Social and Information Networks

Tutorial #1: Maximizing Strong Edges under STC

University of Toronto CSC303
Winter/Spring 2021
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Week 2: January 18-22 (2021)

Today's agenda

In lecture we've covered Chapter 3 of the textbook on "Strong and Weak Ties".

Today:

- Questions from Lecture
- Review of Graph Terminology
- Maximizing Strong Edges under STC
- Quercus Quiz

Questions?

Some Useful Terminology

- Directed vs. undirected graph

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- Weighted vs. unweighted graph

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- (Strongly) connected component
- Giant component
- Triadic closure

Strong Triadic Closure (STC)

We consider a social network where vertices (nodes) are people and edges (links) signify friendship. Edges are weighted, they are either *Strong* (meaning full friendship), or *Weak* (meaning acquaintance).

Definition (Strong Triadic Closure)

A graph satisfies Strong Triadic Closure (STC) if whenever (A, B) and (A, C) are strong ties, then there will be a tie (possibly only a weak tie) between B and C.

Strong Triadic Closure (STC)

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MINSTC is the problem of labeling the edges of a graph so that STC holds and the number of weak edges in the labeling is minimized.

Solving MINSTC

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MINSTC is an NP-Hard problem (the definition is beyond the scope of the course, for this course it means we don't have a polynomial time solution).

We can solve MINSTC by reducing it to the Minimum Vertex Cover Problem, and using an approximation algorithm to solve the vertex cover problem.

Solving MINSTC: "Covering" Open Triangles

For a set of edges E, we can define STC as:

$$\forall (A, B), (B, C) \in E : \text{strong}((A, B)) \wedge \text{strong}((B, C)) \implies (A, C) \in E$$

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$$\forall (A,B),(B,C) \in E : \text{strong}((A,B)) \wedge \text{strong}((B,C)) \implies (A,C) \in E$$

By the contrapositive, we have an equivalent definition:

$$\forall (A,B),(B,C) \in E : (A,C) \notin E \implies \text{weak}((A,B)) \vee \text{weak}((B,C))$$

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i.e. For the "open triangle" A, B, C (i.e. a triangle missing exactly one side), we must cover it (i.e. make one of the present sides weak).

Solving MINSTC: Reduction to Vertex Cover

Therefore consider the graph $G = (V, E)$ that we want to perform MINSTC on. Let us define the corresponding graph $G_T = (V_T, E_T)$ such that:

- $V_T = E$
- $(\alpha, \beta) \in E_T \iff$ the edges α and β are an open triangle in G

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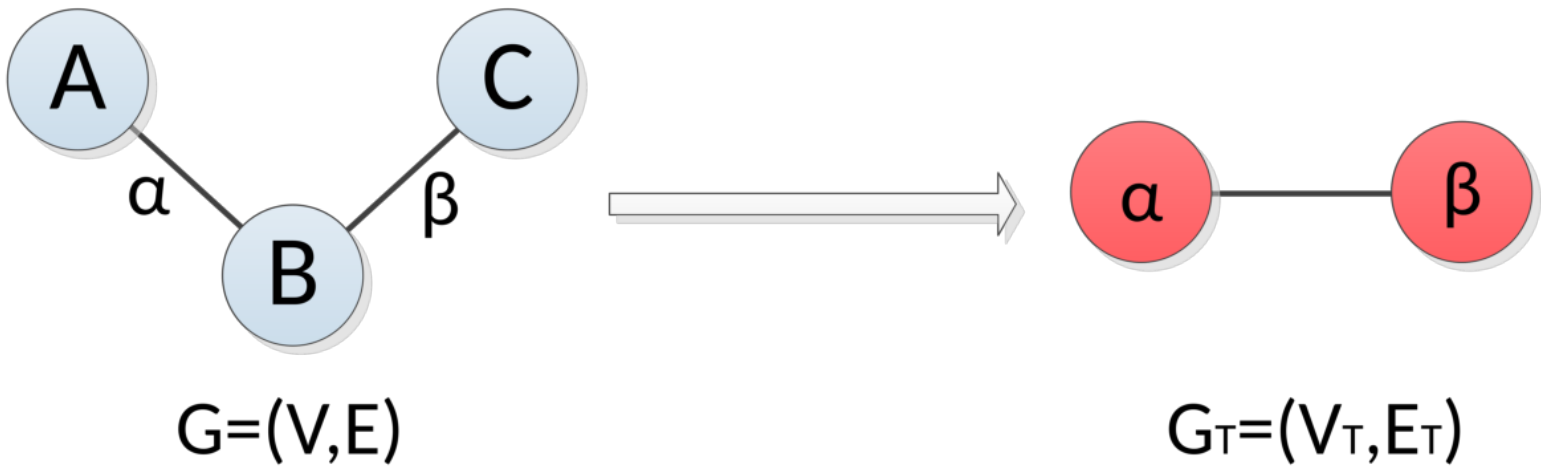

