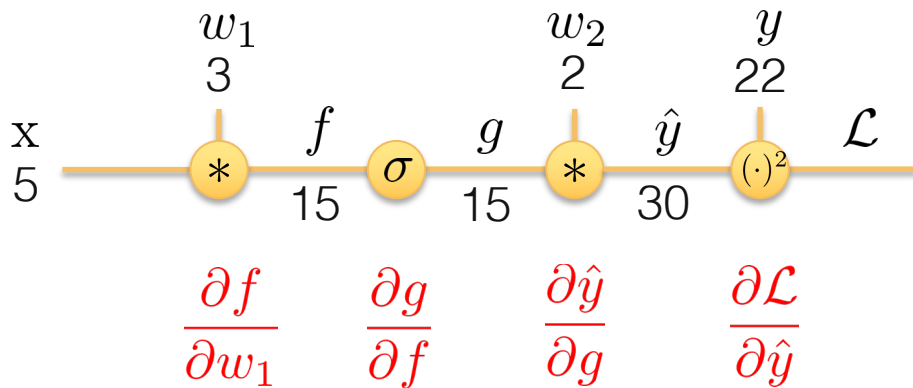
Autodiff Example

Let's plug in the values now...



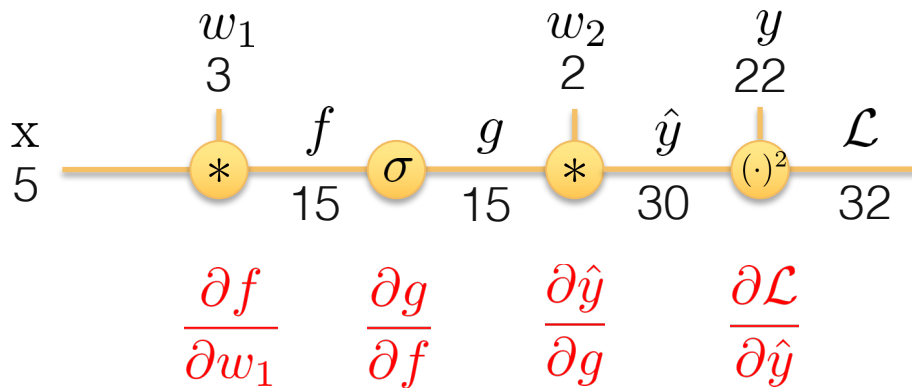
Autodiff Example

Let's plug in the values now...



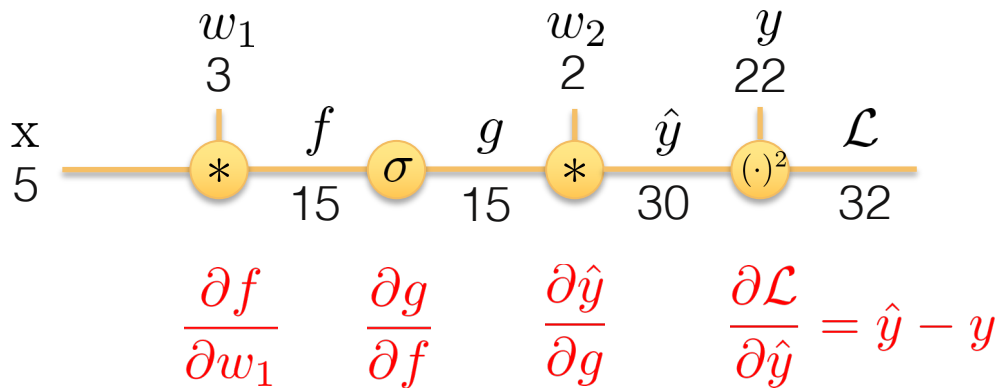
Autodiff Example

Let's plug in the values now...



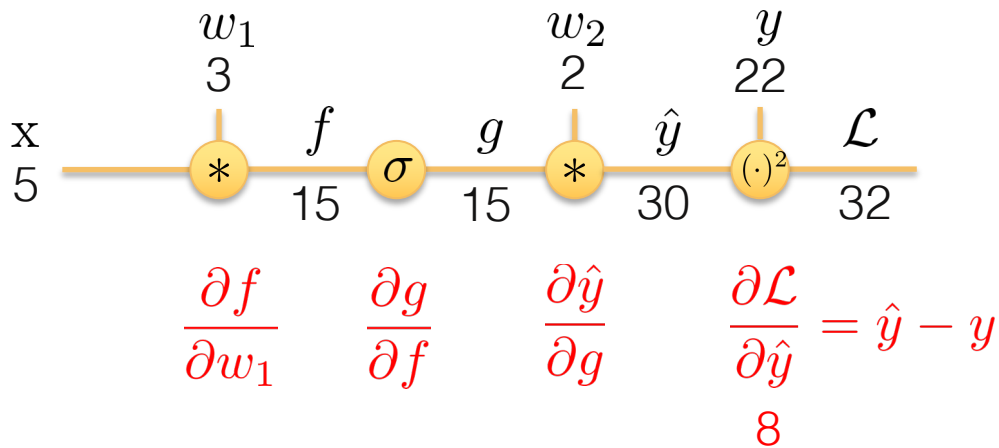
Autodiff Example

Let's plug in the values now...



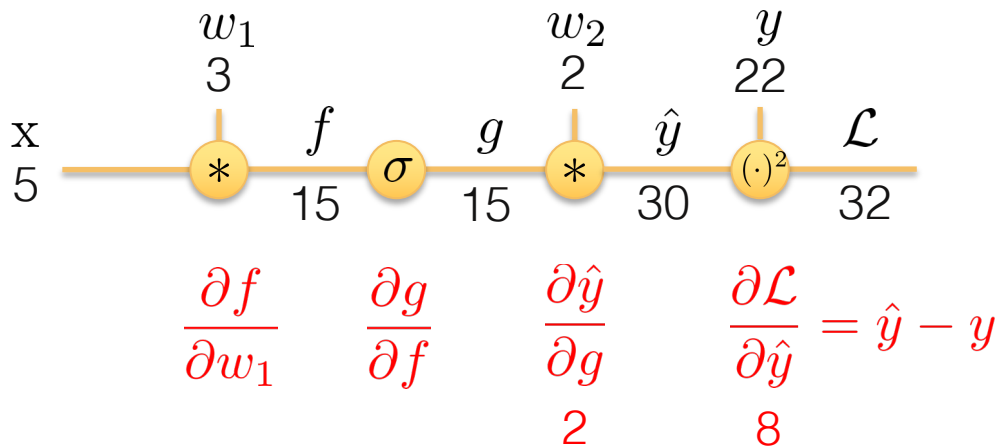
Autodiff Example

Let's plug in the values now...



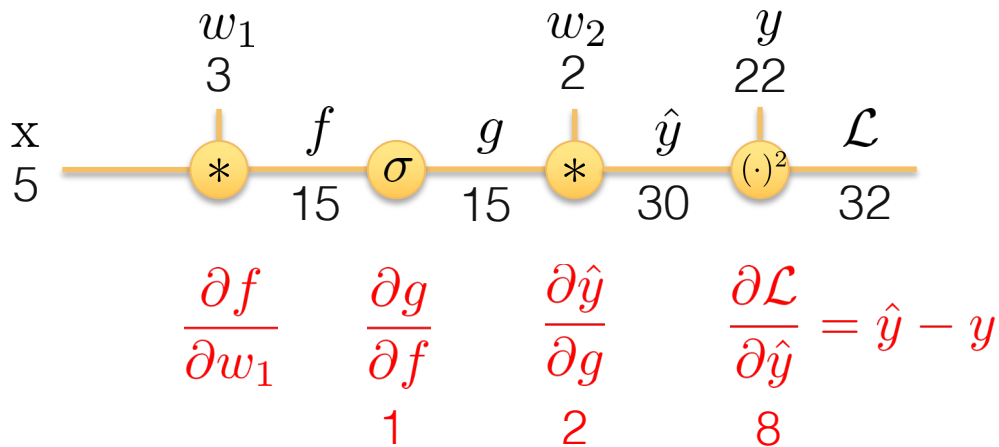
Autodiff Example

Let's plug in the values now...



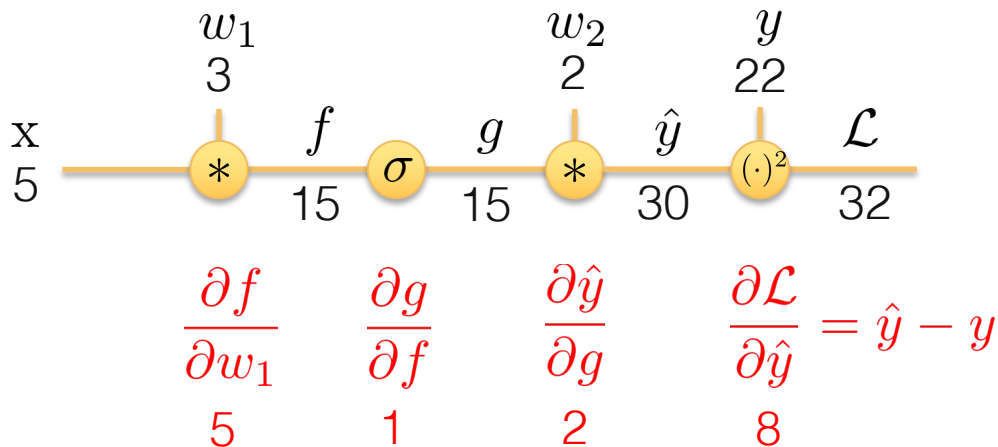
Autodiff Example

Let's plug in the values now...



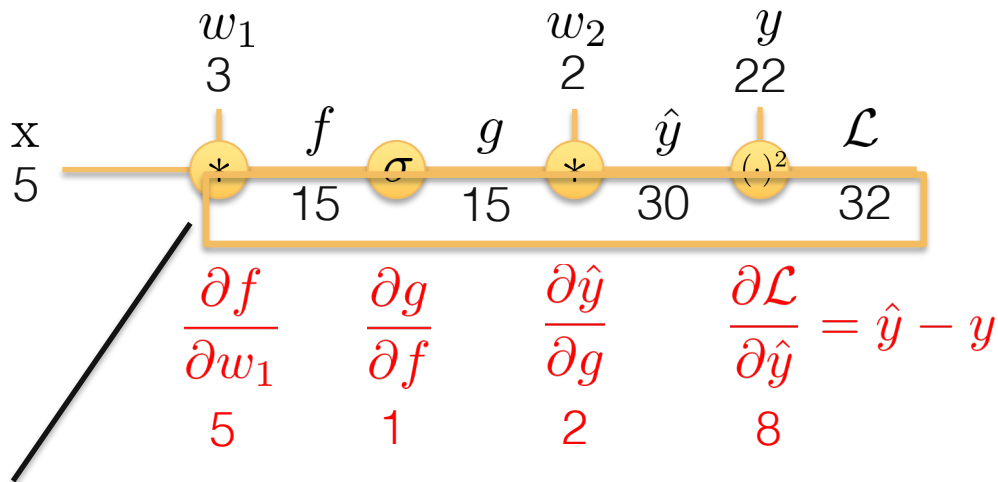
Autodiff Example

Let's plug in the values now...



Autodiff Example

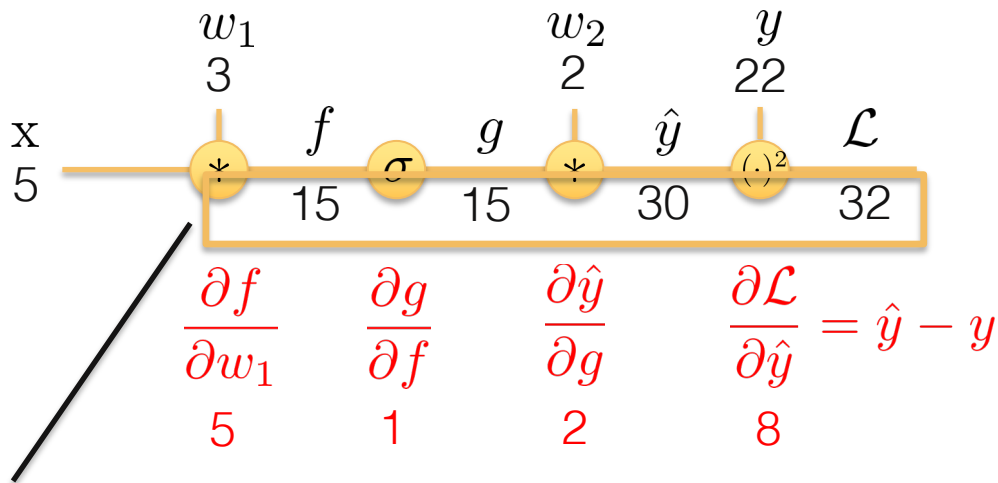
What is backpropagation?



Save these intermediate values during forward computation

Autodiff Example

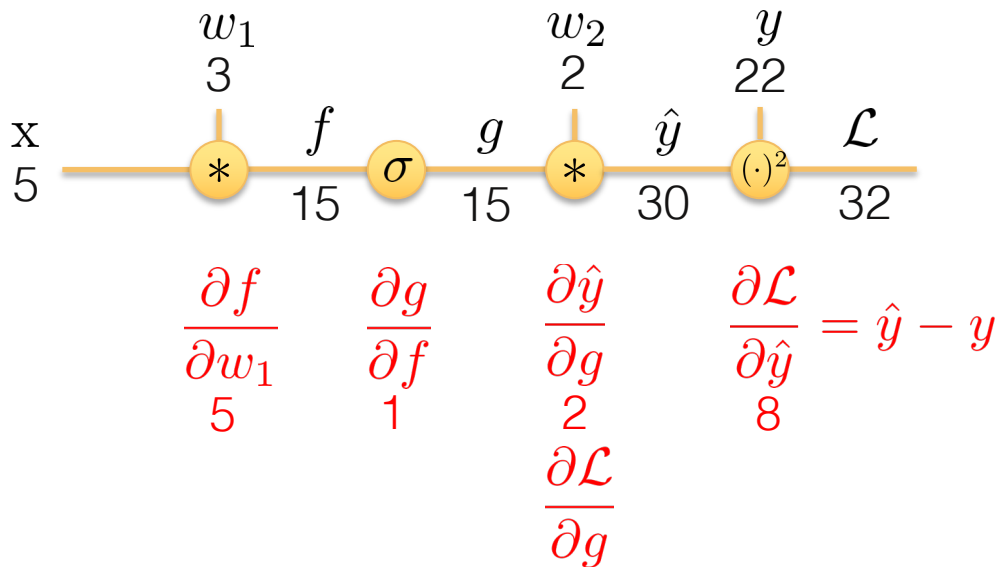
What is backpropagation?



Then we perform a “backward pass”

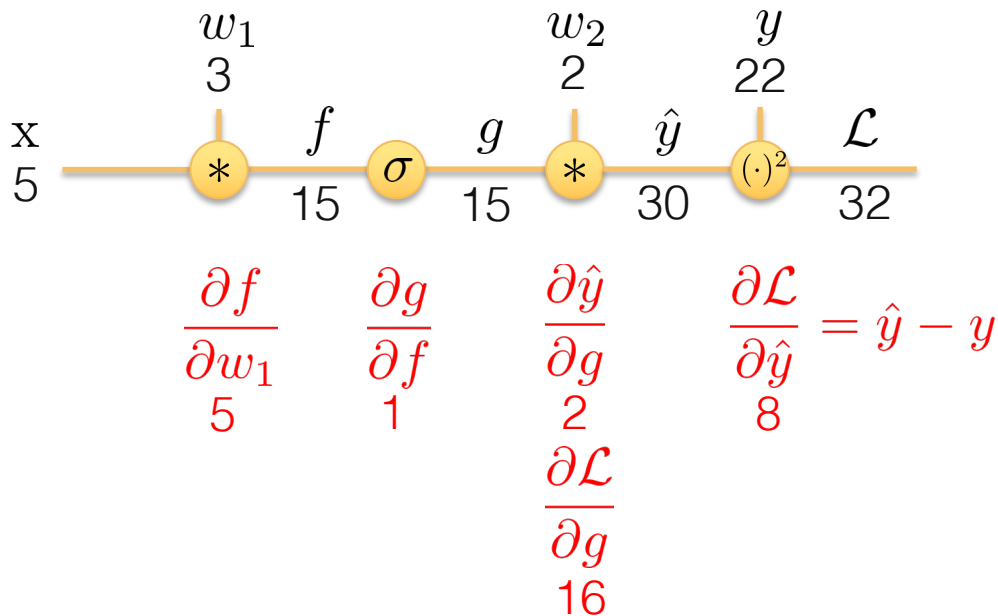
Autodiff Example

What is backpropagation?



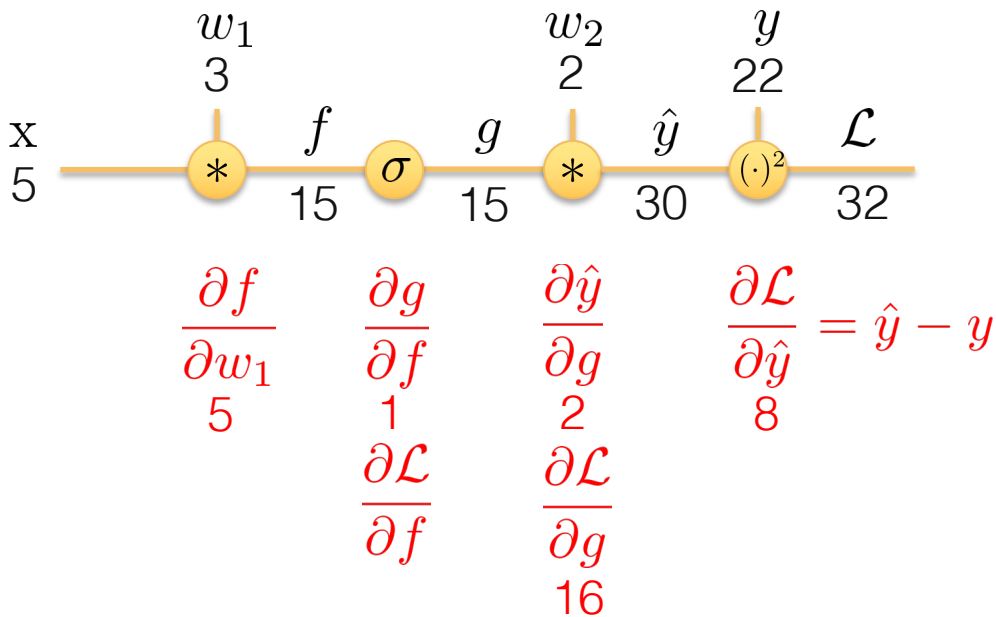
Autodiff Example

What is backpropagation?



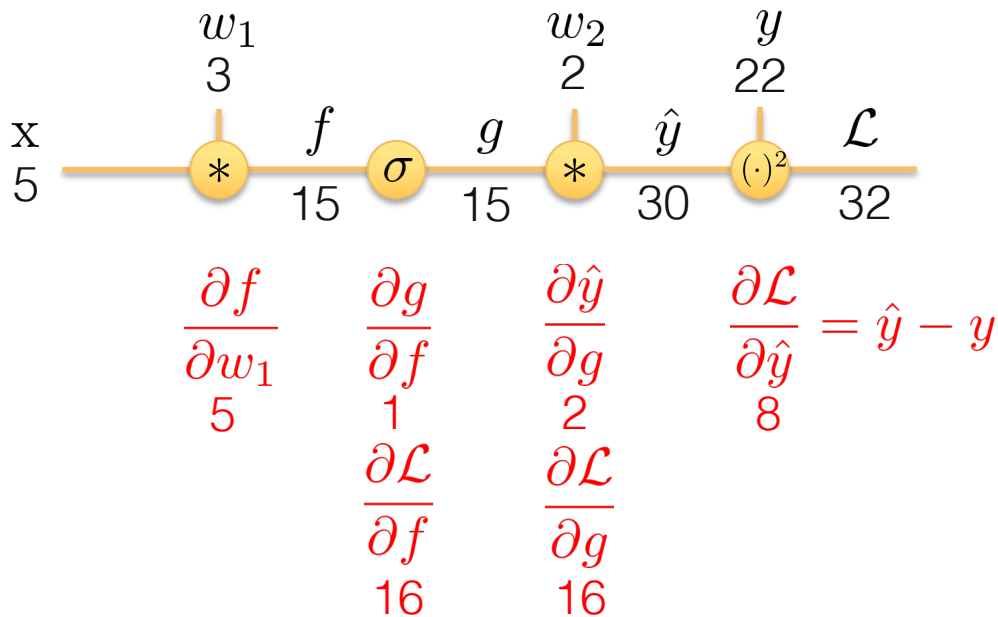
Autodiff Example

What is backpropagation?



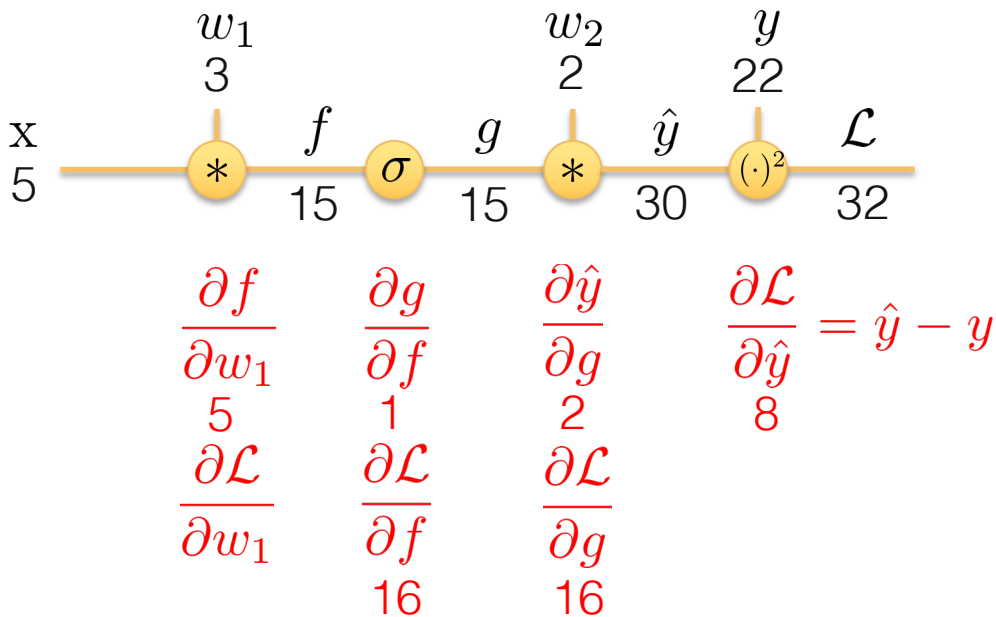
Autodiff Example

What is backpropagation?



