Social and Information Networks

University of Toronto CSC303
Winter/Spring 2023

Week 12: April 3-7

Today's agenda

- Now without the least excuse or the slightest provocation, I'll fondly introduce (for your mental delectation), the third annual X-treme CSC303 review-and/or-additional-topics bingo! (and also Jeopardy)

Rules

- The rules are simple! Just download the list of bingo spots from Quercus, paste the list into `https://jie-luo.github.io/BingoMaker/`, and generate your card!

  Exciting and *dynamic* demo here!

Rules

- As on Jeopardy, there is a grid of questions with various monetary values – a random person will be selected to chose & answer (or skip) a question, and the first person to answer correctly gets the points
- OR, for 50 points you can instead hear highlights from one of last year's critical review papers
- OR, for 250 points you can briefly talk about your own critical review paper!

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- At the same time, if the answer is on your bingo card (or if you can find a connection with either the answer or the paper), then you can fill the space on your card
  - Be creative with connections! :)
- At the end of the day, we'll have winners for Jeopardy and Bingo
  - If you claim Bingo, you'll have to explain your spaces

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Rules

- Feel free to form teams!

EXCITING prizes!

- But what good is a game without prizes? So what can our lucky winners look forwards to?

EXCITING prizes!

- But what good is a game without prizes? So what can our lucky winners look forwards to?

Figure: Exciting prizes! Also, unnatural eyes, hands and arms!

- Nothing but the finest weird attribution free images, emailed directly to your home! (or not, I mean, do you really want images like that one on your hard drive? Ugh, it gives me the shivers)

That grid thing that always shows up in these (only now there's two of them, but not really because the second's a list)

The grid of questions:

Triangles, Triangles, Triangles!	Equilibria	Movin' through a network	????
100 (25)	**100** (37)	**100** (48)	**100** (59)
200 (27)	**200** (39)	**200** (50)	**200** (61)
300 (29)	**300** (41)	**300** (52)	**300** (63)
400 (31)	**400** (43)	**400** (54)	**400** (65)
500 (34)	**500** (45)	**500** (56)	**500** (67)

For 50 points (or 500 if you get your own): Chose a paper, any paper!

Paper #1	Paper #2	Paper #3	Paper #4	Paper #5	Paper #6
(8)	(10)	(12)	(15)	(17)	(21)

PRIZES: B1 (70) B2 (71) B3 (72) J (73)

Paper # 1: The effects of balance on social fragmentation

- https://doi.org/10.1098/rsif.2020.0752
- Interesting paper, looks at finding a consistent explanation for increased connectivity between people (e.g., via the internet), and increased formation of isolated communities

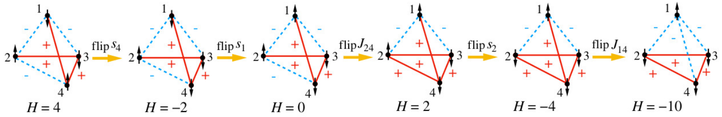


Figure 2. Example for how social stress, H, is gradually lowered in a sequence of changes of opinions and links between four individuals (assuming $g = 1$ for simplicity). Opinions of the nodes are given by ↑ and ↓, positive links are red, negative are blue. In the course of this sequence, the number of balanced triangles changes from two to four. Note that in the second step social stress is temporarily increased. This is a consequence of the stochastic nature of the model, where also unfavourable events happen from time to time.

- Create a model with two opinions, edges can change sign or people can change opinion
- System changes over time to reduce total social tension
  - A combination of unstable triangles, and friends with opposite opinions
  - Therefore model creates stable triangles, and through social influence attempts to create homophily

Paper # 1: The effects of balance on social fragmentation

The authors ran their model on various graphs where all nodes have degree k.
They found that for a fixed amount of randomness T in the model, then there is a sharp threshold where on one side the graph is sharply fragmented, and on the other it is not.

(7)

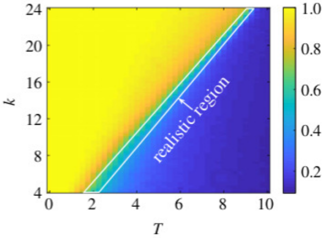


Figure 3. Phase diagram of the stochastic coevolutionary model with social balance. The balance level, f, is shown as a function of the average network degree, k, and social temperature, T. A phase separation line is visible. Below it, for low values of connectivity and high T, there exits a socially coherent phase (blue), above the line there is a phase of social fragmentation (yellow). Empirically reasonable values of f around 0.7 are indicated with a white box. Results were obtained for regular graphs ($\epsilon = 0$), $g = 1$, $N = 400$ and are averaged over 500 realizations. Random initial conditions in links and opinions.

Paper # 2: Social network analysis: Characteristics of online social networks after a disaster

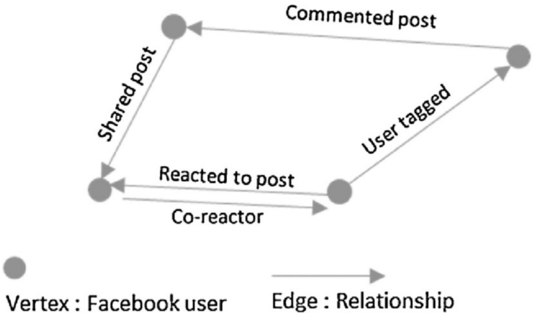
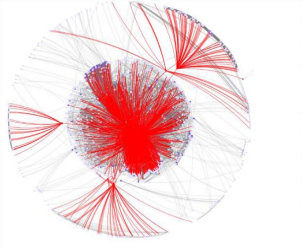


Fig. 4. Illustration of users and relationship.

- `https://doi.org/10.1016/j.ijinfomgt.2017.08.003`
- Authorized analysed Facebook communication patterns during the 2016 Louisiana flood
  - Specifically, interactions with the city of Baton Rouge's Facebook page
  - Network was filtered to only include information relevant to the flood

Paper # 2: Social network analysis: Characteristics of online social networks after a disaster

Table 6
Top 10 betweenness centrality and degree distribution.
Red-colored lines are all edges linked with top 10 betweenness centrality vertices excluding CBR.



