Lec09

March 13, 2019

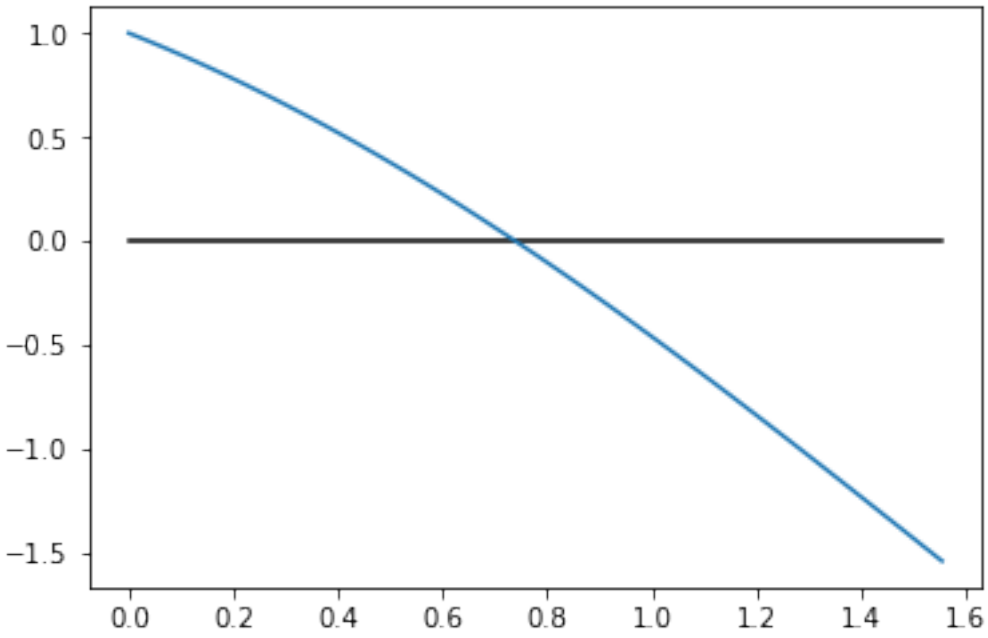
In [1]:
```
from chapter5 import bisect, fixed_point # you will write these functions in A4
```

In [2]:
```
import matplotlib.pyplot as plt
import numpy as np

def plot_function(f, x_min, x_max, resolution=100):
    x = np.arange(x_min, x_max, (x_max - x_min) / resolution)
    y = f(x)
    zero = np.zeros_like(y)
    plt.figure()
    plt.plot(x, zero, '-k') # horizontal axis line y=0
    plt.plot(x, y)
```

In [3]:
```
def f(x):
    return np.cos(x) - x
```

In [4]:
```
plot_function(f, 0, np.pi/2)
```



```
In [5]: bisect(f, 0, np.pi / 2, n=25)
```

