Social and Information Networks

University of Toronto CSC303
Winter/Spring 2023

Week 7: Feb 27 - March 3

Wed. Mar 1: Announcements and Corrections

- Happy March! I for one can't wait for Spring :)
- Critical review papers & groups are due this Friday
- Today's 10PM Zoom office hours have moved to tomorrow at the same time
  - Today's 2PM office hours are unaffected :)

This week's agenda

- Twitter rumour cascades
- Structural virality
- Threshold model
  - Complete cascades
  - Blocking clusters
  - A first look at selecting initial adopters

Chapter 19: Influence spread in a social network

- We begin a study of the spread/diffusion of products/influence in an existing social network (Chapter 19)
  - In contrast to the population wide influence spread covered in Chapter 18 (on power laws)
    - e.g., The Salganik music selection experiment is unaffected by the underlying social network; participants only see total downloads
    - Population wide effects are also discussed in Chapters 16 & 17, which we won't be covering

Social network effects

- Now we wish to consider an existing social network where edges (ties) between individuals represent some sort of friendship/relationship.
- This takes us back to concepts introduced in Chapters 3 and 4.
- There we saw the contrast between
  - selection (we tend to be friends with people of similar backgrounds, geography, interests)
  - social influence (we join clubs, are influenced) by our friends/relations.

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- Rather than link creation (e.g., selection), we will now study spread (e.g., influence)
- The goal (as throughout the course) is to qualitatively understand a process or observed phenomena in a highly stylized (but hopefully still interesting) setting.
- We will (as usual) be interested in what kind of general conclusions can be inferred from such an understanding.

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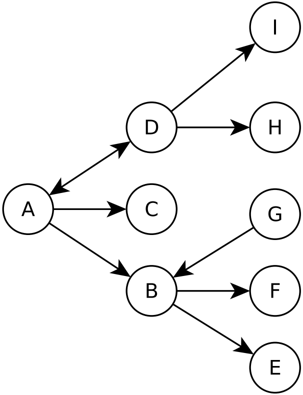

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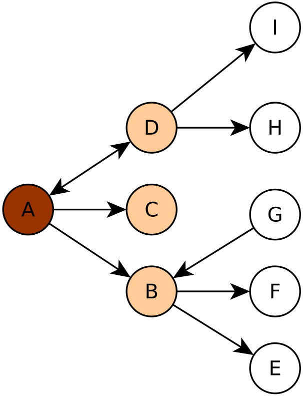

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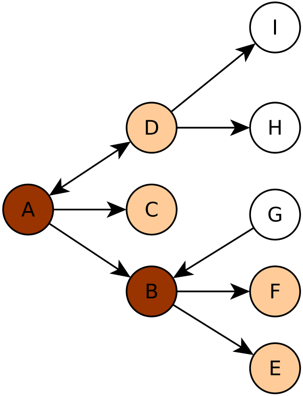

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  "*Falsehood will fly, as it were, on the wings of the wind, and carry its tales to every corner of the earth; whilst truth lags behind; her steps, though sure, are slow and solemn, and she has neither vigour nor activity enough to pursue and overtake her enemy*"
  – Thomas Francklin, 1787

  "*A lie can run round the world before the truth has got its boots on*"
  – Sir Terry Pratchett, The Truth

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- They defined news to be true (resp. false) if it was verified (resp. rejected) by one of six independent fact checking organizations
    - snopes.com, politifact.com, factcheck.org, truthorfiction.com, hoax-slayer.com, urbanlegends.about.com

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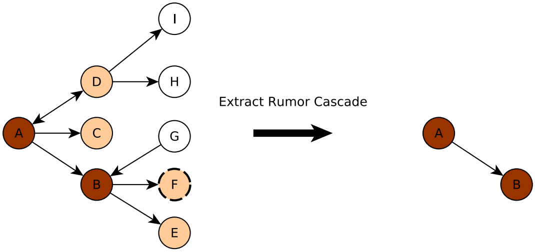

- Their final dataset contained 126,000 stories, tweeted by 3 million people more than 4.5 million times.

