Social and Information Networks

University of Toronto CSC303
Winter/Spring 2022

Week 1: January 10-14 (2022)

This week's agenda

- Course organization
  - People
  - Resources & communication
  - Course structure
  - Survey on office hours & course organization
- Basic graph terminology
- What can graphs represent?
- Why study networks?
  - What are the challenges?
- Example application: Detecting romantic relationships
  - Embeddedness of an edge
  - Dispersion of an edge
- Most likely, we will start next week's material on "the strength of weak ties"

Course Organization

Course Instructor: Ian Berlot-Attwell

- Email: 303s22 HYPHEN instr AT cs DOT toronto DOT edu

Teaching Assistants: Mohammad Mahdi Abdollah Pour, Alexander Chen, Parsa Farinneya, Fengwei Sun

Communications

1. Course Web page: source of first resort
   `https://www.cs.toronto.edu/~ianberlot/303s22/`

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3. Discussion board: **Piazza** for questions of general interest
   `https://piazza.com/utoronto.ca/winter2022/csc303/home`
   - I encourage questions in-class which leads to less confusion especially with regard to technical questions.
   - Questions about extensions to the material, or possible examples are also welcome – they often lead to some of the most interesting discussion!

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4. Office hours (Zoom): regular schedule TBA, also by appointment

Course Materials

Key Resources:

1. Text: D. Easley, J. Kleinberg. Networks, Crowds, and Markets: Reasoning About a Highly Connected World. Cambridge University Press, 2010. Online version available at `http://www.cs.cornell.edu/home/kleinber/networks-book/`
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Aside: CSC303 is based on the text by Easley and Kleinberg, previous parts of (the now discontinued) CSC200 by Borodin and Craig Boutilier, and the current course developed by Ashton Anderson at UTSC.

Lecture/Tutorial/Course Structure

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- Key factors: Student Learning, Student Experience, Public Health
- Note: If I've misjudged your preferences, or learning styles, we can still change things; for this reason I will be running a survey to collect your thoughts
  - Assessments can be changed via the syllabus (requires a class vote)
  - Lecture delivery can be changed with an announcement

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  - Lectures Mondays & Wednesdays
  - Tutorials on Fridays; starting Jan 21rst (this Friday is lecture)

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- More generally
  - Readings posted on web site usually posted in advance.
  - The readings often (but not always!) cover all or most of the lecture material – I suggest doing them in advance if possible
  - Lecture slides (some detailed, some less so) will usually be posted one or two days *after* the class. **You are responsible (i.e., can be tested) for information that occurs in lectures and tutorials.**

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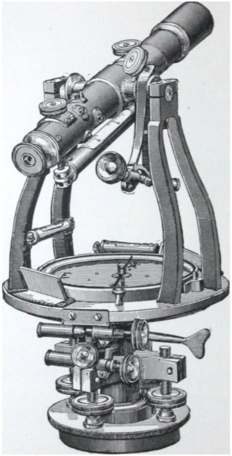
- Feedback, suggestions, & ideas for improving the course are welcome via
  - Email
  - Anonymously via `https://forms.gle/vVHLEsUGKSn41R6Z9`

Survey on Office Hours & Delivery Method

- `https://forms.gle/1dpvZfYh1XUK9rrx9`
  Link also on Quercus
- 5 minutes
- Closes Friday Jan 14th



Preparation, grading scheme and schedule

You should be comfortable with basic probability and discrete math concepts as would be covered in the prerequisites. I have posted a probability primer & linear algebra review on the course web page.

Grading Scheme

1. Participation: 5% – Quercus quizzes
2. Assignments: Two, each worth 15% = 30%
   Due dates: February 18 and March 28
3. Critical review of a current article (groups 3-4): Worth 10%
   Due date: March 25
4. Term Test (take-home): Worth 20%
   Tentative date: March 11-14, should take you 2-4 hours
5. Final Exam (take-home): Worth 35%
   Tentative date: TBD, 48 hour window, should take you 3-5 hours

Be careful! Feb 18 is sooner than you'd think, and a lot of material is due in the last few weeks.

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4. The "20%" rule: For any question or subquestion on any quiz, test, assignment or the final exam, you will receive 20% of the assigned question credit if you state "I do not know how to answer this question". That is, it is important to know what you do not know. If you have partial ideas then provide them; but no credit will be given for answers that do not show any understanding of the question.

What's in a name? Graphs or Networks?

Networks are graphs. Typically, terminology varies:

- graphs have vertices connected by edges
- networks have nodes connected by links

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Many different types of networks

- Social networks
- Information networks
- Transportation networks
- Communication networks
- Biological networks (e.g., protein interactions)
- Neural networks

Visualizing Networks

- nodes: entities (people, countries, companies, organizations, ...)
- links (may be directed or weighted): relationship between entities
  - friendship, classmates, did business together, viewed the same web pages, ...
  - membership in a club, class, political party, ...

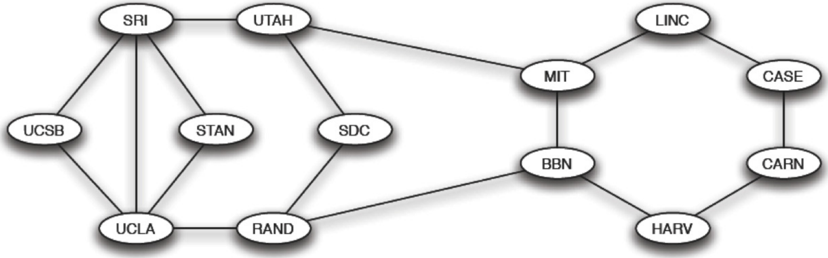
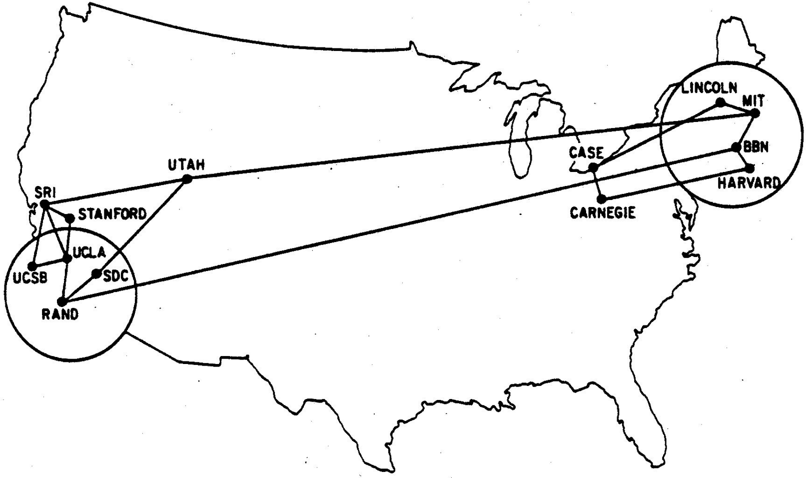


Figure: Initial internet: Dec. 1970 [E&K, Ch.2]

December 1970 internet visualized geographically [Heart et al 1978]



The first social network analysis

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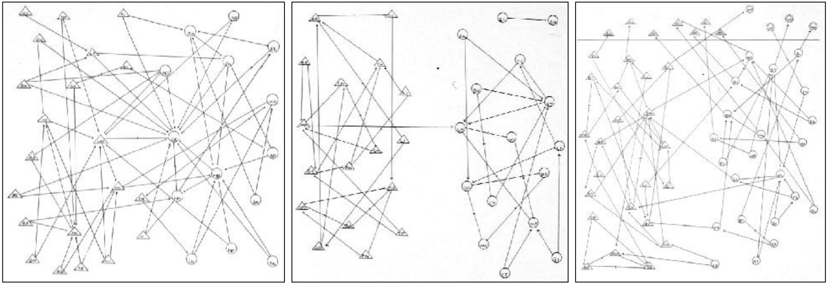

1st grade 4th grade 8th grade

Moreno's sociograms, 1934

A closer look at grade 1 in Moreno sociogram

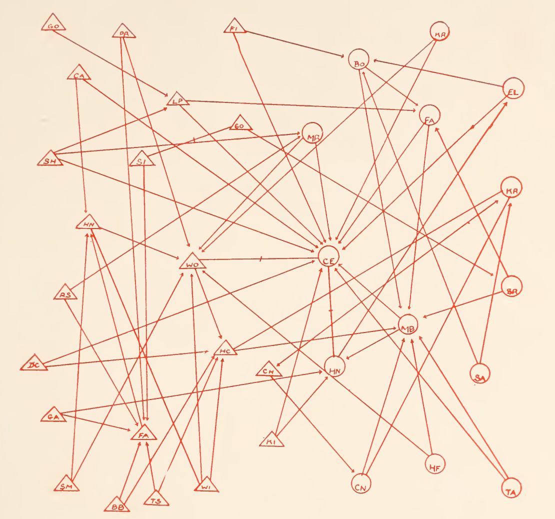

Figure: 21 boys, 14 girls. Directed graph. Most nodes have out-degree 2. 18 are not chosen, thus having in-degree 0. Note also that there are some “stars” with high in-degree.