Users	Betweenness
CBR	1,082,218
Organization#1	166,500
Organization#2	81,549
Organization#3	49,982
User#19	31,711
User#20	29,937
User#21	26,838
Organization#4	22,838
Organization#5	20,572
User#22	13,791

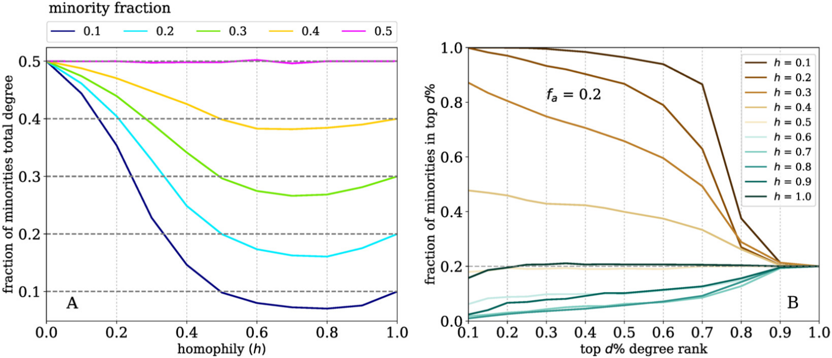
- Organizations have high betweenness centrality (total betweenness going through a node, where betweenness is as in Girvan-Newman algorithm), and function as gatekeeper nodes
- However, with the exception of the local government, the nodes of high out-degree tend to be people (7)

Paper # 3: Homophily influences ranking of minorities in social networks

- https://doi.org/10.1038/s41598-018-29405-7
- Very nicely done paper, studies relationship between homophily (or it's opposite, heterophily) and the degree of nodes
- The authors modified the preferential-attachment model with a homophilily parameter, $h \in [0, 1]$ ($h < 0.5$ is heterophily, and $h > 0.5$ is homophily)

Paper # 3: Homophily influences ranking of minorities in social networks

- Their simulations suggest that moderate homophily causes degree under-representation, both in total degree of nodes in the minority and and the proportion of the highest ranking nodes that are in the minority
- Strong homophily causes poor connectivity with the opposite group
- In contrast, heterophily benefits the minority group



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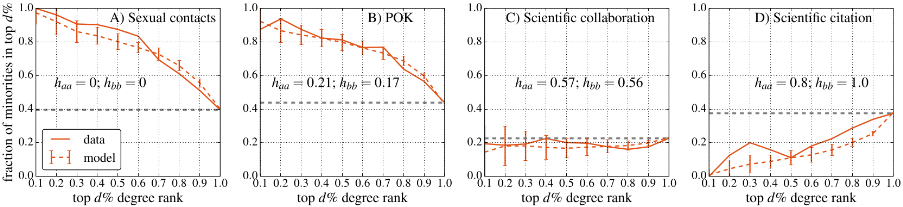
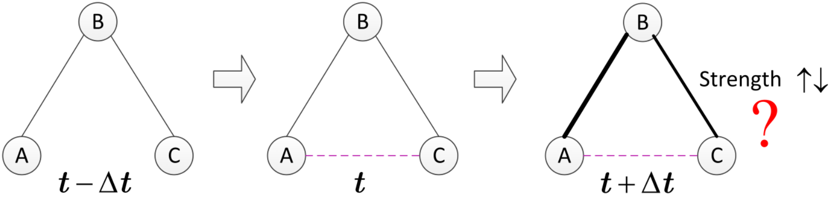


- Model predictions compare favourably with real-life heterophilic networks (dating networks), and homophilic networks (scientific citations between research topics) (7)

Paper # 4: Will Triadic Closure Strengthen Ties in Social Networks?

- https://doi.org/10.1145/3154399
- We've previously talked about the interaction between tie-strength and triadic closure, but this paper seeks to go further by quantifying tie-strength



Paper # 4: Will Triadic Closure Strengthen Ties in Social Networks?

- The authors study two who-talks-to-whom networks
  - anonymized cellphone data (tie strength being number of calls)
  - Weibo data (tie strength being number of comments or retweets)
- Average tie-strength tends to increase over time
- However, in $\approx 80\%$ of cases the original ties weaken

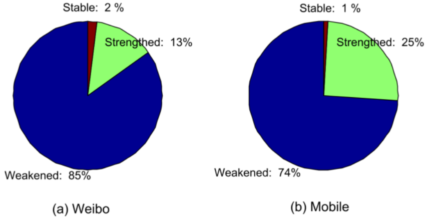


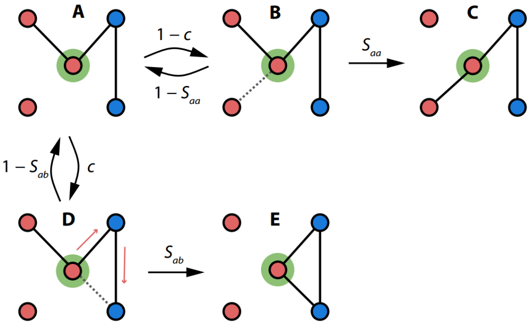
Fig. 3. Tie strength dynamics after triadic closure in social networks. In Weibo, ties in about 85% triads have been weakened, while in Mobile ties in 74% triads have been weakened.

