CSC338 Exam Jam!

CSC338 Exam Jam Format:

- Choose your teams of 4-5
- Take an answer tracking sheet
- Try to get as many points as you can

For each question...

- Discuss with your team members
- Write down your final answer **in pen** on the answer sheet before the countdown
- Keep track of your team points

True or false

- 10 questions
- 30 seconds each per question
- 1 points per question
- Write “true” or “false” on the answer sheet before the 30 seconds are up

Question 1

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Answer: True

Question 2

The condition number for the problem of evaluating a function f(x) is always greater than or equal to 1.

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Answer: False

Question 3

When Newton's method converges, it always converges more quickly than the bisection method.

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Answer: False

Question 4

If x is any n-vector then $||x||_\infty \geq ||x||_2$.

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Answer: False

Question 5

In the IEEE double-precision floating-point number system, the underflow level (i.e., UFL in your textbook) is the smallest positive floating-point number.

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Answer: True

Question 6

If $|f(\hat{x})|$ is small, then $\hat{x}$ must be close to a root of $f(x)$. That is, $\hat{x}$ must be close to some x that satisfies $f(x) = 0$.

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Answer: False

Question 7

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Answer: False

Question 8

The conditioning of the problem of solving a set of linear equations $Ax = b$ is independent of the vector b.

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Answer: True

Question 9

In the gradient descent (steepest descent) algorithm, larger α is usually better.

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Answer: False

Question 10

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Answer: True

How many points does your team have?

Question 11 – 2 minutes [2 pts]

Does the function $f(x_1, x_2) = x_1^2 + x_1 x_2 + x_2^2$ have a:

- local maximum
- local minimum
- saddle point
- none of the above

at $x_1 = 0$, $x_2 = 0$?

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at $x_1 = 0$, $x_2 = 0$?

Answer: local minimum

Hessian matrix is

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Question 12 – 2 minutes [2 pts]

We want to use the bisection method to estimate a root of a function f accurate to 10^{-3} (i.e. $|x_{root} - x_{est}| \leq 10^{-3}$).

We start with the initial bracket of [-0.5, 0.5]. How many bisection iterations are required?

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Answer: 10

- Iteration 0: interval size $= 1$
- Iteration 1: interval size $= \frac{1}{2}$
- Iteration k: interval size $= \frac{1}{2^k}$

Question 13 – 3 minutes [3 pts]

Compute the Cholesky factorization of this matrix:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 2 \\ 0 & 2 & 5 \end{bmatrix}$$

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Answer:

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

Question 14 – 2 minutes [2 pts]

What is the Householder Transform that will eliminate all but the first element of $a = [2, 3, -1, 1, 1]^T$? Find the vector v that describes the transform.

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Answer: $v = [6, 3, -1, 1, 1]^T$ since $||a|| = 4$

Question 15 – 1 minutes [2 pts]

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- $x^2 - y^2$
- $(x + y)(x - y)$

can be evaluated more accurately in floating-point arithmetic?

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Answer: $(x + y)(x - y)$

Question 16 – 1 minutes [1 pts]

Which problem has the **worst** conditioning:

- Finding the minima of $f(x) = x^4 - 1$
- Evaluating $f(x) = x^4 - 1$ at the point $x = 0$
- Finding a root of $f(x) = x^4 - 1$

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- Evaluating $f(x) = x^4 - 1$ at the point $x = 0$
- Finding a root of $f(x) = x^4 - 1$

Answer: Finding the minima of $f(x) = x^4 - 1$