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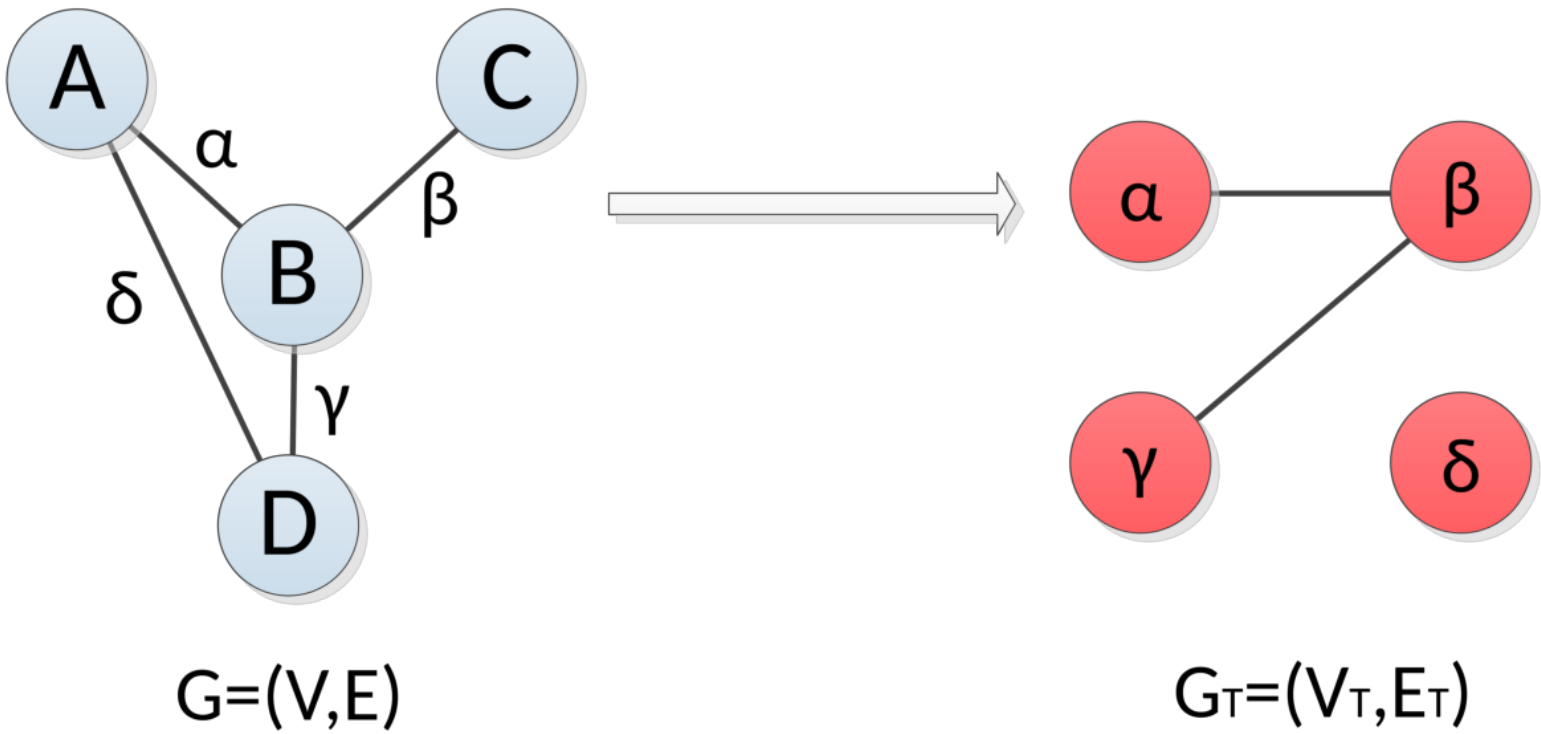

Solving MINSTC: Reduction to Vertex Cover

Therefore consider the graph $G = (V, E)$ that we want to perform MINSTC on. Let us define the complementary graph $G_T = (V_T, E_T)$ such that:

- $V_T = E$
- $(\alpha, \beta) \in E_T \iff$ the edges α and β are an open triangle in G

Therefore, TFAE:

- MINSTC on G
- Finding the minimal set of weak edges that covers all open triangles in G
- Finding the minimal set of vertices of G_T such that all edges have at least one endpoint in the set
- Min Vertex Cover on G_T

Therefore we can solve (or approximate) MINSTC by solving (resp. approximating) Min Vertex Cover

Solving Min Vertex Cover: Greedy Algorithm

A greedy approximation to solve the min vertex cover is to repeatedly add the vertex that covers the most uncovered edges until all edges are covered.

This approach often works well in practice, but it can be shown that if run on a graph $G' = (V', E')$ then it may return a solution that is $\Theta(\log |V'|)$ times larger than the optimal solution.

Solving Min Vertex Cover: Maximal Matching Algorithm

The maximal matching approach iterates over all edges in E'. Whenever it finds an edge $e \in E'$ that is not covered by the current cover, then both endpoints of the edge are added to the current cover.

Note that as the true optimal cover must contain one endpoint for every edge, then the solution returned is at most twice the size of the optimal solution. Thus it is called a 2-approximation.

Quercus Quiz