What Does an Aberrated Photo Tell Us About the Lens and the Scene?

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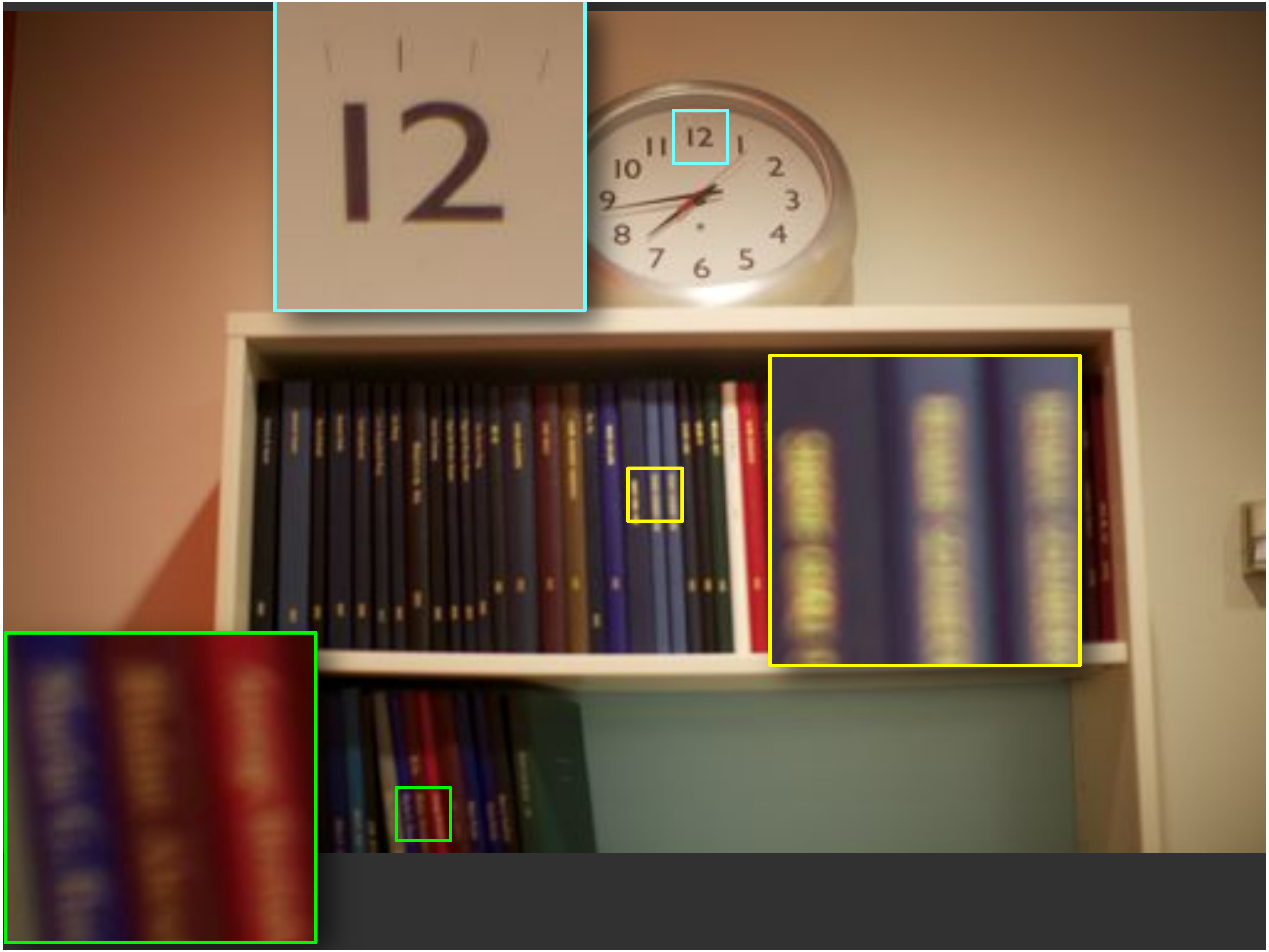
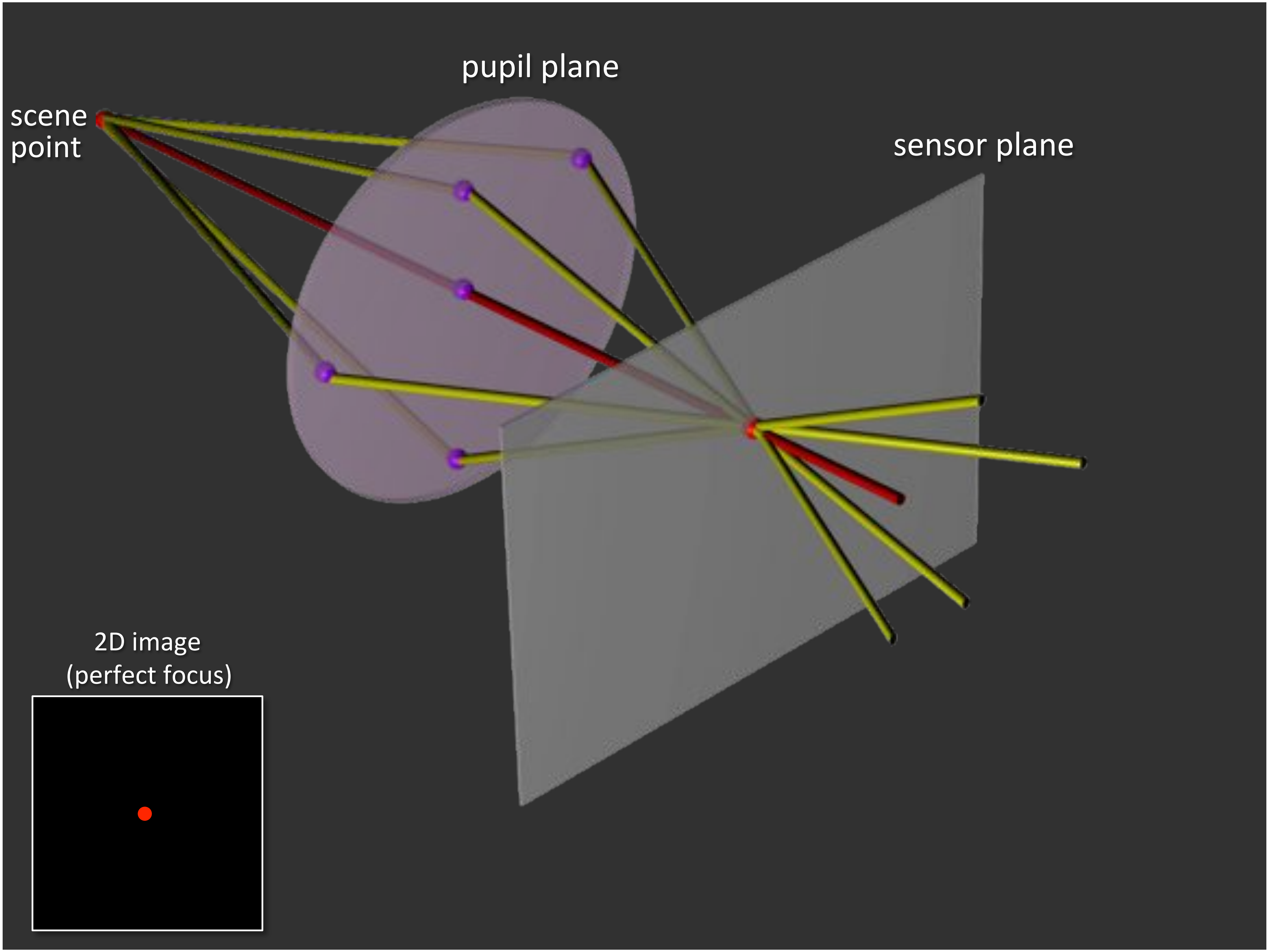