Measuring a Rumour Cascade

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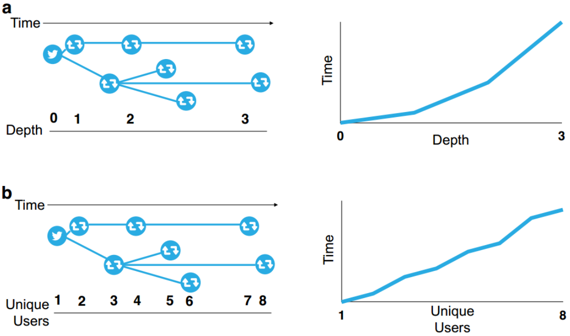

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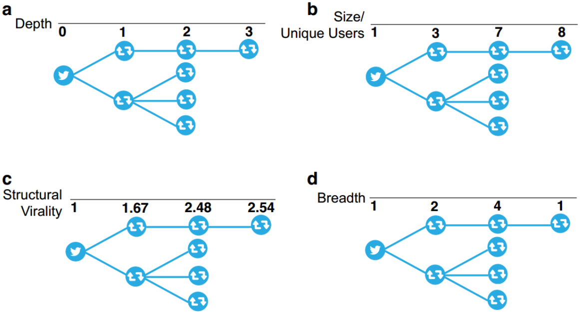

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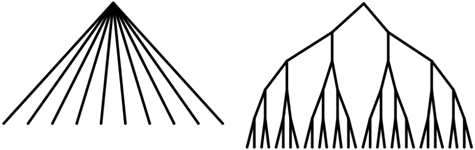
Structural Virality

- Structural virality is a measure meant to distinguish transmission via broadcast (large burst or bursts) or rapid peer-to-peer spreading (exponential growth over time)

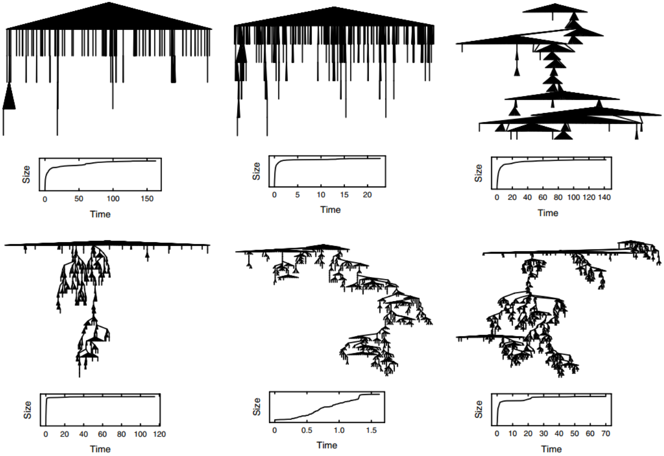
Figure 1 **A Schematic Depiction of Broadcast vs. Viral Diffusion, Where Nodes Represent Individual Adoptions and Edges Indicate Who Adopted from Whom**



[From The Structural Virality of Online Diffusion, Goel et al., 2014]

Structural Virality

Figure 3 A Random Sample of Cascades Stratified and Ordered by Increasing Structural Virality, Ranging from 2 to 50



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How to Measure Structural Virality?

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- Average depth solves chains, but is still problematic

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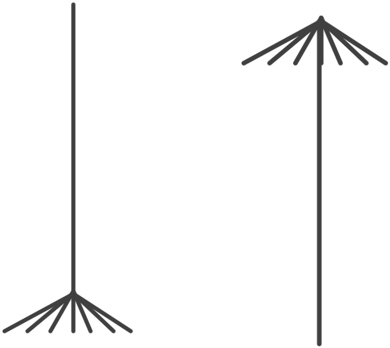

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  - All pure broadcast structures should be equally viral, regardless of their size
- Structural virality is defined as the average *undirected* distance between all pairs of nodes

$$\text{virality}(G) = \frac{1}{|V|(|V|-1)} \sum_{u,v \in V} d(u,v)$$

- Equivalently, it can be viewed as the average undirected distance to a node, averaged over all nodes
- It still has pathological cases, but it is empirically useful

Example: Spread of news through Twitter

- Looking at our static measures, we can see that false news cascades travel deeper, reach more people, and have greater breadth and structural virality
- Looking at our dynamic measures, we can see that, on average, fake news grows in depth and size faster than real news

