Information Retrieval

CSC401/2511 – Natural Language Computing – Spring 2026

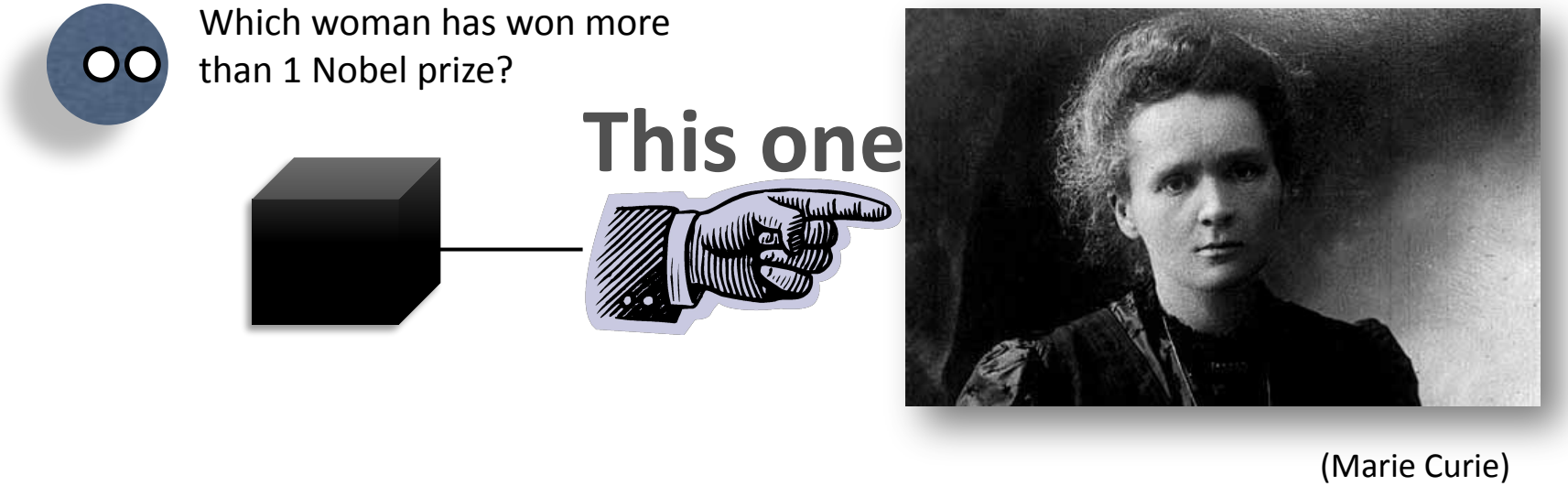
Lecture 14

University of Toronto

UNIVERSITY OF TORONTO

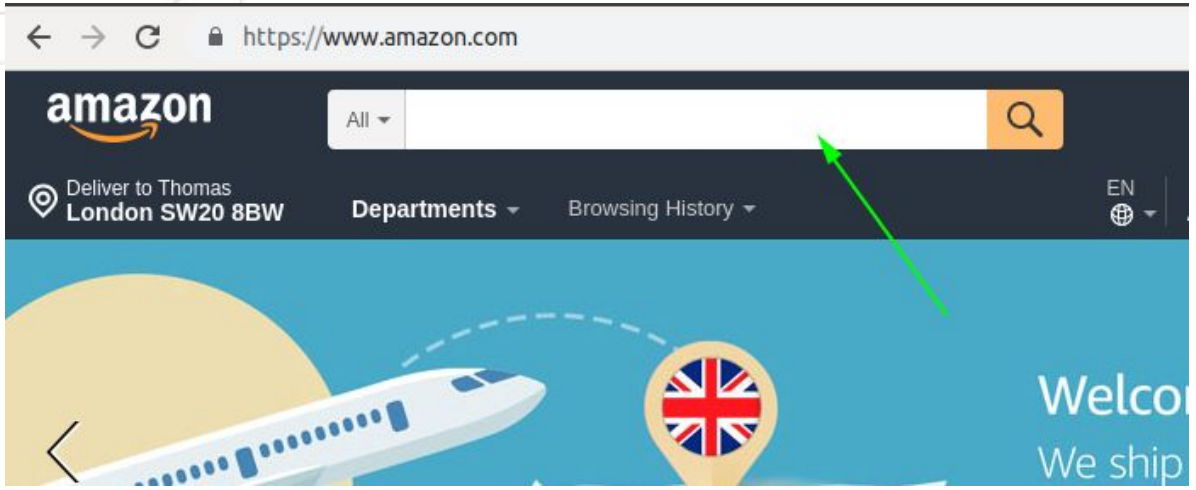
What is Information Retrieval?