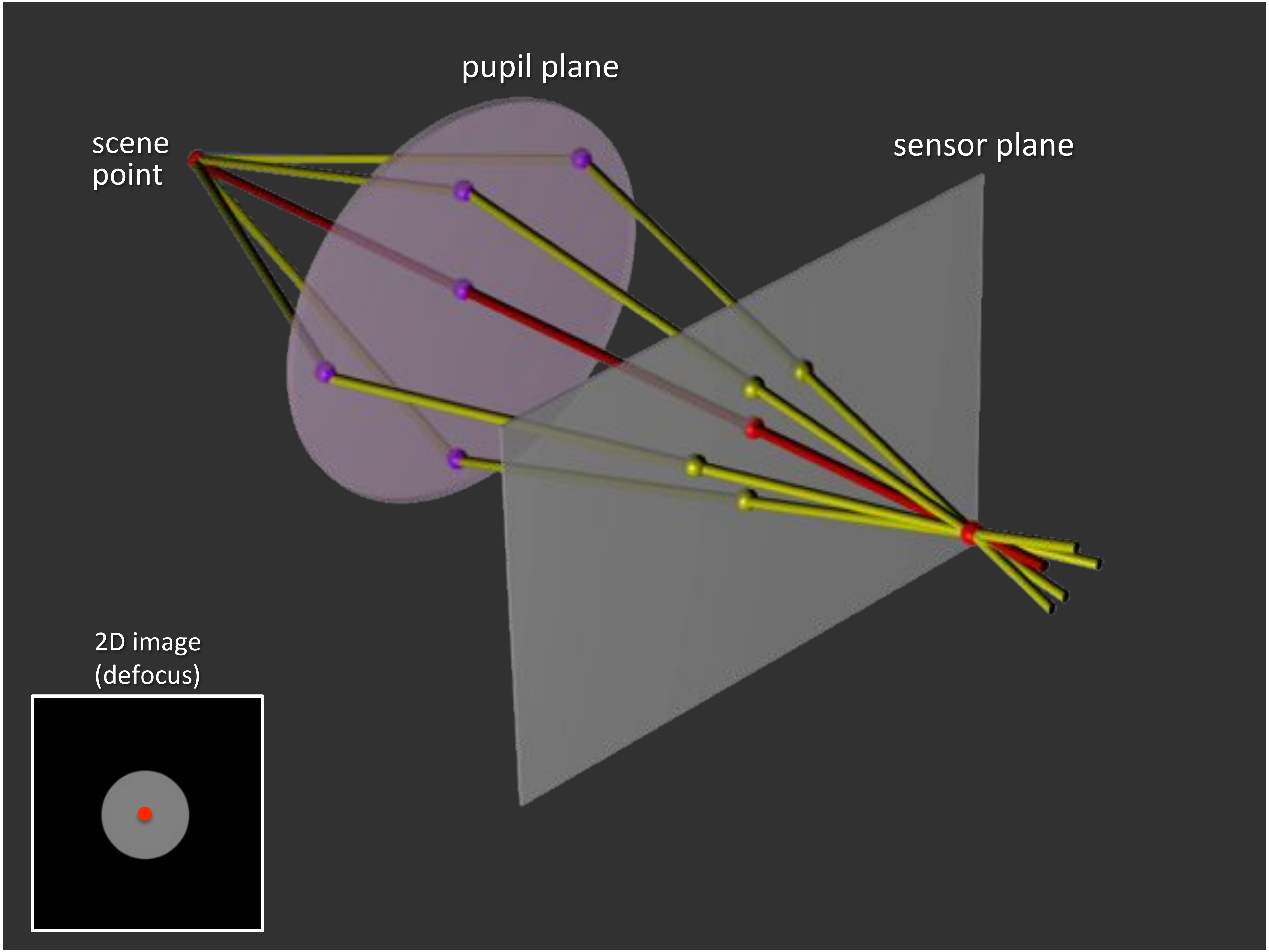
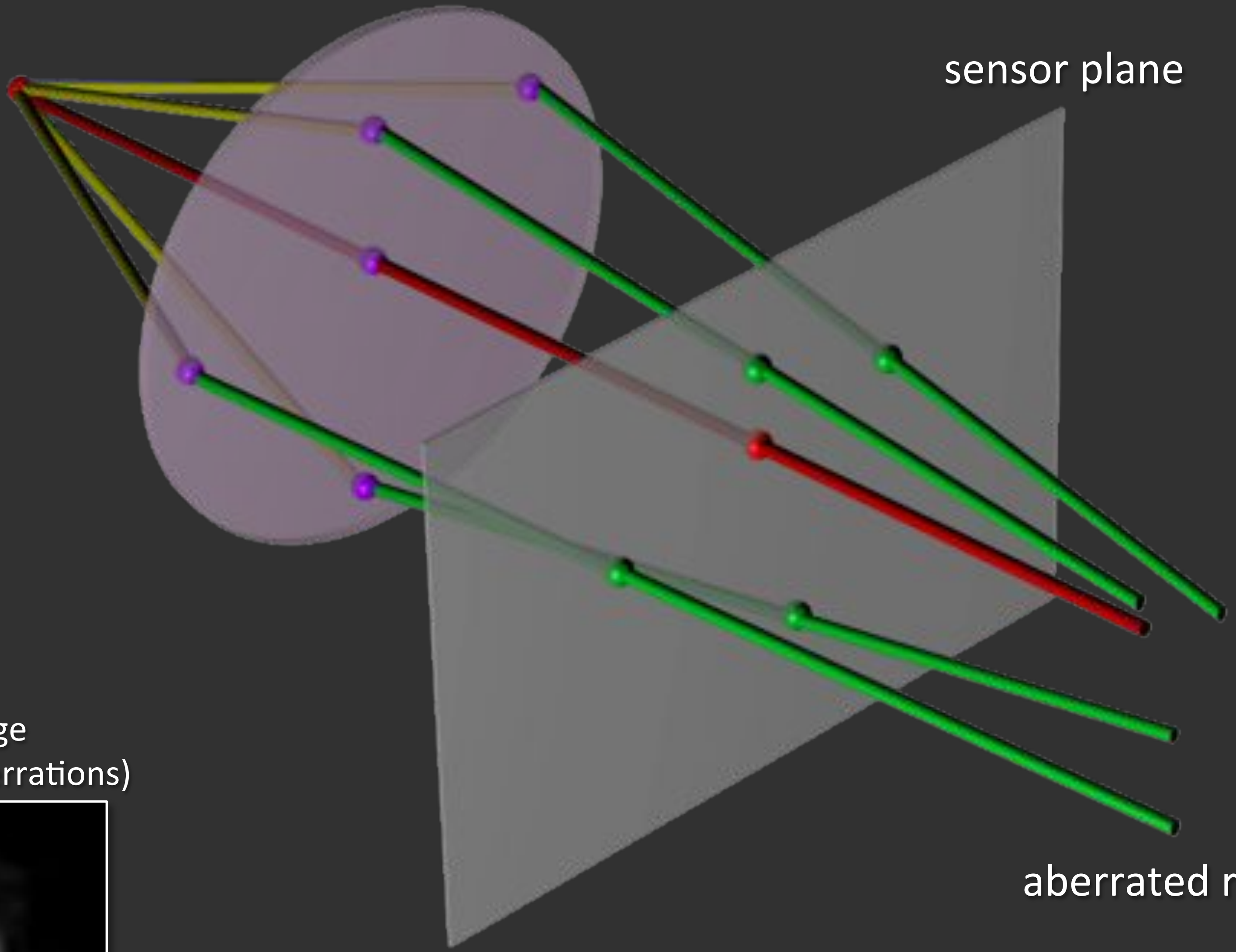



pupil plane
scene point