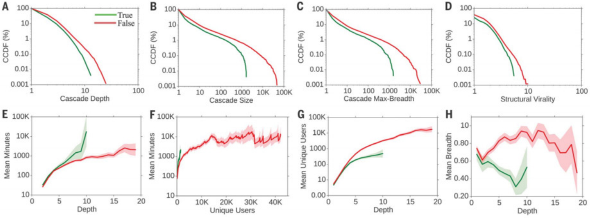

Fig. 2. Complementary cumulative distribution functions (CCDFs) of true and false rumor cascades. (**A**) Depth. (**B**) Size. (**C**) Maximum breadth. (**D**) Structural virality. (**E** and **F**) The number of minutes it takes for true and false rumor cascades to reach any (E) depth and (F) number of unique Twitter users. (**G**) The number of unique Twitter users reached at every depth and (**H**) the mean breadth of true and false rumor cascades at every depth. In (H), plot is lognormal. Standard errors were clustered at the rumor level (i.e., cascades belonging to the same rumor were clustered together; see supplementary materials for additional details).

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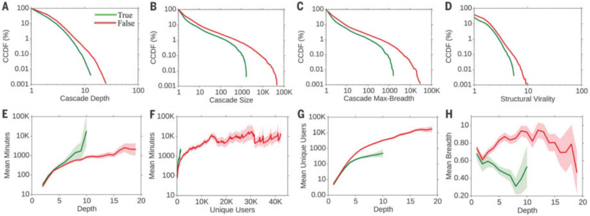

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- All of these features were significantly different, and seem to indicate that Truth does indeed tarry with it's boots
- Question: What could be possible causes of this?

Fake vs. Real news spread: Structural Differences?

- Differences in the underlying following-followee network around nodes prone to spreading falsehood could explain the result
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- Could these nodes have higher out-degree, or in-degree?
- Vosoughi et al. found that the nodes spreading false information tended to have significantly *fewer* followers and followees
- Furthermore, at the individual level they were significantly less active, and on Twitter for significantly less time

Fake vs. Real news spread: Blame the bots?

- Bots in the network could be encouraging spread
- In effect, these nodes would be selectively more contagious
- The authors removed bots from their data and recalculated
- They found that bots increase spread, however both real and fake news were amplified equally

Fake vs. Real news spread

- The authors did an analysis of the text of real and fake news, and found that fake news was consistently more novel under various metrics
- The authors concluded that rather than structural factors or individual characteristics, the greater spread of misinformation comes from individuals being more likely to transmit it
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- More recent work by Meyers et al. (`https://doi.org/10.1007/978-3-030-61841-4_10`) has attempted to exploit these structural differences in rumour cascades to identify fake news
- Furthermore, Meyers et al. found that although the individual cascades may be smaller, true stories tend to have a higher number of cascades, resulting in truth reaching more people overall, and resulting in truth remaining in circulation for longer on Twitter
- So indeed, at least on Twitter, it appears that the steps of truth are sure, if slow and solemn

Recap

- Twitter rumour cascades
- Structural virality

Models of influence spread/diffusion

- One of the most important themes of the text (and CSC303) is that we construct models to gain insight.
  - Our models are often (maybe always) very simplified given the complexity of real social and economic networks.
  - There is always a tradeoff between the adherence to reality and our ability to analyse and gain insight.

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- The main emphasis in Chapter 19 is on a very simple process of diffusion where each person has 2 alternative decisions:
  1. stay with a current "product" B
  2. or switch to a (new) product A.

A simple model of diffusion in a social network

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- A standard example is what messenger application we might choose to use to the extent that we are mostly influenced by our friends rather than by general population wide usage: e.g., do you use SMS, Skype, Zoom, Teamviewer, Jitsi Meet, Microsoft Teams, email, Pidgin (not a pigeon), Slack, Discord, Element, NeoChat, Quercus messages, Snapchat, Steam, Telegram (not a telegram), Instagram Direct, WhatsApp, iMessage, Facebook Messenger, WeChat, Signal, AOL instant messenger, Google Hangouts . . . . What influences you most? Friends or general population information?

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  - Choosing between two weekly television shows that occur at the same time or who to vote for may be other examples, depending on specifics

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- In fact, the model given in this chapter dictates that certain decisions (i.e. to change from B to A) are irreversible.
  - The text calls this a "progressive process" in the sense that it progresses in only one direction. Any good examples of truly (or *essentially*) irreversible decisions?