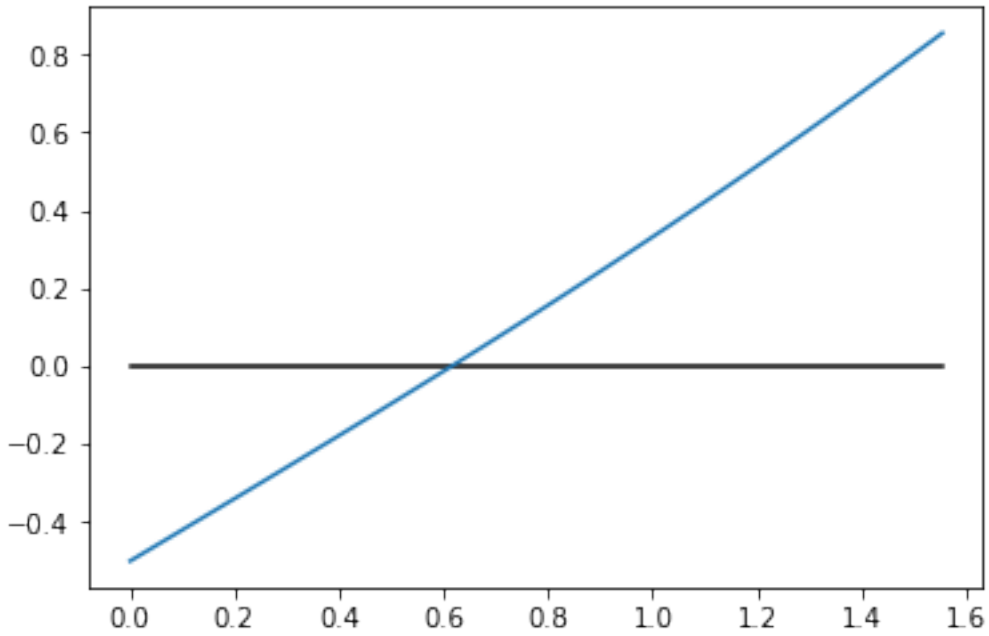
```
Iteration: 1         f(mid) = -0.078291         a = 0.000000         b = 0.785398
Iteration: 2         f(mid) = 0.531180         a = 0.392699         b = 0.785398
Iteration: 3         f(mid) = 0.242421         a = 0.589049         b = 0.785398
Iteration: 4         f(mid) = 0.085787         a = 0.687223         b = 0.785398
Iteration: 5         f(mid) = 0.004640         a = 0.736311         b = 0.785398
Iteration: 6         f(mid) = -0.036607         a = 0.736311         b = 0.760854
Iteration: 7         f(mid) = -0.015928         a = 0.736311         b = 0.748583
Iteration: 8         f(mid) = -0.005630         a = 0.736311         b = 0.742447
Iteration: 9         f(mid) = -0.000491         a = 0.736311         b = 0.739379
Iteration: 10         f(mid) = 0.002075         a = 0.737845         b = 0.739379
Iteration: 11         f(mid) = 0.000792         a = 0.738612         b = 0.739379
Iteration: 12         f(mid) = 0.000150         a = 0.738995         b = 0.739379
Iteration: 13         f(mid) = -0.000170         a = 0.738995         b = 0.739187
Iteration: 14         f(mid) = -0.000010         a = 0.738995         b = 0.739091
Iteration: 15         f(mid) = 0.000070         a = 0.739043         b = 0.739091
Iteration: 16         f(mid) = 0.000030         a = 0.739067         b = 0.739091
Iteration: 17         f(mid) = 0.000010         a = 0.739079         b = 0.739091
Iteration: 18         f(mid) = 0.000000         a = 0.739085         b = 0.739091
Iteration: 19         f(mid) = -0.000005         a = 0.739085         b = 0.739088
Iteration: 20         f(mid) = -0.000002         a = 0.739085         b = 0.739087
Iteration: 21         f(mid) = -0.000001         a = 0.739085         b = 0.739086
Iteration: 22         f(mid) = -0.000001         a = 0.739085         b = 0.739086
Iteration: 23         f(mid) = -0.000000         a = 0.739085         b = 0.739085
Iteration: 24         f(mid) = -0.000000         a = 0.739085         b = 0.739085
Iteration: 25         f(mid) = -0.000000         a = 0.739085         b = 0.739085
```

```
Out[5]: (0.7390851262506977, 0.7390851730640762)
```

```
In [9]: #   f(x) = x - 0.2 * sin(x) - 0.5

        def f(x):
            return x - 0.2 * np.sin(x) - 0.5
```

```
In [10]: plot_function(f, 0, np.pi/2)
```




```
In [11]: bisect(f, 0, np.pi/2, n=25)
```

```
Iteration: 1         f(mid) = 0.143977         a = 0.000000         b = 0.785398
Iteration: 2         f(mid) = -0.183838         a = 0.392699         b = 0.785398
Iteration: 3         f(mid) = -0.022065         a = 0.589049         b = 0.785398
Iteration: 4         f(mid) = 0.060345         a = 0.589049         b = 0.687223
Iteration: 5         f(mid) = 0.018996         a = 0.589049         b = 0.638136
Iteration: 6         f(mid) = -0.001569         a = 0.613592         b = 0.638136
Iteration: 7         f(mid) = 0.008705         a = 0.613592         b = 0.625864
Iteration: 8         f(mid) = 0.003565         a = 0.613592         b = 0.619728
Iteration: 9         f(mid) = 0.000998         a = 0.613592         b = 0.616660
Iteration: 10         f(mid) = -0.000286         a = 0.615126         b = 0.616660
Iteration: 11         f(mid) = 0.000356         a = 0.615126         b = 0.615893
Iteration: 12         f(mid) = 0.000035         a = 0.615126         b = 0.615510
Iteration: 13         f(mid) = -0.000126         a = 0.615318         b = 0.615510