Given a query, search for the most relevant document among a knowledge base.



(Marie Curie)

Search Engines are (mainly) IR systems



Information Retrieval ↔ Information Extraction

Question Answering ↔ Text Summarization

Information Retrieval system

Given a query, search for the most relevant document among a knowledge base.

- Three key problems here:
  - How to represent the query?
  - How to store a knowledge base?
  - How to search efficiently and accurately?
- The problems are closely related. We will look at some popular approaches.

Scenario 1: SQL

- Structured Query Language (SQL) query
- How to represent the query?
  SQL queries.
- How to store a knowledge base?
  Tabular entries with predefined schemas.
- How to search efficiently and accurately?
  Compile and execute the SQL queries.

Scenario 2: Max-similarity search

- Find the document that is the most similar to the query.
- How to represent the query?

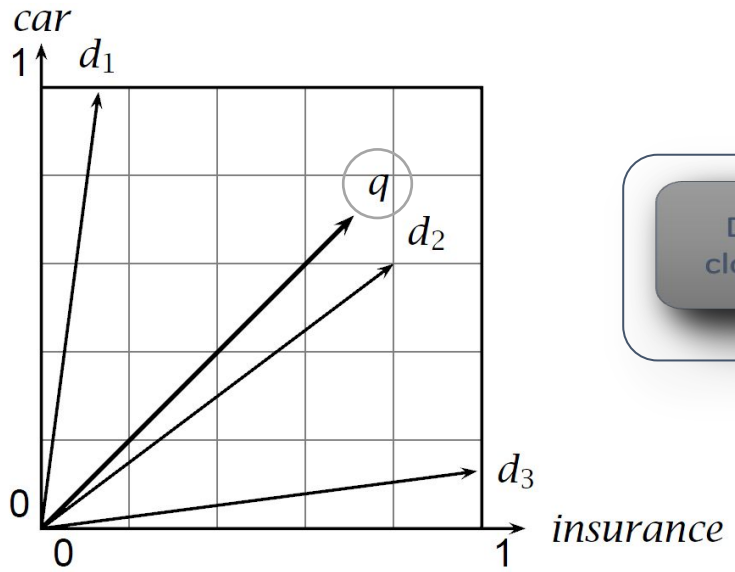
  Query is just another text-based document.
- How to store a knowledge base?

  Vectorized documents.
- How to search efficiently and accurately?

  Compute the **similarity score** between the query and each document. Return the document with the highest similarity score.

Similarity score

- If the query and the available documents can be represented by vectors, we can determine **similarity** according to their **cosine distance**.
  - Vectors that are **near** each other (within a certain **angular radius**) are considered relevant.



Document $\boldsymbol{d_2}$ is closest to query $\boldsymbol{q}$.

Vectorization: *tf.idf*

- *tf.idf* is a traditional method to vectorize the documents.
- It starts by weighting *words* in the *documents*.

  - **Term frequency, tf_{ij}**: number of occurrences of word w_i in document d_j.
  - **Document frequency, df_i**: number of documents in which w_i appears.
  - **Collection frequency, cf_i**: total occurrences of w_i in the collection.

Term frequency

- **Higher** values of tf_{ij} (for contentful words) suggest that word w_i is a **good** indicator of the content of document d_j.
  - When considering the relevance of a document d_j to a keyword w_i, tf_{ij} should be **maximized**.

- We often **dampen** tf_{ij} to temper these comparisons.
  - $tf_{dampen} = 1 + \log(tf)$, if $tf > 0$.

Document frequency

- The **document frequency**, df_i, is the number of documents in which w_i appears.
  - **Meaningful** words may occur repeatedly in a related document, but **functional** (or less meaningful) words may be distributed evenly over all documents.

Word	Collection frequency	Document frequency
kernel	10,440	3997
try	10,422	8760

  - E.g., *kernel* occurs about as often as *try* in total, but it occurs in fewer documents – it is a more **specific** concept.