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  - The text calls this a "progressive process" in the sense that it progresses in only one direction. Any good examples of truly (or *essentially*) irreversible decisions?
  - For example, the decision to get a tattoo.

A threshold model for spread

- We assume that some number of individuals are enticed (at some time $t = 0$) to adopt a new product A.

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- Outside of these “initial adopters”, we assume all other individuals in the network are initially using a different product B (or equivalently this is the first product in a given market).
- This is not really a competitive influence model as B is not really competing. (More comments later.)
- The first model we consider for diffusion is that every node v has a threshold q (in absolute or relative terms) for how many of its neighbors must have adopted product A before v adopts A.

Threshold model (continued)

- For simplicity the text initially assumes that every node v (i.e. individual) in the network has the same threshold but then later explains how to deal with individual thresholds.

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- If the threshold has not been reached then v decides not to adopt A at this time.

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Note

Although it is not explicitly stated, the initial adopters never reverse their adoption.

- Given these model assumptions, adopting A is irreversible for all nodes in the network.

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- This determines a simple coordination game.

$$\begin{array}{cc|c|c|} & & \multicolumn{2}{c}{w} \\ & & A & B \\ \cline{3-4} v & A & a, a & 0, 0 \\ \cline{3-4} & B & 0, 0 & b, b \\ \cline{3-4} \end{array}$$

Figure: $A - B$ coordination [Fig 19.1, E&K]

Coordination game induces threshold

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  - By staying with B, v has payoff

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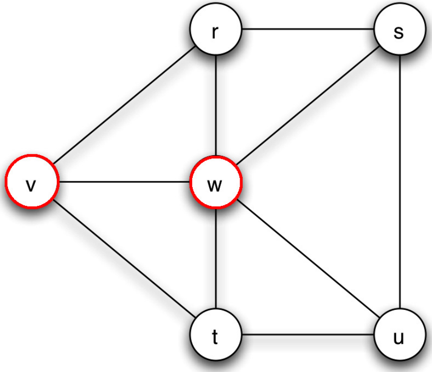
  - Note! This q is *not* $1-p$

Mon. Mar 6: Announcements and Corrections

- Midterm this Friday in tutorial
  - You will be writing the test in the usual tutorial rooms; please go to the tutorial section that you are enrolled in on ACORN
  - Don't forget you can have a handwritten aid sheet, letter size (8.5×11 inches), double-sided
- A1 marks should be out soon, we're working to get those out before the midterm
- The exam date will be released soon
- A2 will be released soon
  - Practice Questions have been released
  - Questions 3, 4 and 6 of the practice questions may help you study for the midterm

The process unfolds (example: $a = 3$ and $b = 2$)

[Fig 19.3, E&K]



$t = 0$

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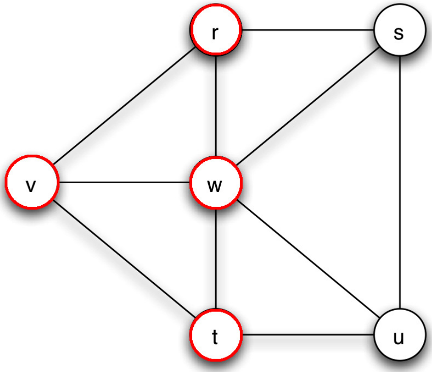
The process unfolds (example: $a = 3$ and $b = 2$)

[Fig 19.3, E&K]



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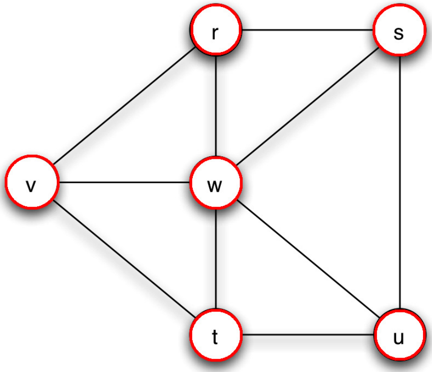

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Complete cascades vs tightly-knit communities (example: $a = 3$, $b = 2$, $q = 2/5$)

