This assignment is due at the **start** of your lecture on Thursday, 3 October 2019.

For the questions that require you to write a MatLab program, hand-in the program and its output as well as any written answers requested in the question. Your program and its output, as well as your written answers, will be marked. Your program should conform to the usual CS standards for comments, good programming style, etc.

When first learning to program in MatLab, students often produce long, messy output. Try to format the output from your program so that it is easy for your TAs to read and to understand your results. For example, if you are asked to print a table of values, print them as a table that fits on one page. To this end, you might find it helpful to read "A short description of fprintf" on the course webpage

`http://www.cs.toronto.edu/~krj/courses/336/`

Marks will be awarded for well-formatted, easy-to-read output.

Also, your TAs will appreciate your using a word processor to write the answers to questions (or parts of questions) that do not require a program. If you do write those answers by hand, make sure that they are easy to read.

1. [6 marks: 2 marks for each part]

   What are the approximate absolute and relative errors in approximating $\sqrt{2}$ by the following values?

   (a) 1.41

   (b) 1.41421

   (c) 1.4142135

   For the purposes of this question, you can assume that the "true" value of $\sqrt{2}$ is 1.4142135623730950488.

   Correctly round each error to three significant digits (using the *round-to-nearest* rounding rule and write it in the normalized form $\pm d_1 . d_2 d_3 \cdot 10^n$, where each d_i, for $i = 1, 2, 3$, is an integer that satisfies $0 \leq d_i \leq 9$, $d_1 \neq 0$ and n is an integer.

   You might find it helpful to use a computer or a calculator to assist you with the required arithmetic for this question.

2. [10 marks: 1 mark for each part]

   In a floating-point number system with parameters $\beta = 10$, $p = 3$, $L = -10$ and $U = +10$ that uses the *round-to-nearest* rounding rule and allows gradual underflow with subnormal numbers, what is the result of each of the following floating-point arithmetic operations?

(a) $5.21 \cdot 10^1 + 3.46 \cdot 10^{-1}$

(b) $4.53 \cdot 10^3 - 6.38 \cdot 10^1$

(c) $2.63 \cdot 10^1 + 2.53 \cdot 10^{-3}$

(d) $2.32 \cdot 10^4 - 8.76 \cdot 10^6$

(e) $3.25 \cdot 10^8 \times 4.51 \cdot 10^{-5}$

(f) $-4.51 \cdot 10^7 \times 3.45 \cdot 10^3$

(g) $5.23 \cdot 10^{-5} \times 4.62 \cdot 10^{-5}$

(h) $-6.12 \cdot 10^{-7} \times 4.14 \cdot 10^{-5}$

(i) $-5.48 \cdot 10^{-7} \times 1.62 \cdot 10^{-6}$

(j) $3.28 \cdot 10^{-6} \times 1.25 \cdot 10^{-7}$

Write each answer as a normalized 3-decimal-digit floating-point number, if possible. If that is not possible, write your answer as either a subnormal 3-decimal-digit floating-point number or zero, if that is the most accurate representation. If that is not possible either, then write your answer as +Inf, −Inf or NAN, whichever best represents your answer.

You might find it helpful to use a computer or a calculator to assist you with the required arithmetic for this question.

3. [10 marks: 5 marks for each part]

Consider the function $f(x) = \log_{\mathrm{e}}(x)$ for real positive x.

(a) Is this function well-conditioned or ill-conditioned in a relative sense with respect to small relative changes in the value of the input argument x for x close to 1?

Is this function well-conditioned or ill-conditioned in a relative sense with respect to small relative changes in the value of the input argument x for x close to 10?

Justify your answer for each case.

(b) Write a little MatLab program to verify your predictions from part (a).

Hand in your MatLab program and its output.

Also include a brief explanation of why you believe your computational results from part (b) support your theoretical predictions from part (a).

4. [10 marks: 5 marks for each part]

Do question 2 on the 2018 final exam.

You can find the 2018 final exam on the webpage

`http://www.cs.toronto.edu/~krj/courses/336/exam.2018.pdf`