A closer look at grade 4 in Moreno sociogram

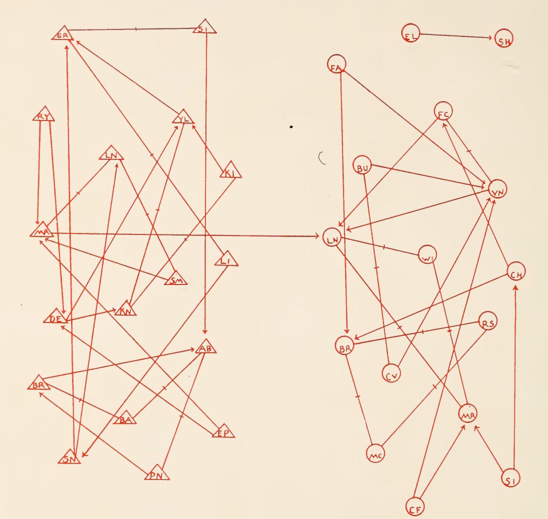

Figure: 17 boys, 16 girls. Directed graph with 6 unchosen having in-degree 0. Moreno depicted his graphs to emphasize inter-gender relations. Note only one edge from a boy to a girl.

A closer look at grade 8 in Moreno sociogram

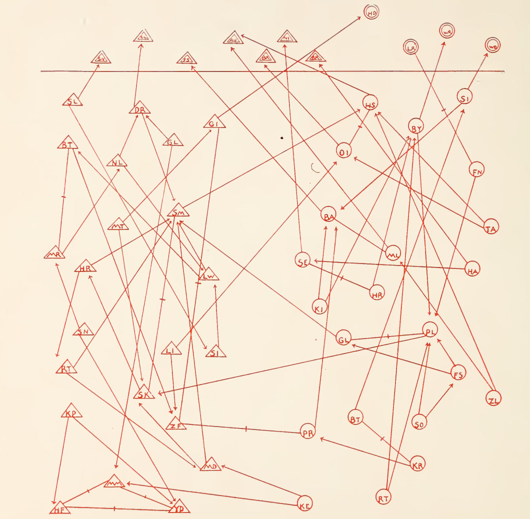

Figure: 22 boys, 22 girls. Directed graph with 12 unchosen having in-degree 0. Some increase in inter-gender relations. Double triangles and circles above line indicate individuals outside of the class.

Romantic Relationships [Bearman et al, 2004]

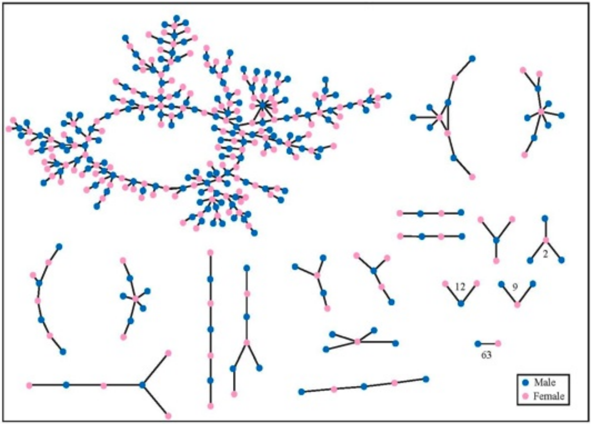

Figure: Dating network in US high school over 18 months.

- Illustrates common "structural" properties of many networks
- What is the benefit of understanding this network structure?

Kidney Exchange: Swap Chains

- Waiting list for kidney donation: approximately 100K in US and growing (i.e., new patients added but many deaths while waiting). The wait for a deceased donor could be 5 years and longer.
- Live kidney donations becoming somewhat more common in N.A. to get around waiting list problems: requires donor-recipient pairs. What if they are incompatible?

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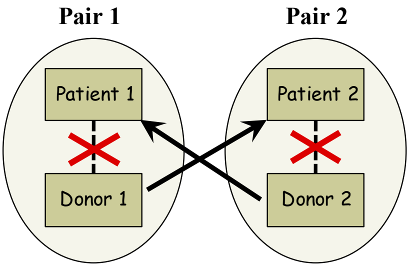

- Exchange: supports willing pairs who are incompatible
  1. allows multiway-exchange
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Kidney Exchange: Swap Chains

- But what if someone renegs? $\Rightarrow$ Cycles require simultaneous transplantation; Paths require an altruistic donor!

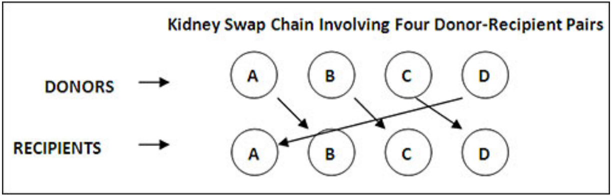


Figure: Dartmouth-Hitchcock Medical Center, NH, 2010

Communities: Karate club division

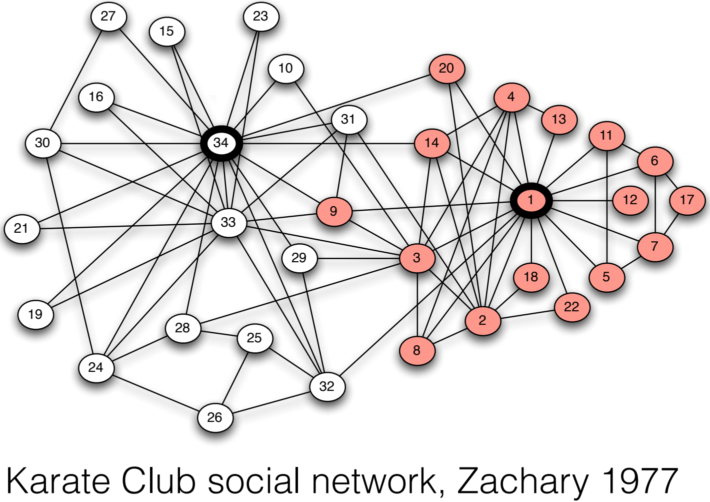


Figure: Karate club splis into two clubs

Communities: 2004 Political blogsphere

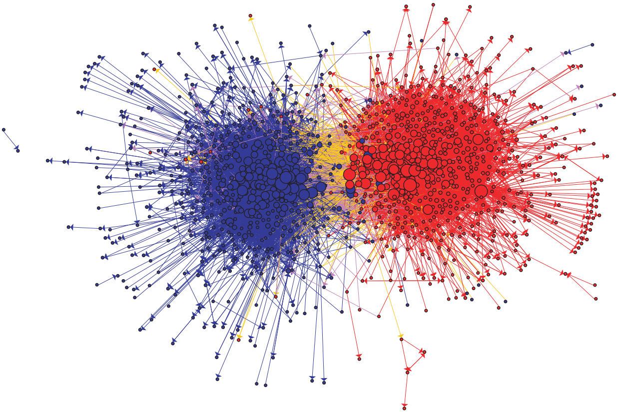


Figure 1: Community structure of political blogs (expanded set), shown using utilizing a GEM layout [11] in the GUESS[3] visualization and analysis tool. The colors reflect political orientation, red for conservative, and blue for liberal. Orange links go from liberal to conservative, and purple ones from conservative to liberal. The size of each blog reflects the number of other blogs that link to it.