Iteration: 14         f(mid) = -0.000045         a = 0.615414         b = 0.615510
Iteration: 15         f(mid) = -0.000005         a = 0.615462         b = 0.615510
Iteration: 16         f(mid) = 0.000015         a = 0.615462         b = 0.615486
Iteration: 17         f(mid) = 0.000005         a = 0.615462         b = 0.615474
Iteration: 18         f(mid) = -0.000000         a = 0.615468         b = 0.615474
Iteration: 19         f(mid) = 0.000002         a = 0.615468         b = 0.615471
Iteration: 20         f(mid) = 0.000001         a = 0.615468         b = 0.615469
Iteration: 21         f(mid) = 0.000000         a = 0.615468         b = 0.615469
Iteration: 22         f(mid) = 0.000000         a = 0.615468         b = 0.615468
Iteration: 23         f(mid) = -0.000000         a = 0.615468         b = 0.615468
Iteration: 24         f(mid) = -0.000000         a = 0.615468         b = 0.615468
```

```
Iteration: 25         f(mid) = 0.000000         a = 0.615468         b = 0.615468
```

```
Out[11]: (0.6154681272322162, 0.6154681740455947)
```

```
In [12]: def g(x):
             return 0.2 * np.sin(x) + 0.5
```

```
In [13]: fixed_point(g, 1.0)
```

```
Iteration: 1         x = 0.668294
Iteration: 2         x = 0.623930
Iteration: 3         x = 0.616846
Iteration: 4         x = 0.615693
Iteration: 5         x = 0.615505
Iteration: 6         x = 0.615474
Iteration: 7         x = 0.615469
Iteration: 8         x = 0.615468
Iteration: 9         x = 0.615468
Iteration: 10         x = 0.615468
Iteration: 11         x = 0.615468
Iteration: 12         x = 0.615468
Iteration: 13         x = 0.615468
Iteration: 14         x = 0.615468
Iteration: 15         x = 0.615468
Iteration: 16         x = 0.615468
Iteration: 17         x = 0.615468
Iteration: 18         x = 0.615468
Iteration: 19         x = 0.615468
Iteration: 20         x = 0.615468
```

```
Out[13]: 0.6154681694899654
```

```
In [14]: fixed_point(g, -1.0)
```

```
Iteration: 1         x = 0.331706
Iteration: 2         x = 0.565131
Iteration: 3         x = 0.607105
Iteration: 4         x = 0.614098
Iteration: 5         x = 0.615244
Iteration: 6         x = 0.615432
Iteration: 7         x = 0.615462
Iteration: 8         x = 0.615467
Iteration: 9         x = 0.615468
Iteration: 10         x = 0.615468
Iteration: 11         x = 0.615468
Iteration: 12         x = 0.615468
Iteration: 13         x = 0.615468
```

```
Iteration: 14         x = 0.615468
Iteration: 15         x = 0.615468
Iteration: 16         x = 0.615468
Iteration: 17         x = 0.615468
Iteration: 18         x = 0.615468
Iteration: 19         x = 0.615468
Iteration: 20         x = 0.615468
```

```
Out[14]: 0.6154681694899651
```

```
In [15]: fixed_point(g, 1000)
```

```
Iteration: 1         x = 0.665376
Iteration: 2         x = 0.623471
Iteration: 3         x = 0.616771
Iteration: 4         x = 0.615681
Iteration: 5         x = 0.615503
Iteration: 6         x = 0.615474
Iteration: 7         x = 0.615469
Iteration: 8         x = 0.615468
Iteration: 9         x = 0.615468
Iteration: 10         x = 0.615468
Iteration: 11         x = 0.615468
Iteration: 12         x = 0.615468
Iteration: 13         x = 0.615468
Iteration: 14         x = 0.615468
Iteration: 15         x = 0.615468
Iteration: 16         x = 0.615468
Iteration: 17         x = 0.615468
Iteration: 18         x = 0.615468
Iteration: 19         x = 0.615468
Iteration: 20         x = 0.615468
```

```
Out[15]: 0.6154681694899654
```

