CSC 338H5 S 2019 Midterm Test
Duration — 60 minutes
Aids allowed: single-sided aid sheet,
non-programmable calculator

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| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 |
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**Student Number:**

**UTORid:** ___________

**Last Name:** ____________________ **First Name:** ____________________

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*Do **not** turn this page until you have received the signal to start.*

Fill out the identification section above, write your name and bubble in the Student Number on the top right, and read the instructions below.

*Good Luck!*

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This test consists of 5 questions on 8 pages (including this page). *When you receive the signal to start, please make sure that your copy is complete.*

- Bubble in your student number on the top right corner of this page using either a pen or a pencil.
- If you use any space for rough work, indicate clearly what you want marked.
- There is one blank page at the back of the exam if you need more space.
- **Do not remove any pages from the exam booklet.**

# 1: ______/ 8

# 2: ______/ 6

# 3: ______/ 4

# 4: ______/10

# 5: ______/12

TOTAL: ______/40

## Question 1. [8 MARKS]

Circle either "True" or "False" for each of the below statements.

1. True False The propagated data error is affected by the stability of the algorithm.
2. True False A well-posed problem can be ill-conditioned.
3. True False The problem of factorizing a non-singular $n \times n$ matrix $A$ into $A = LU$ where $L$ is lower triangular and $U$ is upper triangular is well-posed.
4. True False The inverse of an elementary elimination matrix is upper triangular.
5. True False For an $n \times n$ matrix $A$, if $cond(A) = 1$, then $A = I$ where $I$ is the identity matrix.
6. True False The matrix $A = \begin{bmatrix} 2 & 7 \\ 4 & 15 \end{bmatrix}$ has a large condition number.
7. True False An $n \times n$ matrix $A$ with $cond(A) = 2.13$ is ill-conditioned.
8. True False The product of two upper triangular matrices is upper triangular.

## Question 2. [6 marks]

Consider the normalized floating-point system $F(\beta = 10, p = 3, L = -10, U = 10)$, where chopping is used for rounding.

### Part (a) [1 mark]

What is the representation of $\frac{1}{7}$ in the floating-point system? (What are the values of the mantissa and exponent?)

### Part (b) [1 mark]

What is the representation of $\frac{1}{9}$ in the floating-point system?

### Part (c) [2 marks]

Compute $fl(\frac{1}{7}) - fl(\frac{1}{9})$, where the subtraction is floating-point subtraction.

### Part (d) [2 marks]

What is the relative error of the result of part (c)? Write the relative error as a percentage, rounded to the nearest whole percent.

## Question 3. [4 MARKS]

Consider, again, the normalized floating-point system $F(\beta = 10, p = 3, L = -10, U = 10)$, where chopping is used for rounding.

Suppose that we allow subnormal floating-point numbers in our system. How many subnormal floating-point numbers would we have?

## Question 4. [10 marks]

### Part (a) [5 marks]

Consider the condition number of the function $(f \circ g)(x) = f(g(x))$. Is it true that the condition number of $f \circ g$ is equal to the product of the condition numbers of $f$ and the condition number of g? In other words, prove or disprove the statement $K_{f \circ g}(x) = K_f(g(x)) \cdot K_g(x)$.

### Part (b) [5 marks]

Show that $cond(AB) \leq cond(A)cond(B)$, where $A$ and $B$ are $n \times n$ non-singular matrices.

## Question 5. [12 marks]

### Part (a) [6 marks]

Consider the following matrix. Find the $LU$ factorization of $A$ using Gauss Elimination. You do not need to use pivoting. Show your steps, and write your final result below.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix} \quad L = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \quad U = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

**Part (b)** [3 marks]

Solve the system $Ax = b$ for $b = [7, 4, 2]^T$ using your results from part (a). Show all your work.

**Part (c)** [3 marks]

Compute $||A||_1$, $||x||_1$ and $||b||_1$, where $A$, $b$ and $x$ are from parts (a) and (b). Why would we expect that $\frac{||b||_1}{||x||_1} \leq ||A||_1$?