Inverse document frequency

- Very specific words, w_i, would give **smaller** values of df_i.
- To maximize specificity, the **inverse document frequency** is

$$idf_i = \log\left(\frac{D}{df_i}\right)$$

where D is the total number of documents
and we scale with log (why? next slide)

- This measure gives **full** weight to words that occur in 1 document, and **zero** weight to words that occur in all documents.

Inverse document frequency

- The probability of a document containing word i is:

$$\frac{df_i}{D}$$

"A document containing word i" is an event.

Small p: this event is more surprising.

Therefore, more information

- idf_i is the amount of information provided by observing the event.

tf.idf vectorization of a document

- We combine the **term frequency** and the **inverse document frequency** to give us a joint measure of **relatedness** between words and documents:

$$tf.idf(w_i, d_j) = \begin{cases} (1 + \log(tf_{ij})) \log \frac{D}{df_i} & \text{if } tf_{ij} \geq 1 \\ 0 & \text{if } tf_{ij} = 0 \end{cases}$$

- The j^{th} document is therefore represented by a vector:

$$[tf.idf(w_1, d_j), \\ tf.idf(w_2, d_j), \\ \dots, \\ tf.idf(w_{|W|}, d_j)]$$

Aside: BM25

- BM25 is a baseline algorithm of IR.
- Given query $Q = [q_1, q_2, \ldots, q_n]$, BM25 computes a similarity score for document d_j as:

$$\text{Score}(Q) = \sum_{i=1}^{n} \log \frac{D}{df_i} \times g(tf(q_i, d_j); k_1, b)$$

$g(\cdot)$ is an engineered function that has hyperparameters k_1 and b
The details of $g(\cdot)$ are unimportant to our discussion.

Scenario 3: Semantic Doc2Vec

- IR setting: Also using max-similarity search.
- The idea of word2vec can be applied as well.
- Goal: train a document encoder E.
- Design optimization goals for E so that:
  - If d_A and d_B are close to each other, then $\text{sim}\langle E(d_A), E(d_B)\rangle$ should be large.
  - If d_A and d_B are far from each other, then $\text{sim}\langle E(d_A), E(d_B)\rangle$ should be small.
- The definitions of closeness vary from algorithm to algorithm.

Semantic Doc2Vec

- Example: How does the Contriever paper define closeness?
  - Positive samples d_+ for a document are **augmented** following some heuristics.
  - Negative samples d_- are **randomly sampled** from within the batches.

- A contrastive loss objective is:

$$L(q, d_+, d_-) = \frac{e^{\text{sim}\langle E(q), E(d_+)\rangle/\tau}}{e^{\text{sim}\langle E(q), E(d_+)\rangle/\tau} + \sum_i e^{\text{sim}\langle E(q), E(d_-)\rangle/\tau}}$$

where τ is the temperature of the softmax.

Evaluating the retrieval systems

- Some commonly used metrics include:
  - Precision
  - Recall
  - F-score
  - Precision @ k

Precision and Recall

- **Precision**: $\frac{N_{\text{relevant \& retrieved}}}{N_{\text{retrieved}}}$
  - Among all retrieved documents, how many are relevant?
  - Precision in machine learning: $\frac{TP}{P}$
- **Recall**: $\frac{N_{\text{relevant \& retrieved}}}{N_{\text{relevant}}}$
  - Among all relevant documents, how many are retrieved?
  - Recall in machine learning: $\frac{TP}{T}$
- Note: Precision and recall has some tradeoff.

F-measure

- **F-measure** is the weighted harmonic mean of precision and recall:
  - $F = \frac{1}{\alpha\frac{1}{p}+(1-\alpha)\frac{1}{\mathrm{r}}}$
- Where p is precision, r is recall, and $\alpha \in [0,1]$.
- Notes:
  - When $\alpha = \frac{1}{2}$, we have $F_1 = \frac{2pr}{p+r}$
  - If either of precision or recall is 0 (i.e., true positive count $TP = 0$), then F is arbitrarily set to 0.

Precision at k

- Modern IR systems usually do not just give one result.
  - Even if the 1^{st} result is not relevant, the 2^{nd}, etc. results could be relevant too.
- People sometimes measure the **precision at k (P@k)**:
  - Among the top k results, how many of them are relevant?
- **P@k** has some potential problems:
  - The 1^{st}, 2^{nd}, …, k^{th} locations have no differences.
  - If there are less than k relevant results, then even the best system can't get **P@k**=1.

