CSC338: Nonlinear Equations

1. True or False: If an iterative method for solving a nonlinear equation gains more than one bit of accuracy per iteration, then it is said to have a superlinear convergence rate.

2. Rank the following methods from the slowest convergence rate to the fastest convergence rate:
   - Bisection method
   - Newton's method
   - Secant method

3. If the bisection method for finding a zero of a function f starts with an initial bracket of length 1, what is the length of the interval containing the root after six iterations?

4. Consider the nonlinear equation $f(x) = x^2 - x = 0$. With $x_0 = 1$ as a starting point, what is the value of x_1 if we use Newton's method to solve this problem?

5. Consider the nonlinear equation $f(x) = x^2 - x = 0$. With $x_0 = 1$ and $x_1 = 2$ as starting points, what is the value of x_2 if we use the secant method to solve this problem?

6. What is the convergence rate of an iterative scheme if the magnitudes of the errors at successive iterations are: $e_0 = 10^{-2}$, $e_1 = 10^{-3}$, $e_2 = 10^{-4}$, $e_3 = 10^{-5}$, ...

7. What is the convergence rate of an iterative scheme if the magnitudes of the errors at successive iterations are: $e_0 = 0.022$, $e_1 = 0.00031$, $e_2 = 0.000023$, $e_3 = 0.0000019$, ...

8. What is the convergence rate of an iterative scheme if the magnitudes of the errors at successive iterations are: $e_0 = 0.022$, $e_1 = 0.0031$, $e_2 = 0.000023$, $e_3 = 0.0000000019$, ...