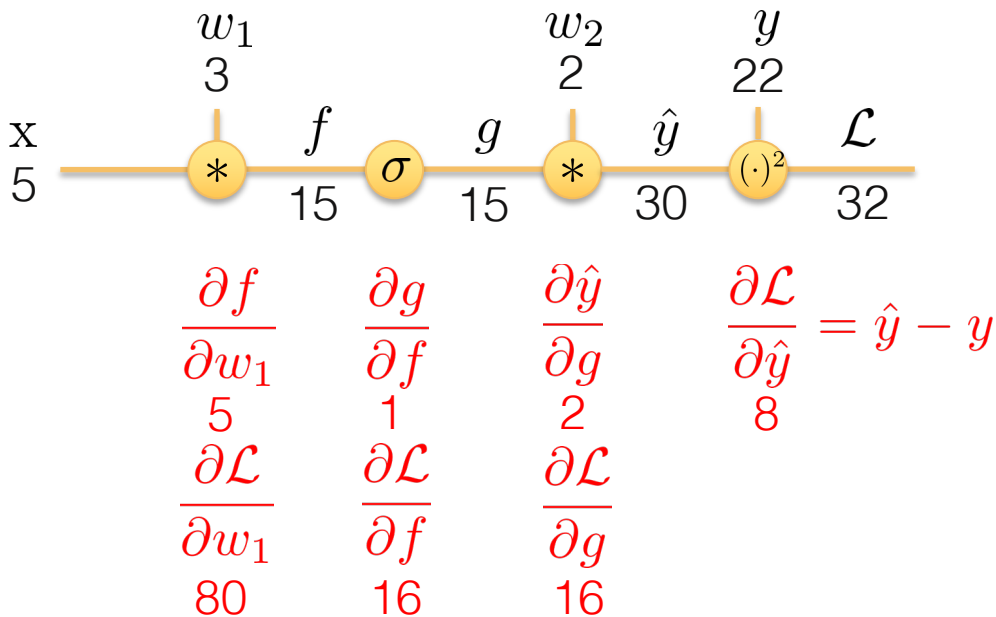
Autodiff Example

What is backpropagation?



Autodiff Example

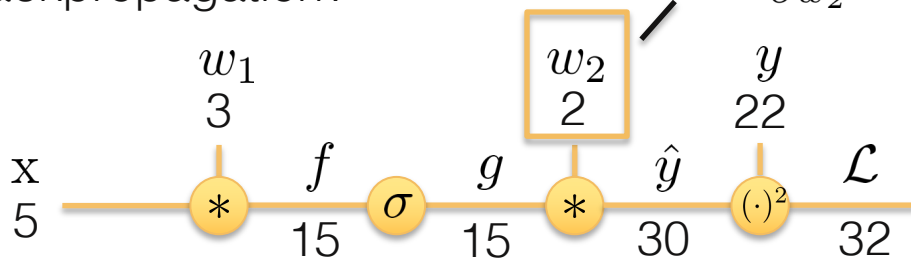
What is backpropagation?



Autodiff Example

What is backpropagation?

What about $\frac{\partial \mathcal{L}}{\partial w_2}$?



$$\frac{\partial f}{\partial w_1}$$
$$5$$

$$\frac{\partial \mathcal{L}}{\partial w_1}$$
$$80$$

$$\frac{\partial g}{\partial f}$$
$$1$$

$$\frac{\partial \mathcal{L}}{\partial f}$$
$$16$$

$$\frac{\partial \hat{y}}{\partial g}$$
$$2$$

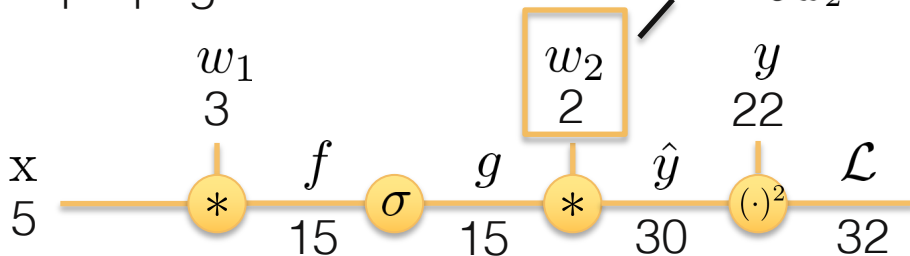
$$\frac{\partial \mathcal{L}}{\partial g}$$
$$16$$

$$\frac{\partial \mathcal{L}}{\partial \hat{y}} = \hat{y} - y$$
$$8$$

Autodiff Example

What is backpropagation?

What about $\frac{\partial \mathcal{L}}{\partial w_2}$?



$$\frac{\partial f}{\partial w_1}$$

$$5$$

$$\frac{\partial \mathcal{L}}{\partial w_1}$$

$$80$$

$$\frac{\partial g}{\partial f}$$

$$1$$

$$\frac{\partial \mathcal{L}}{\partial f}$$

$$16$$

$$\frac{\partial \hat{y}}{\partial g}$$

$$2$$

$$\frac{\partial \mathcal{L}}{\partial g}$$

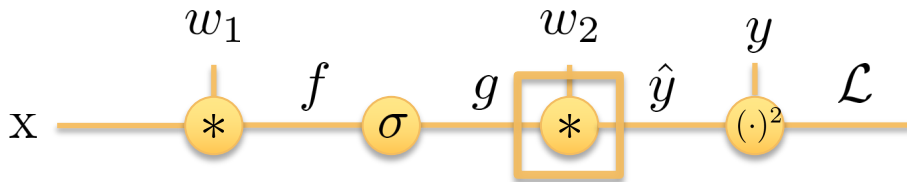
$$16$$

$$\frac{\partial \mathcal{L}}{\partial \hat{y}} = \hat{y} - y$$

We can re-use computation!

$$\frac{\partial \mathcal{L}}{\partial w_2} = \frac{\partial \hat{y}}{\partial w_2} \frac{\partial \mathcal{L}}{\partial \hat{y}}$$

Autodiff Example



PyTorch Code:

```
class Multiply(torch.autograd.Function):  
    @staticmethod  
    def forward(ctx, x, y):  
        ctx.save_for_backward(x, y)  
        z = x * y  
        return z  
    @staticmethod  
    def backward(ctx, grad_z):  
        x, y = ctx.saved_tensors  
        grad_x = y * grad_z # dz/dx * dL/dz  
        grad_y = x * grad_z # dz/dy * dL/dz  
        return grad_x, grad_y
```

Need to stash
some values for
use in backward

Upstream
gradient

Multiply upstream
and local gradients

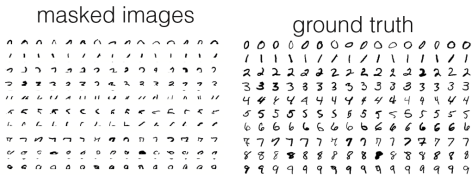
Image Inpainting Training Loop

1. Sample batch of images from dataset

[illegible]

Image Inpainting Training Loop

1. Sample batch of images from dataset



2. Run forward pass to calculate network output for each image

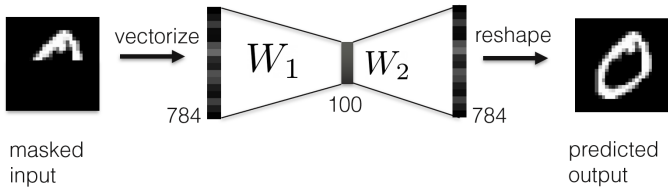
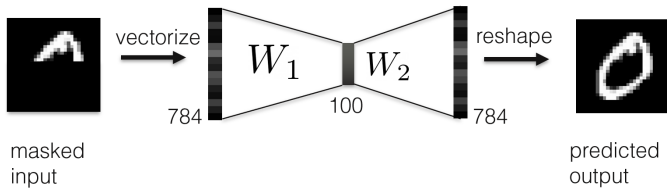


Image Inpainting Training Loop

1. Sample batch of images from dataset



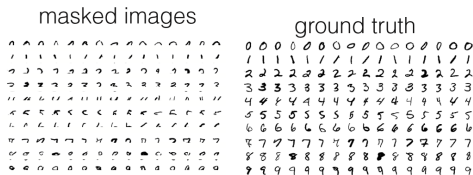
2. Run forward pass to calculate network output for each image



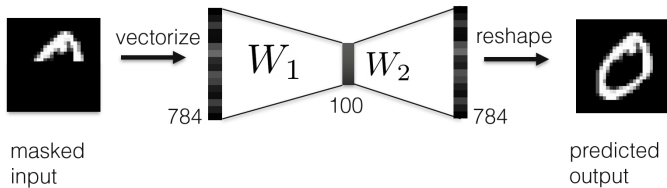
3. Run backward pass to calculate gradients with backpropagation

Image Inpainting Training Loop

1. Sample batch of images from dataset



2. Run forward pass to calculate network output for each image



3. Run backward pass to calculate gradients with backpropagation

4. Update parameters with stochastic gradient descent

4. Update parameters with stochastic gradient descent

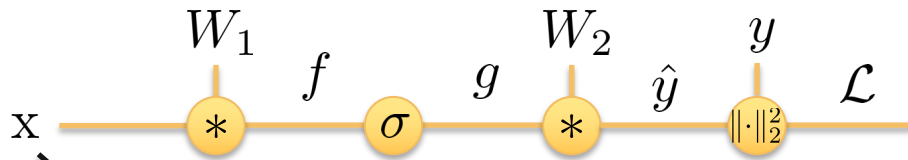
$$\mathcal{L}_\theta = ||y - \hat{y}||_2^2$$

$$W_2^{(k+1)} = W_2^{(k)} - \alpha \frac{\partial \mathcal{L}}{\partial W_2}$$

$$W_1^{(k+1)} = W_1^{(k)} - \alpha \frac{\partial \mathcal{L}}{\partial W_1}$$

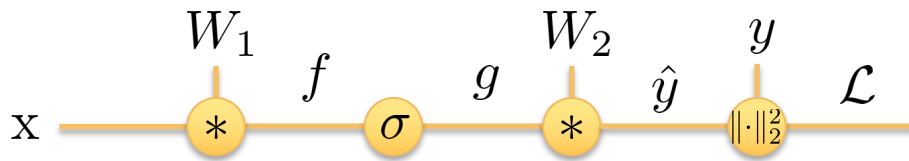


Vector Differentiation



But wait, aren't these vectors?

Vector Differentiation



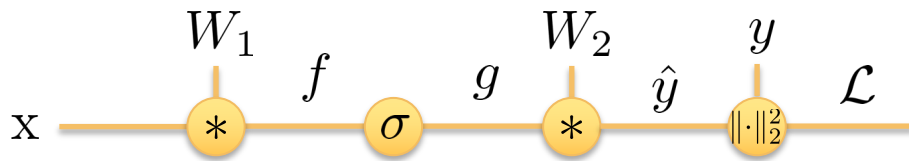
Recap: vector differentiation

Scalar by Scalar

$$x, y \in \mathbb{R}$$

$$\frac{\partial y}{\partial x} \in ?$$

Vector Differentiation



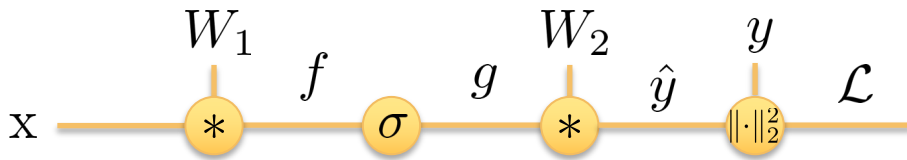
Recap: vector differentiation

Scalar by Scalar

$$x, y \in \mathbb{R}$$

$$\frac{\partial y}{\partial x} \in \mathbb{R}$$

Vector Differentiation



Recap: vector differentiation

Scalar by Scalar

$$x, y \in \mathbb{R}$$

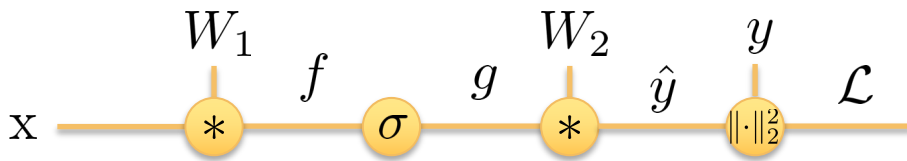
$$\frac{\partial y}{\partial x} \in \mathbb{R}$$

Scalar by Vector

$$x \in \mathbb{R}^N, y \in \mathbb{R}$$

$$\frac{\partial y}{\partial x} \in ?$$

Vector Differentiation



Recap: vector differentiation

Scalar by Scalar

$$x, y \in \mathbb{R}$$

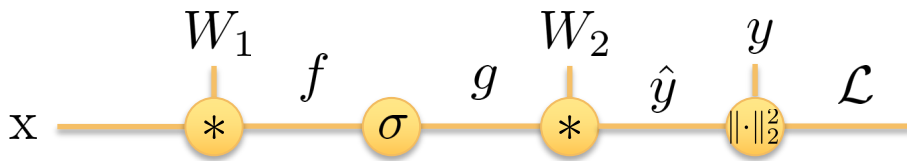
$$\frac{\partial y}{\partial x} \in \mathbb{R}$$

Scalar by Vector

$$x \in \mathbb{R}^N, y \in \mathbb{R}$$

$$\frac{\partial y}{\partial x} \in \mathbb{R}^N$$

Vector Differentiation



Recap: vector differentiation

Scalar by Scalar

$$x, y \in \mathbb{R}$$

$$\frac{\partial y}{\partial x} \in \mathbb{R}$$

Scalar by Vector

$$x \in \mathbb{R}^N, y \in \mathbb{R}$$

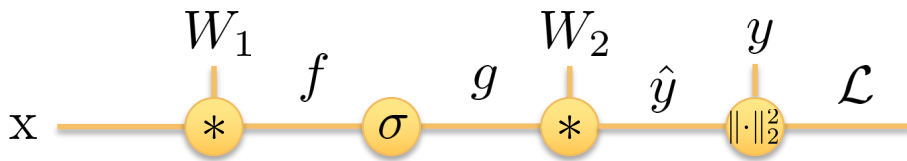
$$\frac{\partial y}{\partial x} \in \mathbb{R}^N$$

Vector by Vector

$$x \in \mathbb{R}^N, y \in \mathbb{R}^M$$

$$\frac{\partial y}{\partial x} \in \text{?}$$

Vector Differentiation



Recap: vector differentiation

Scalar by Scalar

$$x, y \in \mathbb{R}$$

$$\frac{\partial y}{\partial x} \in \mathbb{R}$$

Scalar by Vector

$$x \in \mathbb{R}^N, y \in \mathbb{R}$$

$$\frac{\partial y}{\partial x} \in \mathbb{R}^N$$

Vector by Vector

$$x \in \mathbb{R}^N, y \in \mathbb{R}^M$$

$$\frac{\partial y}{\partial x} \in \mathbb{R}^{N \times M}$$

Vector Differentiation

$$\overset{\in \mathbb{R}^P}{w} \longrightarrow \overset{\in \mathbb{R}^N}{x} \longrightarrow \overset{\in \mathbb{R}^M}{y}$$

$$\frac{\partial x}{\partial w} \in \mathbb{R}^{P \times N}$$

$$\frac{\partial y}{\partial x} \in \mathbb{R}^{N \times M}$$

Vector Differentiation

$$\overset{\in \mathbb{R}^P}{w} \longrightarrow \overset{\in \mathbb{R}^N}{x} \longrightarrow \overset{\in \mathbb{R}^M}{y}$$

$$\frac{\partial x}{\partial w} \in \mathbb{R}^{P \times N}$$

$$\frac{\partial y}{\partial x} \in \mathbb{R}^{N \times M}$$

$$\frac{\partial y}{\partial w} = \frac{\partial x}{\partial w} \frac{\partial y}{\partial x} \in \mathbb{R}^{P \times M}$$

Vector Differentiation

$$\overset{\in \mathbb{R}^P}{w} \longrightarrow \overset{\in \mathbb{R}^N}{x} \longrightarrow \overset{\in \mathbb{R}^M}{y}$$

$$\frac{\partial x}{\partial w} \in \mathbb{R}^{P \times N}$$

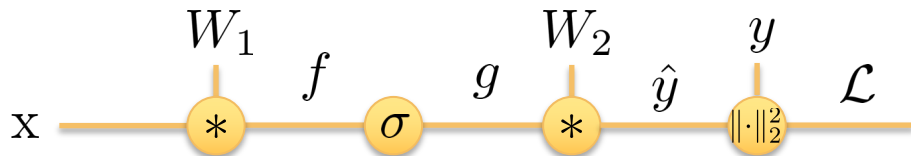
$$\frac{\partial y}{\partial x} \in \mathbb{R}^{N \times M}$$

$$\frac{\partial y}{\partial w} = \frac{\partial x}{\partial w} \frac{\partial y}{\partial x} \in \mathbb{R}^{P \times M}$$

sometimes the Jacobian is defined as the transpose of this,
depending on whether you left or right multiply

(I like to left multiply because it aligns with the direction of the computational graph)

Recap: vector differentiation



Example 1: matrix multiply

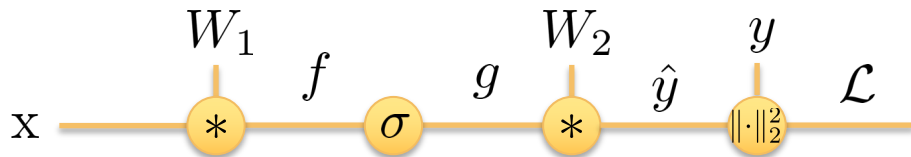
$$\frac{\partial \hat{y}}{\partial g} = \frac{\partial}{\partial g} W_2 g$$

$$W_2 \in \mathbb{R}^{M \times N}$$

$$g \in \mathbb{R}^N$$

$$\frac{\partial \hat{y}}{\partial g} \in \mathbb{R}^{N \times M}$$

Recap: vector differentiation

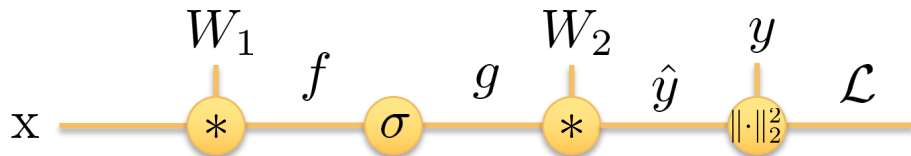


Example 1: matrix multiply

$$\left. \begin{aligned} \frac{\partial \hat{y}}{\partial g} &= \frac{\partial}{\partial g} W_2 g \\ W_2 &\in \mathbb{R}^{M \times N} \\ g &\in \mathbb{R}^N \\ \frac{\partial \hat{y}}{\partial g} &\in \mathbb{R}^{N \times M} \end{aligned} \right|$$

$$\begin{aligned} &\frac{\partial}{\partial g} W_2 g \\ &= \frac{\partial}{\partial g} \begin{bmatrix} w_{11}g_1 + \cdots + w_{1n}g_n \\ \vdots \quad \ddots \quad \vdots \\ w_{m1}g_1 + \cdots + w_{mn}g_n \end{bmatrix} \end{aligned}$$

Recap: vector differentiation



Example 1: matrix multiply

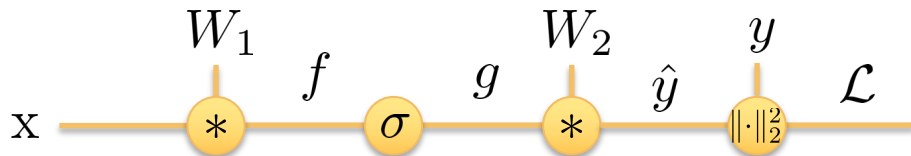
$$\left. \begin{aligned} \frac{\partial \hat{y}}{\partial g} &= \frac{\partial}{\partial g} W_2 g \\ W_2 &\in \mathbb{R}^{M \times N} \\ g &\in \mathbb{R}^N \\ \frac{\partial \hat{y}}{\partial g} &\in \mathbb{R}^{N \times M} \end{aligned} \right|$$

$$\frac{\partial}{\partial g} W_2 g$$

$$\frac{\partial \hat{y}}{\partial g} = \begin{bmatrix} \frac{\partial \hat{y}_1}{\partial g_1} & \cdots & \frac{\partial \hat{y}_m}{\partial g_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial \hat{y}_1}{\partial g_n} & \cdots & \frac{\partial \hat{y}_m}{\partial g_n} \end{bmatrix}$$

$$= \frac{\partial}{\partial g} \begin{bmatrix} w_{11}g_1 + \cdots + w_{1n}g_n \\ \vdots & \ddots & \vdots \\ w_{m1}g_1 + \cdots + w_{mn}g_n \end{bmatrix}$$

Recap: vector differentiation



Example 1: matrix multiply

$$\frac{\partial \hat{y}}{\partial g} = \frac{\partial}{\partial g} W_2 g \quad \left| \quad \frac{\partial}{\partial g} W_2 g \right.$$

$$W_2 \in \mathbb{R}^{M \times N}$$

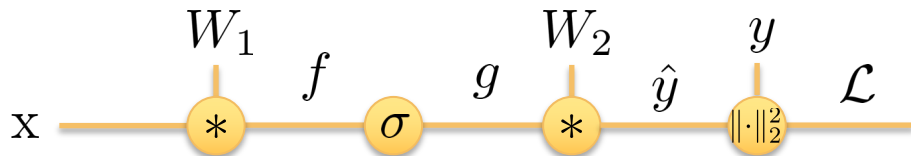
$$g \in \mathbb{R}^N$$

$$\frac{\partial \hat{y}}{\partial g} \in \mathbb{R}^{N \times M}$$

$$= \frac{\partial}{\partial g} \begin{bmatrix} w_{11}g_1 + \cdots + w_{1n}g_n \\ \vdots \\ w_{m1}g_1 + \cdots + w_{mn}g_n \end{bmatrix} = \begin{bmatrix} w_{11} & \cdots & w_{m1} \\ \vdots & \ddots & \vdots \\ w_{1n} & \cdots & w_{mn} \end{bmatrix}$$

$$\frac{\partial \hat{y}}{\partial g} = \begin{bmatrix} \frac{\partial \hat{y}_1}{\partial g_1} & \cdots & \frac{\partial \hat{y}_m}{\partial g_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial \hat{y}_1}{\partial g_n} & \cdots & \frac{\partial \hat{y}_m}{\partial g_n} \end{bmatrix}$$

Recap: vector differentiation



Example 1: matrix multiply

$$\frac{\partial \hat{y}}{\partial g} = \frac{\partial}{\partial g} W_2 g$$

$$W_2 \in \mathbb{R}^{M \times N}$$

$$g \in \mathbb{R}^N$$

$$\frac{\partial \hat{y}}{\partial g} \in \mathbb{R}^{N \times M}$$

$$\frac{\partial}{\partial g} W_2 g$$

$$= \frac{\partial}{\partial g} \begin{bmatrix} w_{11}g_1 + \cdots + w_{1n}g_n \\ \vdots \\ w_{m1}g_1 + \cdots + w_{mn}g_n \end{bmatrix} = \begin{bmatrix} w_{11} & \cdots & w_{m1} \\ \vdots & \ddots & \vdots \\ w_{1n} & \cdots & w_{mn} \end{bmatrix}$$

$$= W_2^T$$

$$\frac{\partial \hat{y}}{\partial g} = \begin{bmatrix} \frac{\partial \hat{y}_1}{\partial g_1} & \cdots & \frac{\partial \hat{y}_m}{\partial g_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial \hat{y}_1}{\partial g_n} & \cdots & \frac{\partial \hat{y}_m}{\partial g_n} \end{bmatrix}$$

Recap: vector differentiation

Example 2: elementwise functions

$$h = f \odot g$$

$$f \in \mathbb{R}^N$$

$$g \in \mathbb{R}^N$$

$$\frac{\partial h}{\partial f} \in \mathbb{R}^{N \times N}$$

Recap: vector differentiation

Example 2: elementwise functions

$$h = f \odot g$$

$$f \in \mathbb{R}^N$$

$$g \in \mathbb{R}^N$$

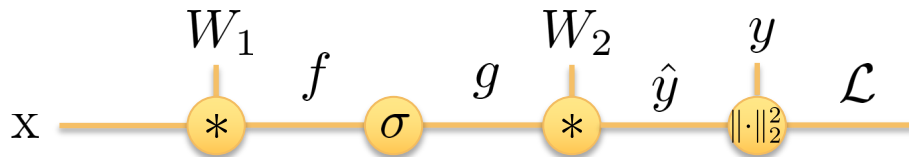
$$\frac{\partial h}{\partial f} \in \mathbb{R}^{N \times N}$$

$$\frac{\partial h}{\partial f} = \begin{bmatrix} g_1 & & 0 \\ & \ddots & \\ 0 & & g_n \end{bmatrix} = \text{diag}(g)$$

$$\frac{\partial h}{\partial f} = \begin{bmatrix} \frac{\partial h_1}{\partial f_1} & \cdots & \frac{\partial h_n}{\partial f_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial h_1}{\partial f_n} & \cdots & \frac{\partial h_n}{\partial f_n} \end{bmatrix}$$

Recap: vector differentiation

Final hint: dimensions should always match up!