Figure: [E&K, Fig 1.4]

Communities: 2017 Twitter online discourse regarding Black Lives Matter

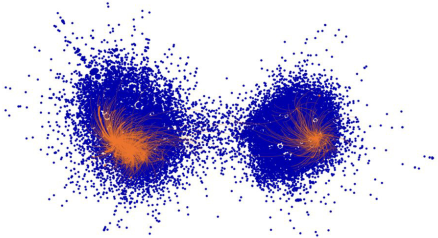


Fig. 1. Retweet Network Graph: RU-IRA Agents in #BlackLivesMatter Discourse. The graph (originally published [3]) shows accounts active in Twitter conversations about #BlackLivesMatter and shooting events in 2016. Each node is an account. Accounts are closer together when one account retweeted another account. The structural graph shows two distinct communities (pro-BlackLivesMatter on the left; anti-BlackLivesMatter on the right).

Accounts colored orange were determined by Twitter to have been operated by Russia's Internet Research Agency. Orange lines represent retweets of those account, showing how their content echoed across the different communities.
The graph shows IRA agents active in both "sides" of that discourse.

Figure: From Starbird et al [2017, 2019]

Communities and hierarchical structure: Email communication

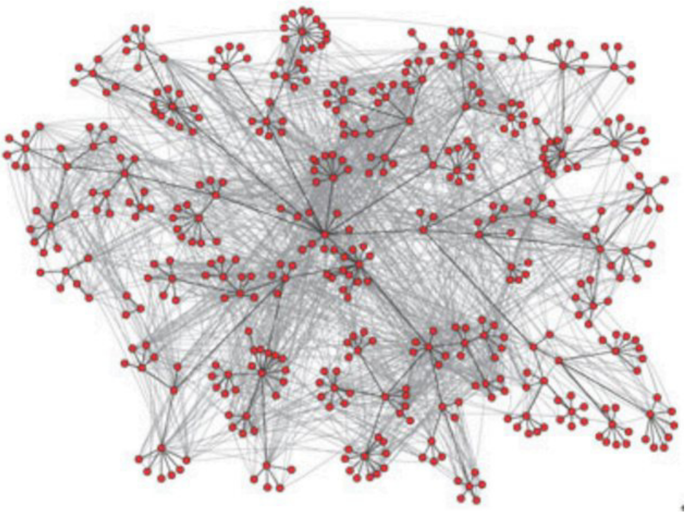
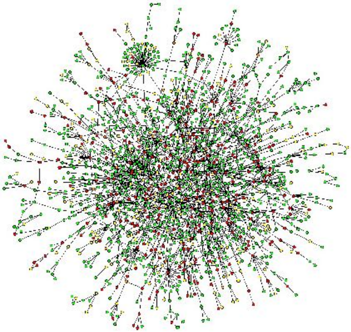


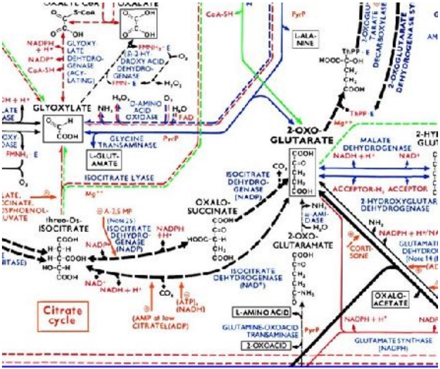
Figure: Email communication among 436 employees of Hewlett Packard Research Lab, superimposed on the organizational hierarchy [Fig 1.2, EK textbook]

Protein-protein interaction network



Protein-Protein Interaction Networks
Nodes: Proteins
Edges: 'physical' interactions

Metabolic network



Metabolic networks

Nodes: Metabolites and enzymes

Edges: Chemical reactions

The web as a directed graph of hyperlinks

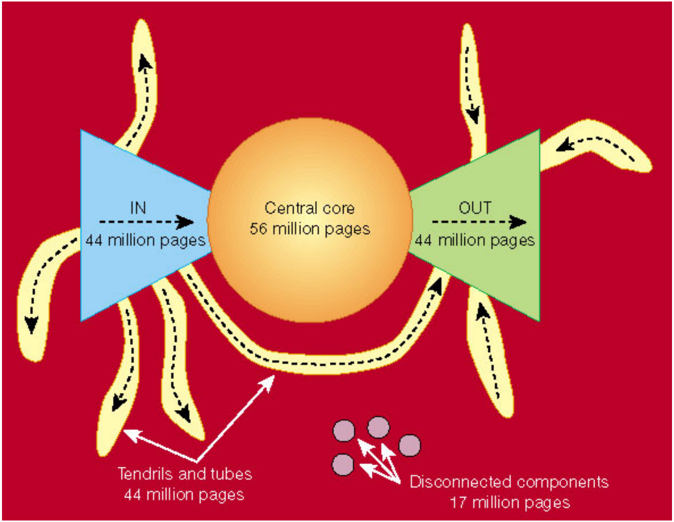


Figure: A schematic picture of the bow tie structure of the 1999 Web. Although the numbers are outdated, the structure has persisted. [Fig 13.7, EK textbook]

The current interest in networks

- Clearly there are complex systems and networks that we are in contact with daily.
- The population of the world can be thought of as social network of approximately 7.9 billion people. As of the second quarter of 2020, the people on Facebook are a *subnetwork* of approximately 2.7 billion active monthly users.
- The language of networks and graph analysis provides a common language and framework to study systems in diverse disciplines. Moreover, networks relating to diverse disciplines may sometimes share common features and analysis.
- The current impact of social and information networks will almost surely continue to escalate (even if Facebook and other social networks are under increasing pressure to protect privacy and eliminate "bad actors").

What can one accomplish by studying networks

We use networks as **a model** of real systems. As such, we always have to keep in mind the goals of any model which necessarily simplifies things to make analysis possible.
In studying social and information networks we can hopefully

- Discover interesting phenomena and statistical properties of the network and the system it attempts to model.
- Formulate hypotheses as to say how networks form and evolve over time
- Predict behaviour for the system being modeled.

And how do we accomplish stated goals

Much of what people do in this field is empirical analysis. We formulate our network model, hypotheses and predictions and then compare against real world (or sometimes synthetically generated) data.

Sometimes we can *theoretically* analyse properties of a network, and then compare to real or synthetic data.

What are the challenges?

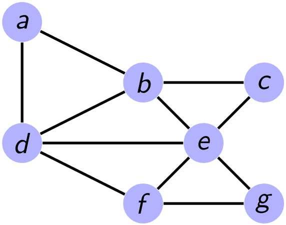
- Real world data is sometimes hard to obtain. For example, search engine companies treat much of what they do as proprietary.
- Many graph theory problems are known to be computationally difficult (i.e., NP hard) and given the size of many networks, results can often only be approximated and even then this may require a significant amount of specialized heuristics and approaches to help overcome (to some extent) computational limitations.
- And we are always faced with the difficulty of bridging the simplification of a model with that of the many real world details that are lost in the abstraction.

Network concepts used in this course

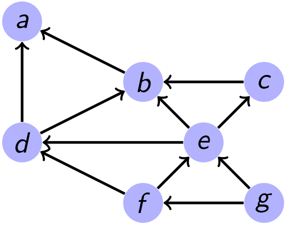
- Two main mathematical subjects of primary relevance to this course:
  1. graph theoretic concepts
  2. probability
- In motivating the course, we have already seen a number of examples of networks and hinted at some basic graph-theoretic concepts. We will now continue that discussion (i.e. material from Chapter 2 of the text) and for part of the next lecture before moving on to Chapter 3.
- We use the previous examples and some new ones to illustrate the basic graph concepts and terminology we will be using.

Graphs: come in two varieties

1. undirected graphs ("graph" usually means an undirected graph.)



2. directed graphs (often called di-graphs).



Visualizing Networks as Graphs

- nodes: entities (people, countries, companies, organizations, ...)
- links (may be directed or weighted): relationship between entities
  - friendship, classmates, did business together, viewed the same web pages, ...
  - membership in a club, class, political party, ...