- Demographics have a strong impact; all-male triads and celebrity triads are more strongly affected
- Men and celebrities are also more likely to maintain weaker triads (7)

Paper # 5: Cumulative effects of triadic closure and homophily in social networks

- This paper looks at the causes of homophily
  - Homophily due to selection (which they call *choice homophily*)
  - Homophily due to restrictions in who you have contact with (which they call *induced homophily*)
- The authors study the question of whether triadic closure can drive *induced homophily*

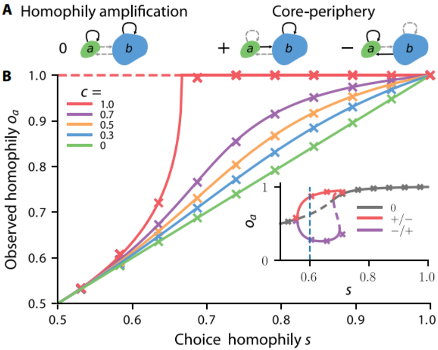
Paper # 5



- The model selects a focal node uniformly at random
  - with probability c it attempts to close a triangle
  - with probability $1 - c$ it attempts to form a friendship with a random node
  - The proposal succeeds or fails with a probability determined by the endpoints' group
  - On success, the edge forms and a pre-existing edge connected to the focus is deleted

Paper # 5

- The model tends to one of two types of stable configuration (illustrated in A), homophily amplification or core-periphery
- We can see that as the probability of triadic closure, c, increases (line colour), the observed homophily is amplified from the choice homophily
- Observed homophily: proportion of neighbours of the same group
- Choice homophily: s is the probability of accepting a member of the same group, and $1 - s$ is the probability of accepting an opposite member
- Results consistent with datasets



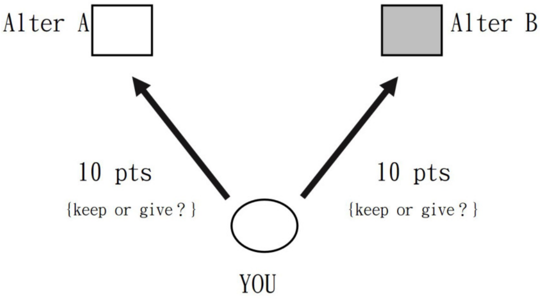
(7)

Paper # 6: Triadic balance in the brain: Seeking brain evidence for Heider's structural balance theory

- https://doi.org/10.1016/j.socnet.2020.05.003
- This paper attempted to find physical evidence of mental stress arising from unstable triangles
- Participants where placed in an fMRI machine, and played a 3-player variant of the prisoner's dilemma for money
  - More points = More money

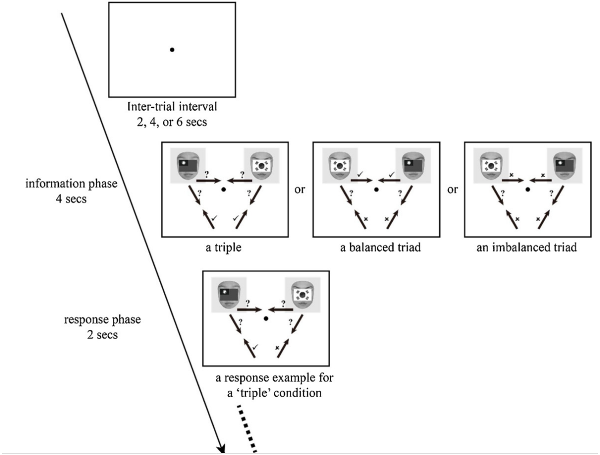
Paper # 6: Triadic balance in the brain: Seeking brain evidence for Heider's structural balance theory

- There are 3 players, they can defect or cooperate, the best outcome is when all players cooperate
- You can chose to give alters A and/or B 10 points, in which case they receive 20 points
- The alters also decide whether to keep or give points



Paper # 6:

- Participants where shown a triangle, and given the choice to change their decisions



Paper # 6:

- Participants have a weak, but significant, preference to change to balanced triads

Table 1
Participants' decisions of triads given a specific initial triadic scenario.

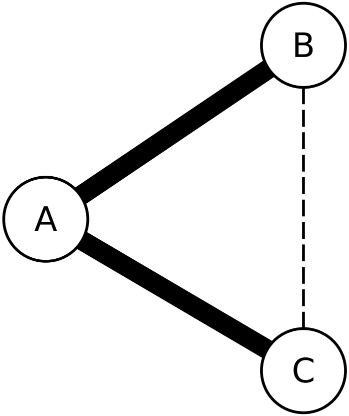
Decision Stimuli	Balanced Triads	Unbalanced Triads
Balanced Triads ($N = 2083$)	1470 (71 %)	613 (29 %)
Unbalanced Triads ($N = 2042$)	721 (35 %)	1321 (65 %)

- "Among the active brain areas ... two of them were significantly correlated with participants' probability of settling on unbalanced triads: the left superior frontal gyrus ... and the right supramarginal gyrus ... intensity of activation of the two brain areas in unbalanced triads is even stronger for people who have stronger preferences for structural imbalance. The result suggests not only that cognitive dissonance occurs when people are confronted with structural imbalance, but also that it intensifies when people choose to settle on structural imbalance." (7)

Triangles, Triangles, Triangles!

Triangles, Triangles, Triangles! for 100

- Does this graph satisfy strong triadic closure?



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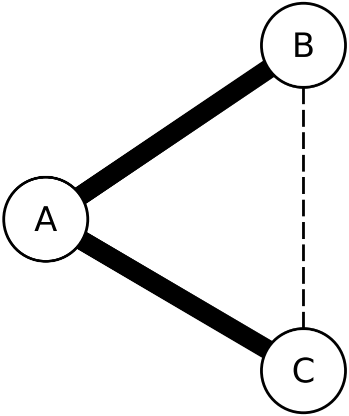

- Yes, it only has the strong edges (A,B) and (A,C). STC implies that we should have the edge (B,C), and indeed it is present

Triangles, Triangles, Triangles! for 100

- From the bounty of "triangle" search results, your prize is...

Triangles, Triangles, Triangles! for 100

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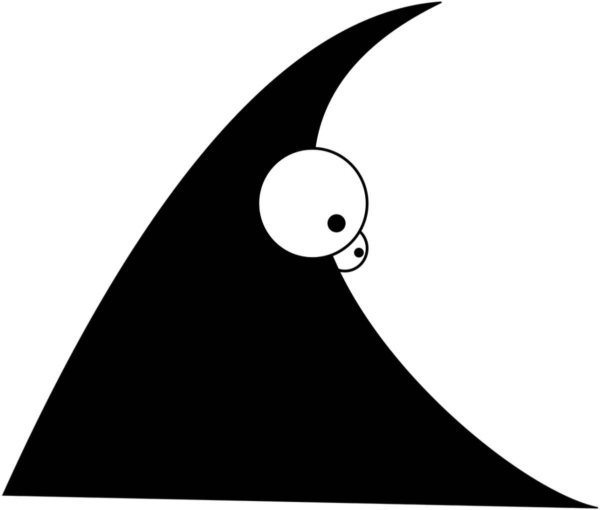

- absolutely adorable! Can we keep it? ;) (7)

Triangles, Triangles, Triangles! for 200

- What are the three types of closure in a social-affiliation network?