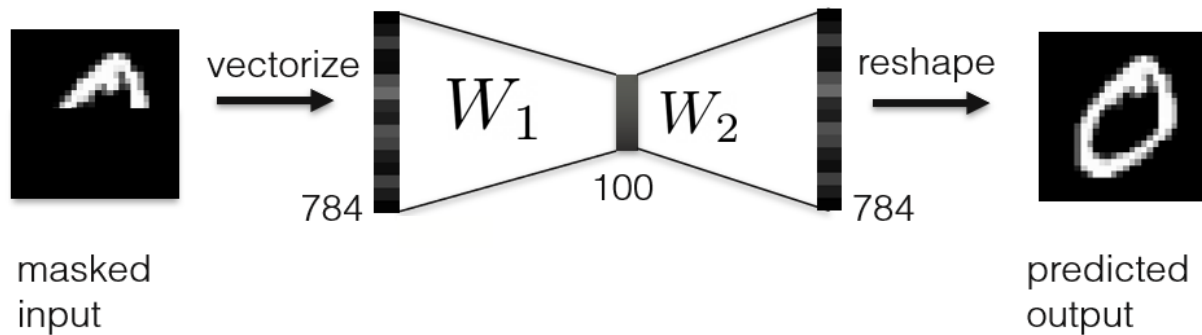
$$\frac{\partial \mathcal{L}}{\partial W_1} = \frac{\partial f}{\partial W_1} \frac{\partial g}{\partial f} \frac{\partial \hat{y}}{\partial g} \frac{\partial \mathcal{L}}{\partial \hat{y}}$$

You should be able to calculate derivatives of each of these terms and then perform matrix multiplications without issues

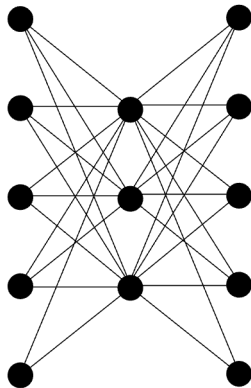
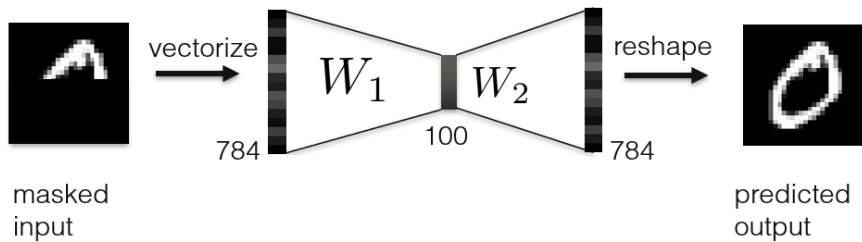
Today

- What is a neural network?
- Training/optimizing neural nets
- Why “neural”?
- Convolutional neural networks
- Applications & inverse problems

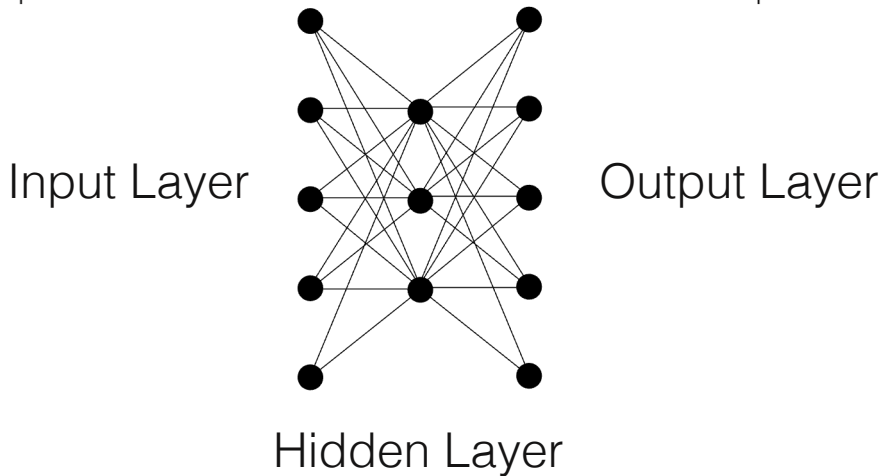
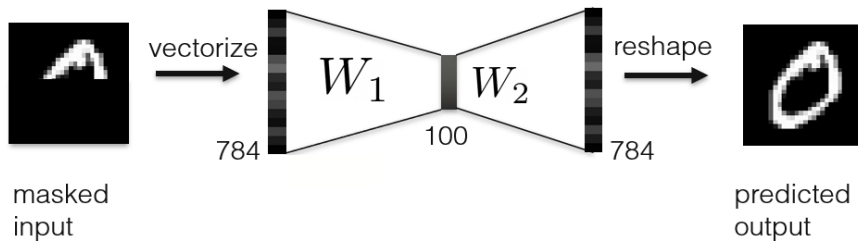
Why “neural” network?



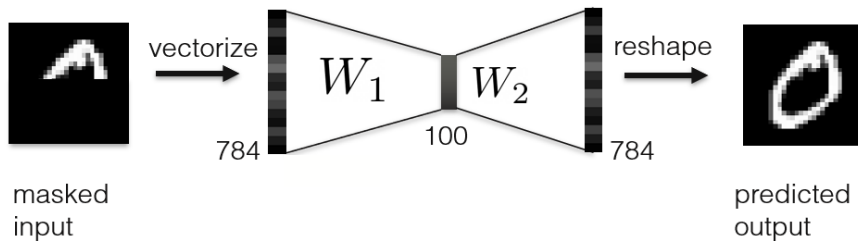
Why “neural” network?



Why “neural” network?



Why “neural” network?

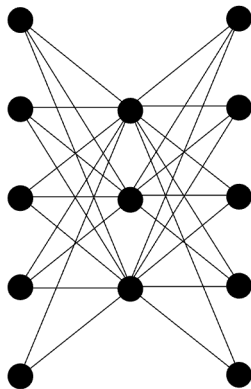


Input Layer

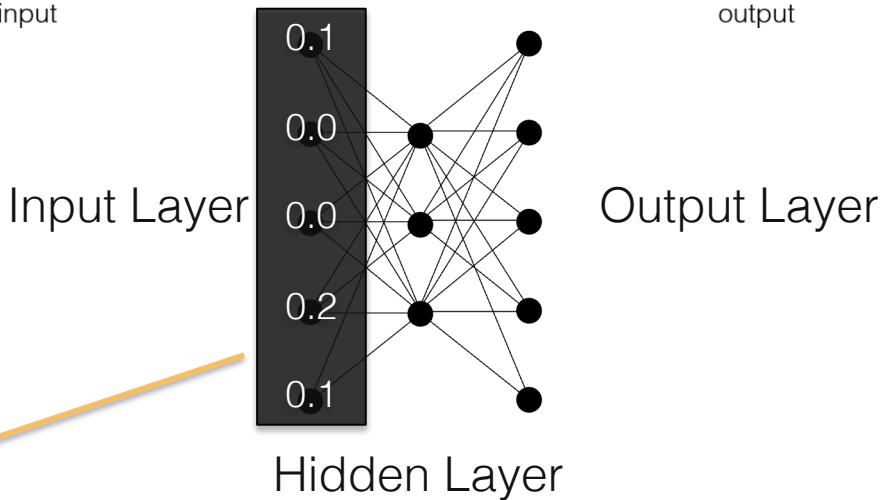
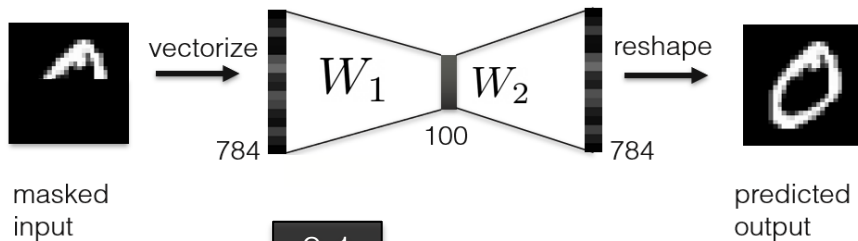
Output Layer

“Neuron”

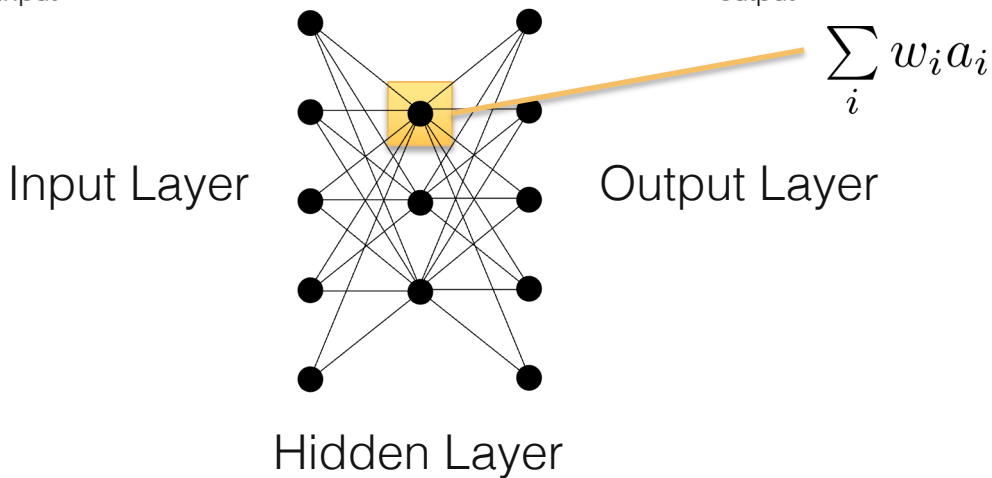
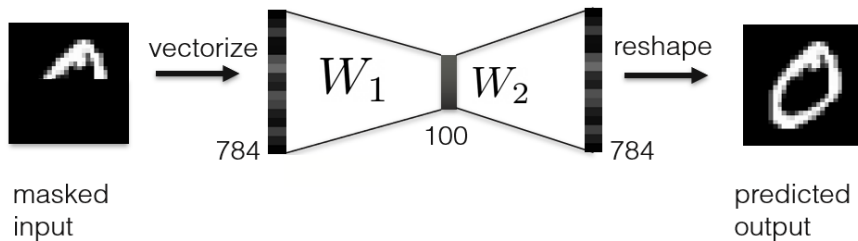
Hidden Layer

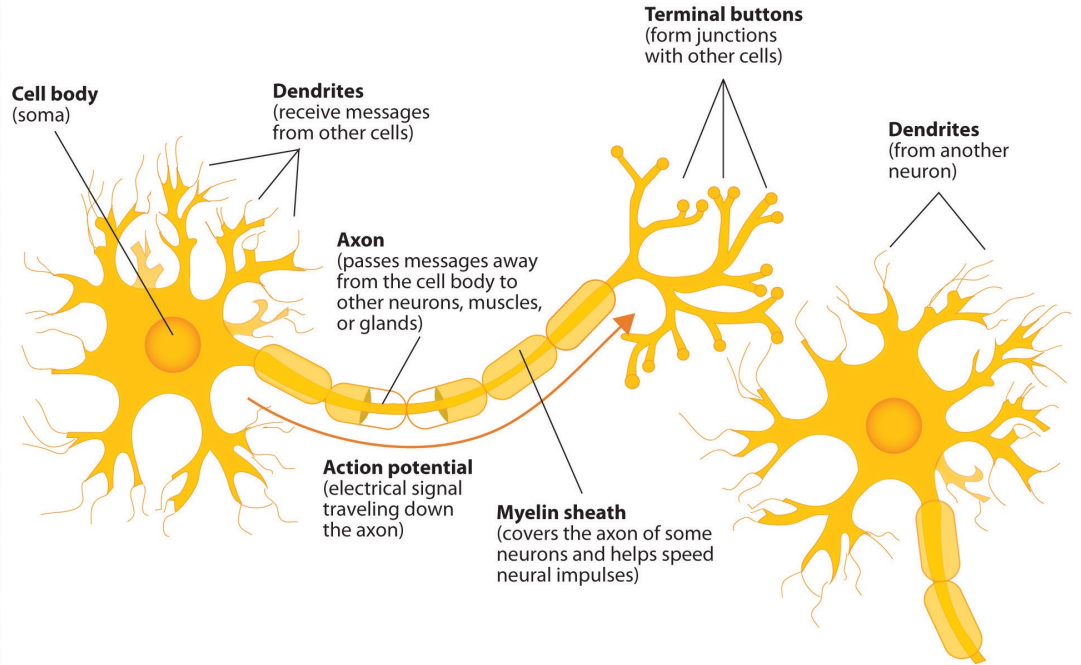
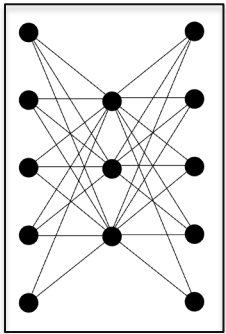


Why “neural” network?



Why “neural” network?





Loose analogy!

- Neurons have activation potentials, all-or-none firing behavior
- Interconnectivity between actual neurons is dense and complicated
- Connection between neurons is complex non-linear dynamical system

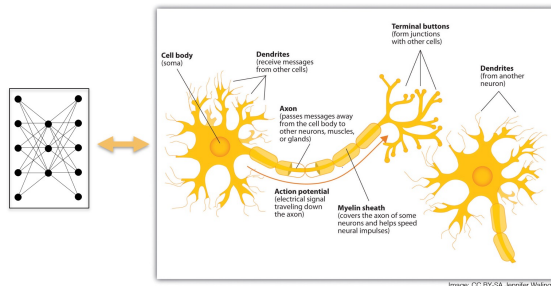
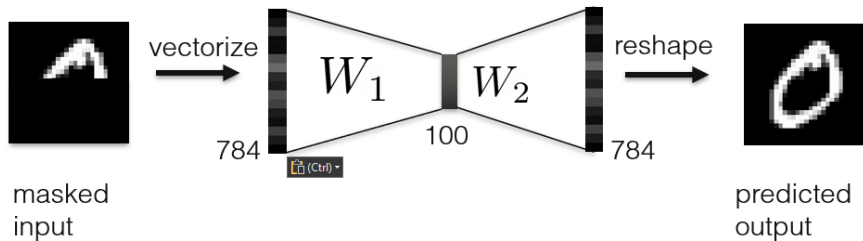


Image: CC BY-SA, Jennifer Walzing

Today

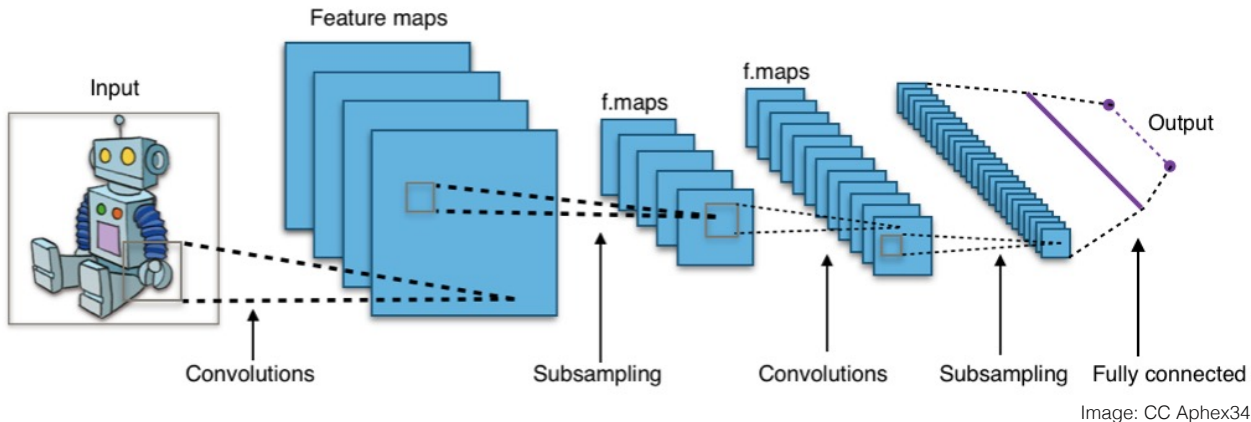
- What is a neural network?
- Training/optimizing neural nets
- Why “neural”?
- Convolutional neural networks
- Applications & inverse problems

Drawbacks of fully-connected networks



- spatial structure is destroyed
- fully-connected weights do not scale

Convolutional Neural Networks



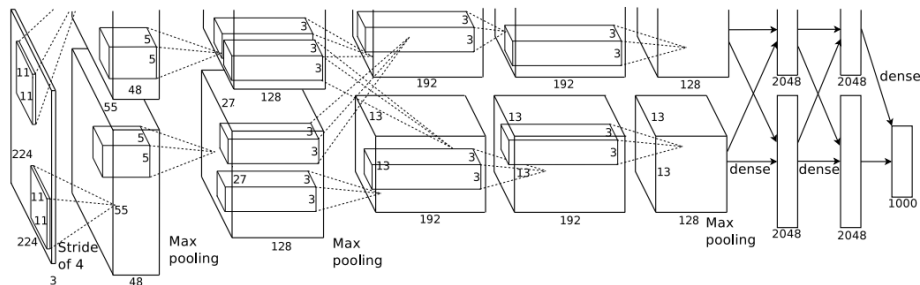
- Exploit spatial structure
- Scale to large inputs with fewer parameters
- Remarkable performance for processing visual data

AlexNet & surge in popularity

2010: ImageNet Large Scale Visual Recognition Challenge

- 10 million labeled images

First convolutional
network for image
classification



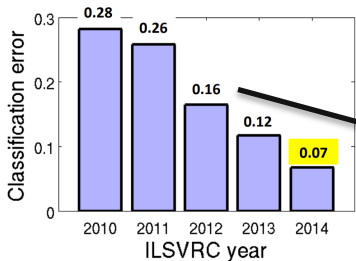
AlexNet [Krizhevsky '12]

AlexNet & surge in popularity

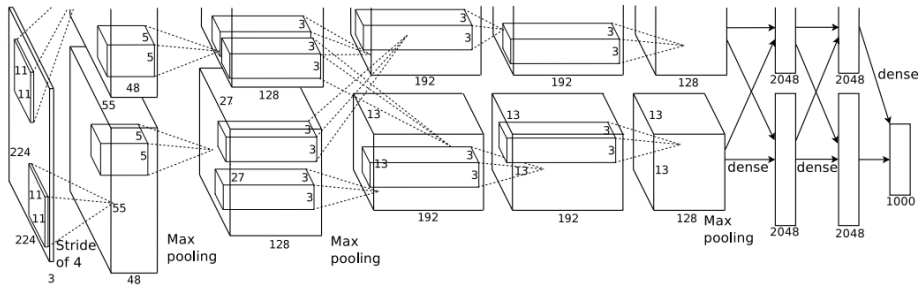
2010: ImageNet Large Scale Visual Recognition Challenge

- 10 million labeled images

First convolutional network for image classification



[Russakovsky '15]



AlexNet [Krizhevsky '12]

CNNs

AlexNet & surge in popularity

2010: ImageNet Large Scale Visual Recognition Challenge

- 10 million labeled images



Deep networks



VGG

[Simonyan '14]



GoogLeNet

[Szegedy '14]



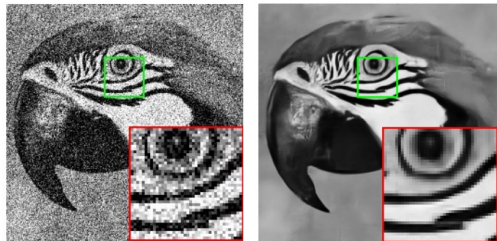
ResNet

[He '15]

Imaging & Image processing

Image Denoising

[Zhang '17]



Learned ISPs

[Chen '18]



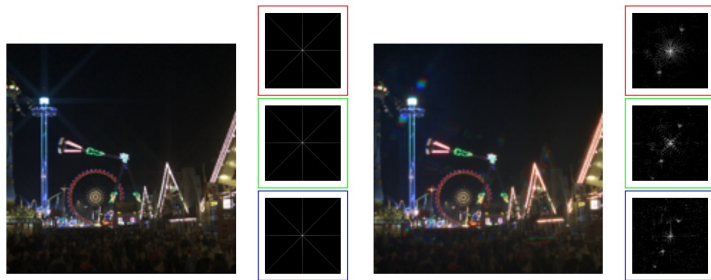
Image Deblurring

[Nah '17]