Term-document Matrix

- Based on all the terms and documents, we can have a term-document matrix X, where each entry $X_{i,j}$ is tf.idf(w_i, d_j)

- What can we do about this matrix? (Think of its property: large, sparse, noisy…)

Singular Value Decomposition

(some content borrowed from Professor Stark Draper)

Singular Value Decomposition (SVD)

Any matrix $A \in \mathbb{R}^{m \times n}$ can be expressed as

$$A = U\tilde{\Sigma}V^{\mathrm{T}}$$

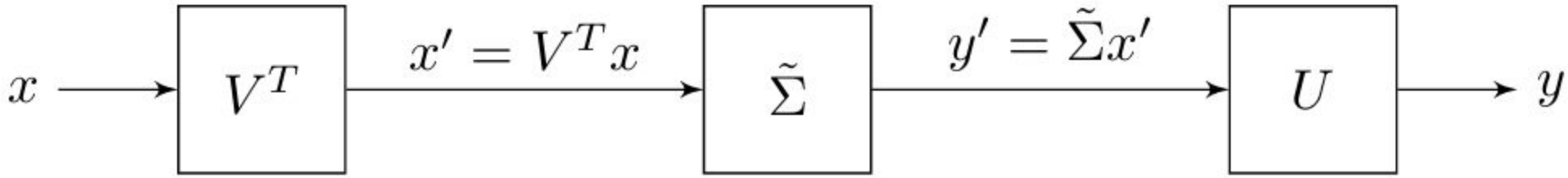
where

- $U \in \mathbb{R}^{m \times m}$ is an orthogonal matrix (so $UU^{\mathrm{T}} = U^{\mathrm{T}}U = I_m$)
- $V \in \mathbb{R}^{n \times n}$ is an orthogonal matrix (so $VV^{\mathrm{T}} = V^{\mathrm{T}}V = I_n$)
- $$\tilde{\Sigma} = \left[\begin{array}{c|c} \Sigma & 0_{r,n-r} \\ \hline 0_{m-r,r} & 0_{m-r,n-r} \end{array}\right] \in \mathbb{R}^{m \times n}$$

  where $\Sigma = \mathrm{diag}(\sigma_1, \ldots, \sigma_r)$, each $\sigma_i > 0$, $i \in [r]$, r is $\mathrm{rank}(A)$, and $0_{k,l}$ denotes the $k \times l$ all-zeros matrix.

Intuition behind SVD

What does this operation mean “physically”?