Triangles, Triangles, Triangles! for 200

- What are the three types of closure in a social-affiliation network?
  1. Triadic Closure: Two people become friends, because of a mutual friend
     - ★ May correspond to either selection, or influence
  2. Focal Closure: Two people become friends, because they are both members of the same focus
     - ★ Corresponds to selection
  3. Membership Closure: A person joins a focus, because they are friends with a member
     - ★ Corresponds to influence

Triangles, Triangles, Triangles! for 200

- For getting this question right, you're prize is...

Triangles, Triangles, Triangles! for 200

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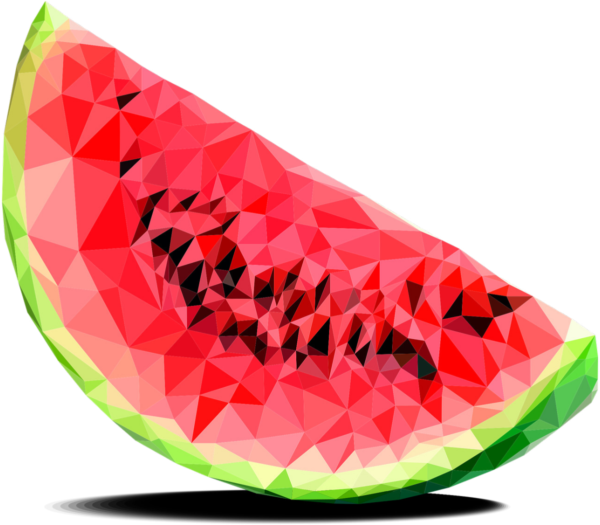
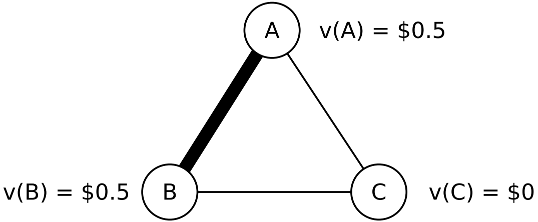


- Polygonal watermelon! (Caution: sharp seeds) (7)

Triangles, Triangles, Triangles! for 300

- In class we saw bargaining networks, in which adjacent nodes can enter a deal to split \$1 between themselves, and each node can only make a deal with one of its neighbours
- Is the following outcome stable?



Triangles, Triangles, Triangles! for 300

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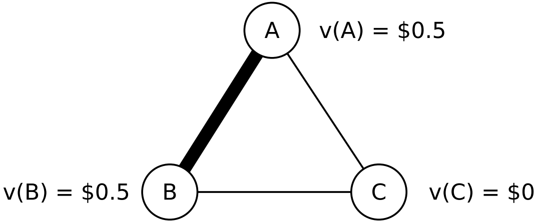

- No, a matching is stable iff there is no pair of nodes which can enter a deal that strictly improves both of their payouts
  - Node *C* can offer node *A* \$0.60
- More generally, we saw in class that this bargaining network has no stable solution

Triangles, Triangles, Triangles! for 300

- How have the triangles rewarded you? You're the proud owner of...

Triangles, Triangles, Triangles! for 300

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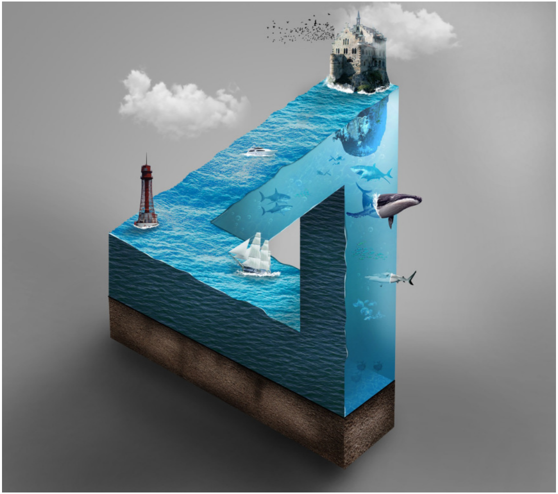


- A maritime-themed optimal illusion! May contain small parts unsuitable for children, spare whales sold separately (7)

Triangles, Triangles, Triangles! for 400

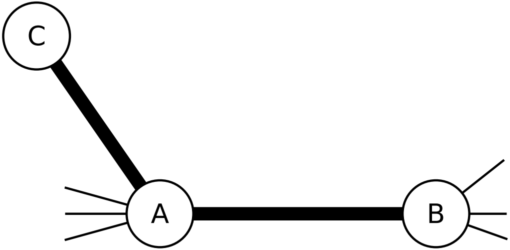
- Recall from class, that Strong Triadic Closure (STC) requires that for any strong edges (A, B) and (B, C), then there must exist the edge (A, C)
- What is the connection between STC and inter-community edges?

Triangles, Triangles, Triangles! for 400

- Recall from class, that Strong Triadic Closure (STC) requires that for any strong edges (A, B) and (B, C), then there must exist the edge (A, C)
- What is the connection between STC and inter-community edges?
- At a high level, STC tells us that inter-community edges tend to be weak

Triangles, Triangles, Triangles! for 400

- More rigorously, we proved that for any node A with at least 2 strong edges, then any local bridge adjacent to A must be weak
  - Recall that an edge is a local bridge if, upon the removal of the edge, the shortest path between the endpoints is of length at least 3 (or no such path exists)



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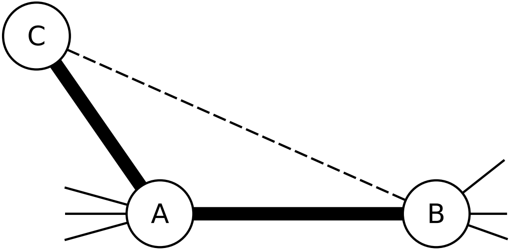

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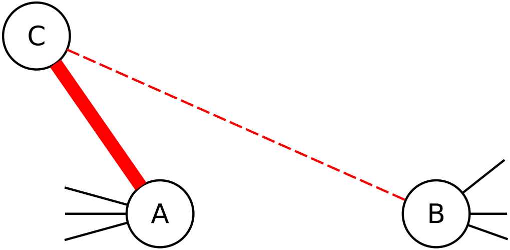


Triangles, Triangles, Triangles! for 400

- Congratulations, you've just won a brand new...