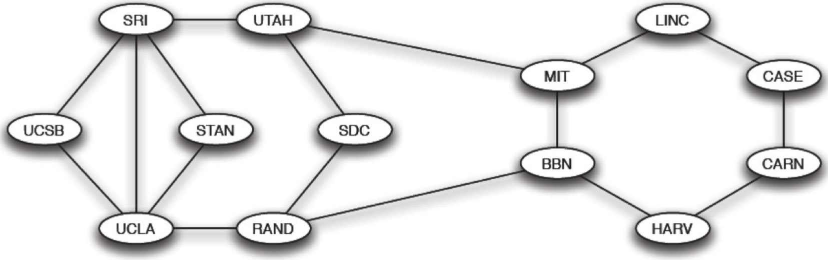


Figure: Internet: Dec. 1970 [E&K, Ch.2]

Adjacency matrix for graph induced by eastern sites in alphabetical order) in 1970 internet graph: another way to represent a graph

$$A(G) = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

- This node induced subgraph is a 6 node regular graph of degree 2. It is a simple graph in that there are no self-loops or multiple edges (two or more edges between the same two nodes).
- Note that the adjacency matrix of an (undirected) simple graph is a symmetric matrix (i.e. $A_{i,j} = A_{j,i}$) with $\{0,1\}$ entries.
- To specify distances, we would need to give weights to the edges to represent the distances.

Directed Graph Example: Kidney Exchange

- Live kidney donation common in North America to get around waiting list problems: donor-recipient pairs are nodes and links are directed.
- Exchange: supports willing pairs who are incompatible
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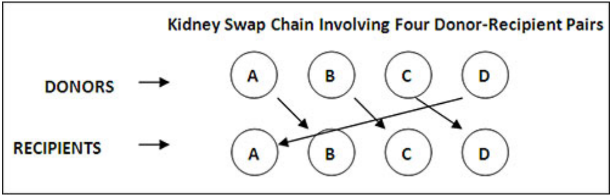
- But what if someone reneges? $\Rightarrow$ require simultaneous transplantation! Non-cyclic paths can be started by an altruistic donor!



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More definitions and terminology

- In order to refer to the nodes and edges of a graph, we define graph $G = (V, E)$, where
  - V is the set of nodes (often called vertices)
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- Undirected graph: an edge (u, v) is an unordered pair of nodes.

- Directed graph: a directed edge (u, v) is an ordered pair of nodes $\langle u, v \rangle$.
  - However, we usually know when we have a directed graph and just write (u, v).

Basic definitions continued

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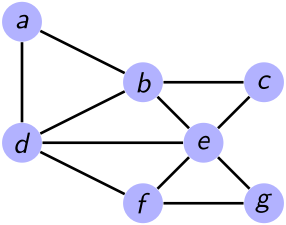
- The length of a path is the number of edges on that path.
- A graph is a connected if there is a path between every pair of nodes. For example, the following graph is connected.



Romantic Relationships [Bearman et al, 2004]

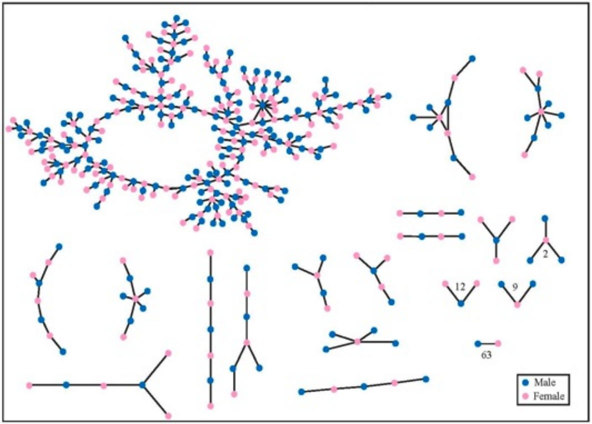
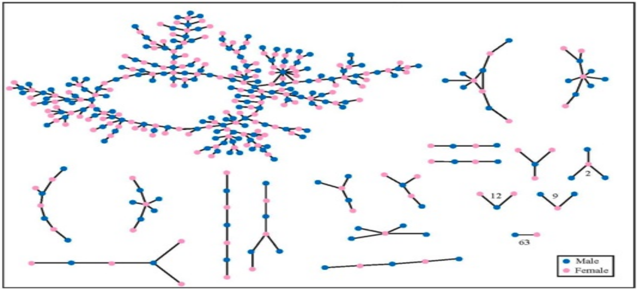


Figure: Dating network in US high school over 18 months.

- Illustrates common “structural” properties of many networks
- What predictions could you use this for?

More basic definitions



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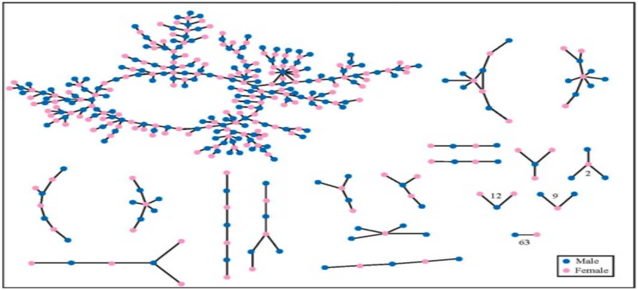
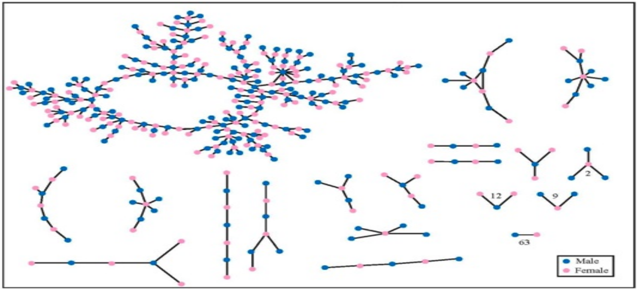


Observation

Many connected components including one "giant component"

More basic definitions



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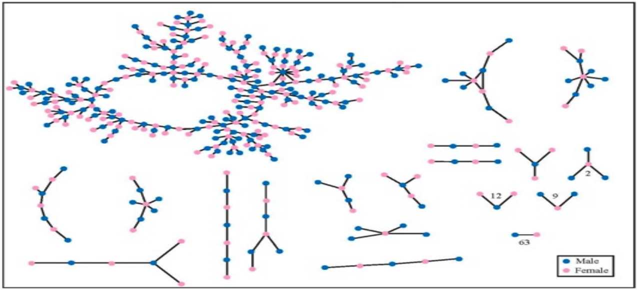
Many connected components including one "giant component"

- We will use this same graph to illustrate some other basic concepts.
- A cycle is path $u_1, u_2, \ldots, u_k$ such that $u_1 = u_k$; that is, the path starts and ends at the same node.

Simple paths and simple cycles

- Usually only consider simple paths and simple cycles: no repeated nodes (other than the start and end nodes in a simple cycle.)



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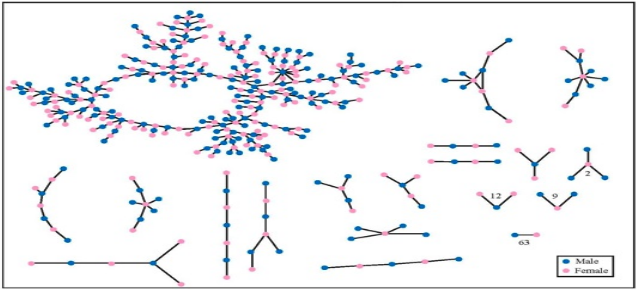

Observation

- There is one big simple cycle and (as far as I can see) three small simple cycles in the "giant component".
- Only one other connected component has a cycle: a triangle having three nodes. Note: this graph is "almost" bipartite and "almost" acyclic.

Wed. Jan 12th: Announcements & Corrections

- Recording of Monday's lecture is up (see course website). Anonymized chat logs and a downloadable link are both Quercus
- A friendly reminder, I strongly encourage all of you to unmute and talk, but do make sure to mute yourselves again afterwards – otherwise it can interfere with the Zoom recording
- A couple Zoom hiccups (joining from browser & some name issues) are now *hopefully* resolved (if not, please do let me know)

Wed. Jan 12th: Announcements & Corrections

- Are you interested in studying online with other students in the class? You may be interested in joining, or even leading, a registered study group of up to 8 students!
  - `https://sidneysmithcommons.artsci.utoronto.ca/recognized-study-groups/`
  - Consider volunteering as a registered study group leader today!