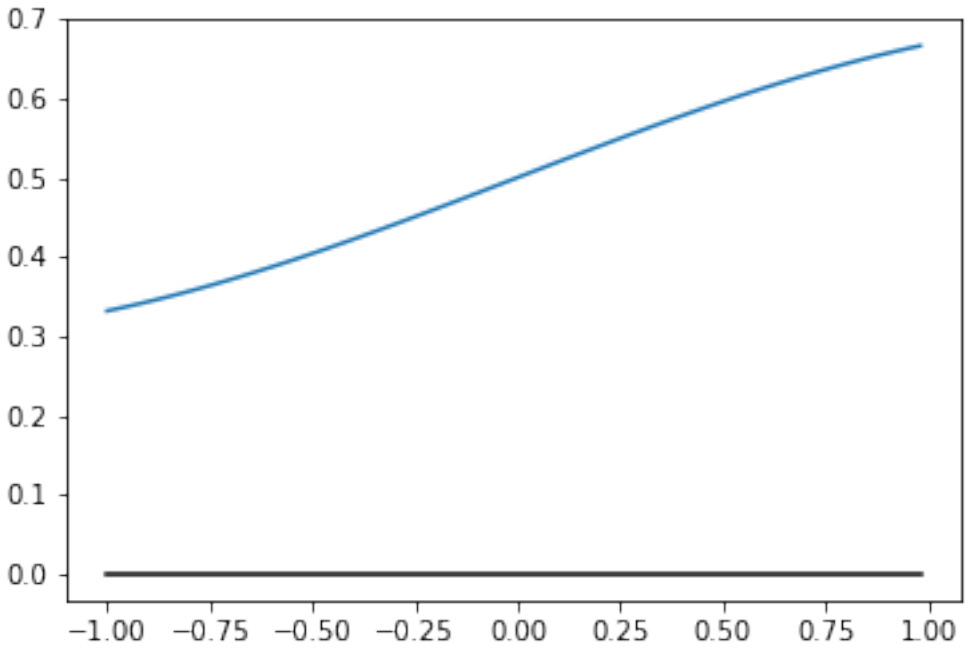
```
In [16]: fixed_point(g, -1000)
```

```
Iteration: 1         x = 0.334624
Iteration: 2         x = 0.565683
Iteration: 3         x = 0.607198
Iteration: 4         x = 0.614114
Iteration: 5         x = 0.615247
Iteration: 6         x = 0.615432
Iteration: 7         x = 0.615462
Iteration: 8         x = 0.615467
Iteration: 9         x = 0.615468
Iteration: 10         x = 0.615468
```

```
Iteration: 11         x = 0.615468
Iteration: 12         x = 0.615468
Iteration: 13         x = 0.615468
Iteration: 14         x = 0.615468
Iteration: 15         x = 0.615468
Iteration: 16         x = 0.615468
Iteration: 17         x = 0.615468
Iteration: 18         x = 0.615468
Iteration: 19         x = 0.615468
Iteration: 20         x = 0.615468
```