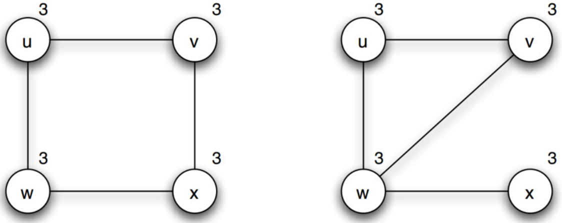
Triangles, Triangles, Triangles! for 400

- Congratulations, you've just won a brand new...

- Industrialized ancient Egypt? Remodeled Louvre? Well sue me, *I* think it looks pretty anyhow (7)

Triangles, Triangles, Triangles! for 500

- In class we saw a revolt model in which nodes have a threshold t and will revolt if they know that t nodes (including themselves) will revolt
- Why is it impossible for a revolt to occur in b), yet a revolt will occur if we create the triangle in c)?



(b) *An uprising will not occur* (c) *An uprising can occur*

[Modified from 19.14 of E&K]

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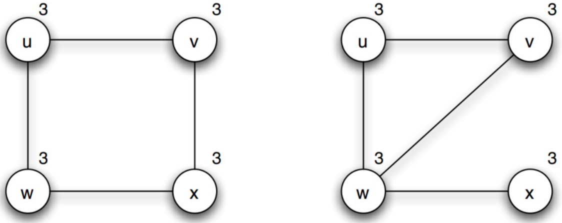

(b) *An uprising will not occur* (c) *An uprising can occur*

[Modified from 19.14 of E&K]

- The triangle in (c) makes willingness to revolt shared knowledge, whereas in (b) no node knows that it's neighbours know that there enough willing people to meet their thresholds

Triangles, Triangles, Triangles! for 500

- With a search term of "triangles", you are the lucky winner of...

Triangles, Triangles, Triangles! for 500

- With a search term of “triangles”, you are the lucky winner of...

- An antique, highly concerned (and concerning) cow-penguin! May you cherish it always. (7)

Equilibria

Equilibria for 100

- Consider the Schelling model with a threshold of $t = 3$
- What does an equilibrium in this model look like?

Equilibria for 100

- Consider the Schelling model with a threshold of $t = 3$
- What does an equilibrium in this model look like?
- Each node (either of class A or B) has at least 3 neighbours of the same class
- The Schelling model tends to converge to highly segregated solutions, this homophily is driven purely by (relatively mild) local choices

Equilibria for 100

- And your prize is...

Equilibria for 100

- And your prize is...

- “airship”! Methinks having sails perpendicular to propellers might not be the best arrangement, but at least the piece looks nice :) (7)

Equilibria for 200

- Recall that for a game, the price of anarchy (POA) is defined as $\frac{cost(S)}{cost(OPT)}$ where *cost* is the social cost of a solution to the game, S is the worst Nash equilbrium for the game, and OPT is the optimal solution to the game
- In a congestion network, *cost* is the total ammount of time people spend commuting
- From class we know that the price of anarchy in a congestion network with linear cost functions is at most $\frac{4}{3}$
- Given that such a congestion network has an optimal social cost of 900 minutes, then what is the range of values that the real social cost would be in? (Hint: assume real traffic converges to the Nash Equilibrium)

Equilibria for 200

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- Given that such a congestion network has an optimal social cost of 900 minutes, then what is the range of values that the real social cost would be in? (Hint: assume real traffic converges to the Nash Equilibrium)
- The cost would be at least 900, and at most $900 \times \frac{4}{3} = 1200$

Equilibria for 200

- With a search term of "equilibria", you are the lucky winner of...

Equilibria for 200

- With a search term of “equilibria”, you are the lucky winner of...

- Disco Justice! (7)

Equilibria for 300

- Recall the stable marriage problem from lecture
- What is the male-pessimal solution, and how can it be found?

Equilibria for 300

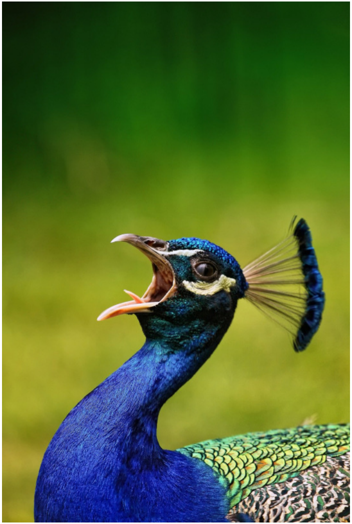
- Recall the stable marriage problem from lecture
- What is the male-pessimal solution, and how can it be found?
- Recall that an outcome to the stable marriage problem is a matching between women W and men M
  - An outcome is stable iff it contains no blocking pair (m, w) where both prefer the other member of the blocking pair over their current match
- The male-pessimal solution is the outcome where each man is matched with their least-preferred woman, out of all woman that the man can be matched to in a stable matching
- The male-pessimal solution is stable, and can be found using the Gale-Shapely algorithm
  - Specifically, FPDA (Female Proposing, Deferred Acceptance)

Equilibria for 300

- Your prize for getting this question right is...

Equilibria for 300

- Your prize for getting this question right is...