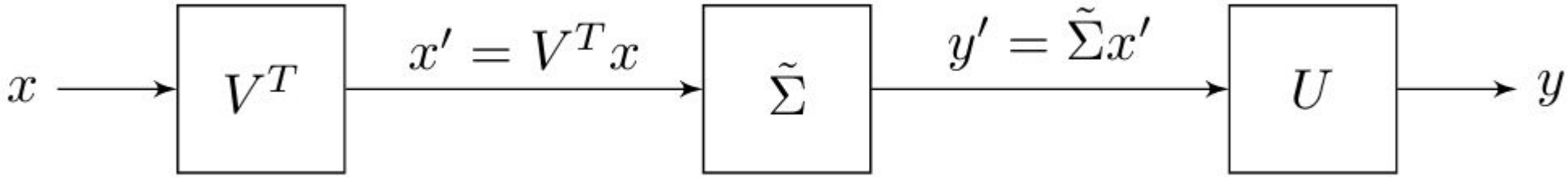
Numerical Example

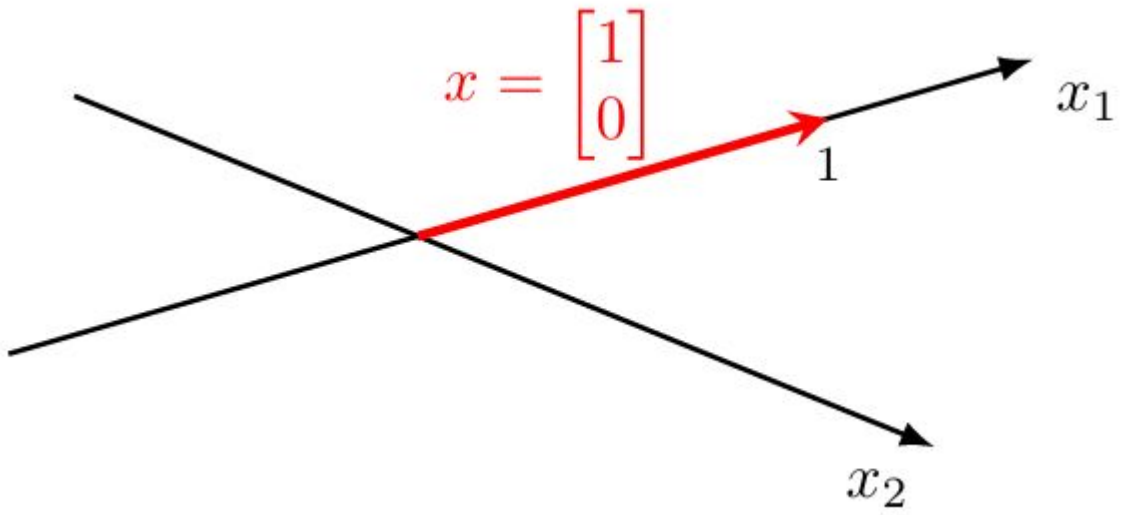
$$y = Ax = U\tilde{\Sigma}V^{\mathrm{T}}x$$

$$U = \left[\begin{array}{c:cc} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & -\frac{2}{\sqrt{6}} \end{array}\right], \tilde{\Sigma} = \left[\begin{array}{c:c} 2 & 0 \\ \hdashline 0 & 0 \\ 0 & 0 \end{array}\right], V = \left[\begin{array}{c:c} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \hdashline \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{array}\right]$$

$$A = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & -\frac{2}{\sqrt{6}} \end{bmatrix} \begin{bmatrix} -\frac{2}{\sqrt{2}} & \frac{2}{\sqrt{2}} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} -\frac{2}{\sqrt{6}} & \frac{2}{\sqrt{6}} \\ -\frac{2}{\sqrt{6}} & \frac{2}{\sqrt{6}} \\ -\frac{2}{\sqrt{6}} & \frac{2}{\sqrt{6}} \end{bmatrix}$$