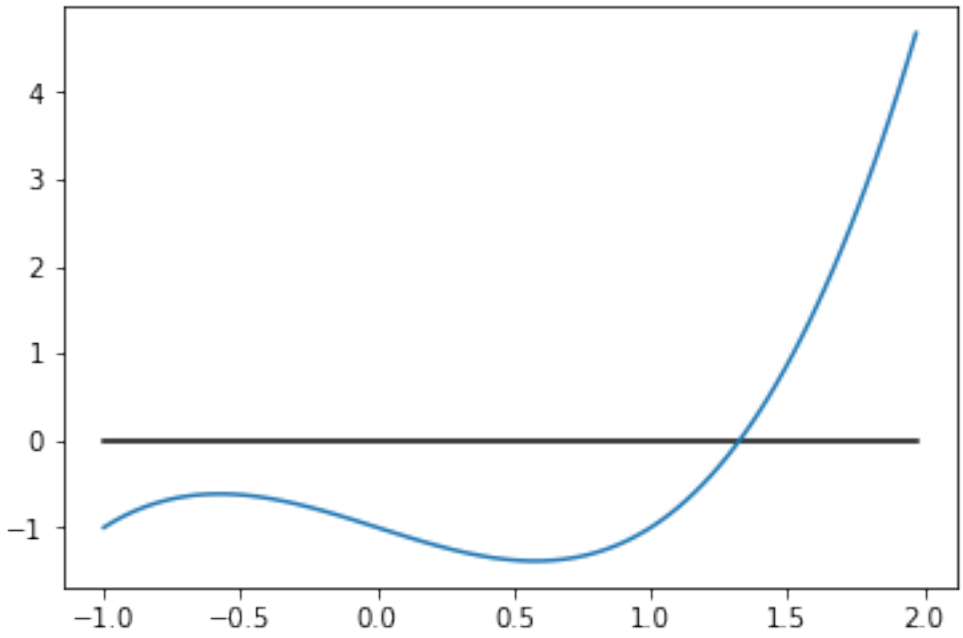
Out[16]: 0.6154681694899651

In [22]:
```
plot_function(g, -1, 1)
```



In [26]:
```
def f(x):
    return x ** 3 - x - 1
def g(x):
    return x ** 3 - 1
```