- The strangest looking picture of a peacock I've ever seen! (7)

Equilibria for 400

- Recall that the hubs and authorities algorithm converges to an equilibrium, in which further updates no longer affect the value
- Given the vector of authority values at time t, $\mathbf{a}_t$, what is the matrix M such that we can compute our updated hub values as $\mathbf{h}_{t+1} = M\mathbf{a}_t$

Equilibria for 400

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- Given the vector of authority values at time t, $\mathbf{a}_t$, what is the matrix M such that we can compute our updated hub values as $\mathbf{h}_{t+1} = M\mathbf{a}_t$
- M is the adjacency matrix of the graph
  - i.e., M_{ij} is 1 iff we have a hyperlink on page i pointing to page j

Equilibria for 400

- With a search term of “equilibria”, you can now possess...

Equilibria for 400

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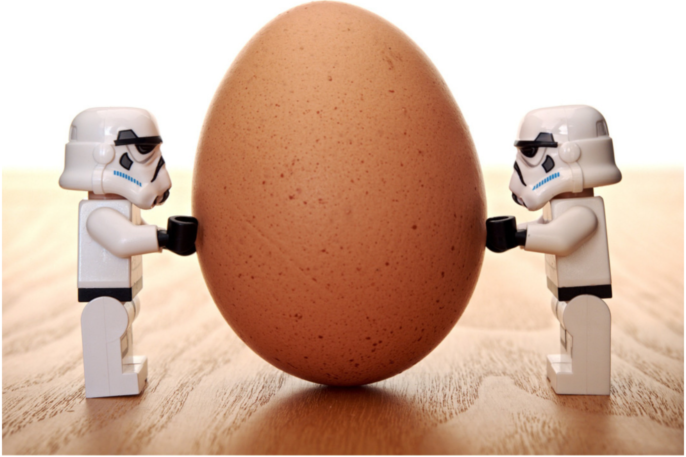


- a pair of Lego stormtroopers making a *very* large breakfast (7)

Equilibria for 500

- What information can we extract from the eigenvalues and eigenvectors of the signed Laplacian matrix of a graph, G?

Equilibria for 500

- What information can we extract from the eigenvalues and eigenvectors of the signed Laplacian matrix of a graph, G?
- If we let λ_1 to be the smallest eigenvalue, and x be a corresponding eigenvector, then:
  - If $\lambda_1 = 0$ then the graph is completable to a strongly balanced network
    - ★ If none of x's elements are zero (which will be the case if G is connected), then G is completable to two communities comprised of the positive and negative elements of x respectively
  - Otherwise, smaller values of λ_1 indicate that the graph is closer to being completable to a strongly balanced network

Equilibria for 500

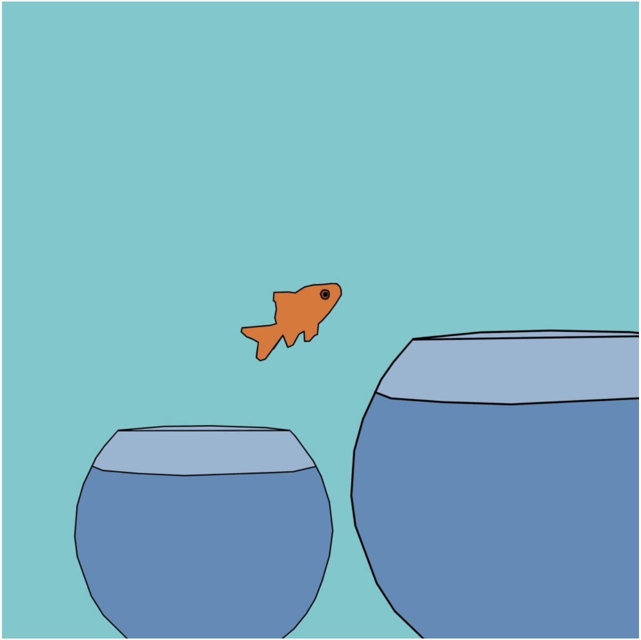
- For your marvelous memory of the matrix, your prize is...

Equilibria for 500

- For your marvelous memory of the matrix, your prize is...

- the Lego stormtrooper's circus act! (7)

Movin' through a network



Movin' through a network for 100

- What is the basic reproductive number, R_0? (Hint: we covered this in the section on disease spread)

Movin' through a network for 100

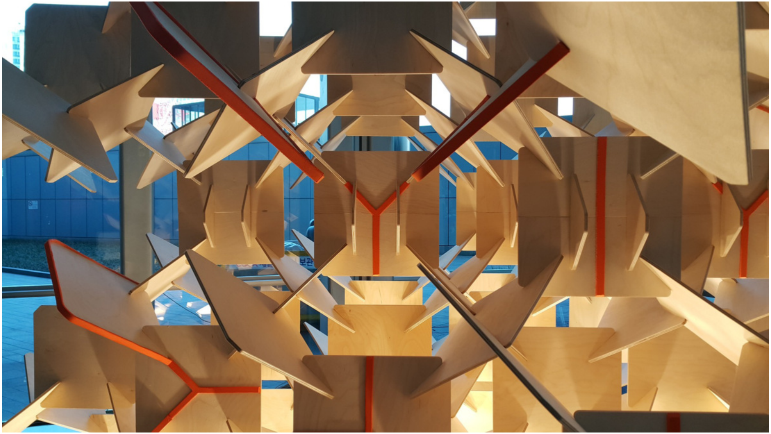
- What is the basic reproductive number, R_0? (Hint: we covered this in the section on disease spread)
- R_0 is the expected number of new cases created by one case of the disease
- In a branching process where each person is connected to k new individuals and the disease has a probability p of spreading, then $R_0 = pk$

Movin' through a network for 100

- With that, you now have...

Movin' through a network for 100

- With that, you now have...



- Your own... uh... hmmm. Well, it's pretty in a headache inducing kind of way anyways. (7)

Movin' through a network for 200

- Describe the unscaled PageRank update

Movin' through a network for 200

- Describe the unscaled PageRank update
- Each node's rank is evenly divided among it's outgoing edges
  - Nodes with no outgoing edges keep all of their rank
- Recall that in the limit, this update is equivalent to looking at proportion of time spend in each node by a random walk on the graph

Movin' through a network for 200

- With that, you have the dubious privilege of taking home with you...

Movin' through a network for 200

- With that, you have the dubious privilege of taking home with you...