sensor plane
2D image
(perfect focus)


pupil plane
scene point
sensor plane
2D image (defocus)



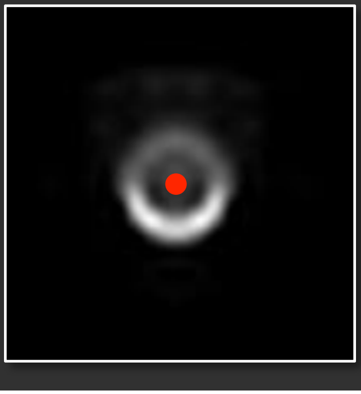
pupil plane

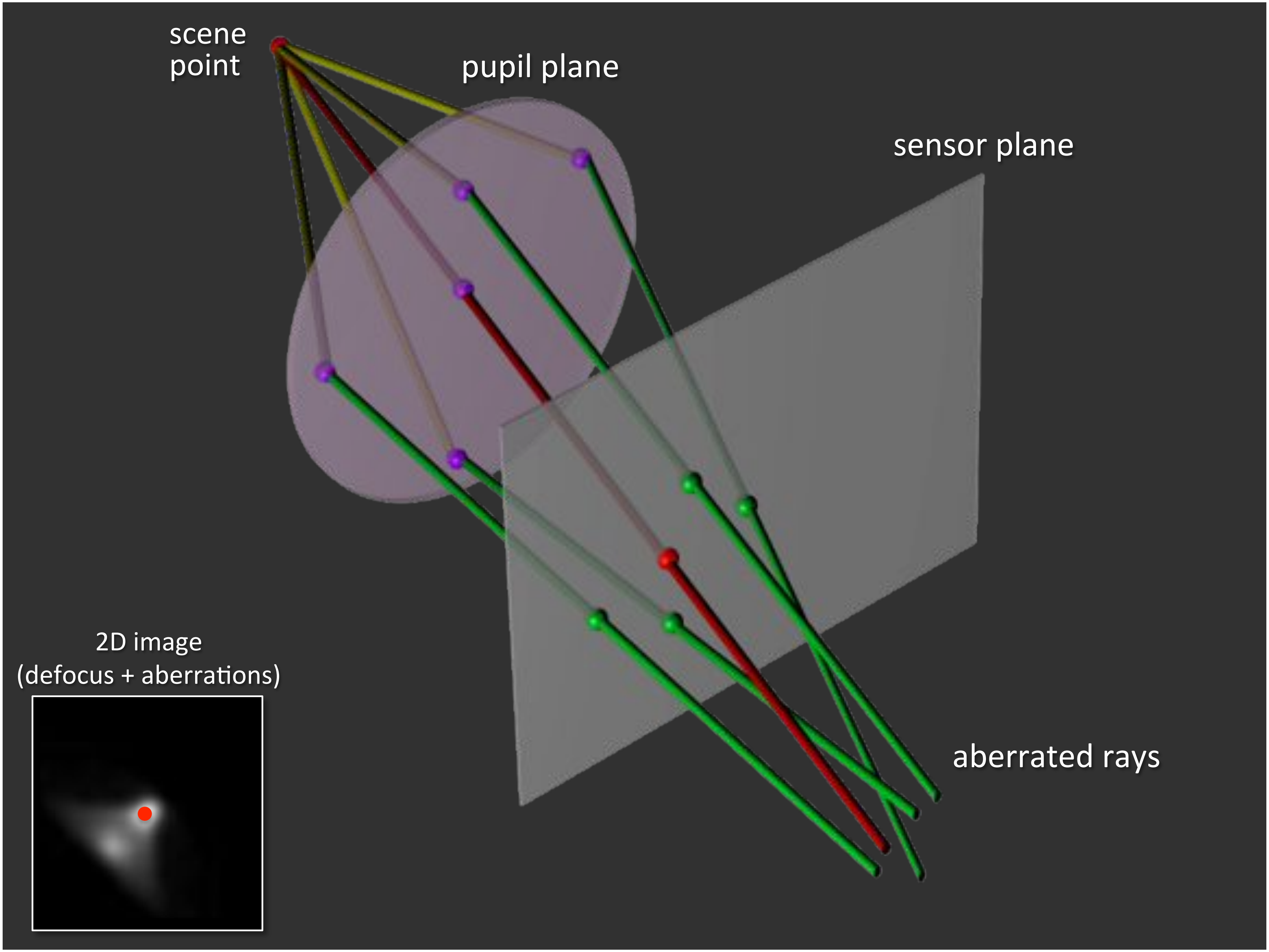
scene
point