Wed. Jan 12th: Announcements & Corrections

- Do you know an awesome teaching assistant, or course instructor? If so, consider nominating them for a Teaching Excellence Award!
  - TA Teaching Excellence Award: `https://tatp.utoronto.ca/awards/ta-awards/`
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- Seriously, the TA award been running since 2004 and I don't think anyone in the UTSG CS department has ever won or been shortlisted! The closest we've gotten is some computer engineers, and computer scientists from UTSC/UTM

Wed. Jan 12th: Announcements & Corrections

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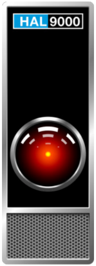
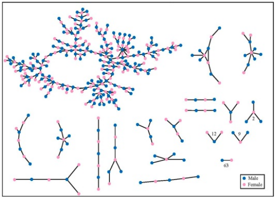


- Happy 25th (30th?) Birthday Hal!

Wed. Jan 12th: Announcements & Corrections

- In Monday's lecture, someone asked: Do dating apps use something like our dating network?



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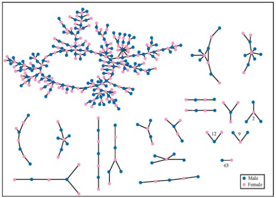

- Unclear! (But probably not)
  - Dating algorithms tend to be quite secretive and proprietary with their algorithms – same as web-search algorithms

Wed. Jan 12th: Announcements & Corrections

- The only concrete information I was able to find, is that circa 2016, Tinder was assigning users a numeric score, internally called the "Elo score", presumably after the Elo rating commonly used to measure comparative chess skill
- Score increased or decreased based on who they "beat" in a match, with score increase depending on the "skill" (i.e. score) of the players
- Although the article doesn't go into further detail, presumably the system would then create matches between players of similar skill, with some random variation
- It also appears that this system is no longer in use

Based on a Fast Company piece that can be found here:
`https://www.fastcompany.com/3054871/whats-your-tinder-score-inside-the-apps-internal-ranking-system`

Wed. Jan 12th: Announcements & Corrections

- There are some rumours on the internet that a variant on the Gale-Shapely algorithm is being used, but for practical reasons I doubt that's true (at least, without considerable modification) – there are also some very bad implications for users, but we'll see that when we cover the material in class

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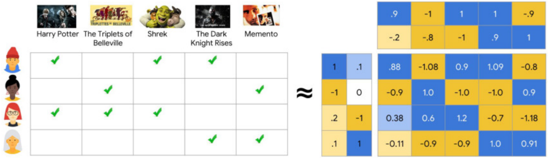

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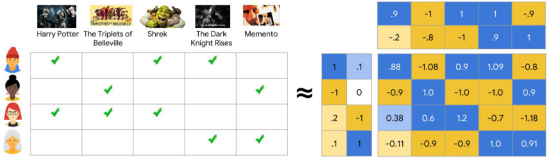

- Can any part of this be viewed as a graph?
  - Yes! But it's a very boring graph; the weights matrix can be interpreted as the adjacency matrix of a fully connected graph – we just don't exploit this fact.

Diagram from https://developers.google.com/machine-learning/recommendation/collaborative/basics

Wed. Jan 12th: Announcements & Corrections

- Allegedly, some apps do use information from social media networks such as Facebook – however assuming that is true, it's still unclear if it's using the actual social network, or instead exploiting the text using NLP
  - We will see later in this lecture, that network structure is quite powerful in the *opposite direction*

Example of an acyclic bipartite graph

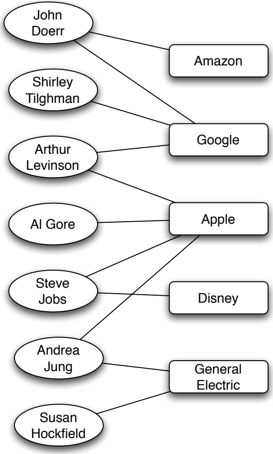

Figure: [E&K, Fig 4.4] One type of affiliation network that has been widely studied is the memberships of people on corporate boards of directors. A very small portion of this network (as of mid-2009) is shown here.

Florentine marriages and "centrality"

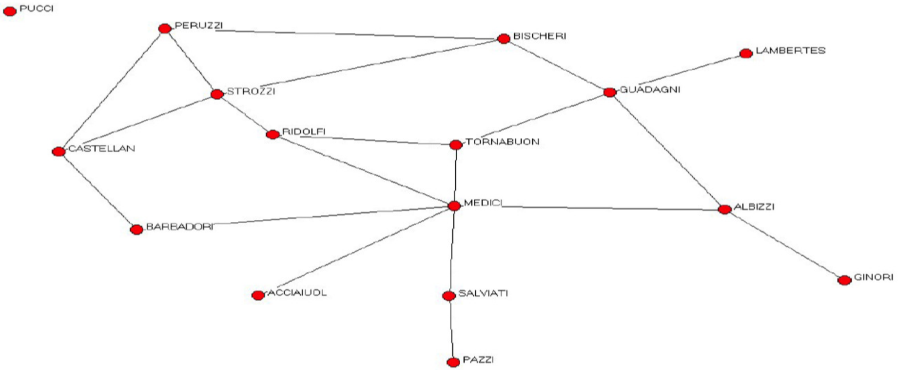

Figure: see [Jackson, Ch 1]

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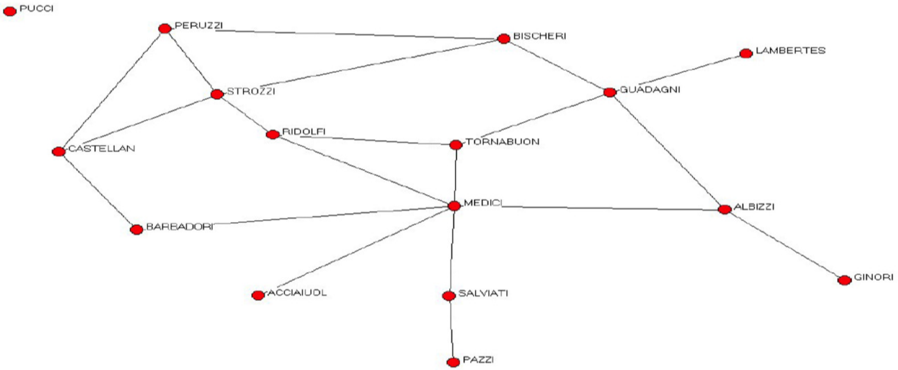

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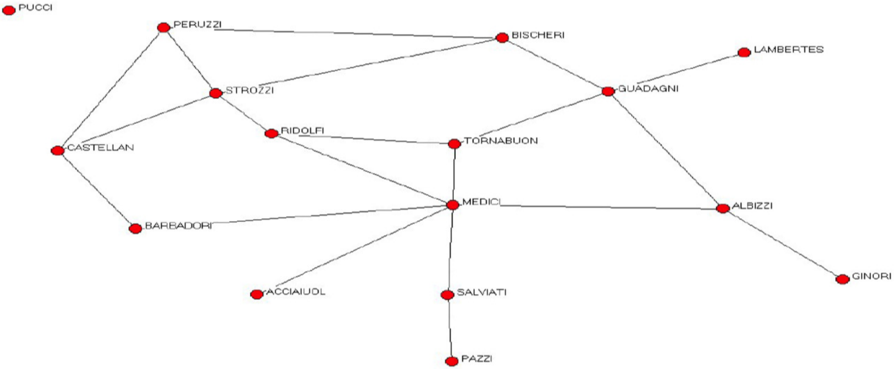

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- Medici connected to more families, but not by much
- More importantly: lie between most pairs of families
  - shortest paths between two families: coordination, communication
  - Medici lie on 52% of all shortest paths; Guadagni 25%; Strozzi 10%

Breadth first search and path lengths [E&K, Fig 2.8]

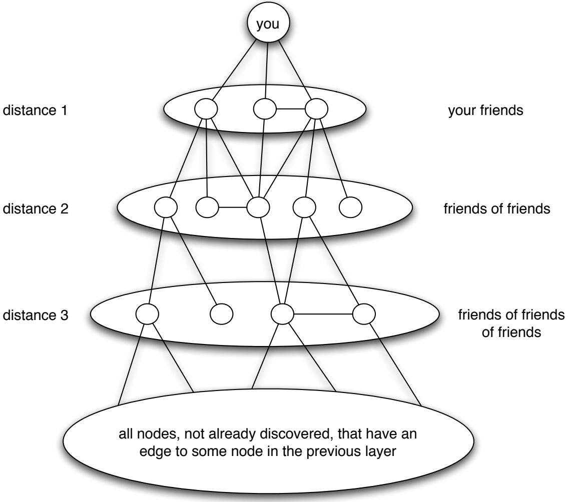


Figure: Breadth-first search discovers distances to nodes one "layer" at a time. Each layer is built of nodes adjacent to at least one node in the previous layer.