- The march of the bananas! (7)

Movin' through a network for 300

- Recall the modified Watts-Strogatz model from class:
  - where every node u is on a grid
  - u is connected to its neighbours
  - u has a random long-distance edge to some node v
  - v is randomly chosen with probability proportional to $d(u, v)^{-2}$, where d is the grid distance
- Recall also decentralized search, where a node tries to pass a message to a target node t by passing it to neighbour with the lowest grid distance to t
- How does the length of the average path length grow with n?

Movin' through a network for 300

- Recall the modified Watts-Strogatz model from class:
  - where every node u is on a grid
  - u is connected to its neighbours
  - u has a random long-distance edge to some node v
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- Recall also decentralized search, where a node tries to pass a message to a target node t by passing it to neighbour with the lowest grid distance to t
- How does the length of the average path length grow with n?
- $O(\log n)$ from lecture
  - We covered a proof of the 1D case in tutorial, which was $O((\log n)^2)$

Movin' through a network for 300

- With a search term of "decentralized", you're the proud owner of...

Movin' through a network for 300

- With a search term of "decentralized", you're the proud owner of...

- An evaporating physical Bitcoin – Bitcoin smoke, don't breath this! (7)

Movin' through a network for 400

- In class we saw that we can have efficient decentralized search in a modified Watts-Strogatz model by selecting v, the endpoint of the long-distance edges of a node u with probability proportional $d(u, v)^{-2}$
- In practice however, this doesn't work as human population is not uniformly dense
- What where the two alternatives to distance that we saw in class?

Movin' through a network for 400

- In class we saw that we can have efficient decentralized search in a modified Watts-Strogatz model by selecting v, the endpoint of the long-distance edges of a node u with probability proportional $d(u,v)^{-2}$
- In practice however, this doesn't work as human population is not uniformly dense
- What where the two alternatives to distance that we saw in class?
- The alternatives we saw were:
  - Rank
    - How many people are closer
    - We have efficient decentralized search with $r_u(v)^{-1}$
  - Social Distance
    - The size of the smallest shared social focus
    - We have efficient decentralized search with $s(u,v)^{-1}$

Movin' through a network for 400

- For getting this question right, your prize is...

Movin' through a network for 400

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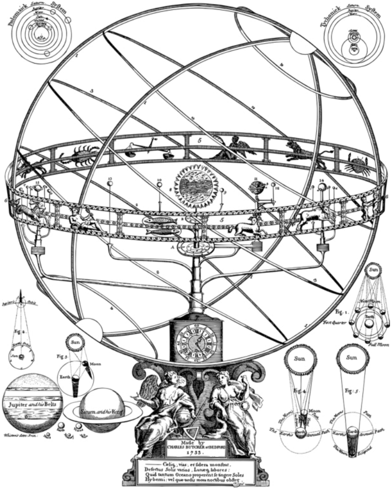

- A diagram of an armillary sphere! (7)

Movin' through a network for 500

- In class we covered various models for the diffusion of an idea or product through a network
- This led to the problem of finding an optimal initial set of adopters
- Given $f(S)$ (the expected final number of adopters given the initial set of adopters, S), we can find a candidate solution by a greedy marginal improvement method:
  - We start with $S = \varnothing$, and repeatedly add the node that causes the greatest increase in f
- What were the properties of f that we saw are sufficient to ensure a constant approximation ratio of $1 - 1/e$?

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- What were the properties of f that we saw are sufficient to ensure a constant approximation ratio of $1 - 1/e$?
- f must be:
  1. Normalized: $f(\varnothing) = 0$
  2. Monotone: $\forall S, T \subseteq V : S \subseteq T \implies f(S) \leq f(T)$
  3. Submodular (i.e. diminishing returns):

$$\forall S, T \subseteq V : \forall v \in V : S \subseteq T \implies f(S \cup \{v\}) - f(S) \geq f(T \cup \{v\}) - f(T)$$

Movin' through a network for 500

- For your exemplary memory and dedication, you shall receive...

Movin' through a network for 500

- For your exemplary memory and dedication, you shall receive...

- a picture of a portrait of the Right Honourable Pug-face himself! (7)

????

???? for 100

- In class, we demonstrated that processes driven by _________, often result in power laws

???? for 100

- In class, we demonstrated that processes driven by ________, often result in power laws
- Popularity
- More specifically:
  - We demonstrated that power laws have to emerge from correlated individual decisions
  - We saw examples where power laws emerge, that could be partially explained by popularity (e.g., sizes of cities, book sales, website in-links)
  - We analyzed the preferential attachment model, which is a model driven by popularity which produces power laws

???? for 100

- What "????" of your own have you won? Well...

???? for 100

- What "????" of your own have you won? Well...

- You've won your very own train hypnotist! (don't make eye contact and keep away from hypnotoads) (7)

???? for 200

- What is the clustering coefficient of a node? You can provide a mathematical definition, or a description

???? for 200

- What is the clustering coefficient of a node? You can provide a mathematical definition, or a description
- For a node A, the clustering coefficient is the probability that any randomly chosen pair of friends of A are themselves friends. Equivalently, it is the extent of triadic closure, measured as the proportion of possible triangles that are closed.
  For undirected edges E then:

$$\mathrm{cc}(A) := \frac{|\{(u,v) \in E | u, v \in N(A), u \neq v\}|}{|\{\{u,v\} | u, v \in N(A) : u \neq v\}|} \in [0,1]$$

- Recall that a low clustering coefficient can be suggestive of mental stress, as show in the Bearman & Moody study from week 1

???? for 200

- For getting this question right, you'll get to take home with you nothing but the finest result from searching "????"...

???? for 200

- For getting this question right, you'll get to take home with you nothing but the finest result from searching "????"...

- your very own bundle of raw asparagus! (now with strawberries and flowers). Enjoy! (7)

???? for 300

- Recall the Wright-Fisher model that we analyzed when studying at mitochondrial Eve
- Suppose that we have 10 single-cellular organisms in a petri-dish, each of which reproduces asexually (more specifically, by mitosis)
- A classmate proposes applying the Wright-Fisher model, and using it to argue that eventually all organisms in the dish will be the same species. What are some possible concerns with applying the Wright-Fisher model?