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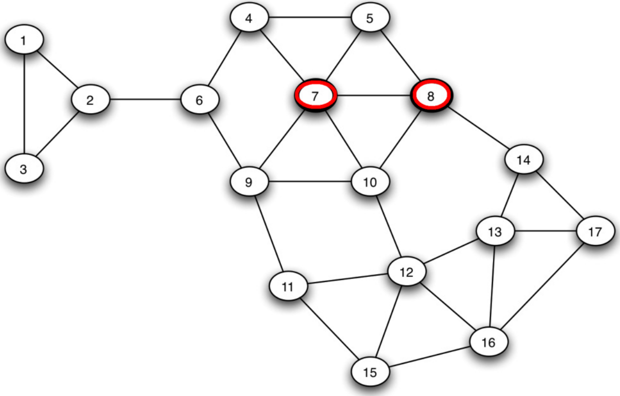
- In the next example, "tightly-knit communities" block the spread.



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[Fig 19.4, E&K]

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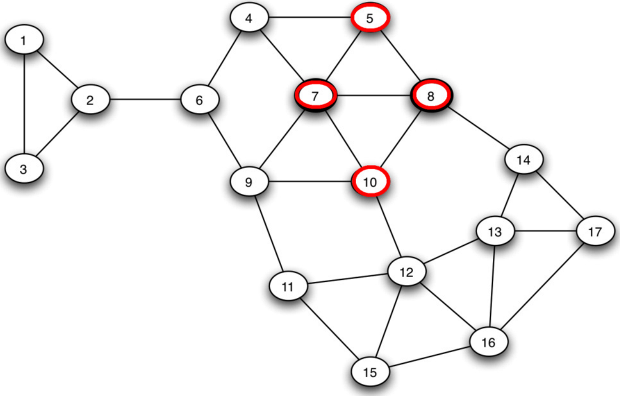

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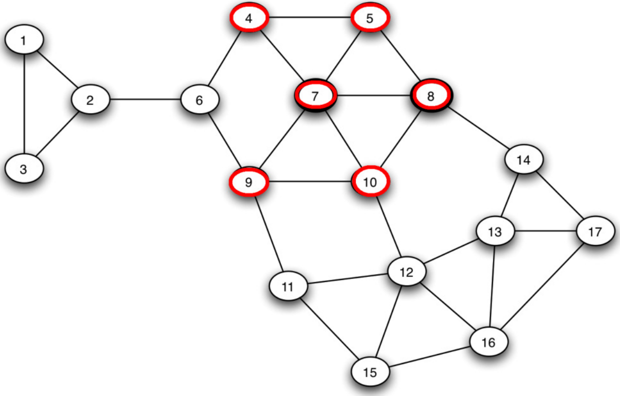

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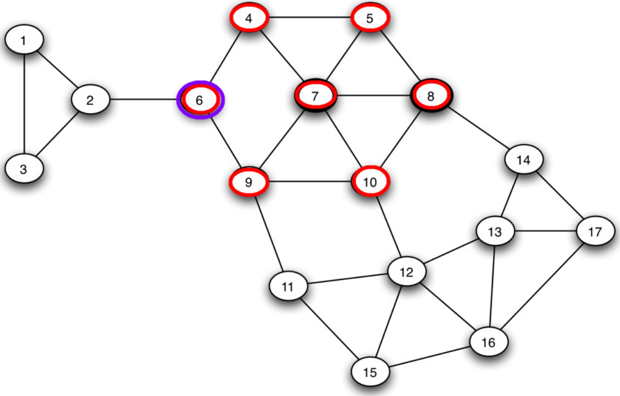

$t = 3$

[Fig 19.4, E&K]

Factors determining the rate and extent of diffusion in a social network

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   - We could also add intrinsic values for A and B to both sides of the above inequality to determine the threshold for v adopting A.

3. The choice of initial adopters.
   - This raises an interesting computational question as to how to select the most influential nodes (within some budgetary constraint).

Defining a tightly-knit community

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Definition

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Aside

- Clustering is a pervasive concept in many fields and contexts (beyond networks).
- It is an intuitive concept that can be defined in many ways.
- There does not appear to be any one definition that is always (or even usually) most preferred.

Clusters at different levels of granularity

- The given definition of a blocking cluster does not imply a unique way of clustering the nodes.

- Indeed if S and T are both clusters of density p, then the union of S and T is a cluster of density p.
  - Note: this is not generally true of the intersection of S and T.

- This clustering definition also implies that the set of all nodes is a cluster of density 1.

