CSC338: Tutorial 5

1. Show that the two definitions of the induced matrix norm are equivalent, i.e.

$$\max_{||\mathbf{x}||=1} ||A\mathbf{x}|| = \max_{||\mathbf{x}||\neq 0} \frac{||A\mathbf{x}||}{||\mathbf{x}||}$$

2. Show that for an $n \times n$ matrix A, its L_1 norm is given by

$$||A||_1 = \max_j \sum_{i=1}^{n} |a_{ij}|$$

3. Suppose the matrix M is $m \times n$ where $m > n$, and M is full-rank. Show that $A = M^T M$ is symmetric positive definite.

4. Show that the Sherman-Morrison formula holds, that for an $n \times n$ invertible matrix A, and vectors u and v, we have

$$(A - \mathbf{u}\mathbf{v}^T)^{-1} = A^{-1} + \frac{A^{-1}\mathbf{u}\mathbf{v}^T A^{-1}}{1 - \mathbf{v}^T A^{-1}\mathbf{u}}$$

Hint: multiply both sides by $(A - \mathbf{u}\mathbf{v}^T)$.