The Small World Phenomena

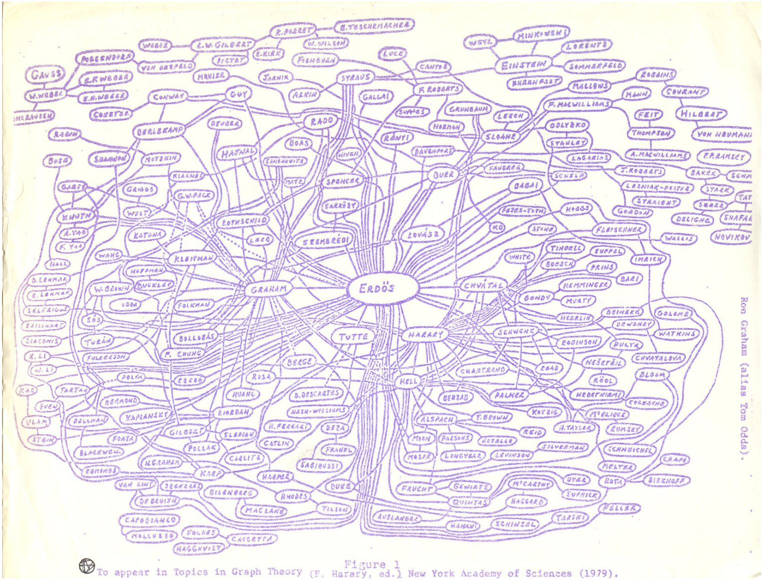
The small world phenomena suggests that in a connected social network any two individuals are likely to be connected (i.e. know each other indirectly) by a short path.

Later in the course we will study 1967 Milgram's small world experiment where he asked random people in Omaha Nebraska to forward a letter to a specified individual in a suburb of Boston which became the origin of the idea of six degrees of separation.

Small Collaboration Worlds

For now let us just consider collaboration networks like that of mathematicians or actors. For mathematicians (or more generally say scientists) we co-authorship on a published paper. For actors, we can form a collaboration network where an edge represents actors performing in the same movie. For mathematicians one considers their Erdos number which is the length of the shortest path to Paul Erdos. For actors, a popular notion is ones Bacon number, the shortest path to Kevin Bacon.

Erdos collaboration graph drawn by Ron Graham [http:/www.oakland.edu/enp/cgraph.jpg]



Analogous concepts for directed graphs

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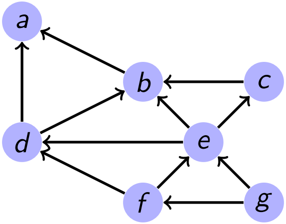
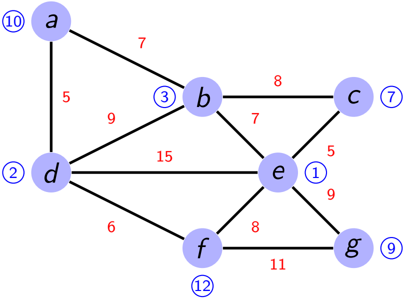
- We now have directed paths and directed cycles. Instead of connected components, we have strongly connected components.



Weighted graphs

- We will often consider weighted graphs. Lets consider a (directed or undirected) graph $G = (V, E)$. Example:



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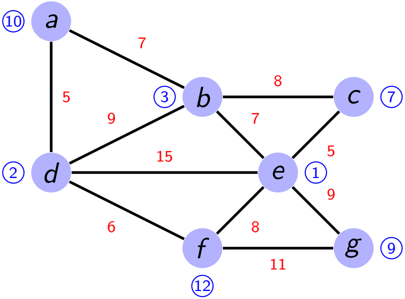

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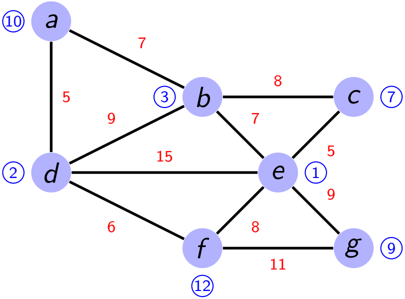

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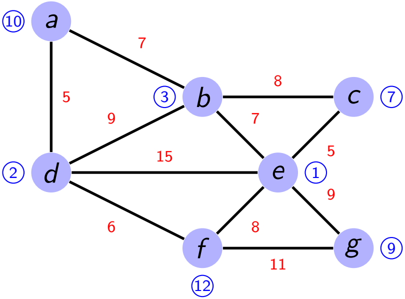

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- For example, in a social network whose nodes represent people, the weight $w(v)$ of node v might indicate the importance of this person.
- The weight $w(e)$ of edge e might reflect the strength of a friendship.

Edge weighted graphs

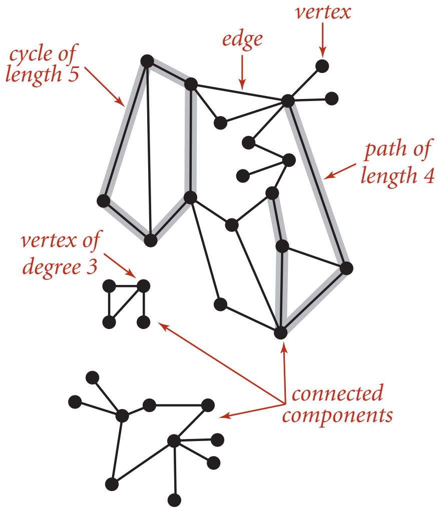
- When considering edge weighted graphs, we often have edge weights $w(e) = w(u, v)$ which are non negative (with $w(e) = 0$ or $w(e) = \infty$ meaning no edge depending on the context).

- In some cases, weights can be either positive or negative. A positive (resp. negative) weight reflects the intensity of connection (resp. repulsion) between two nodes (with $w(e) = 0$ being a neutral relation).

- Sometimes (as in Chapter 3) we will only have a qualitative (rather than quantitative) weight, to reflect a strong or weak relation (tie).

- Analogous to shortest paths in an unweighted graph, we often wish to compute least cost paths, where the cost of a path is the sum of weights of edges in the path.

Graph anatomy: summary thus far

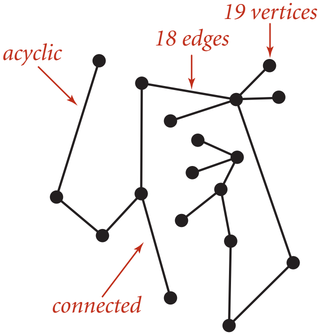


[from Algorithms, 4th Edition by Sedgewick and Wayne]

Acyclic graphs (forests)

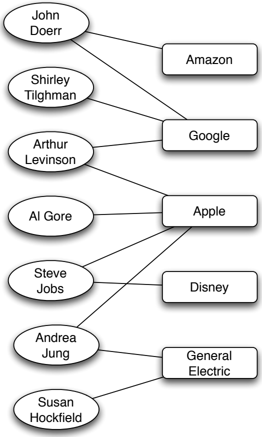
- A graph that has no cycles is called a forest.
- Each connected component of a forest is a tree.

  - A tree is a connected acyclic graph.
  - Question: Why are such graphs called trees?
  - Fact: There are always $n - 1$ edges in an n node tree.