In [27]:
```
plot_function(f, -1, 2)
```

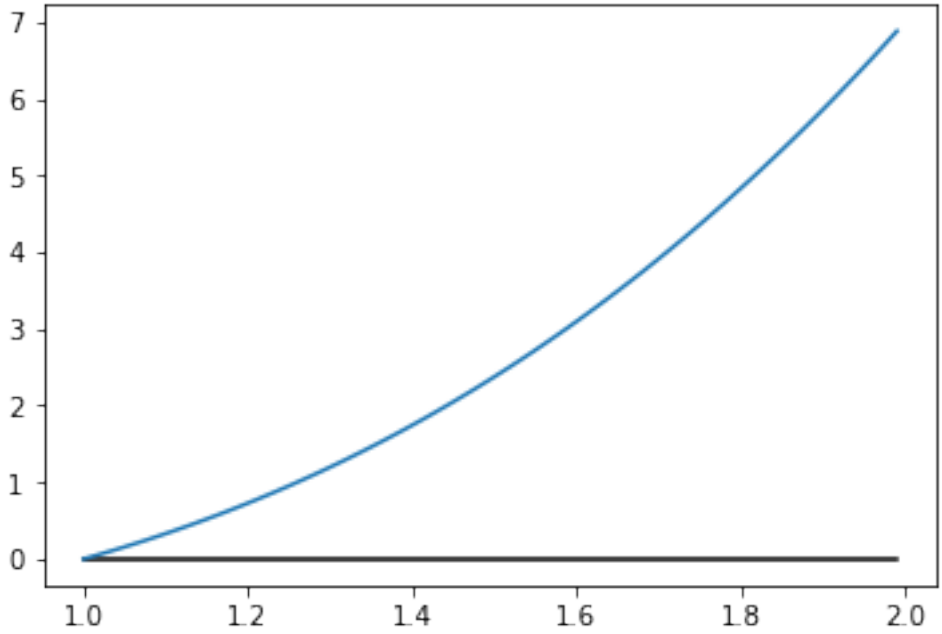



```
In [29]: bisect(f, 1, 2)
```

```
Iteration: 1         f(mid) = 0.875000         a = 1.000000         b = 1.500000
Iteration: 2         f(mid) = -0.296875         a = 1.250000         b = 1.500000
Iteration: 3         f(mid) = 0.224609         a = 1.250000         b = 1.375000
Iteration: 4         f(mid) = -0.051514         a = 1.312500         b = 1.375000
Iteration: 5         f(mid) = 0.082611         a = 1.312500         b = 1.343750
Iteration: 6         f(mid) = 0.014576         a = 1.312500         b = 1.328125
Iteration: 7         f(mid) = -0.018711         a = 1.320312         b = 1.328125
Iteration: 8         f(mid) = -0.002128         a = 1.324219         b = 1.328125
Iteration: 9         f(mid) = 0.006209         a = 1.324219         b = 1.326172
Iteration: 10         f(mid) = 0.002037         a = 1.324219         b = 1.325195
Iteration: 11         f(mid) = -0.000047         a = 1.324707         b = 1.325195
Iteration: 12         f(mid) = 0.000995         a = 1.324707         b = 1.324951
Iteration: 13         f(mid) = 0.000474         a = 1.324707         b = 1.324829
Iteration: 14         f(mid) = 0.000214         a = 1.324707         b = 1.324768
Iteration: 15         f(mid) = 0.000084         a = 1.324707         b = 1.324738
Iteration: 16         f(mid) = 0.000018         a = 1.324707         b = 1.324722
Iteration: 17         f(mid) = -0.000014         a = 1.324715         b = 1.324722
Iteration: 18         f(mid) = 0.000002         a = 1.324715         b = 1.324718
Iteration: 19         f(mid) = -0.000006         a = 1.324717         b = 1.324718
Iteration: 20         f(mid) = -0.000002         a = 1.324718         b = 1.324718
```

```
Out[29]: (1.3247175216674805, 1.3247184753417969)
```

```
In [31]: plot_function(g, 1, 2)
```




```
In [37]: fixed_point(g, 1)

Iteration: 1         x = 0.000000
Iteration: 2         x = -1.000000
Iteration: 3         x = -2.000000
Iteration: 4         x = -9.000000
Iteration: 5         x = -730.000000
Iteration: 6         x = -389017001.000000
Iteration: 7         x = -588715871622270591457689600.000000
Iteration: 8         x = -204040901322752672700486967824357518198151303514804197710939331810075
Iteration: 9         x = -849477147223739044906874308959312156369714874952778644735242512570620
```

```
---------------------------------------------------------------------------
OverflowError                             Traceback (most recent call last)

<ipython-input-37-6b9c1ac41b96> in <module>
----> 1 fixed_point(g, 1)


~/git/csc338/homework/a4/chapter5.py in fixed_point(f, x, n, verbose)
 17     for i in range(n):
```

```
     18         x = f(x)
---> 19         print("Iteration: %d\t x = %f" % (i+1, x))
     20     return x


OverflowError: int too large to convert to float
```

```
In [38]: def g(x):
             return (x + 1) ** (1/3)
```