End-to-End Optimization

[Metzler '19]



Imaging & Image processing

Monocular Depth Estimation

[Eigen '14]

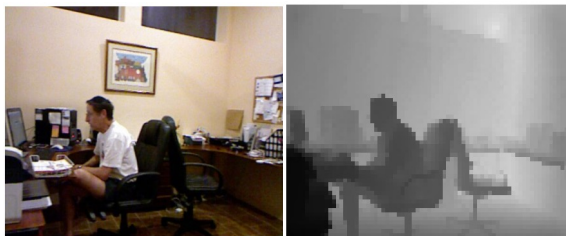


Image Super-resolution

[Lim '17]

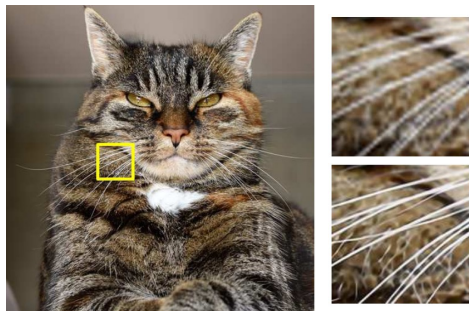
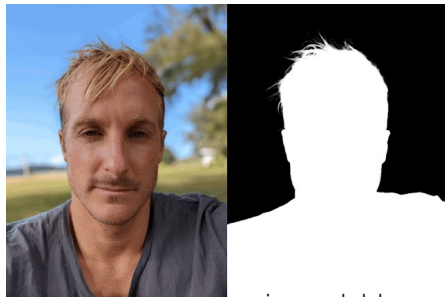


Image Relighting

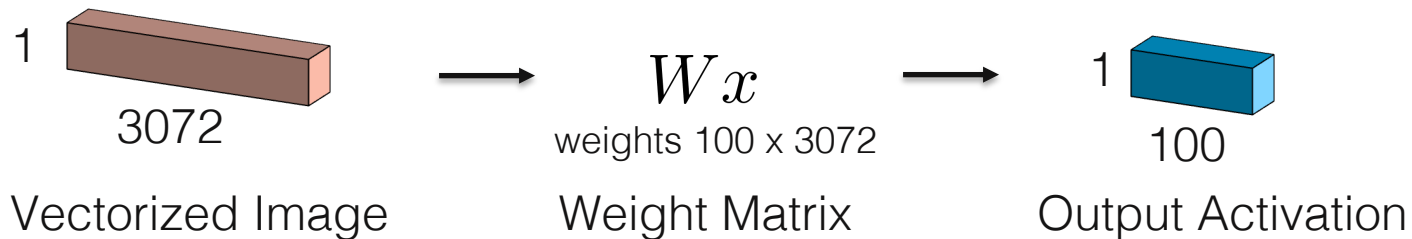
[Sun '19]



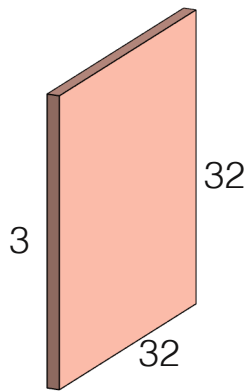
Synthetic Depth-of-Field



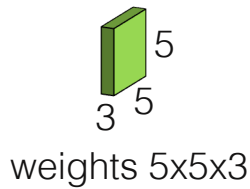
Fully-Connected Layer



Convolutional Layer

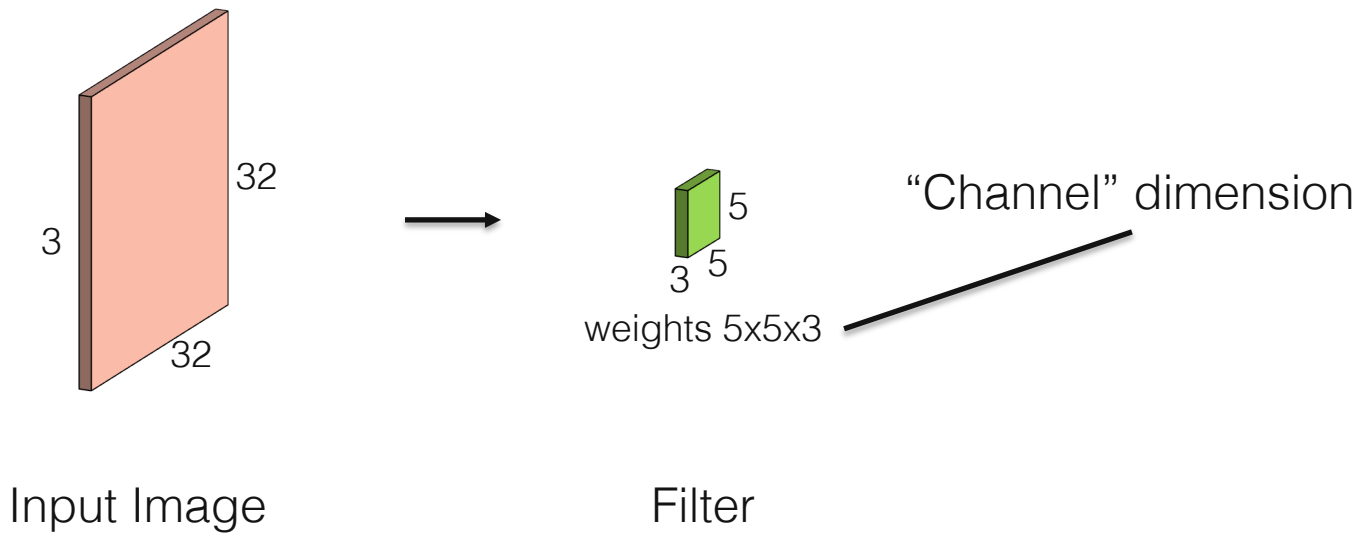


Input Image

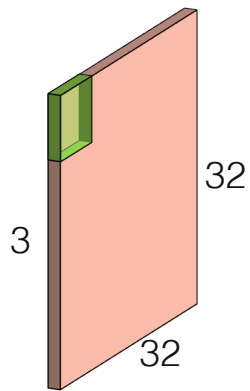


Filter

Convolutional Layer

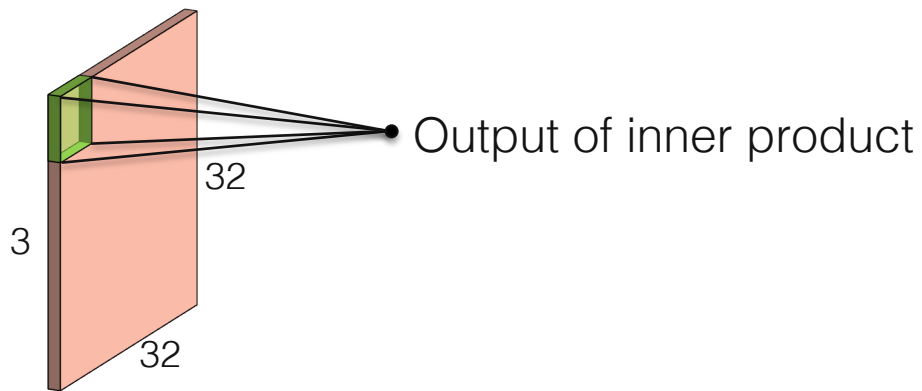


Convolutional Layer



Input Image

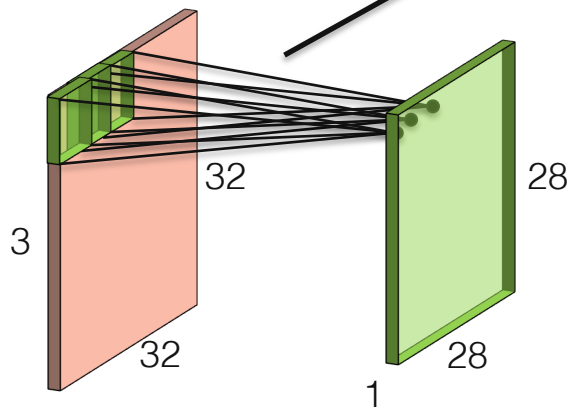
Convolutional Layer



Input Image

Convolutional Layer

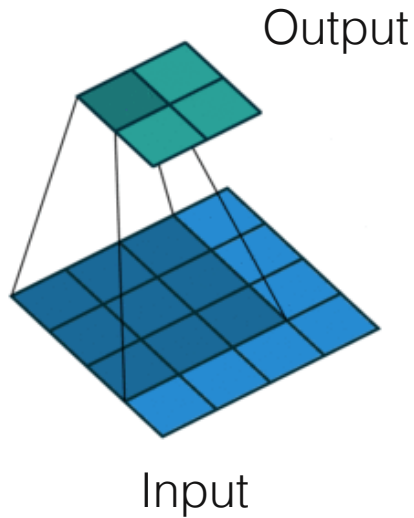
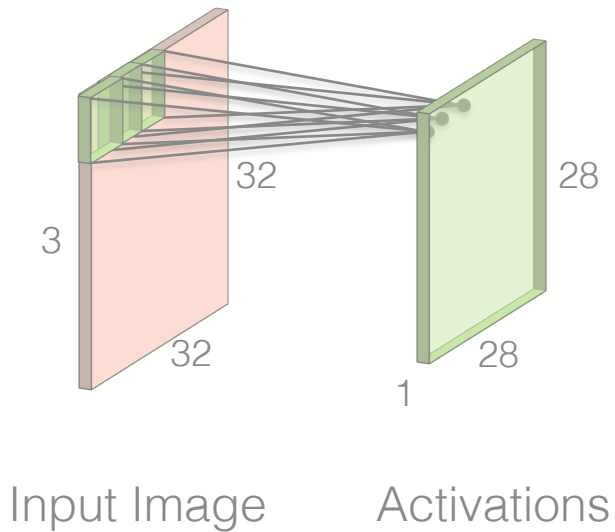
Convolution = sliding window + inner product



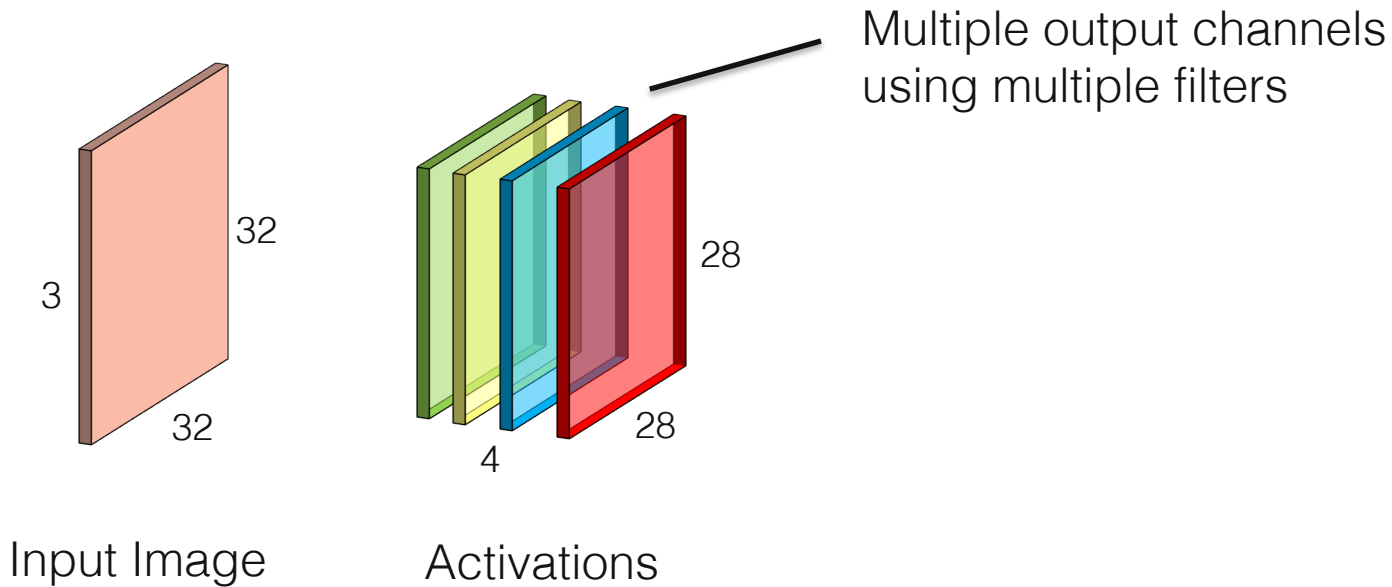
Input Image

Activations

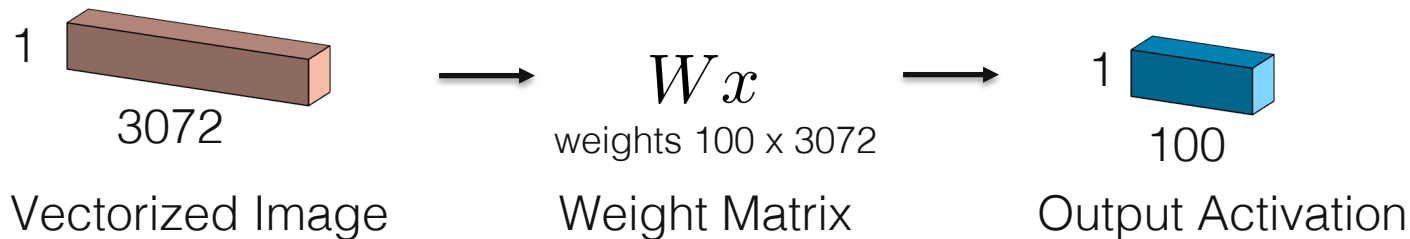
Convolutional Layer



Convolutional Layer

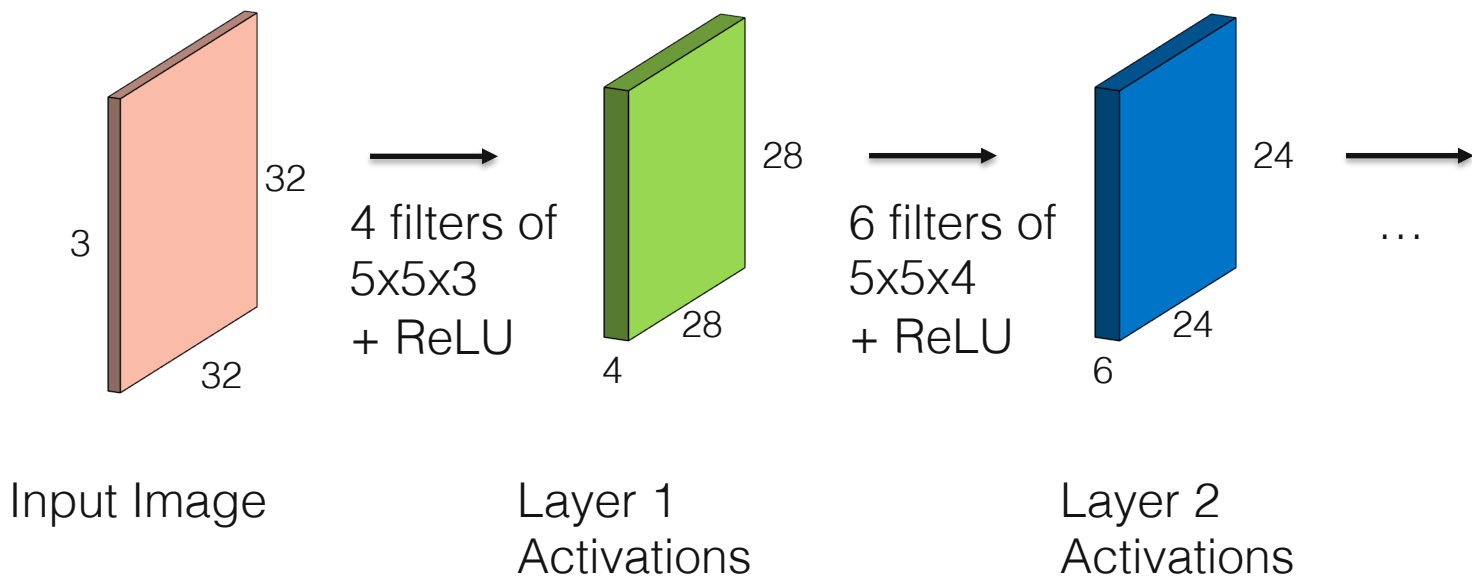


Fully-Connected Layer



Special case of convolutional layer when filter size = input size!

Convolutional Neural Network

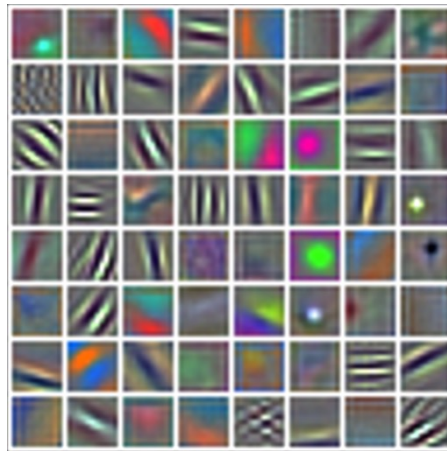


Case Study: AlexNet

Input Image



First-layer Filters

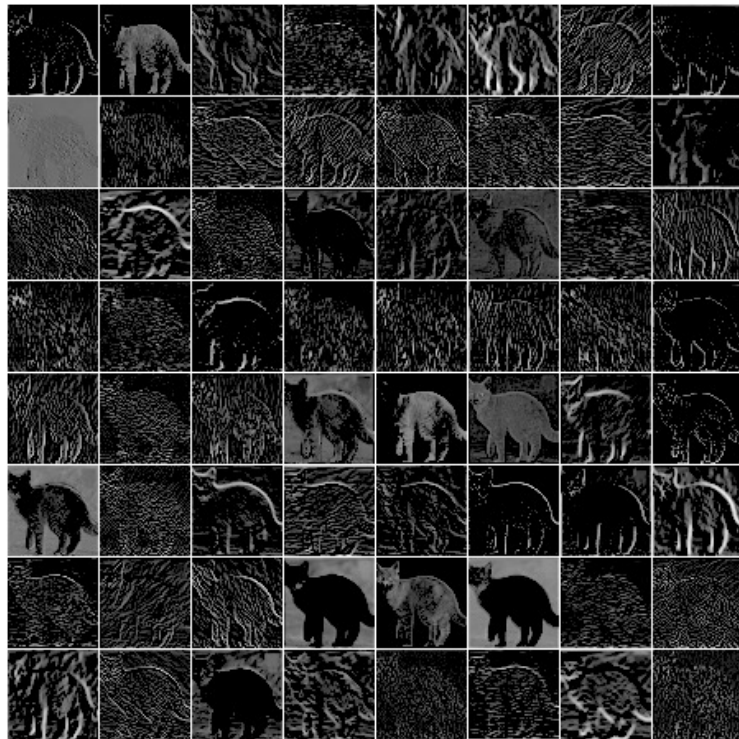
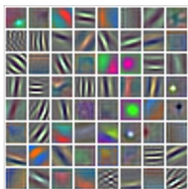


Case Study: AlexNet

Input Image

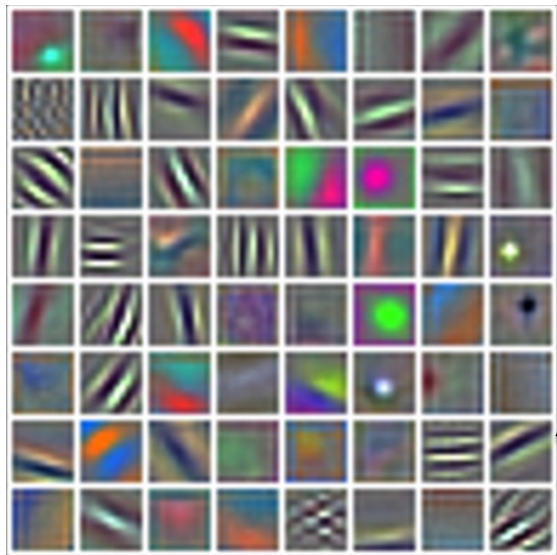


First-layer Filters



Activations

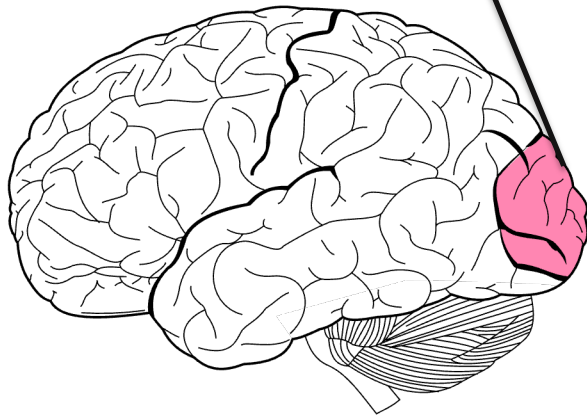
Case Study: AlexNet



First-layer Filters

Similar to simple cells
in visual cortex!