Clusters vs complete cascades

- Suppose we have a network threshold spread model with threshold q, an initial set of A adopters I and $V' = V - I$ is the set of nodes that are not initial adopters.

- Then we have the following (provable) intuitive result that characterizes when complete cascades will or will not occur:

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- Then we have the following (provable) intuitive result that characterizes when complete cascades will or will not occur:

  - If V' contains a cluster C of density greater than $1 - q$, then the initial adopters will not cause a complete cascade. Furthermore, no node in C will adopt A.
  - If in a network with threshold q and an initial set I of adopters does not cause a complete cascade, then the non initial adopters nodes $V' = V - I$ must contain a cluster of density greater than $1 - q$.

When nodes have different thresholds

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- If node v has not yet adopted A at time t, but a fraction p of the $d(v)$ neighbours of v have already adopted A, then:
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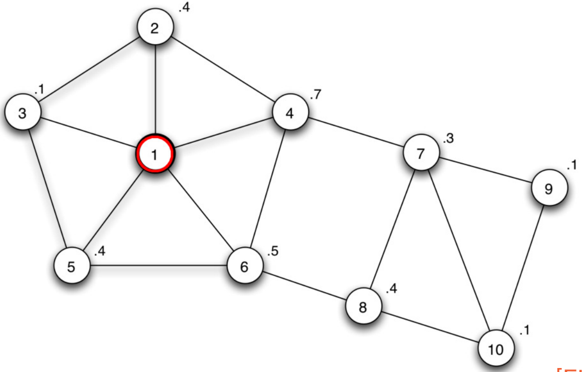
- This is then equivalent to saying that v will switch whenever

$$p \geq \frac{b(v)}{a(v) + b(v)} = q(v)$$

which is then the threshold for node v.

Redefining blocking clusters

- A blocking cluster is now a set of nodes C such that every node $v \in C$ has more than a fraction $1 - q(v)$ of its adjacent nodes in C.
  - Equivalently, every $v \in C$ has less than a fraction $q(v)$ of its adjacent nodes out of C
- It follows (as in the case of homogeneous threshold nodes) that a given set of adopters I in a network will not cause a complete cascade iff $V - I$ contains a blocking cluster C.



[Fig 19.13, E&K]

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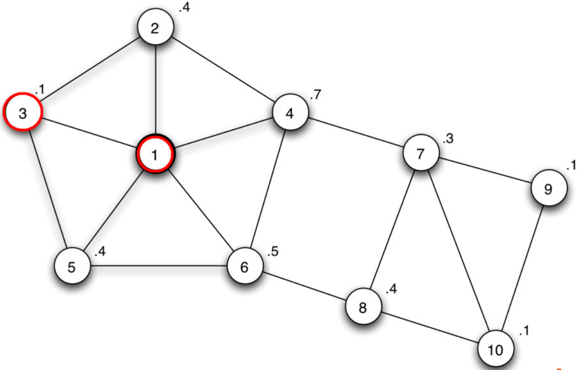

[Fig 19.13, E&K]

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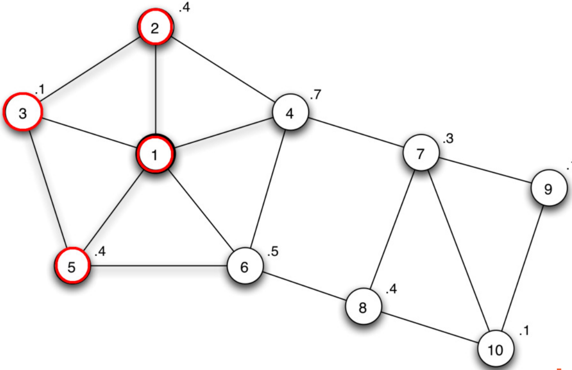

[Fig 19.13, E&K]

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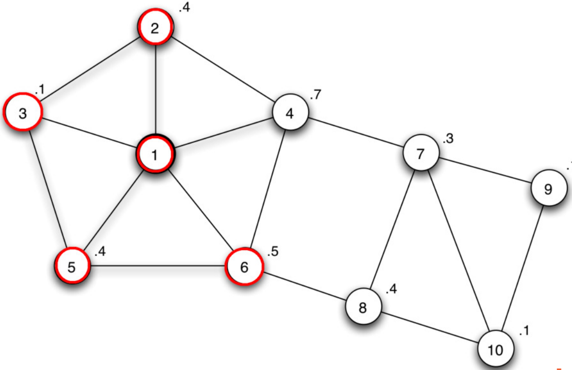