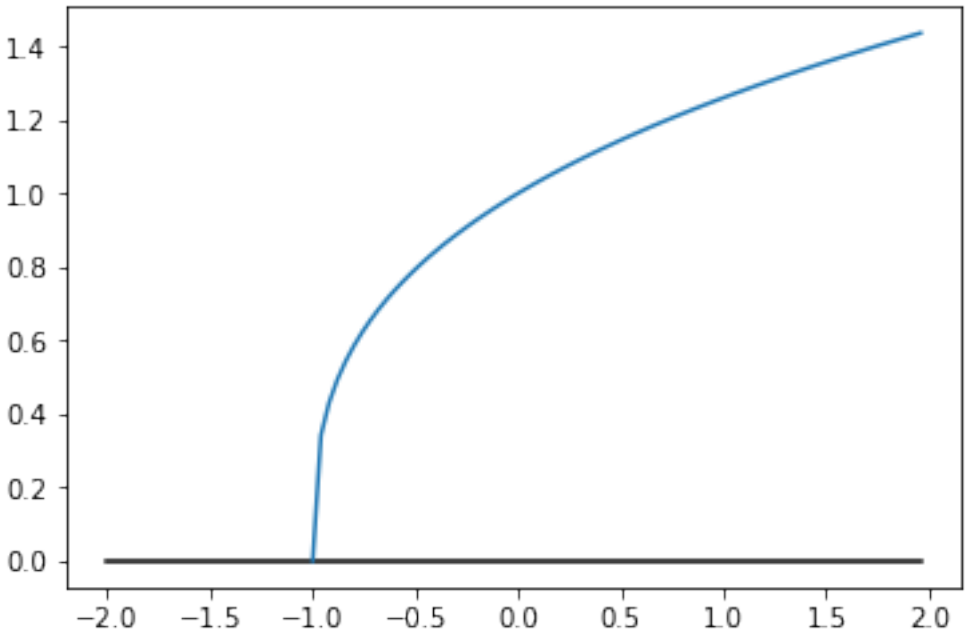
```
In [39]: fixed_point(g, 1)
```

```
Iteration: 1         x = 1.259921
Iteration: 2         x = 1.312294
Iteration: 3         x = 1.322354
Iteration: 4         x = 1.324269
Iteration: 5         x = 1.324633
Iteration: 6         x = 1.324702
Iteration: 7         x = 1.324715
Iteration: 8         x = 1.324717
Iteration: 9         x = 1.324718
Iteration: 10         x = 1.324718
Iteration: 11         x = 1.324718
Iteration: 12         x = 1.324718
Iteration: 13         x = 1.324718
Iteration: 14         x = 1.324718
Iteration: 15         x = 1.324718
Iteration: 16         x = 1.324718
Iteration: 17         x = 1.324718
Iteration: 18         x = 1.324718
Iteration: 19         x = 1.324718
Iteration: 20         x = 1.324718
```

```
Out[39]: 1.3247179572447447
```

```
In [48]: plot_function(g, -2, 2)
```

```
/Users/xuexue/miniconda3/lib/python3.6/site-packages/ipykernel_launcher.py:2: RuntimeWarning: in
```




```
In [47]: fixed_point(g, -1)

Iteration: 1         x = 0.000000
Iteration: 2         x = 1.000000
Iteration: 3         x = 1.259921
Iteration: 4         x = 1.312294
Iteration: 5         x = 1.322354
Iteration: 6         x = 1.324269
Iteration: 7         x = 1.324633
Iteration: 8         x = 1.324702
Iteration: 9         x = 1.324715
Iteration: 10         x = 1.324717
Iteration: 11         x = 1.324718
Iteration: 12         x = 1.324718
Iteration: 13         x = 1.324718
Iteration: 14         x = 1.324718
Iteration: 15         x = 1.324718
Iteration: 16         x = 1.324718
Iteration: 17         x = 1.324718
Iteration: 18         x = 1.324718
Iteration: 19         x = 1.324718
Iteration: 20         x = 1.324718


Out[47]: 1.3247179572447103
```