- Thus, a forest is simply a collection of trees.

Another tree [E&K Figure 4.4]



- The bipartite graph from before (depicting membership on corporate boards) is also an example of a tree.
- In general, bipartite graphs can have cycles.
- Question: is an acyclic graph always bipartite?

Facts

- It is computationally easy to decide if a graph is acyclic or bipartite.
- However, we (in CS) strongly "believe" it is not easy to determine if a graph is tripartite (i.e. 3-colourable).

Analogous concepts for directed graphs

- We now have directed paths and directed cycles.
- Instead of the degree of a node, we have the in-degree and out-degree of a node.

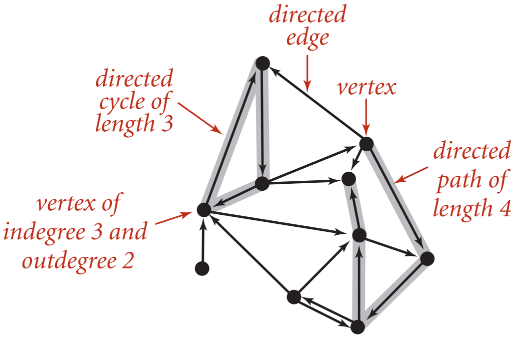
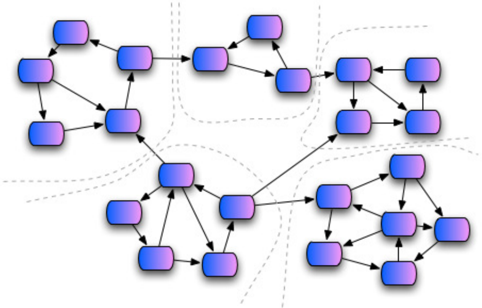


Figure: Directed graph antonomy [from Sedgewick and Wayne]

More analogous concepts for directed graphs

- Acyclic mean no directed cycles.
- Instead of connected components, we have strongly connected components.

[from `http://scientopia.org/blogs/goodmath/`]



- Instead of trees, we have directed (i.e. rooted) trees which have a unique root node with in-degree 0 and having a unique path from the root to every other node.
- Question: What is a natural example of a rooted tree?

Jan 14th: Announcements & Corrections

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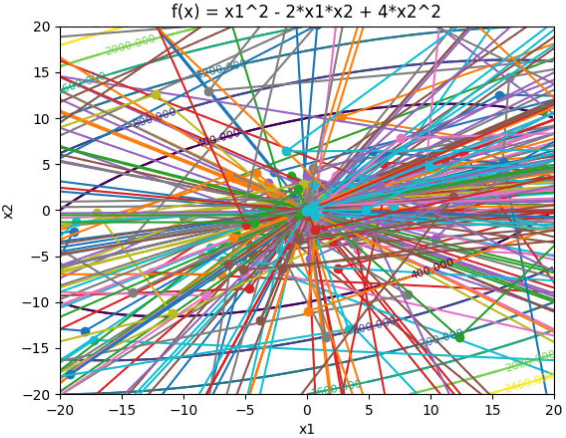
- Happy Friday! You've survived your first week back ;)



  - Hopefully you've been having a good week, and can look forwards to a change of pace with the weekend!

Fri. Jan 14th: Announcements & Corrections

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  - I generally try to keep the website fairly up to date, so if there's anything missing that isn't fixed by a reload/cache clear, then please do ask on Piazza

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- We are currently looking for volunteer note takers for CSC303!
  - To learn more see: `https://studentlife.utoronto.ca/program/volunteer-note-taking/`
  - To volunteer go to: `https://studentlife.utoronto.ca/program/volunteer-note-taking/`
- For further details, see Accessibility Service's call for volunteer note takers in the Quercus announcements

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  - I said you should be able to disprove that with a sufficiently large clique; but here are the details:

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N-colourable

A graph, $G = (V, E)$ is N-colourable if there exists a weight function $w : V \rightarrow \{c_1, c_2, \ldots c_N\}$ such that $\forall (u, v) \in E : w(u) \neq w(v)$

  - i.e.: Colour each node in the graph so that all edges are between nodes of different colours (note, you don't have to use all the colours)

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- A clique with N+1 nodes is not N-colourable: There must be 2 nodes with the same colour (pigeonhole principle). By definition of a clique there is an edge between them (i.e., endpoints of same colour)

Fri. Jan 14th: Announcements & Corrections

- Therefore, all graphs are not 4-colourable

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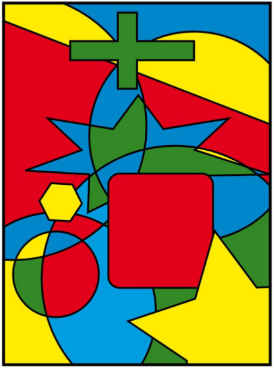
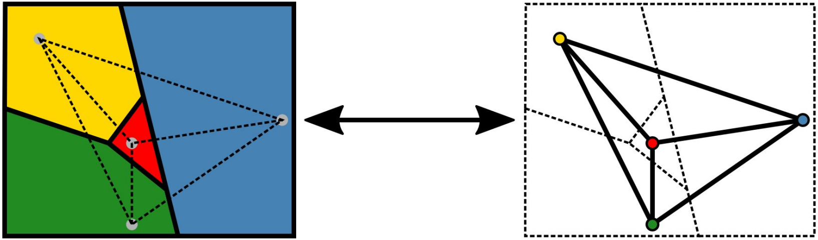
- Therefore, all graphs are not 4-colourable
- ... but, 4 colour theorem informally states that any map can be coloured with 4 colours so that no bordering regions have the same colour! This looks like the same kind of problem?



Fri. Jan 14th: Announcements & Corrections

- In fact, map colouring can be represented as graph colouring! How can this be???



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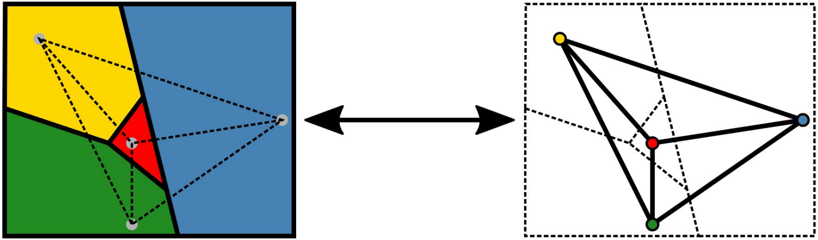

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- The devil is in the details! Only a specific graphs, *planar graphs* correspond to maps
  - Cliques with 4 or more nodes are not planar graphs
- Therefore: All *planar* graphs are 4 colourable

Detecting the romantic relation in Facebook: Course motivation and a lead in to Chapter 3 of text

- There is an interesting paper by Backstrom and Kleinberg (http://arxiv.org/abs/1310.6753) on detecting "the" romantic relation in a subgraph of Facebook users who specify that they are in such a relationship.

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- Backstrom and Kleinberg construct two datasets of randomly sampled Facebook users:
  - (i) an extended data set consisting of 1.3 million users declaring a spouse or relationship partner, each with between 50 and 2000 friends
  - (ii) a smaller data set extracted from neighbourhoods of the above data set (used for the more computationally demanding experimental studies)
- The main experimental results are nearly identical for both data sets.

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- The main experimental results are nearly identical for both data sets.
- Question: How would you go about identifying someone's spouse given their Facebook profile & feed?

Detecting the romantic relation (continued)

- They consider various "interaction features" including
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Detecting the romantic relation (continued)

- They consider various "interaction features" including
  1. the number of photos in which both A and B appear.
  2. the number of profile views within the last 90 days.
- Their focus was various graph structural features of edges, including
  1. the *embeddedness* of an edge (A, B) which is the number of mutual friends of A and B.
  2. various forms of a new *dispersion* measure of an edge (A, B) where high dispersion intuitively means that the mutual neighbours of A and B are not "well-connected" to each other (in the graph without A and B).
  3. One definition of dispersion given in the paper is the number of pairs (s, t) of mutual friends of A and B such that $(s, t) \notin E$ and s, t have no common neighbours except for A and B.