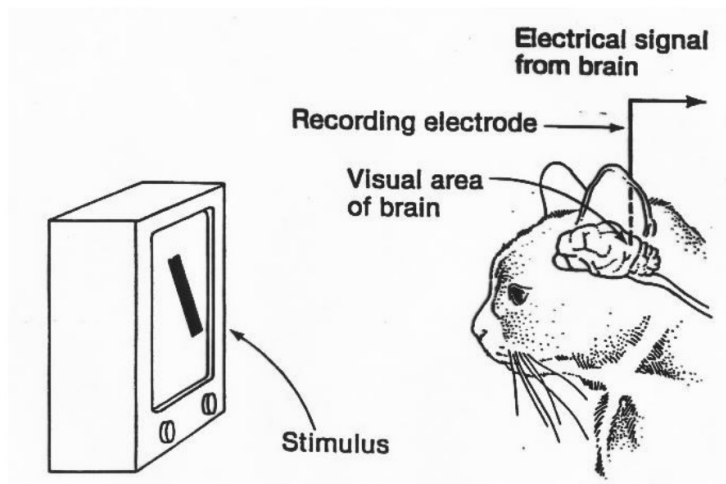
- Edge detectors



Case Study: AlexNet

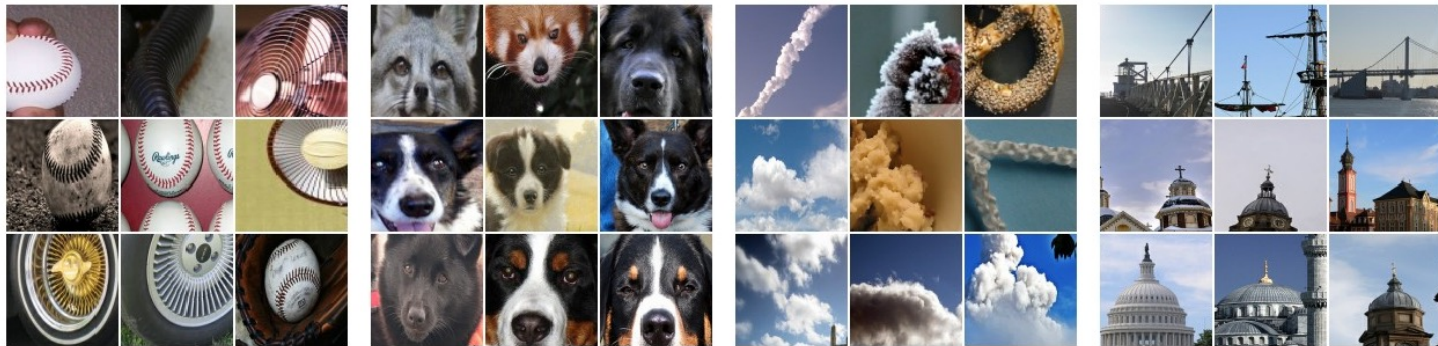


[Hubel & Wiesel 1959]



Simple cells in visual cortex detect edges, complex cells compose earlier responses

CNN higher layer filters



Dataset examples that maximize neuron outputs

[Olah '17]

CNN Building Blocks

Design choices:

- filter size
- number of filters
- padding
- stride

Layer types:

- pooling
- transpose convolutions
- upsampling layers

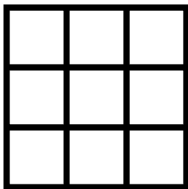
CNN Building Blocks

Filter size

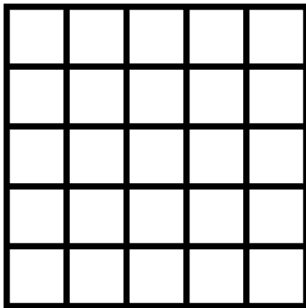
1x1



3x3

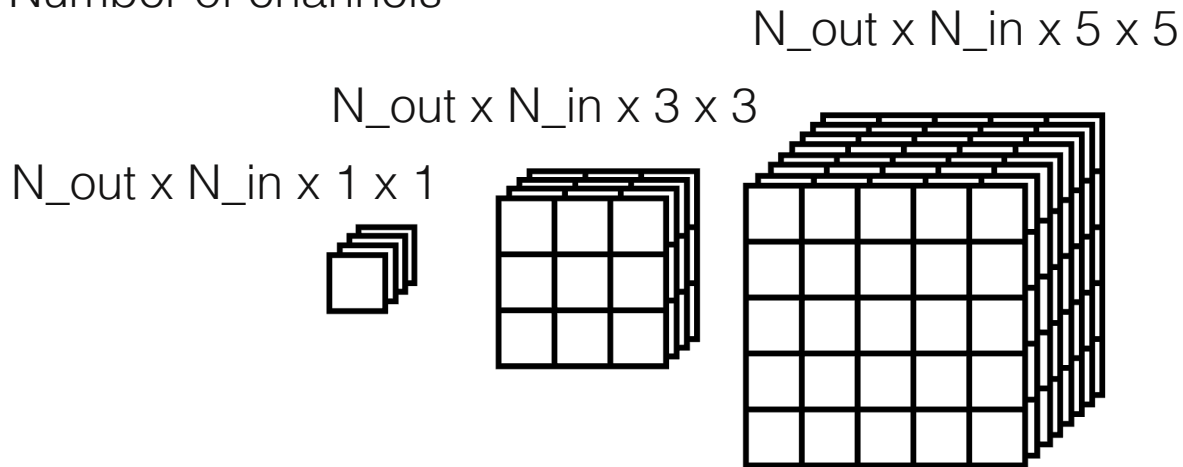


5x5



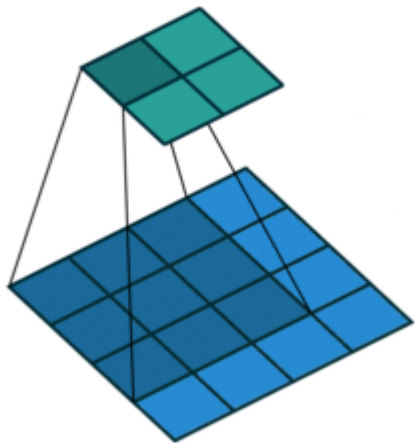
CNN Building Blocks

Number of channels

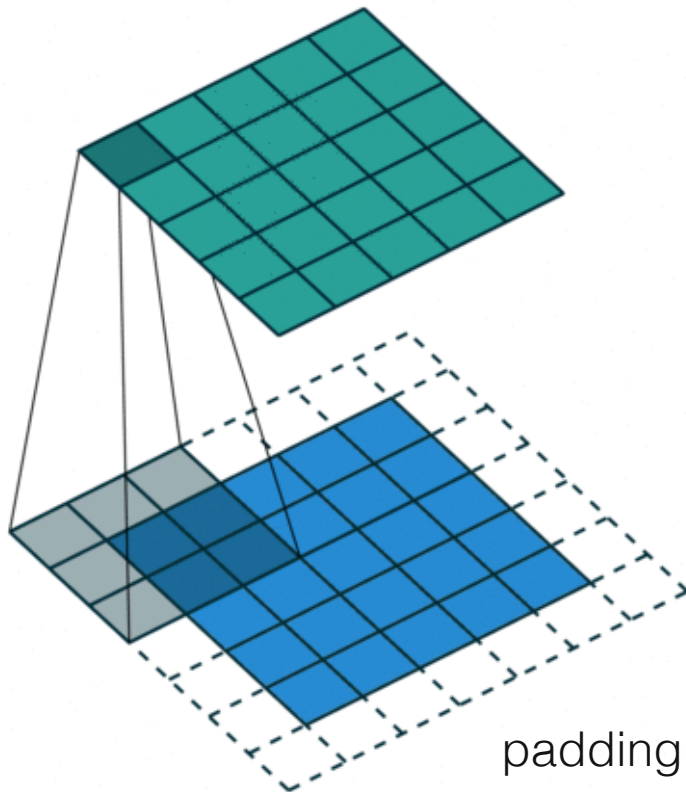


CNN Building Blocks

padding



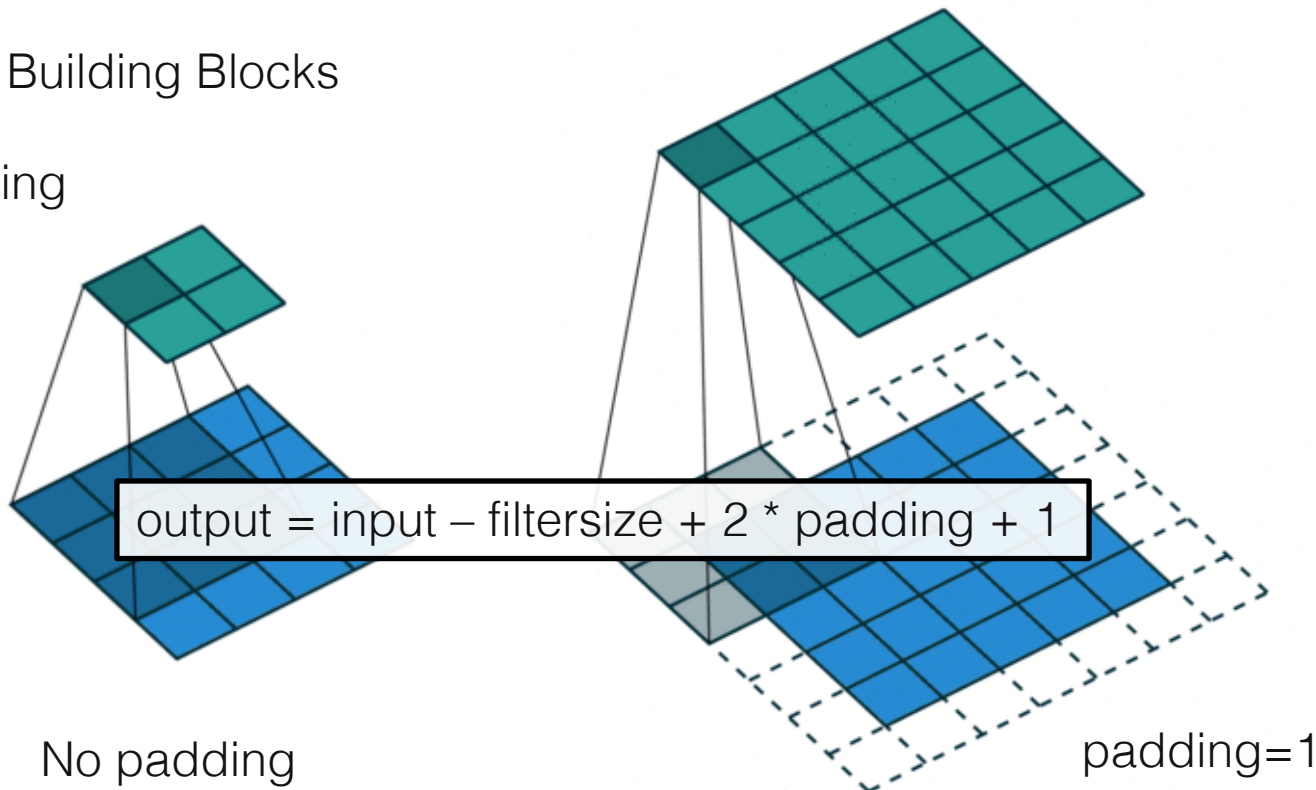
No padding



padding=1

CNN Building Blocks

padding

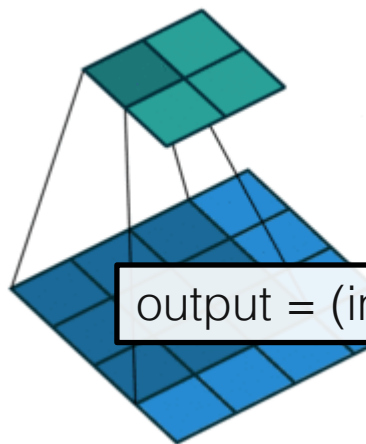


No padding

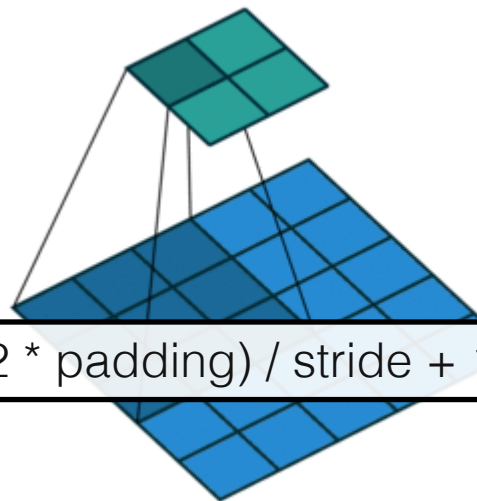
padding=1

CNN Building Blocks

stride



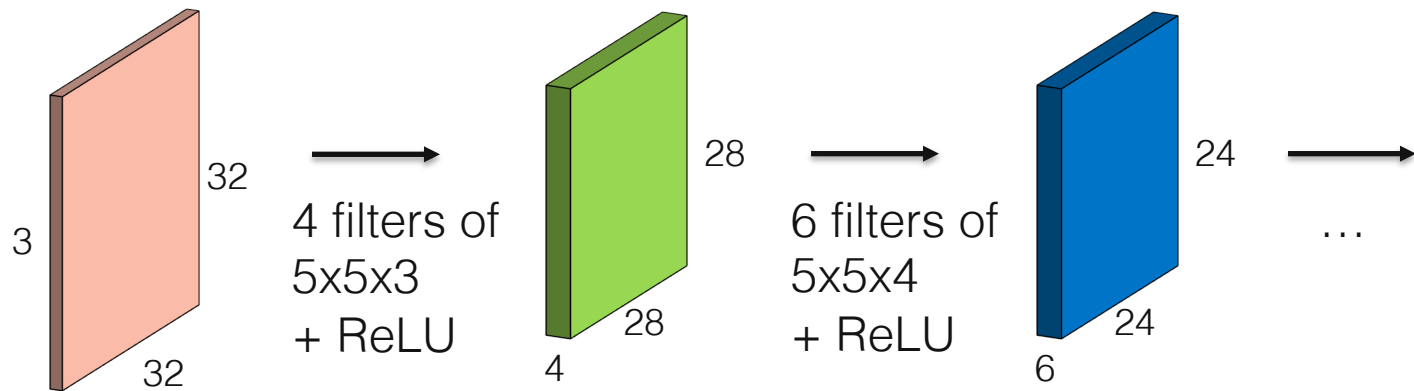
stride = 1



stride = 2

$$\text{output} = (\text{input} - \text{filtersize} + 2 * \text{padding}) / \text{stride} + 1$$

Convolutional Neural Network

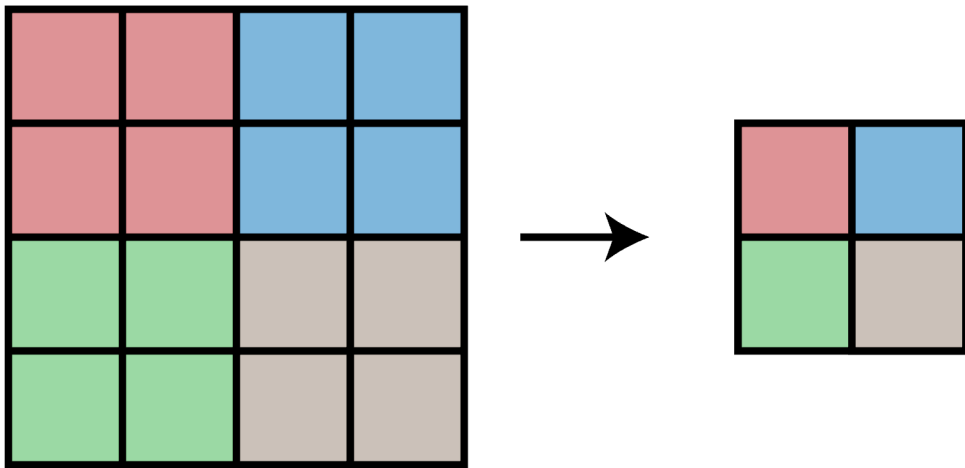


Input Image

Layer 1
Activations

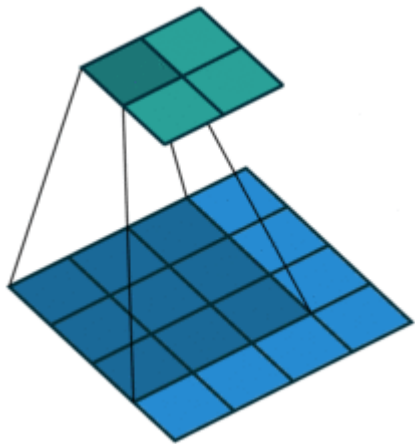
Layer 2
Activations

Layer types: Pooling

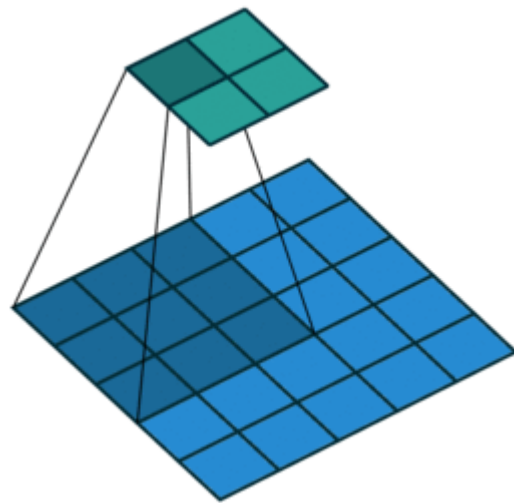


e.g., max pool size=2, stride=2

Transpose Convolution

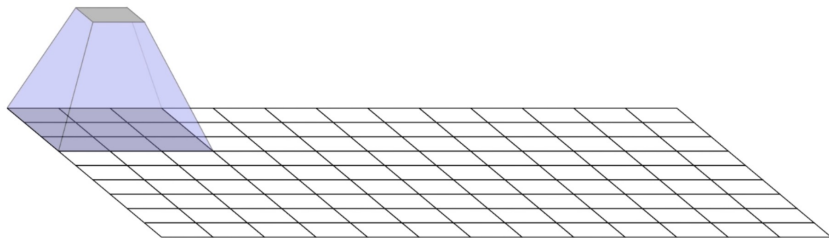


stride=1

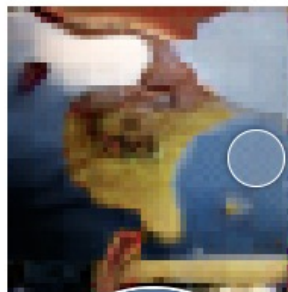


stride=2

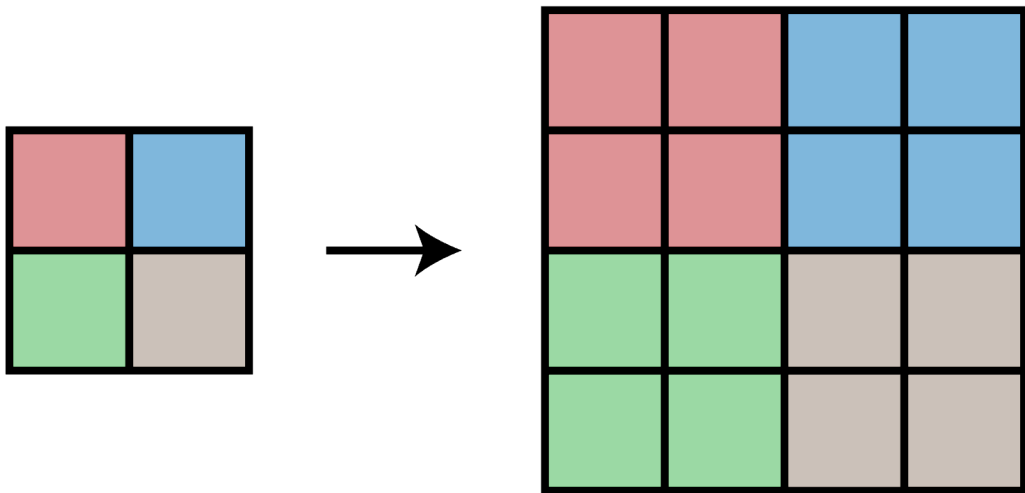
Transpose Convolution (checkerboard artifacts)



[Odena '16]



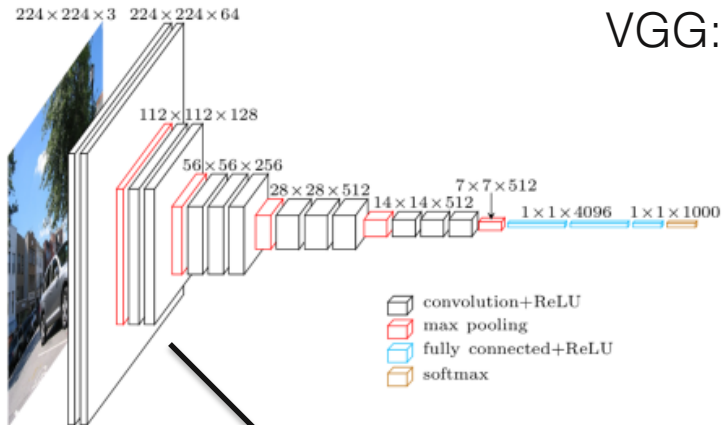
Upsampling layers



e.g., bilinear upsampling, nearest neighbor upsampling

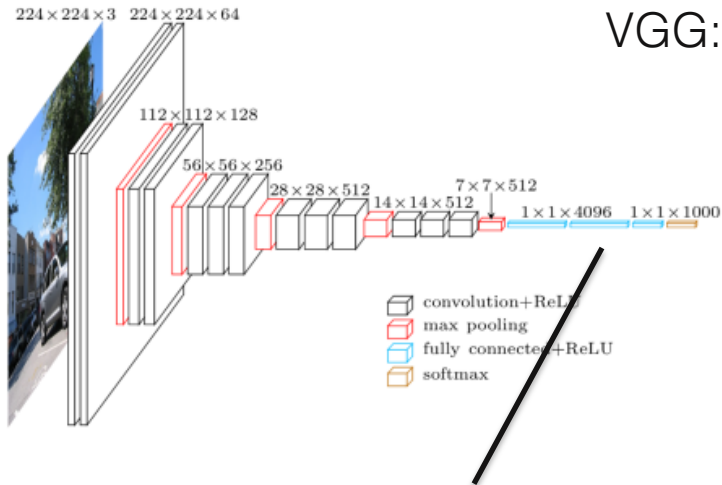
Common Network Architectures

VGG: one of the first “deep” CNNs



downsampling with max pooling

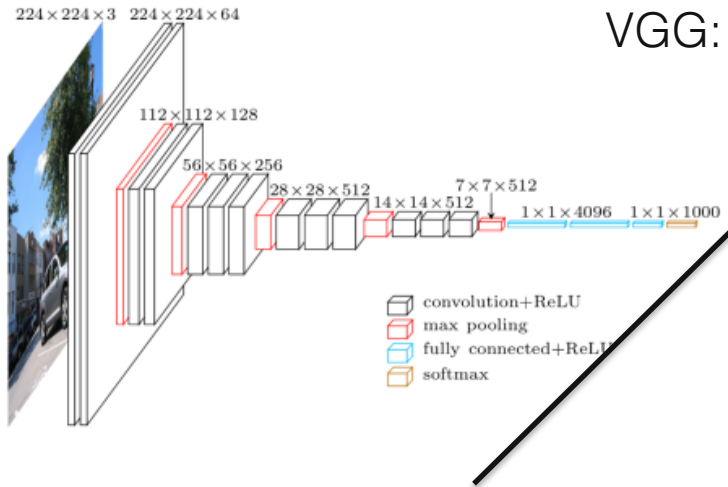
Common Network Architectures



VGG: one of the first “deep” CNNs

Classification scores output with fully-connected layers

Common Network Architectures



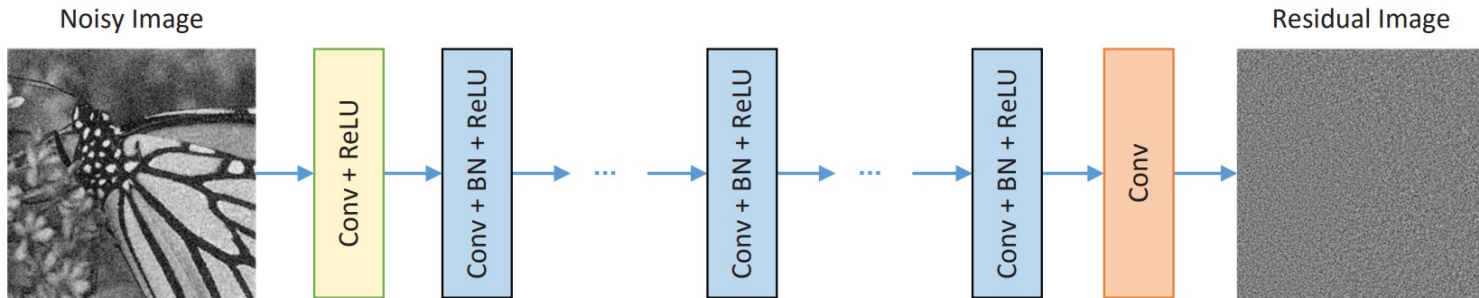
VGG: one of the first “deep” CNNs

Not suitable for image processing...