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- Recall the Wright-Fisher model that we analyzed when studying at mitochondrial Eve
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- A classmate proposes applying the Wright-Fisher model, and using it to argue that eventually all organisms in the dish will be the same species. What are some possible concerns with applying the Wright-Fisher model?
- Some possible concerns are that the population will not be static (it will probably follow some sort of geometric progression), and some species may be better suited to the environment than others
- To apply the model, we would have to address these points

???? for 300

- Congratulations! You've won...

???? for 300

- Congratulations! You've won...

- What appears to be a stained glass window of a rooster surrounded by some *rather* grouchy teddy-bears! (7)

???? for 400

- Briefly describe the Rozenstein algorithm for minimizing weak edges, while maintaining strong connectivity of provided communities

???? for 400

- Briefly describe the Rozenstein algorithm for minimizing weak edges, while maintaining strong connectivity of provided communities
- The algorithm considers the edge causing the most STC violations, and makes it weak if it can do so without disconnecting one of the provided communities – once considered an edge will not be considered again

???? for 400

- For getting this question right, your prize is...

???? for 400

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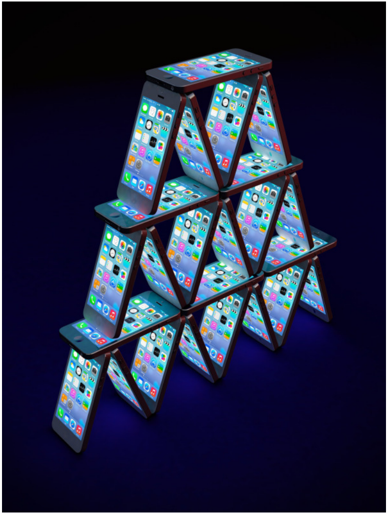

- a house of cards made of iphones! (7)

???? for 500

- Recall that we consider an arbitrary graph to be balanced iff it can be completed into a complete balanced graph
  - A complete graph is balanced if all triangles are balanced (either 3 or 2 positive edges)
- What is the compact characterization of whether an arbitrary graph is balanced? (Hint: it's closely related to the characterization of whether or not a graph is bipartite)

???? for 500

- Recall that we consider an arbitrary graph to be balanced iff it can be completed into a complete balanced graph
  - A complete graph is balanced if all triangles are balanced (either 3 or 2 positive edges)
- What is the compact characterization of whether an arbitrary graph is balanced? (Hint: it's closely related to the characterization of whether or not a graph is bipartite)
- An arbitrary graph is strongly balanced if it contains no cycles with an odd number of negative edges

???? for 500

- Brief proof:
  - By Harary Balance theorem, a balanced complete graph has no negative edges, or we can be divided into two camps (X and Y) which are internally positive, and dislike eachother
  - If the graph is completable then the completed graph contain no cycles with an odd number of negative edges. Therefore, the original cannot

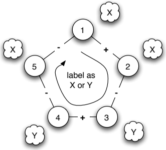

Figure 5.10: If a signed graph contains a cycle with an odd number of negative edges, then it is not balanced. Indeed, if we pick one of the nodes and try to place it in X, then following the set of friend/enemy relations around the cycle will produce a conflict by the time we get to the starting node.

  - If the graph doesn't contain a cycle with an odd number of negative edges then we can find the communities connected by positive edges (they can't contain a negative edges as that would make an odd cycle). We can collapse these communities into supernodes connected by negative edges. This graph is bipartite as there are no cycles with an odd number of negative edges. We now do a 2-colouring using BFS.

???? for 500

- Are you ready for your prize? You are the proud winner of…

???? for 500

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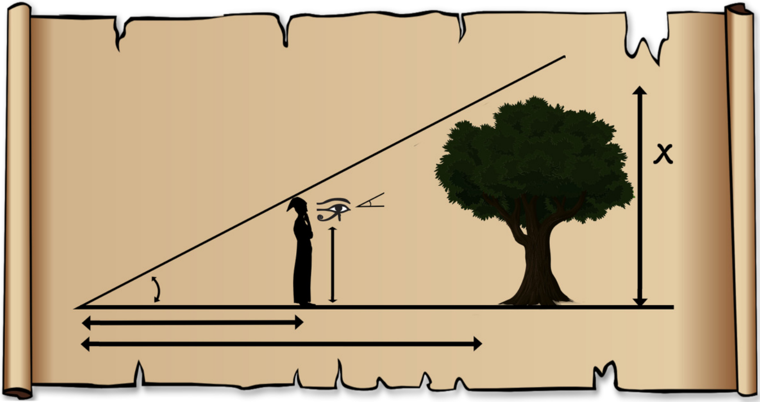

- The eye of Horus compelling a man in a night cap to do trigonometry! (7)

Bingo Prize: 1rst Place

- Our lucky, first place Bingo winner gets Gold Medal...

Bingo Prize: 1rst Place

- Our lucky, first place Bingo winner gets Gold Medal...

- flour. I for one, blame the search results ;) (7)

Bingo Prize: 2nd Place

- Our Bingo runner-up gets...

Bingo Prize: 2nd Place

- Our Bingo runner-up gets...

- Concerned fennel! (7)

Bingo Prize: 3rd Place

- Our third place Bingo winner gets...

Bingo Prize: 3rd Place

- Our third place Bingo winner gets...

- A small toy bulldozer pushing caviar off of (buttered?) toast! (7)

Jeopardy Prize: 3rd Place

- Our third place Jeopardy winner gets...

Jeopardy Prize: 3rd Place

- Our third place Jeopardy winner gets...

- Parent-child bonding on a frosty morning! (7)

Jeopardy Prize: 2nd Place

- Our Jeopardy runner-up gets...

Jeopardy Prize: 2nd Place

- Our Jeopardy runner-up gets...

- Scenic hiking! (7)

Jeopardy Prize: 1rst Place

- Our lucky, first place Jeopardy winner gets...

Jeopardy Prize: 1rst Place

- Our lucky, first place Jeopardy winner gets...

- Lego Star Wars bohemian rhapsody! What you've never known you've always wanted! (7)