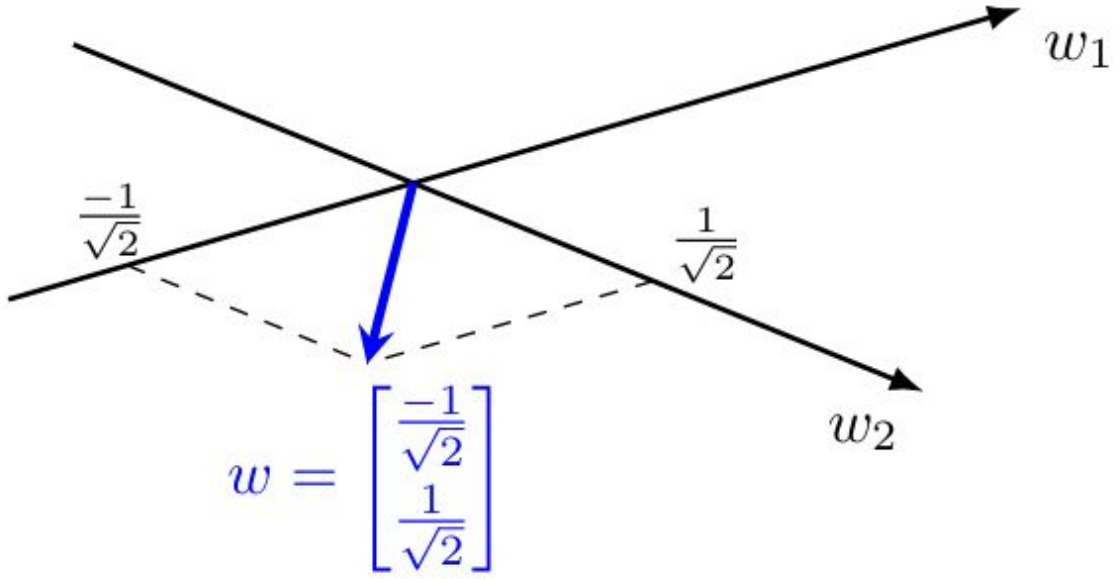
Numerical Example cont'd





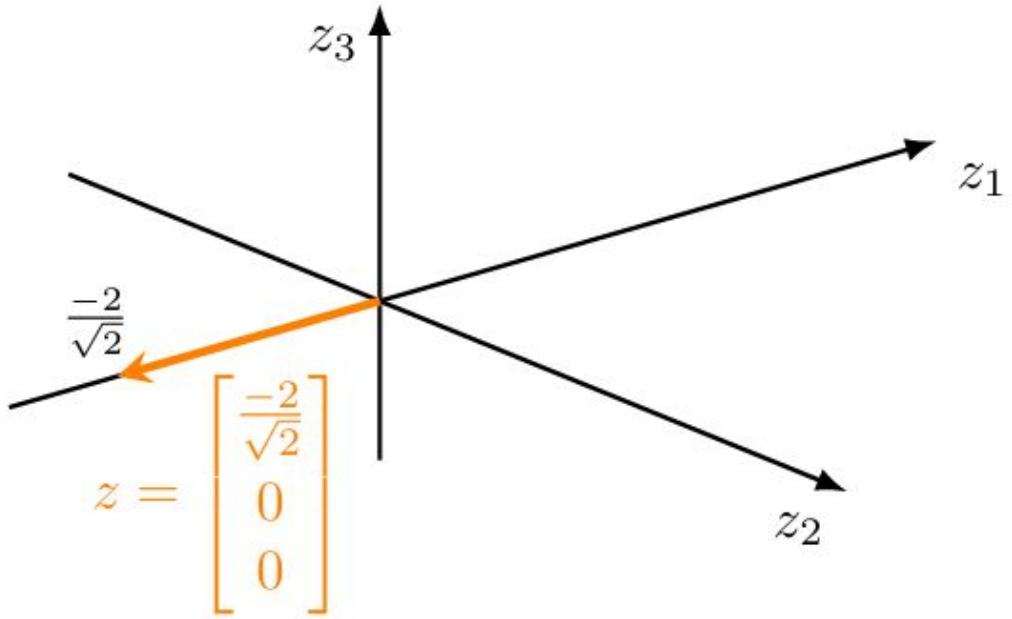
Numerical Example cont'd

$$w = V^{\mathrm{T}} x = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$



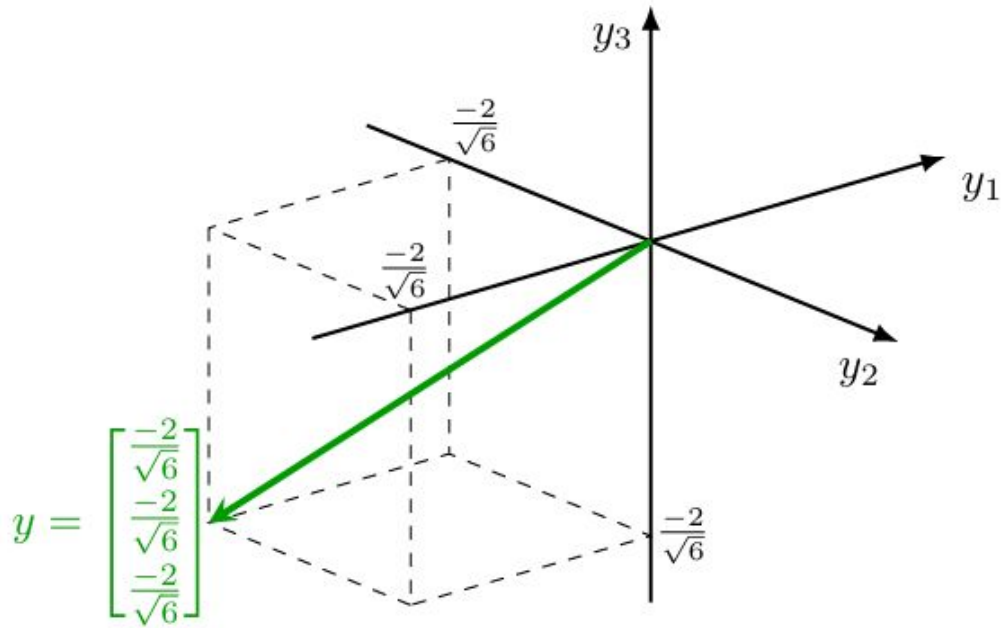
Numerical Example cont'd

$$z = \tilde{\Sigma} w = \begin{bmatrix} 2 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} -\frac{2}{\sqrt{2}} \\ 0 \\ 0 \end{bmatrix}$$