Today

- What is a neural network?
- Training/optimizing neural nets
- Why “neural”?
- Convolutional neural networks
- Applications & inverse problems

Image denoising with DnCNN



[Zhang '16]

Key ideas: residual learning & batch normalization

Residual Learning



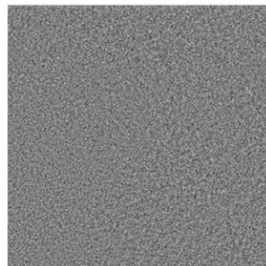
Clean image

=



noisy image

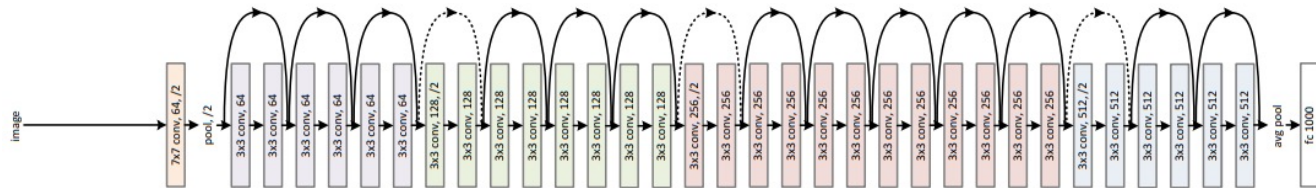
-



estimated noise

[Zhang '16]

Residual Learning



[He '15]

Popularized by residual nets “ResNets” for image classification

- Usually easier to optimize
- Better classification accuracy, good for many tasks!

Batch Normalization

Normalizes layer activations to zero mean, unit variance, preventing distribution shifts during training

- can speed up and stabilize training
- seems to smooth out loss landscape

BATCHNORM2D

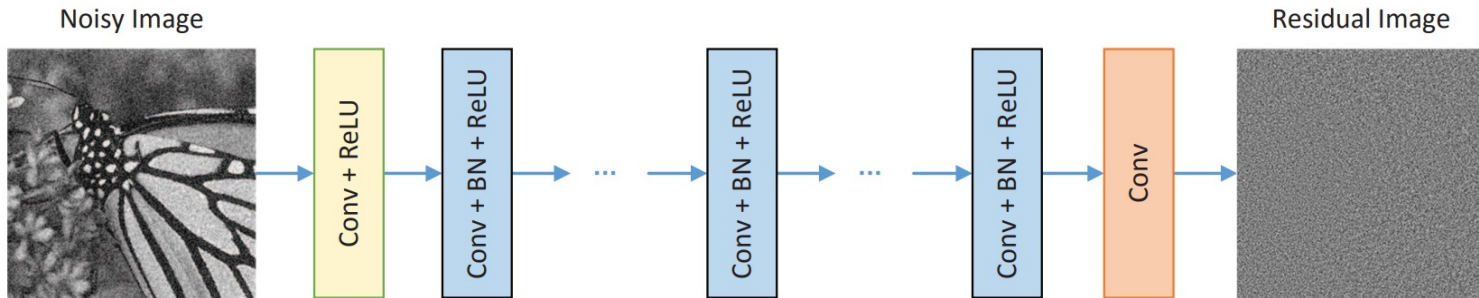
```
CLASS torch.nn.BatchNorm2d(num_features, eps=1e-05, momentum=0.1, affine=True,  
track_running_stats=True, device=None, dtype=None) \[SOURCE\]
```

Applies Batch Normalization over a 4D input (a mini-batch of 2D inputs with additional channel dimension) as described in the paper [Batch Normalization: Accelerating Deep Network Training by Reducing Internal Covariate Shift](#).

$$y = \frac{x - \mathbb{E}[x]}{\sqrt{\text{Var}[x] + \epsilon}} * \gamma + \beta$$

<https://pytorch.org/docs/stable/generated/torch.nn.BatchNorm2d.html>

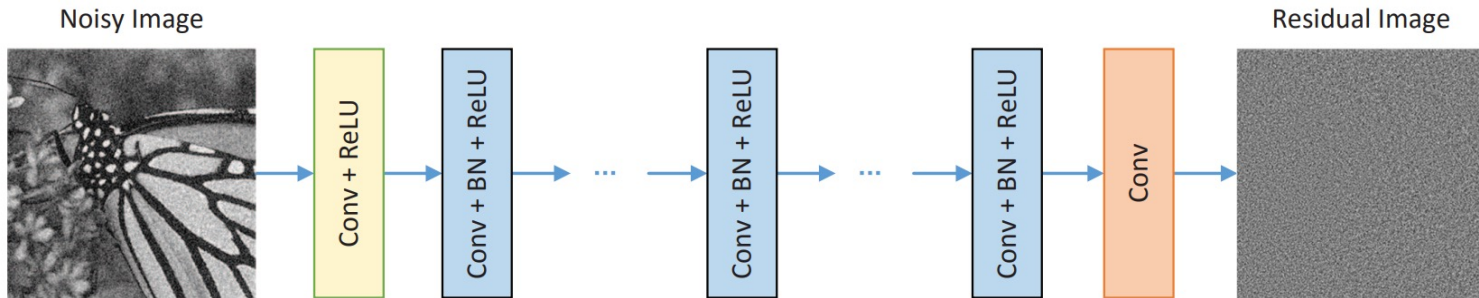
Image denoising with DnCNN



[Zhang '16]

(Remember to disable the bias in your conv layer)

Image denoising with DnCNN



[Zhang '16]

No fully connected layers – can be applied to any input size



(a) Ground-truth



(b) Noisy / 17.25dB

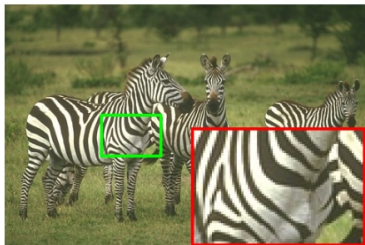


(c) CBM3D / 25.93dB

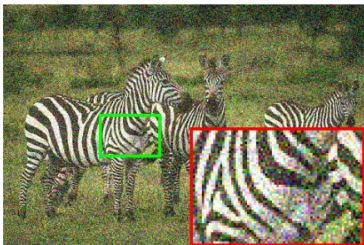


(d) CDnCNN-B / 26.58dB

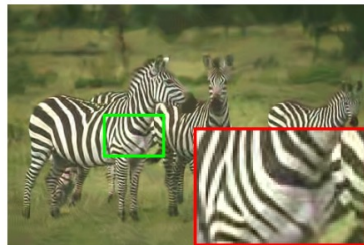
[Zhang '16]



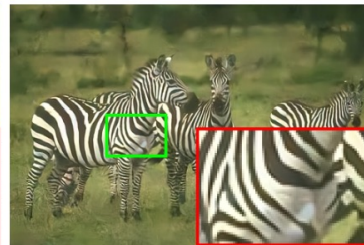
(a) Ground-truth



(b) Noisy / 15.07dB



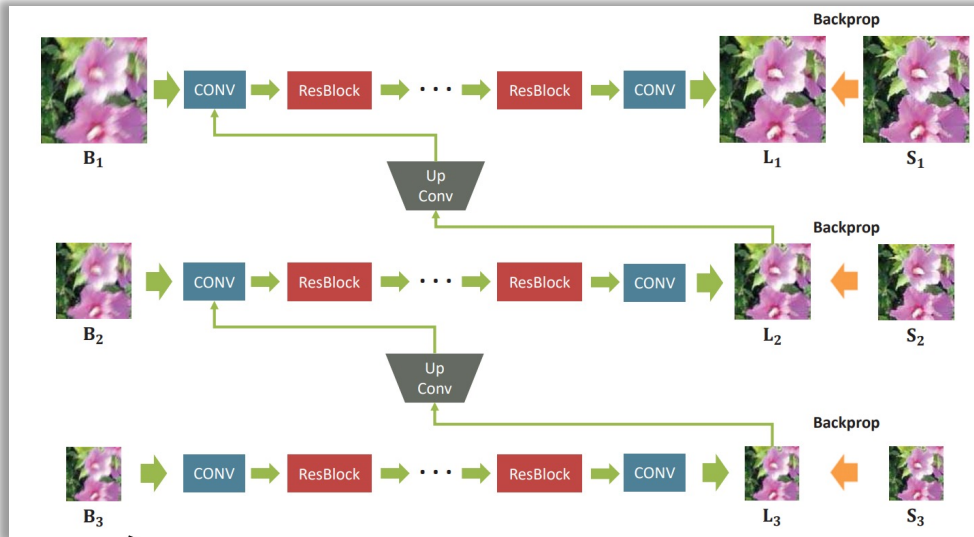
(c) CBM3D / 26.97dB



(d) CDnCNN-B / 27.87dB

[Zhang '16]

Multi-Scale Architectures



[Nah '18]

Uses image pyramid to process & deblur

Multi-Scale Architectures

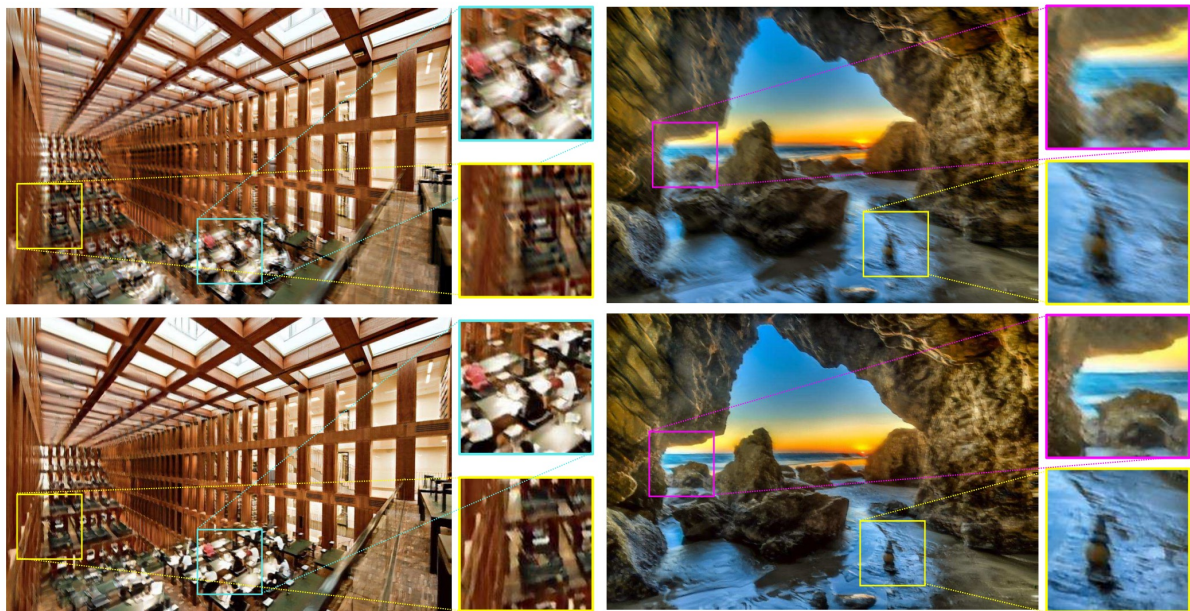
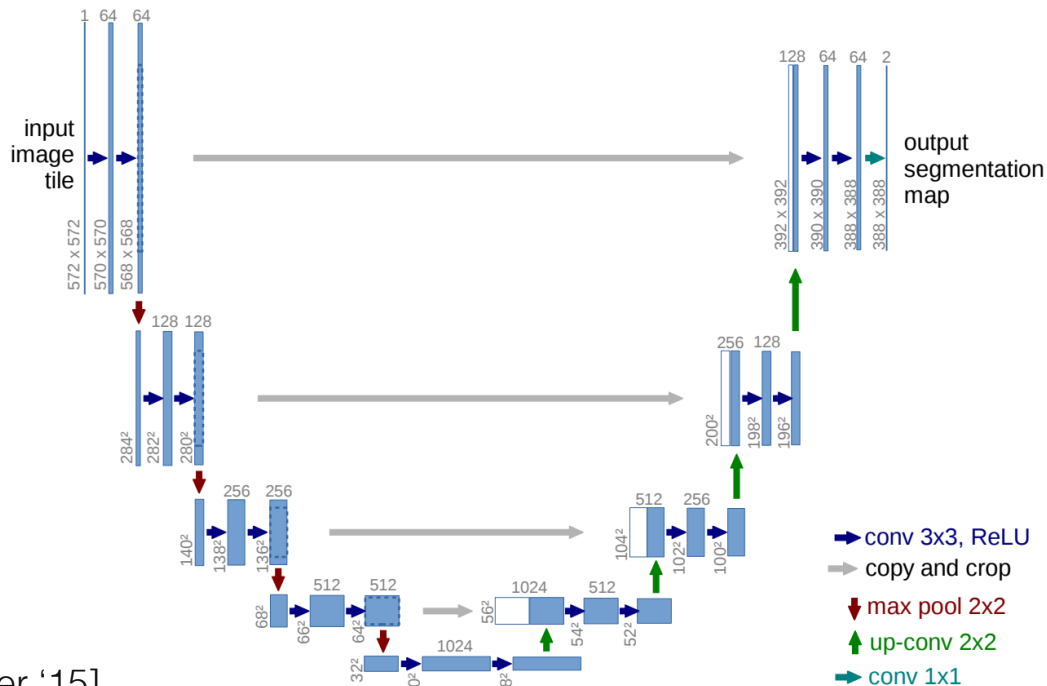


Figure 6. Deblurring results on the dataset [20]. The top row shows results of results of Sun et al. [26] and the bottom row shows our results.

U-Net: General-purpose architecture

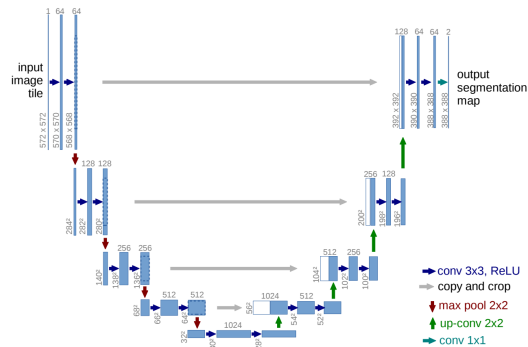


[Ronneberger '15]

U-Net: General-purpose architecture

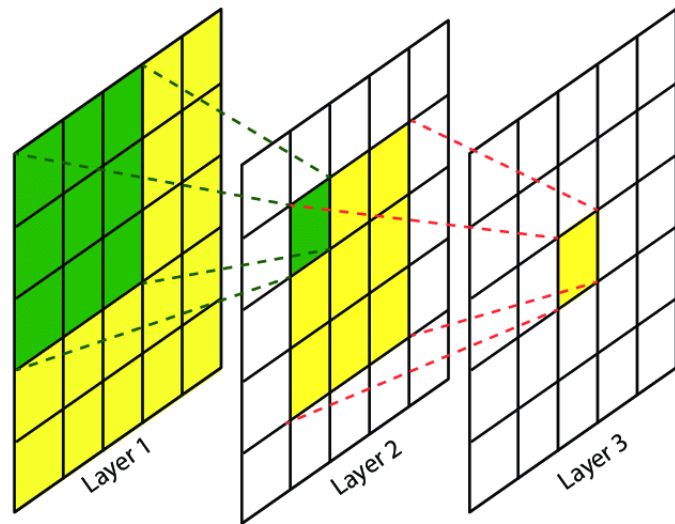
Introduced for biomedical image segmentation

- Uses residual connections
- Multi-scale processing (captures details at different scales)
- Large receptive field!



U-Net: General-purpose architecture

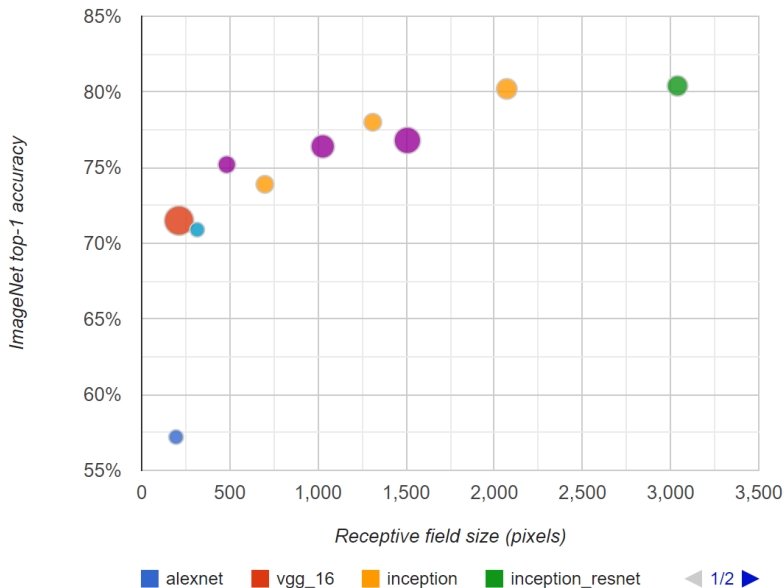
Receptive field: size of the input that contributes to the activation/output value



[Lin '17]

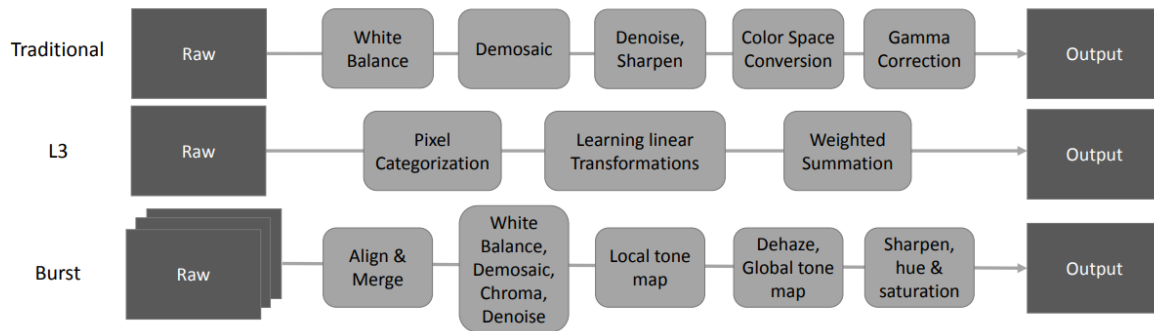
U-Net: General-purpose architecture

Large receptive field is important for high-level vision tasks and semantic understanding

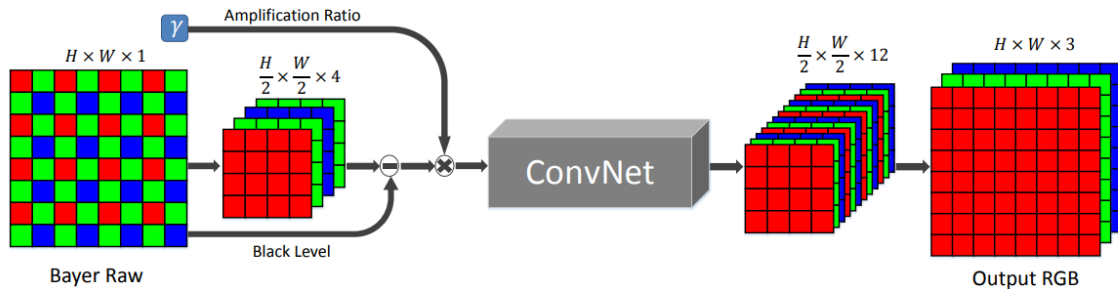


[Araujo '19]

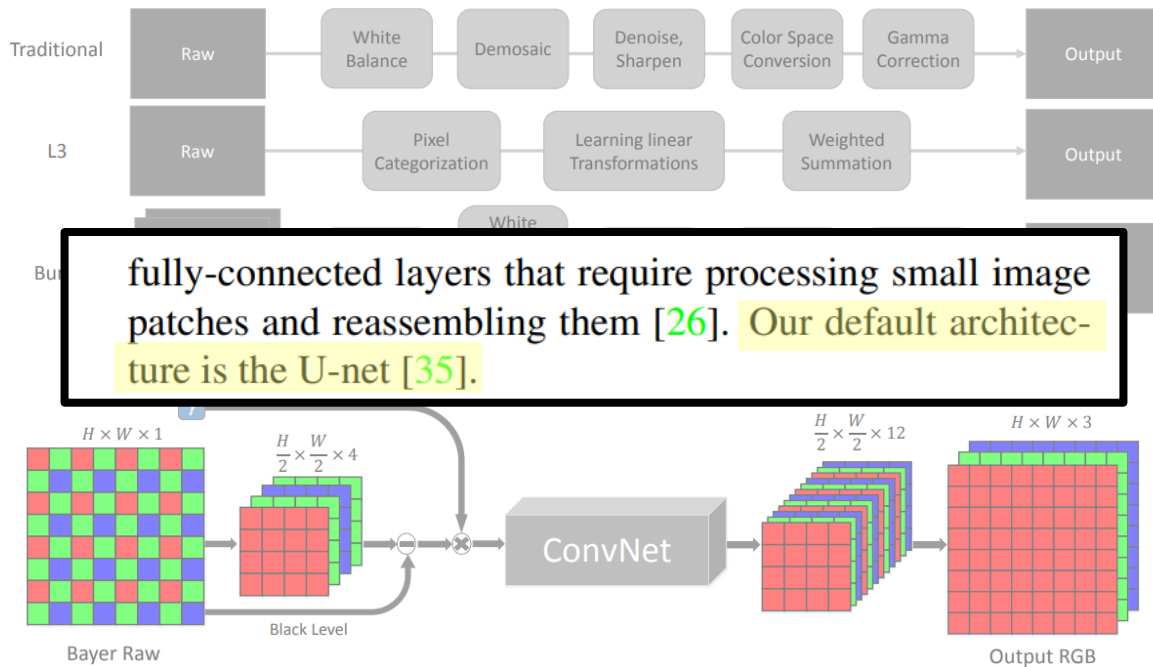
Learned ISP



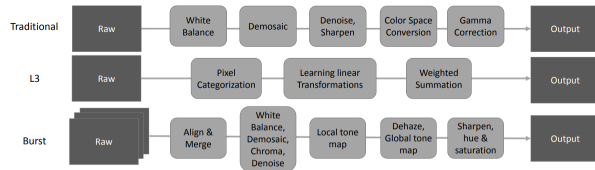
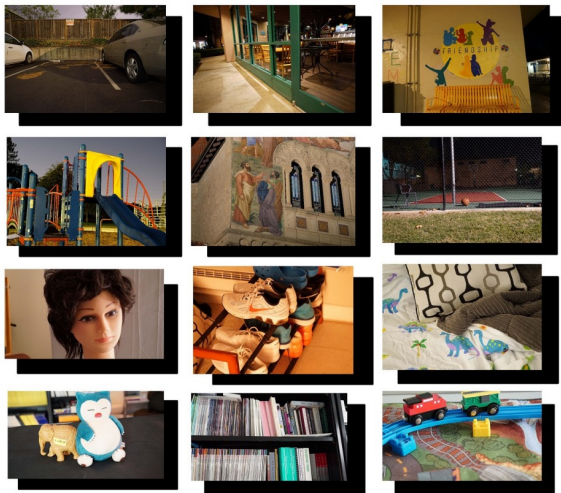
(a)



