CSC 2508S
Spring, 2002

October 30, 2002
DUE: November 13, 2002

Information: Quantification, Representation and Manipulation

Assignment #3

Progressive Encoding of Geographical Entities

Progressive Encoding

In a *progressive encoding* scheme, the bits of the encoding are transmitted sequentially. As the bits are received, the uncertainty in the encoded "message" diminishes monotonically, until asymptotically all uncertainty in the encoded "message" is removed. Ideally the code is structured so that the early bits are most informative about the content of the "message", while later bits convey details at lower and lower levels.

For example, if the "messages" were binary fractions, then the obvious (perfect) progressive encoding would consist of simply transmitting the bits in the order from most significant to least significant. Similarly, if the "messages" were the locations of points in the unit square, then a good (possibly optimal) progressive encoding scheme would be to (i) interleave the bits of the binary fractions representing x and y, and (ii) transmit bit pairs from most significant to least significant.

Problem Context

The situation you are to address is where each message is the shape of a geographic entity (e.g., continent, country, province, state, sea, lake, etc.). Assume that the initial definition of entity j is as a sequence of n_j pairs:

$$[(x_1,y_1), (x_2,y_2), (x_3,y_3), \cdots, (x_{n_j},y_{n_j})]$$

where the n_j values are of the order 100 to 1000, and x_i and y_i are 32-bit binary fractions (i.e., $0 \le x_i < 1$ and $0 \le y_i < 1$). Assume that the sizes of the shapes have been scaled to fill the unit square as much as possible, while being placed close to the origin. As a result of this scaling and placement, the following condition will hold:

There exist i_{btm}, i_{lft}, and i_{ext}, each an integer between 1 and n_j, such that:

(i) $y_{i_{btm}} = 0$ and $x_{i_{lft}} = 0$ and either $x_{i_{ext}} = 1-\varepsilon$ or $y_{i_{ext}} = 1-\varepsilon$ where $\varepsilon \approx 2^{-32}$.

(Intuitively, that means that the lowest point of the shape has $y_i = 0$, the leftmost point of the shape has $x_i = 0$, and either the uppermost point has $y_i \approx 1$ or the rightmost point has $x_i \approx 1$.)

Task

Your task is to propose and evaluate a scheme for carrying out progressive encoding of the shapes of the entities, so that, as more and more bits of the encoding are received, the shape of the entity is known more and more precisely.

Three strategies mentioned in class for representing closed curves concisely were:

i. Connect the dots
   The encoding is also a sequence of points with implicit line segments joining each one to the next.

ii. Treasure map
The encoding consists of an initial point, followed by a sequence of direction and distance pairs.

iii. Blurring
The encoding consists of the original sequence of points, but represented at reduced precision.

However, these approaches address only the need for a *concise* representation, and not the problem of obtaining a progressive encoding. So they can be used as ideas for approaches in your solution, but they don't suffice in themselves.

Objectives

The quality of the progressive encoding can be evaluated as follows. After *b* bits have been received, measure the extent to which the perceived approximation based on the bits received so far differs from the actual shape as defined by the full initial description. Some measures that could be compared between the actual shape and its estimation after *b* bits are:

- length of perimeter
- area
- length of max diameter
- max width at any point perpendicular to the max diameter
- max deviation of any estimated point from the nearest point to it in the actual shape
- area of symmetric difference between the estimated shape and the actual shape.

One goal of the progressive encoding scheme would be to make it possible for the code receiver to distinguish quickly between very different shapes (like Italy versus New York state), and to eventually be able to distinguish even between quite similar shapes (like Saskatchewan and Alberta).

The initial encodings of each shape have sizes roughly in the range 1 KB to 8 KB. In your discussion pay particular attention to the levels of accuracy of approximation achievable after the first 200 (or so) bits, and after the first 1000 bits. Don't devote much effort or attention to the parts of the progressive encoding beyond the first 1000 bits.

Write-up

Propose an overall approach (not a detailed algorithm) to transforming the initial shape definition into a progressive encoding of it. Your approach may be anywhere along the spectrum from (i) quite efficient to form but yielding only a reasonable approximation, to (ii) quite expensive and producing a good approximation.

Your write-up for this assignment should be legibly presented in approximately five to eight pages.