Numerical Example cont'd

$$y = Uz = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & -\frac{2}{\sqrt{6}} \end{bmatrix} \begin{bmatrix} -\frac{2}{\sqrt{2}} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{2}{\sqrt{6}} \\ -\frac{2}{\sqrt{6}} \\ -\frac{2}{\sqrt{6}} \end{bmatrix}$$



Applying that to many values:

Truncated SVD

How does SVD relate to IR? Well…

- Recall our complaint: X is too large, sparse and noisy

- Could we potentially truncate it? (i.e. keep only the top k singular values?)

Latent Semantic Indexing

- Instead of using the entire word space (large, sparse, noisy), let's bring it down to a smaller "semantic"-ish space.

- Truncate the matrix to dimension k, for example.

- Projecting Query Vectors and Document Vectors onto the same latent space of dimension k

Example: Truncated SVD

Suppose we have 3 terms and 3 documents:

w1: car
w2: automobile
w3: cat

d1: car automobile
d2: car automobile
d3: cat

Example: Truncated SVD cont'd

w1: car
w2: automobile
w3: cat

d1: car automobile
d2: car automobile
d3: cat

This will give us the following term-document matrix:

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Example: Truncated SVD cont'd

When we perform the full SVD
(compute singular values, find singular vectors, decompose it),
we have:

$$A = U\Sigma V^\top = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$, where:

$$U = V = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Example: Truncated SVD cont'd

Let's truncate A by keeping only the top k singular values.

If k = 1:

$$A_{k=1} = U\Sigma_{k=1}V^\top = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

If k = 2:

$$A_{k=2} = U\Sigma_{k=2}V^\top = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Example: Truncated SVD

What does the truncation at k = 1 mean in our example?

w1: car	d1: car automobile
w2: automobile	d2: car automobile
w3: cat	d3: cat

- the strong shared structure involving **car**, **automobile** was preserved;
- the smaller separate topic involving **cat** was dropped

Aside: Stop List

Stop List: a list of stop words that the system ignores during indexing and/or retrieval.

e.g. : the, is, of, and, to…

*Do we always use a big universal stop list **today**?*

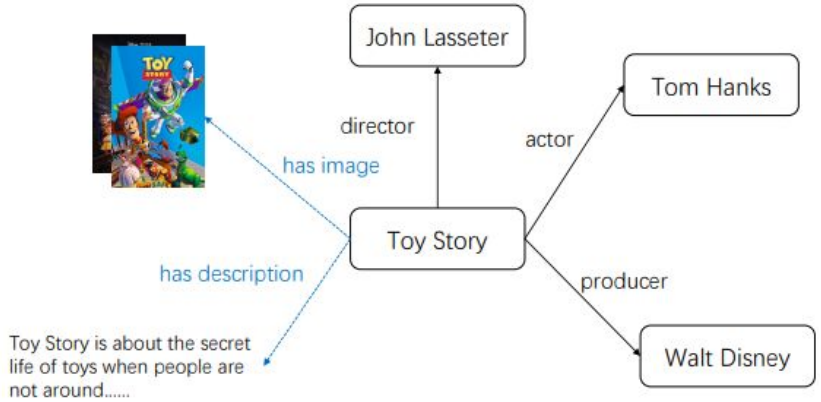
Appendix: Recent challenges of IR

- Structured, relational data
- Multi-modal data

Structured relational data

- Plain texts are **unstructured**.
- Many modern IR systems use **structured** data.
  - E.g., docs vectorized to the same dimensions.
  - E.g., relational data.
- Benefits & challenges of structured data

{“name”: “Toy Story”,
“director”: “John Lasseter”,
… }



Storing structured data

- Saving each complex object as a database entry is one option.

- We can also store (or embed) the $\{R, S, T\}$ triplets.
  - R is the **relation** (e.g., "has-director") between:
  - the **source** S (e.g., "Toy Story") and
  - the **target** T (e.g., "John Lasseter")

Multimodal data

- Most modern IR systems are **multimodal**.
- The objects contain more than texts.
  - Images, sounds, even videos are stored too.
  - Choosing the right schemas is very important!