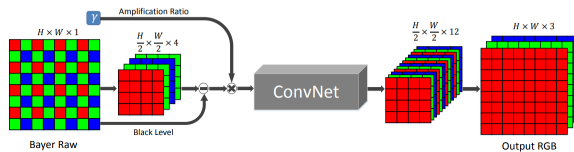
Learned ISP



Learned ISP



(a)



Trained on short-exposure (noisy) / long-exposure image pairs

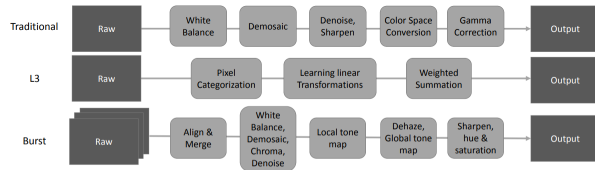
[Chen '18]

Learned ISP

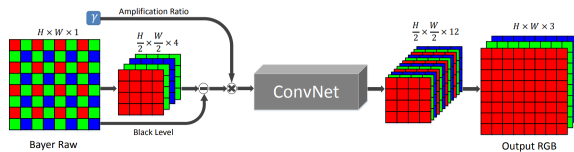


(a) Traditional pipeline

(b) Our result



(a)

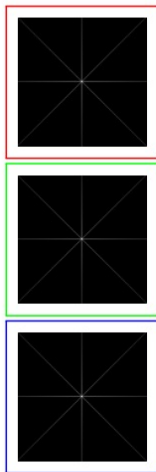
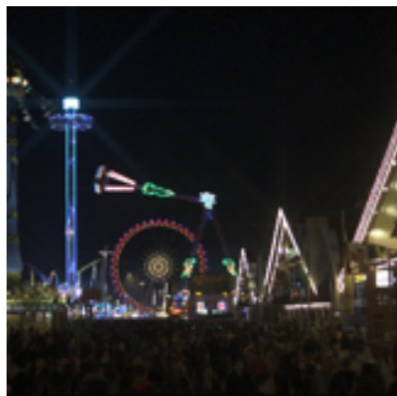


Deep optics for HDR imaging

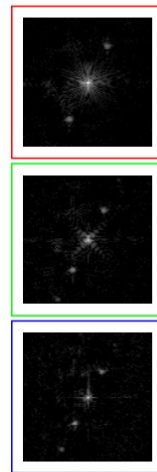
What kind of PSF would be good for HDR imaging?

- Should preserve fine details
- Should help to avoid saturation

Deep optics for HDR imaging

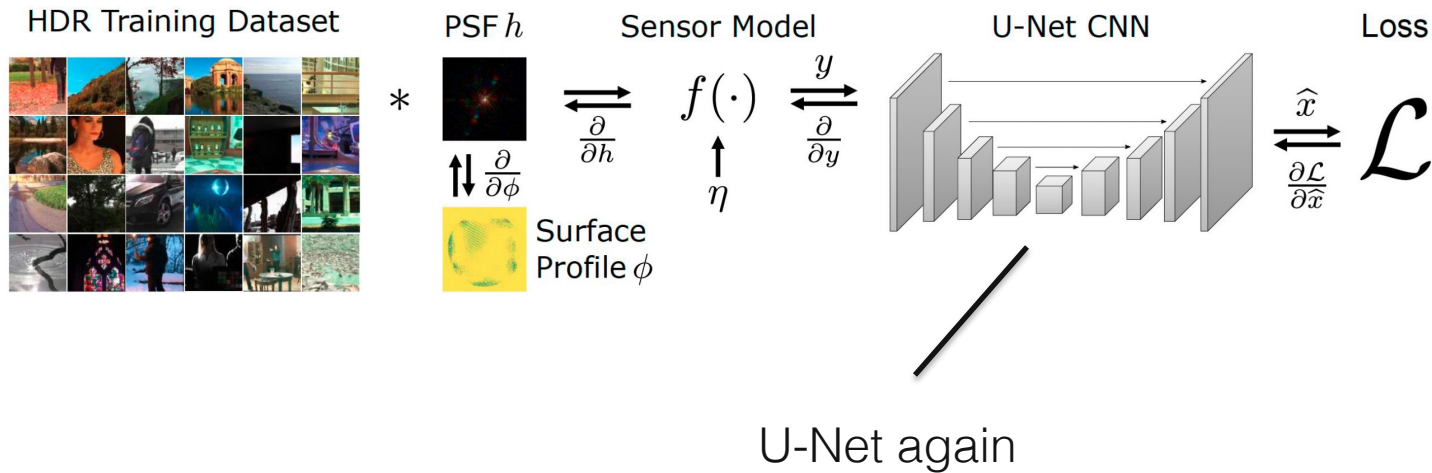


(a) Star PSF

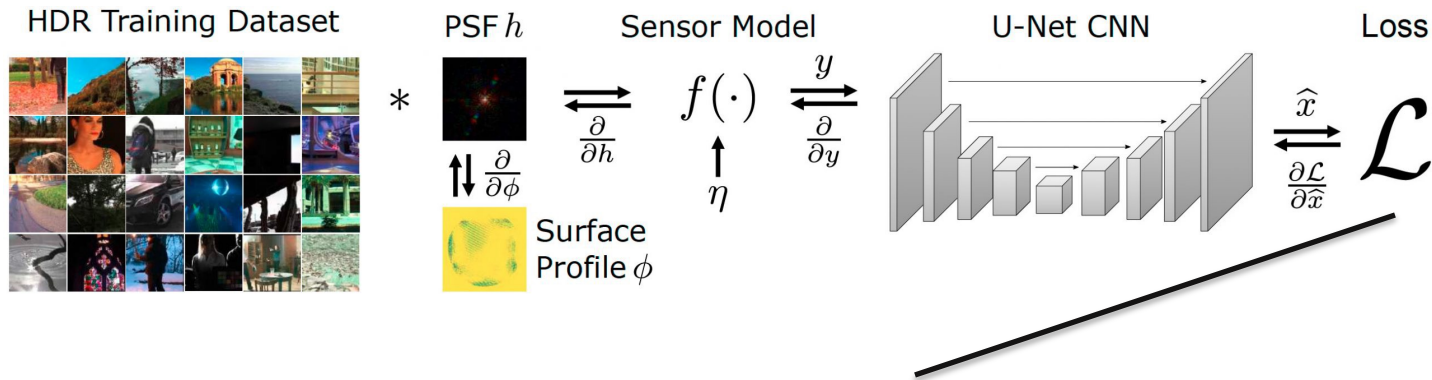


(b) E2E PSF

Deep optics for HDR imaging



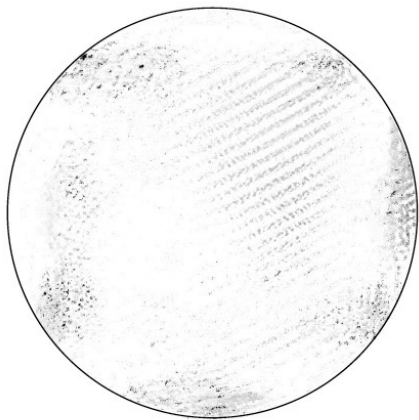
Deep optics for HDR imaging



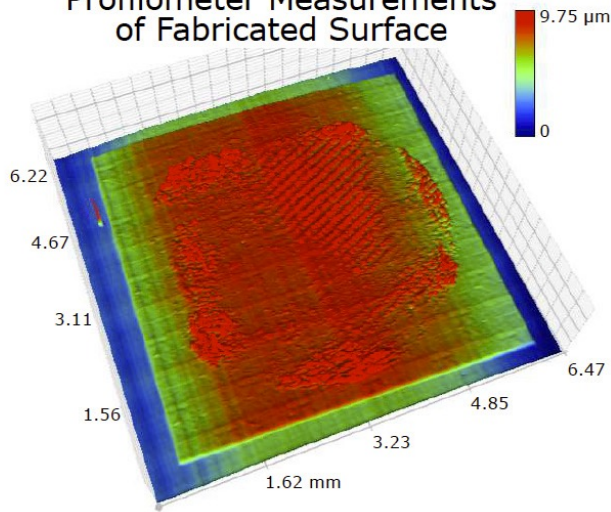
Minimize difference between
reconstruction and tone-mapped
GT images

Deep optics for HDR imaging

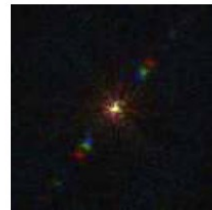
Optimized Height Profile



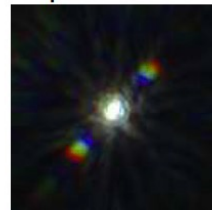
Profilometer Measurements of Fabricated Surface



Simulated PSF



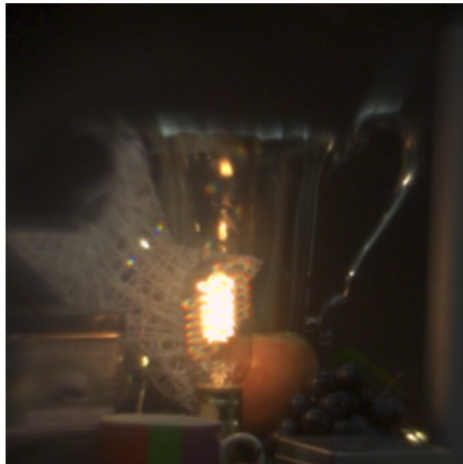
Captured PSF



LDR Image



E2E Measurement



E2E Reconstruction

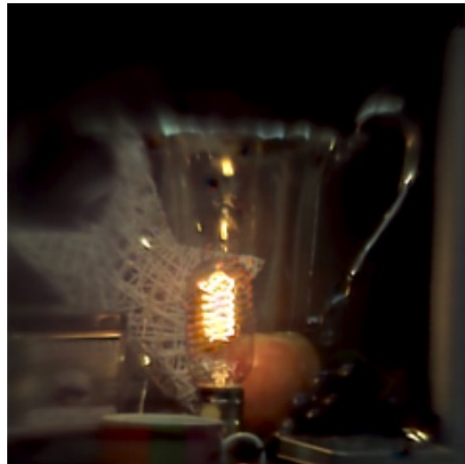
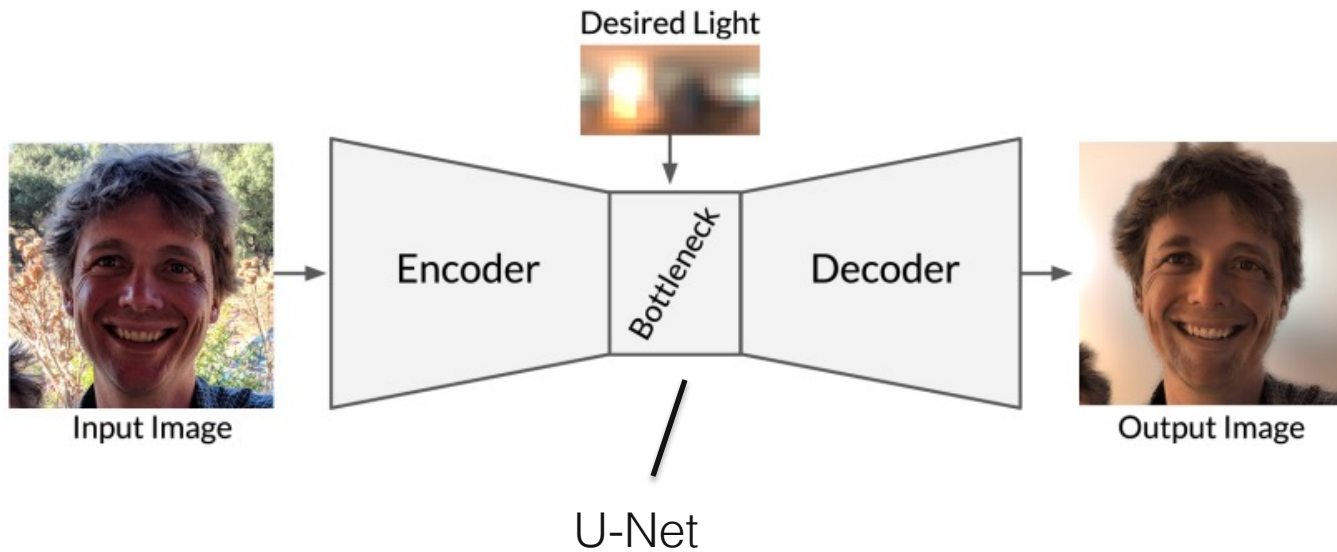


Image Relighting



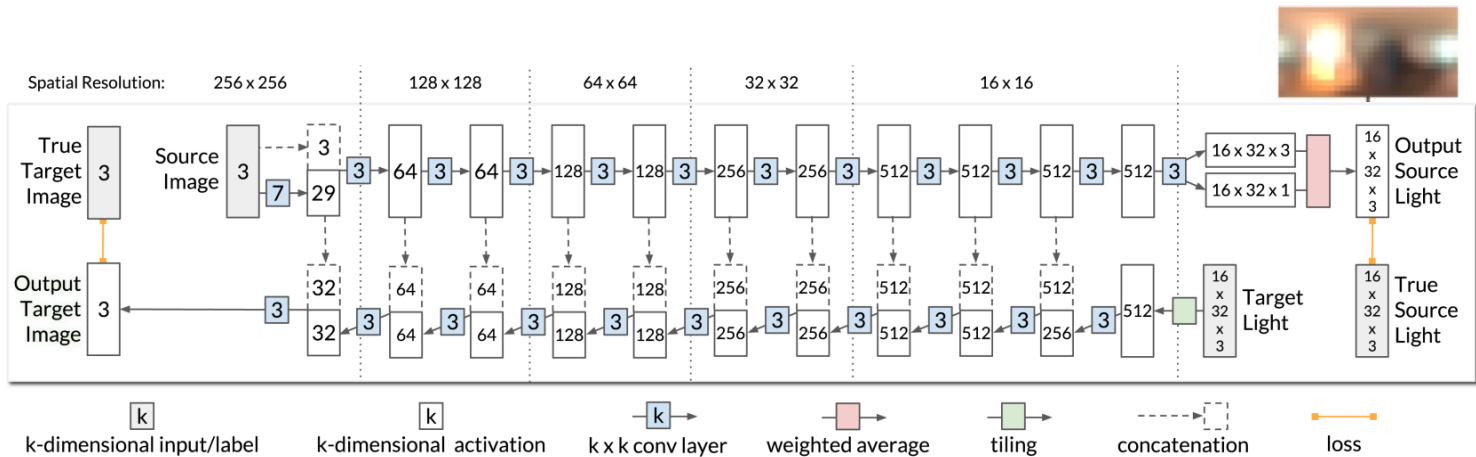
Spatial Resolution: 256 x 256, 128 x 128, 64 x 64, 32 x 32, 16 x 16

True Target Image, Source Image, Output Target Image, Output Source Light, True Source Light, Target Light

Legend:

- k : k -dimensional input/label
- k : k -dimensional activation
- $k \times k$: $k \times k$ conv layer
- \rightarrow : weighted average
- \rightarrow : tiling
- \rightarrow : concatenation
- \rightarrow : loss

Image Relighting



How would you train this network?

Image Relighting



Light-stage dataset capture (Google)

[Sun '19]

Image Relighting

OLAT photos
(columns)

$$\underset{\substack{\text{Re-rendered} \\ \text{image}}}{b} = \underset{\substack{\text{Environment} \\ \text{map}}}{A} \underset{\substack{\text{Environment} \\ \text{map}}}{x}$$

Re-rendered
image

Environment
map

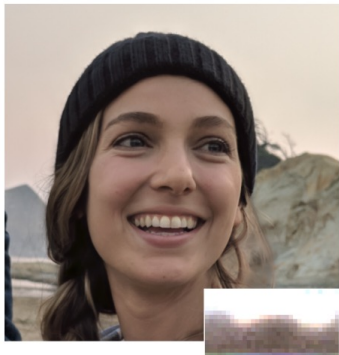


(a) OLAT images (7 cameras).



(b) Ground-truth renderings.

Image Relighting



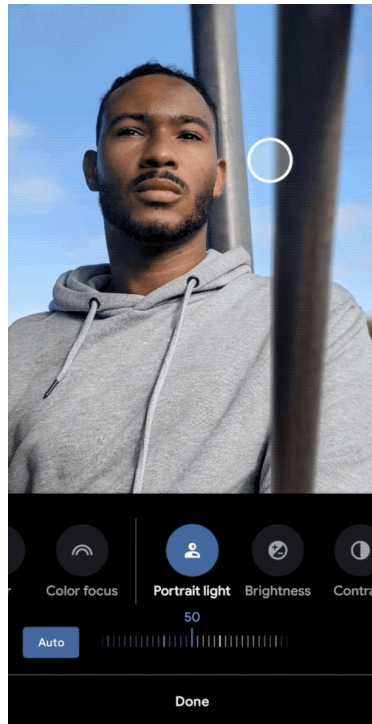
(a) Input image and estimated lighting



(b) Rendered images from our method under three novel illuminations

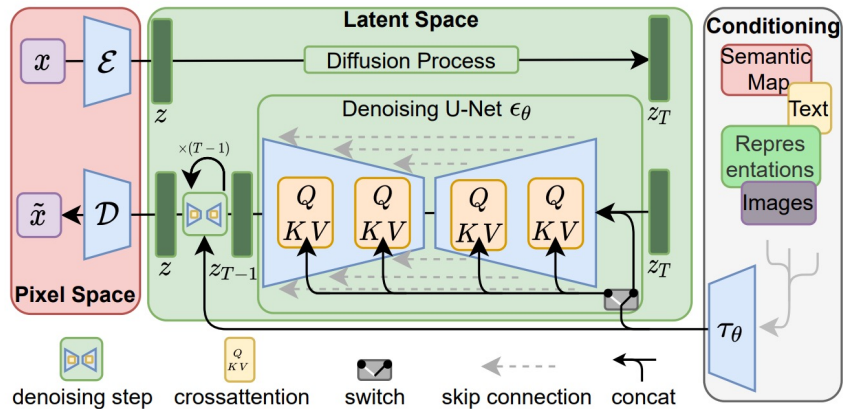
Image Relighting

Now a feature in pixel phones

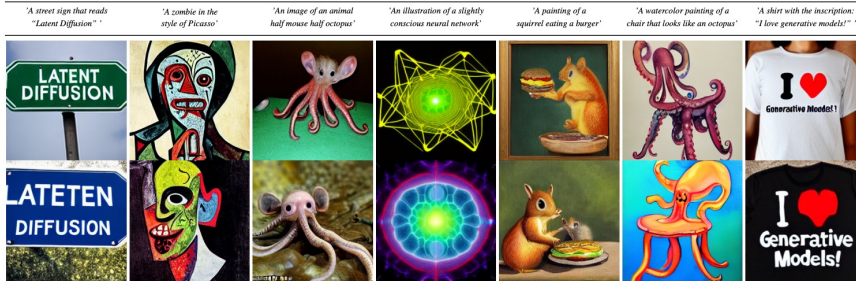


[Sun '19]

Image Generation



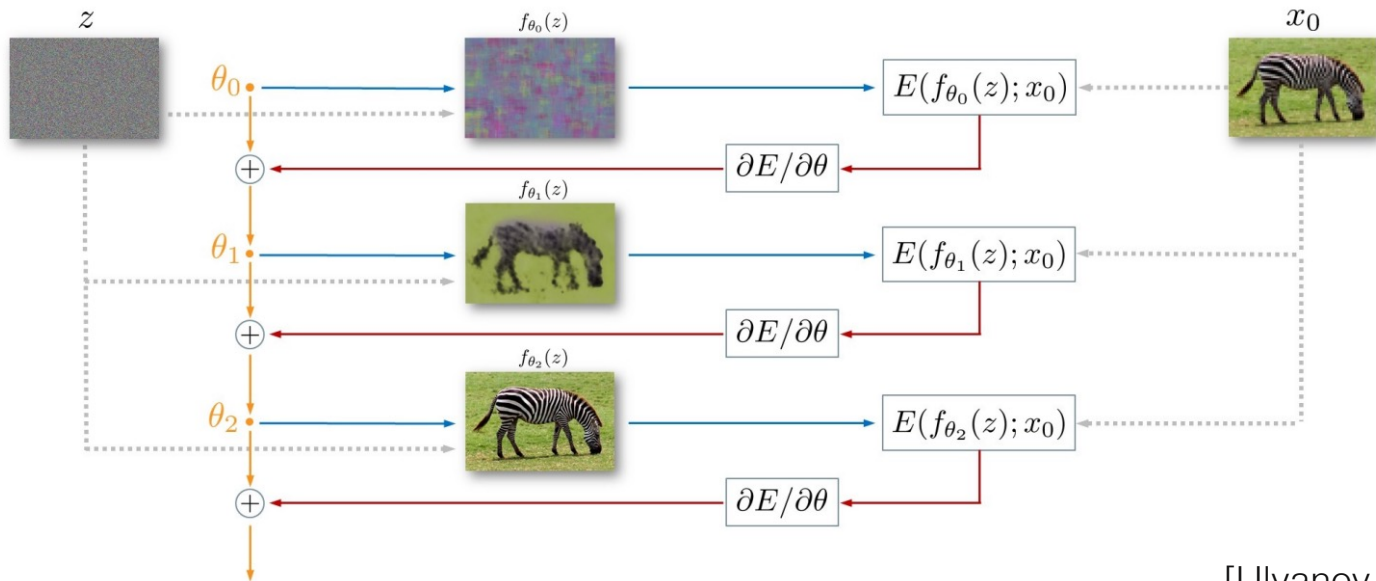
Text-to-Image Synthesis on LAION. 1.45B Model.