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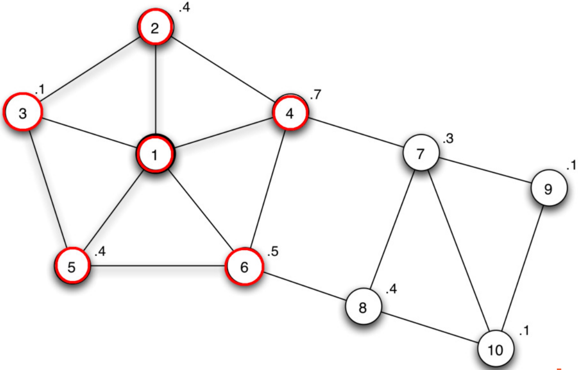

[Fig 19.13, E&K]

$t = 4$

This week's agenda

- Twitter rumour cascades
- Structural virality
- Threshold model
  - Complete cascades
  - Blocking clusters

Further considerations: the "bilingual option"

- In the advanced material (Section 19.7C), the possibility of a third option is considered.
- Here the model allows an individual to maintain both technologies (languages, ideologies, cultural practices) but at a cost c.
- Every individual now can choose to be unilingual (adopting just A or just B) or to be bilingual adopting both (denoted AB).
- The coordination benefit (for each edge) is represented in Figure 19.18. The cost is subtracted from the total benefit over all edges

		w: A	w: B	w: AB
v	A	a, a	$0, 0$	a, a
	B	$0, 0$	b, b	b, b
	AB	a, a	b, b	$(a, b)^+, (a, b)^+$

Figure: A Coordination Game with a bilingual option. Here the notation $(a, b)^+$ denotes the larger of a and b. [Fig 19.18, E&K]

Choosing influential adopters

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- Even in this simple model of (non-competitive) influence spread, and even if we have complete knowledge of the social network, it is not at all clear how to chose an initial set of adopters so as to achieve the largest spread.

- Furthermore the spread process could be much more sophisticated.
  - For example, adoption by a node might be a more random process (say adopting with some probability relative to the nodes threshold) and maybe the influence of neighbors first increases and then decreases over time.

Choosing influential adopters continued

- Suppose we have funds/ability to influence k nodes to become initial adopters.
  - We can try all possible subsets of the entire $n = |V|$ nodes and for each such subset simulate the spread process.
  - But clearly as k gets larger, this "brute force" becomes prohibitive for large (and not even massive) networks.

- It turns out that the problem of the optimum set of initial adopters in many settings is an NP-hard problem.

Can we determine a “good” set of initial adopters?

- For even simple models of information spread similar to those being discussed here, it can be computationally difficult (NP-Hard) to obtain an approximation within a factor n^c for any $c < 1$.

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- Instead we will identify properties of a spread process that will allow a good approximation: a good set of initial adopters that will do “almost as well” as the best set.

Note: What follows is a discussion as to how to choose a set of initial adopters by a relatively efficient approximation algorithm when making some assumptions on the spread process. However, we would need much more efficient methods for massive networks.

Influence maximization models; monotone submodular set functions

- Some spread models have the following nice properties.

For any initial set of adopters, S, let $f(S)$ be size (or more generally a real value benefit since some nodes may be more valuable) of the final set of adopters. Furthermore, let f satisfy:

1. Monotonicity: $f(S) \leq f(T)$ if S is a subset of T
2. Submodularity: $f(S+v) - f(S) \geq f(T+v) - f(T)$ if S is a subset of T

- We also usually assume that $f(\varnothing) = 0$. Such normalized, monotone, submodular functions arise in many applications.

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- We also usually assume that $f(\varnothing) = 0$. Such normalized, monotone, submodular functions arise in many applications.
- The simple threshold examples considered thus far are monotone processes but are not submodular in general. Are these contrived worst case network examples?
- Some variants of the threshold model and related models do satisfy these properties. Next week, we consider two such stochastic models.

Recap

- Twitter rumour cascades
- Structural virality
- Threshold model
  - Complete cascades
  - Blocking clusters
  - A first look at selecting initial adopters