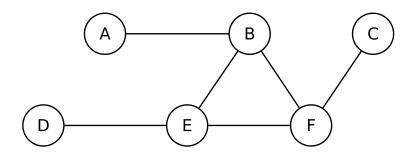
## CSC303: Practice Questions

## We'll be covering solutions in-tutorial on April 7th

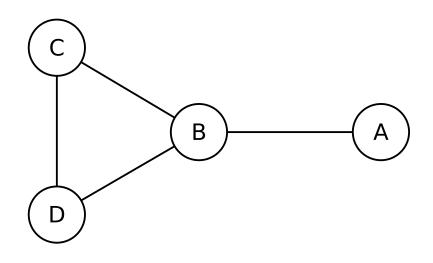
## Question 1:

(a) Run the Sintos & Tsaparas algorithm on the following graph G to find the solution to the MINSTC problem. Recall, in MINSTC our goal is to producing a labeling of strong & weak edges such that STC is preserved and the number of weak edges in minimized.



Make sure to draw the complementary graph  $G_T$ . Find the optimal vertex covering of  $G_T$  manually.

 $Question\ 2:$  Cluster the following graph using the Girvan-Newman algorithm



Question 3: A classmate of yours is studying homophily with respect to yodeling in a network. In this network, there exists data on 1-way friendships. In the network, the directed edge  $\langle A, B \rangle$  signifies that A views B as a friend, but the feeling is not mutual. Your classmate finds that given that B is a yodeler, there is no change in the probability that A yodels based on direction of friendship (i.e. the probability that A yodels is the same when either  $\langle A, B \rangle$  or  $\langle B, A \rangle$  is in the network). Is this evidence for or against social influence causing homophily with respect to yodeling? Briefly justify.

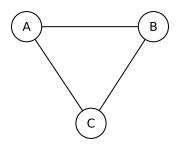
Question 4: Are all labellings of all undirected trees completable to a strongly balanced network? Either provide a proof or a counterexample.

Question 5: Recall the product diffusion process described in class. Recall that each node using product A has a reward of a per neighbour also using A, and that each node using product B has a reward of b per neighbour also using B.

Assume that we change our model so that mismatching neighbours each get a reward of c < a, b. For a node u using B, what proportion of u's neighbours must be using A for it to be non-detrimental for u to switch?

Question 6: In class we saw how to represent SIS using SIR, and in tutorial you saw the SEIR model. How can you represent SEIR as a SIR model with  $t_E = 1$ ? Assume the SEIR contact network is directed.

 $Question\ 7:$  Prove that no solution to the following bargaining network is stable



Question 8: The bargaining networks we have seen assign \$1 to each edge. Under this constraint we've seen that a solution (M, v) is stable iff  $\forall (A, B) \in E \setminus M : v(A) + v(B) \ge 1$ , where G = (V, E) is the underlying network.

If we allow for edges to have different weights (e.g., A and B can split \$1.5, and B and C can split \$0.75, and so on), then does our previous theorem still capture stable solutions (i.e., solutions in which no two nodes out of the matching can make a strictly better deail among themselves)?

Justify your answer.

Question 9: Consider the following matching problem

 $\begin{array}{c} m_1 \succ_{w_1} m_2 \succ_{w_1} m_3 \\ m_2 \succ_{w_2} m_1 \succ_{w_2} m_3 \\ m_1 \succ_{w_3} m_2 \succ_{w_3} m_3 \\ w_2 \succ_{m_1} w_1 \succ_{m_1} w_3 \\ w_1 \succ_{m_2} w_2 \succ_{m_2} w_3 \\ w_1 \succ_{m_3} w_2 \succ_{m_3} w_3 \end{array}$ 

- (a) Run MPDA and FPDA on the given preferences
- (b) Which solution is female-pessimal, and which is male-pessimal?

*Question 10:* If we add a new line in a subway system, then can Braess' paradox emerge? Ignore the time it requires to load/unload travelers. Is it important whether we're considering the travel time of people or subway cars? Is it important whether we consider subway cars to have finite or infinite capacity?

*Question 11:* Consider the problem of decentralized search. Assume that instead of only providing a node with it's neighbours and their grid-distance to the target, we also provide a node with their neighbours' weak links. How could you improve the decentralized search heuristic with this information?

*Question 12:* Assume you run a fast food restaurant. The sales of your products roughly follow a power law distribution. To take advantage of bulk purchases, you would like to maximize the inequality in the sales distribution. How could you do this?

Question 13: Compute the structural virality of a graph consisting of a node  $u_0$  with n children,  $u_1, \ldots u_n$ .