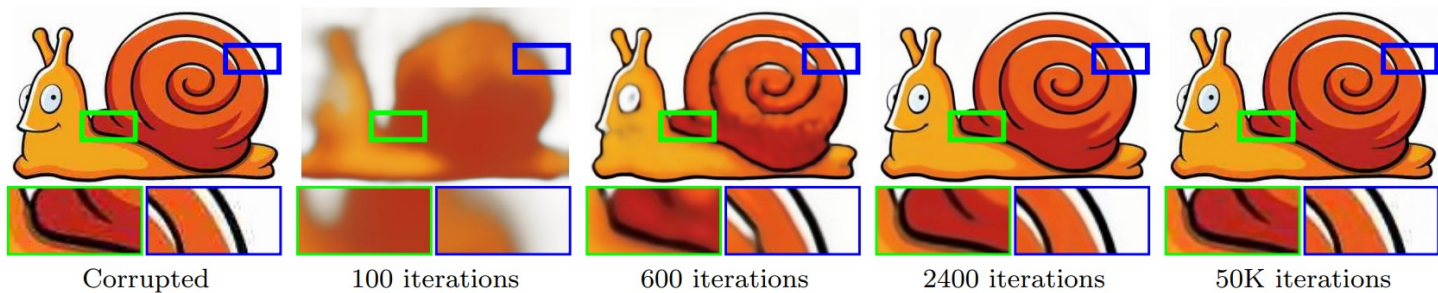
Do we always need training datasets?

Deep image prior

Idea: Overfit a U-Net to a noisy image, but stop training early

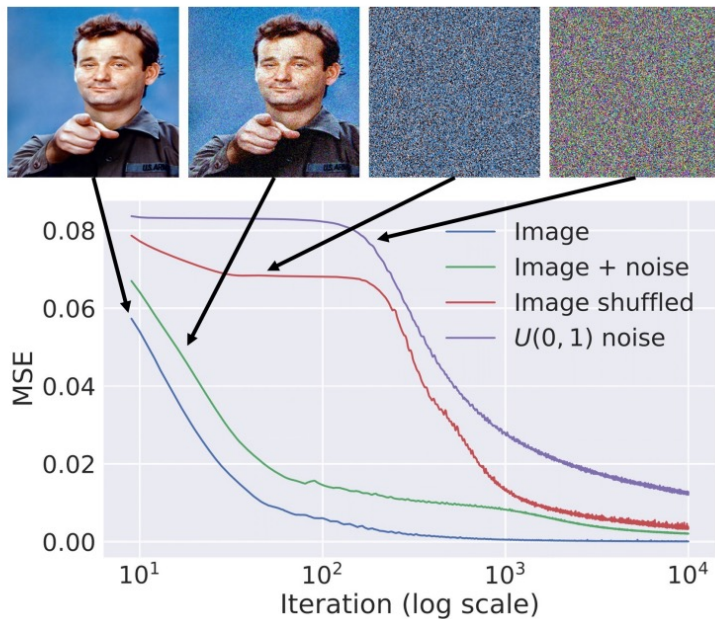


Deep image prior



The CNN itself is a good prior for natural images

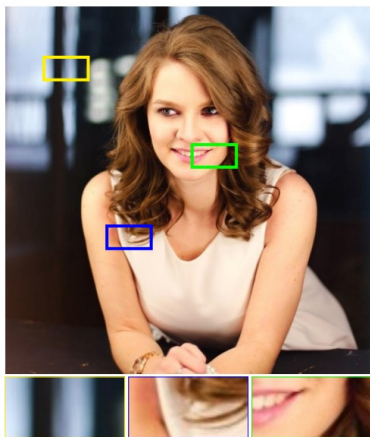
Deep image prior



During training, the network fits the image before noise

[Ulyanov '20]

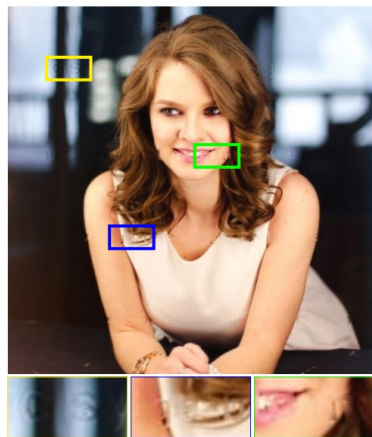
Deep image prior



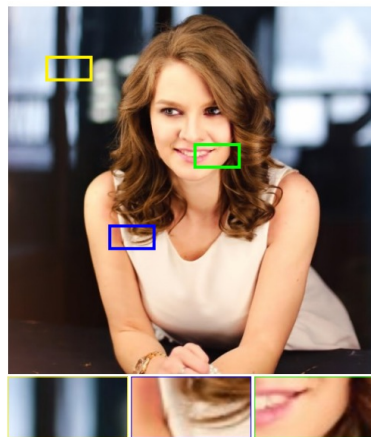
GT



Corrupted



Trained CNN



DIP

Summary

- “Neural” Networks & CNNs
- Building blocks of CNNs and deep networks
- Applications & inverse problems
- Just scratches the surface!
 - GANs, diffusion models, neural fields, neural rendering, text-to-image models, autoregressive models, transformers...

Next Time

- Optimization using alternating direction method of multipliers
- Hybrid techniques!

References and Further Reading

slides adapted from Stanford CS231N: <http://cs231n.stanford.edu/slides/>

CS229/CS231n notes on linear classifiers

<https://cs231n.github.io/linear-classify/>

<https://cs229.stanford.edu/notes2021fall/cs229-notes1.pdf>

CS231n Notes on backprop

<http://cs231n.stanford.edu/handouts/linear-backprop.pdf>

<https://cs231n.github.io/optimization-2/>

Intro to pytorch autograd

https://pytorch.org/tutorials/beginner/blitz/autograd_tutorial.html

Extending pytorch autograd functions

<https://pytorch.org/docs/stable/notes/extending.html>

References and Further Reading

slides adapted from Stanford CS231N: <http://cs231n.stanford.edu/slides/>

Araujo, André, Wade Norris, and Jack Sim. "Computing receptive fields of convolutional neural networks." *Distill* 4.11 (2019)

Chen, Chen, et al. "Learning to see in the dark." *Proc. CVPR*. 2018.

Eigen, David, Christian Puhrsch, and Rob Fergus. "Depth map prediction from a single image using a multi-scale deep network." *Proc. NeurIPS*. (2014).

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Hubel, David H., and Torsten N. Wiesel. "Receptive fields of single neurones in the cat's striate cortex." *The Journal of physiology* 148.3 (1959): 574-591.

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Odena, Augustus, Vincent Dumoulin, and Chris Olah. "Deconvolution and checkerboard artifacts." *Distill* 1.10 (2016)

Olah, Chris, Alexander Mordvintsev, and Ludwig Schubert. "Feature visualization." *Distill* 2.11 (2017)

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Russakovsky, Olga, et al. "Imagenet large scale visual recognition challenge." *IJCV* 115.3 (2015): 211-252.

Simonyan, Karen, and Andrew Zisserman. "Very deep convolutional networks for large-scale image recognition." *Proc. ICLR* (2014).

Sun, Tiancheng, et al. "Single image portrait relighting." *ACM Trans. Graph.* 38.4 (2019): 79-1.

Toshev, Alexander, and Christian Szegedy. "DeepPose: Human pose estimation via deep neural networks." *Proc. CVPR*. 2014.

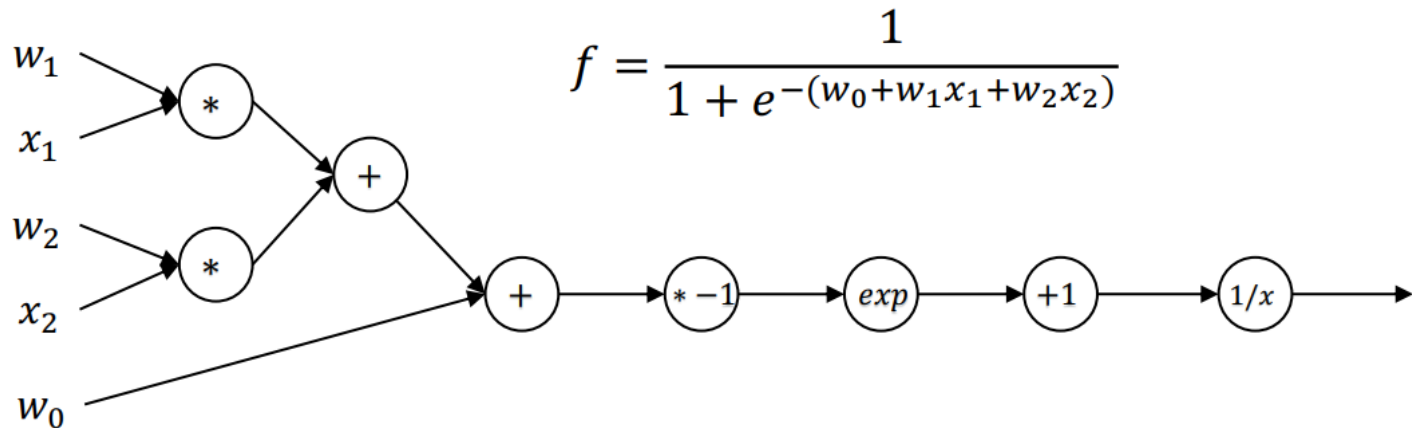
Ulyanov, Dmitry, Andrea Vedaldi, and Victor Lempitsky. "Deep image prior." *Proc. CVPR*. 2018.

Zhang, Kai, et al. "Beyond a gaussian denoiser: Residual learning of deep cnn for image denoising." *IEEE Trans. Imag. Proc.* 26.7 (2017): 3142-3155.

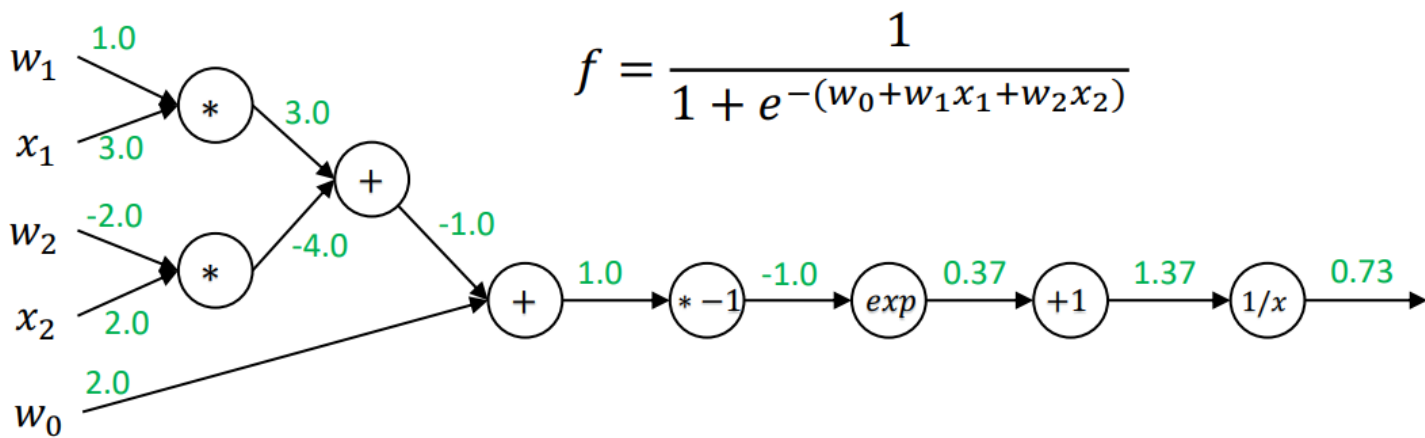
Extra backpropagation example (from Stanford CS231n)

$$f = \frac{1}{1 + e^{-(w_0 + w_1 x_1 + w_2 x_2)}}$$

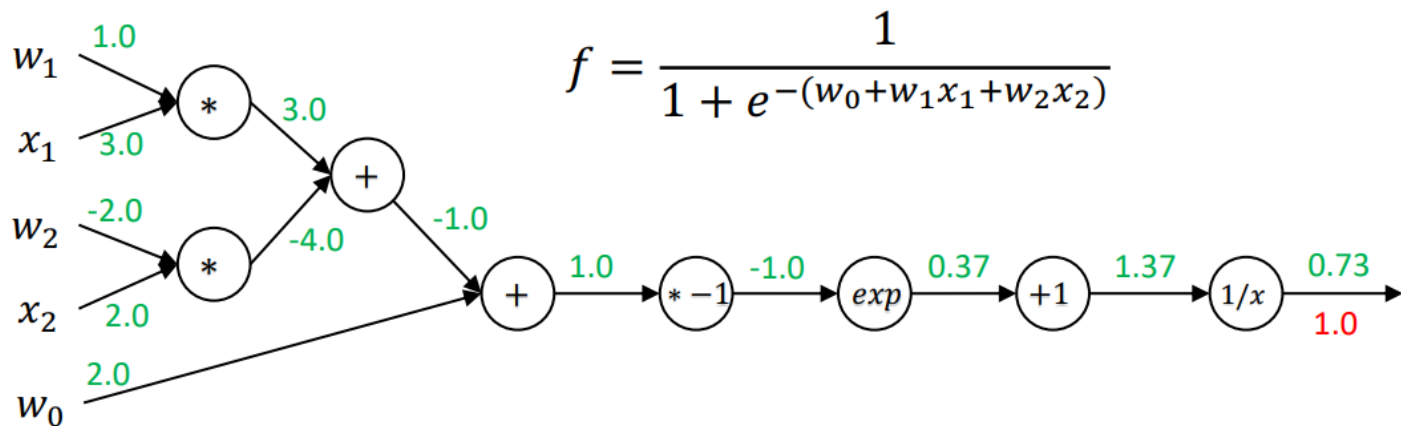
Extra backpropagation example



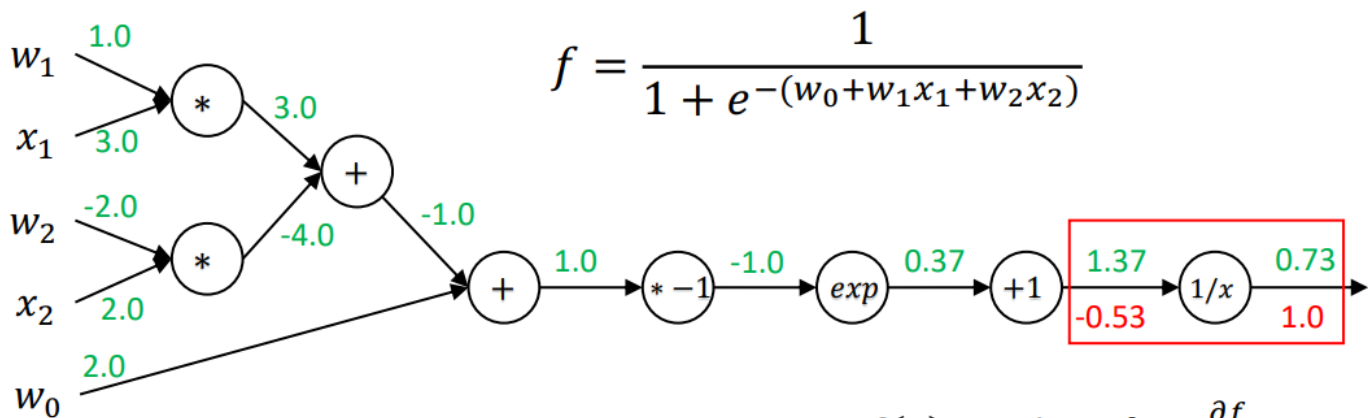
Extra backpropagation example



Extra backpropagation example



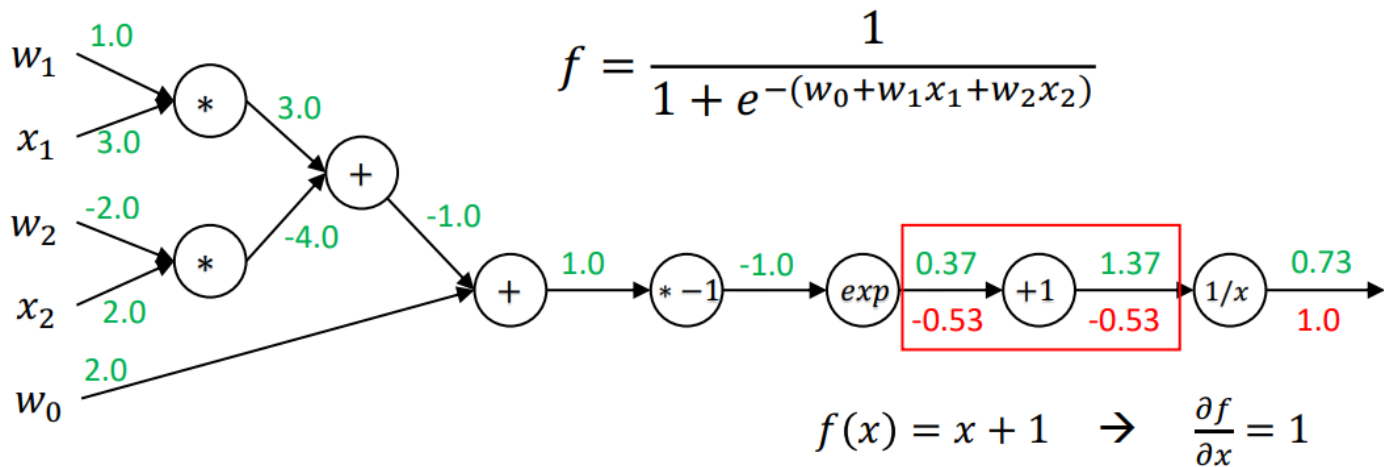
Extra backpropagation example



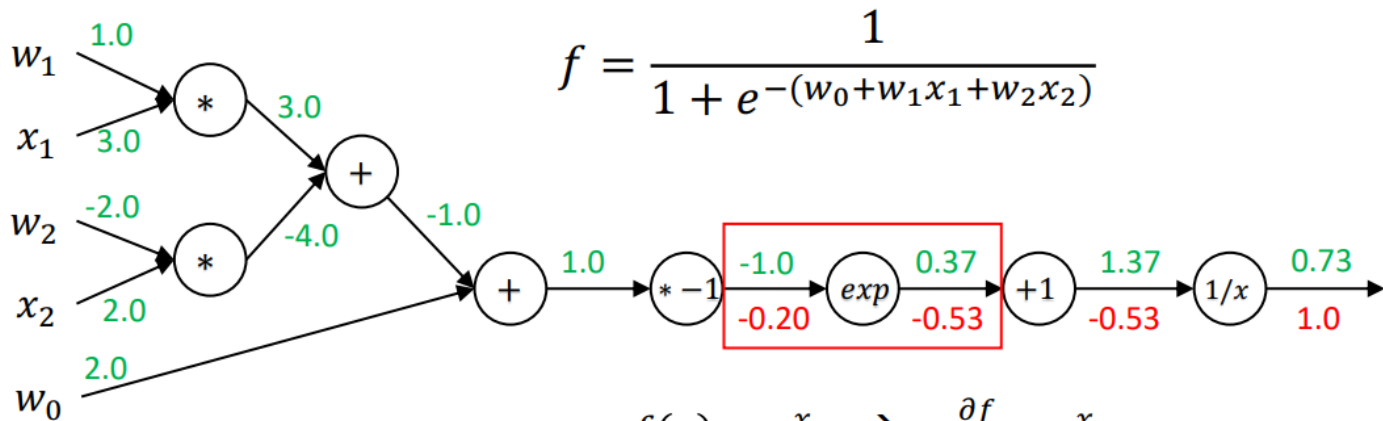
$$f(x) = 1/x \rightarrow \frac{\partial f}{\partial x} = -1/x^2$$

$$\frac{\partial J}{\partial x} = \frac{\partial J}{\partial f} \frac{\partial f}{\partial x} = -1/x^2$$

Extra backpropagation example



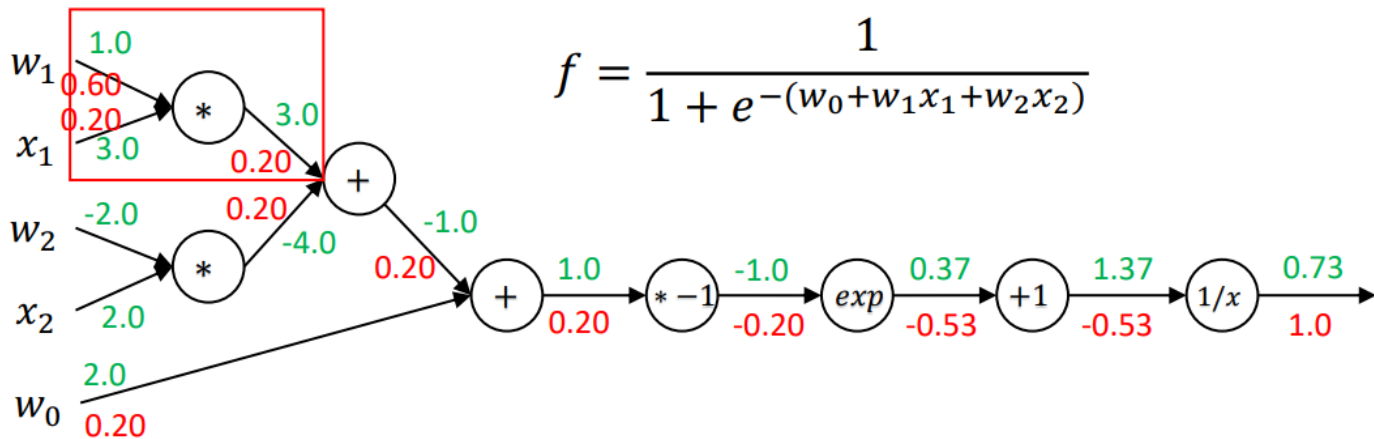
Extra backpropagation example



$$f(x) = e^x \rightarrow \frac{\partial f}{\partial x} = e^x$$

$$\frac{\partial J}{\partial x} = \frac{\partial J}{\partial f} \frac{\partial f}{\partial x} = \frac{\partial J}{\partial f} \cdot e^x$$

Extra backpropagation example



$$f(x, w) = xw \quad \rightarrow \quad \frac{\partial f}{\partial x} = w, \quad \frac{\partial f}{\partial w} = x$$

Extra backpropagation example