Embeddedness and dispersion example from paper

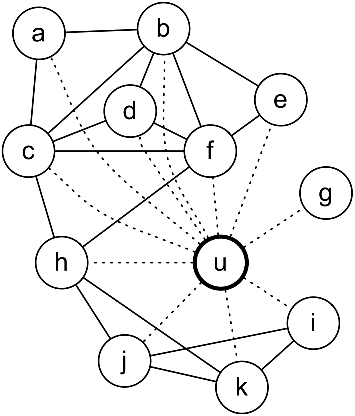

Figure 2. A synthetic example network neighborhood for a user u; the links from u to b, c, and f all have embeddedness 5 (the highest value in this neighborhood), whereas the link from u to h has an embeddedness of 4. On the other hand, nodes u and h are the unique pair of intermediaries from the nodes c and f to the nodes j and k; the u-h link has greater dispersion than the links from u to b, c, and f.

Qualitative results from Backstrom and Kleinberg

- The goal is to predict (for each user in the data set) which of their friendship edges is the romantic relation. Note that each user has between 50 and 2000 friends and assuming say a median of 200 friends, a random guess would have prediction accuracy of $1/200 = .5\%$

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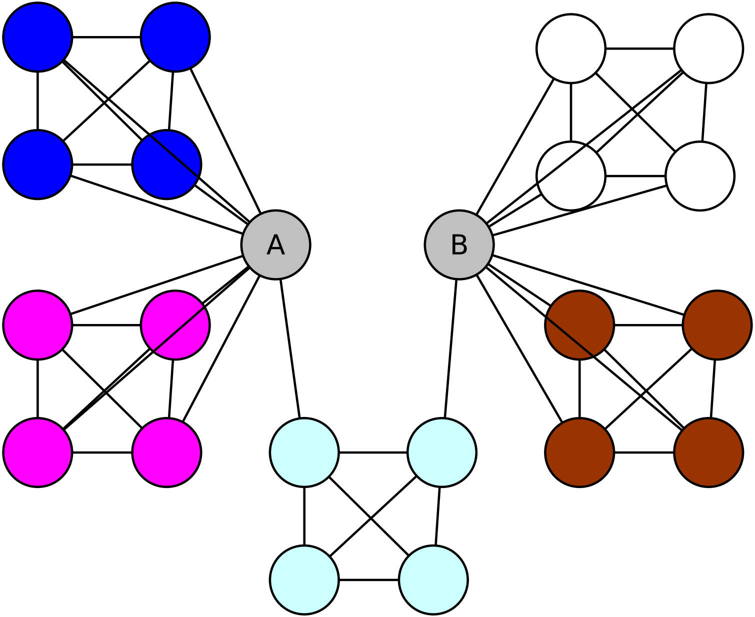
married	0.321	0.607	0.449	0.210
married (fem)	0.296	0.551	0.391	0.202
married (male)	0.347	0.667	0.511	0.220
engaged	0.179	0.446	0.442	0.391
engaged (fem)	0.171	0.399	0.386	0.401
engaged (male)	0.185	0.490	0.495	0.381
relationship	0.132	0.344	0.347	0.441
relationship (fem)	0.139	0.316	0.290	0.467
relationship (male)	0.125	0.369	0.399	0.418

- Various disperson measures do better than the embeddedness measure in its ability to predict the correct romantic relationship.
- By itself, dispersion outperforms various interaction features.
- Why would high dispersion be a better measure than high embeddedness?

Dispersion Example



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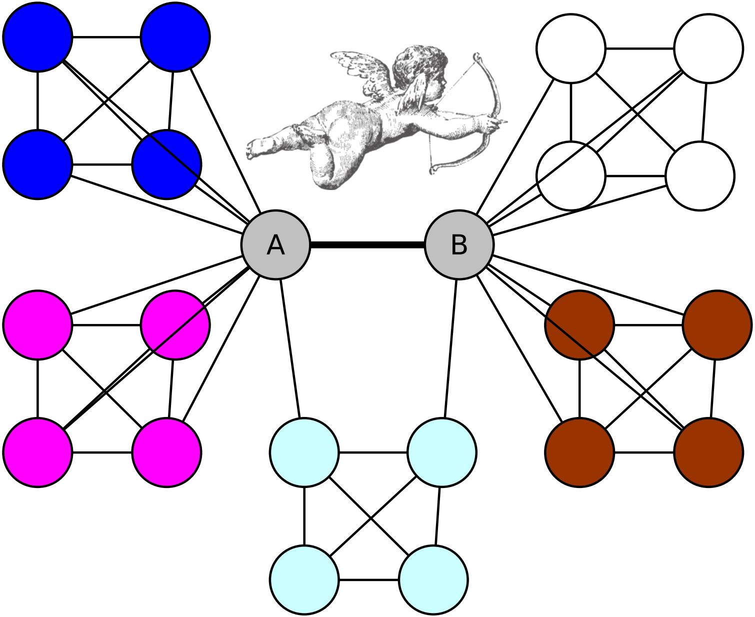

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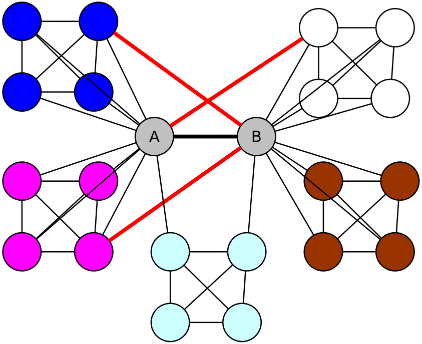
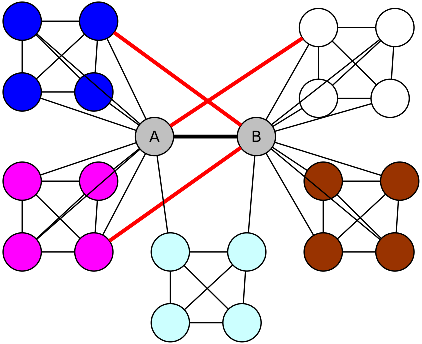


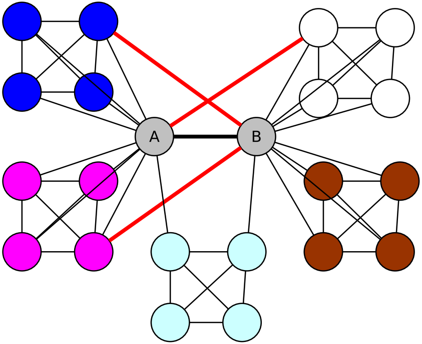
- Embededness of (A, B) is their number of mutual friends
- Easily skewed by dense groups (e.g. white nodes are chess club where everyone is eachothers' friend)

Dispersion Example



- Dispersion is how poorly connected A and B's mutual friends are if A and B are removed from the graph
- Romantic relationship can create pairs of mutual friends in different "social focii" (e.g. A's friend in chess club & B's dark blue friend)

Dispersion Example



- Dispersion is how poorly connected A and B's mutual friends are if A and B are removed from the graph
- Normal friendships are less likely to form these connections (e.g. unlikely to be connections from white nodes to brown nodes if brown nodes are extended family)

Qualitative results from Backstrom and Kleinberg

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- By combining many features, structural and interaction, the best performance is achieved using machine learning classification algorithms based on these many features.

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- By itself, dispersion outperforms various interaction features.
- Why would high dispersion be a better measure than high embeddedness?
- By combining many features, structural and interaction, the best performance is achieved using machine learning classification algorithms based on these many features.
- There are a number of other interesting observations but for me the main result is the predictive power provided by graph structure although there will generally be a limit to what can be learned solely from graph structure.

Some experimental results for the fraction of correct predictions

Recall that we argue that the fraction might be .005 when randomly choosing an edge. Do you find anything surprising?

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married	0.607	0.449	0.624	0.526	0.716
engaged	0.446	0.442	0.472	0.615	0.708
relationship	0.344	0.441	0.377	0.605	0.682

Recap

- Course organization
  - People
  - Resources & communication
  - Course structure
  - Survey on office Hours & course organization
- Basic graph terminology
- What can graphs represent?
- Why study networks?
  - What are the challenges?
- Example application: Detecting romantic relationships
  - Embeddedness of an edge
  - Dispersion of an edge