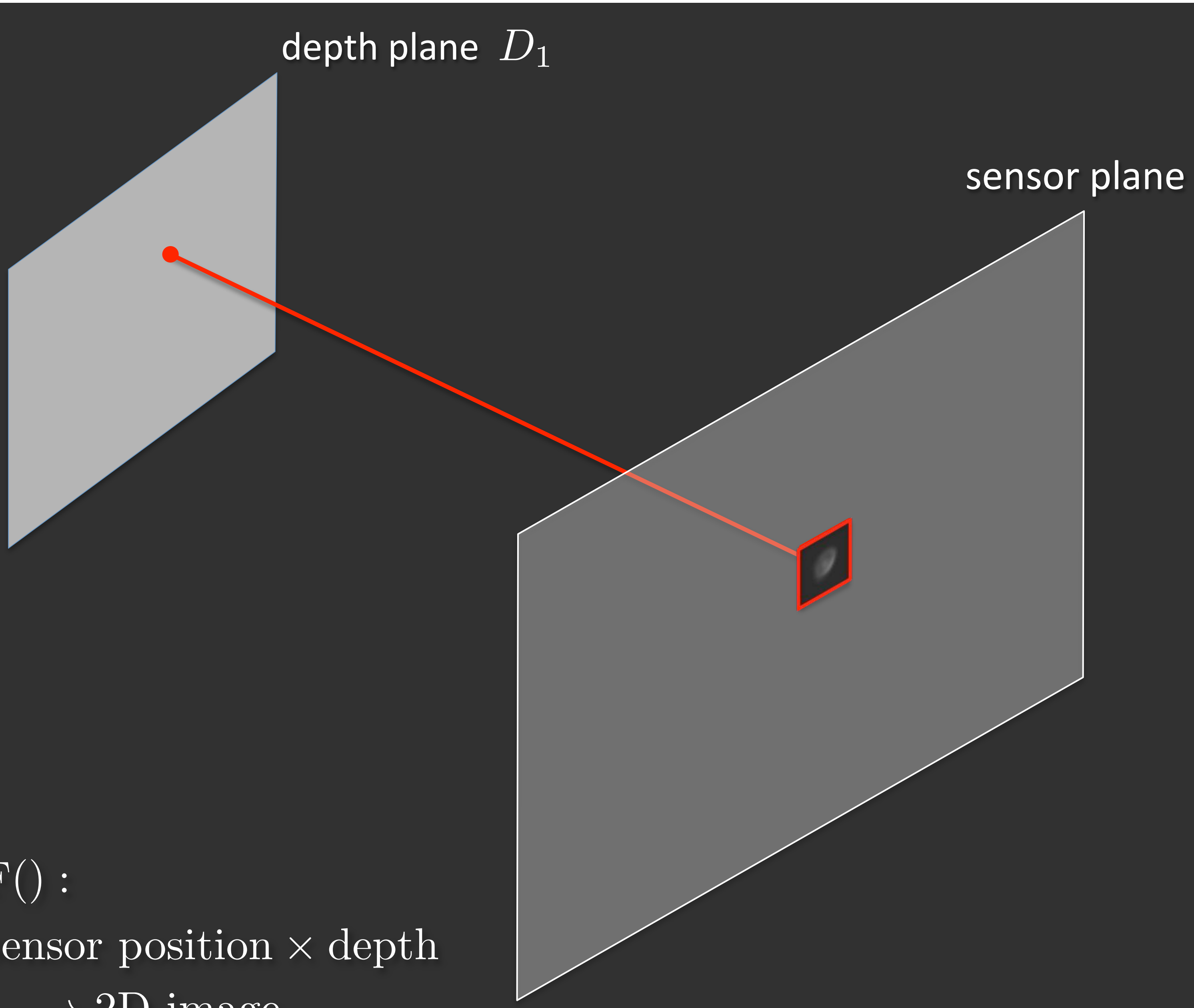
sensor plane

aberrated rays

2D image
(defocus + aberrations)



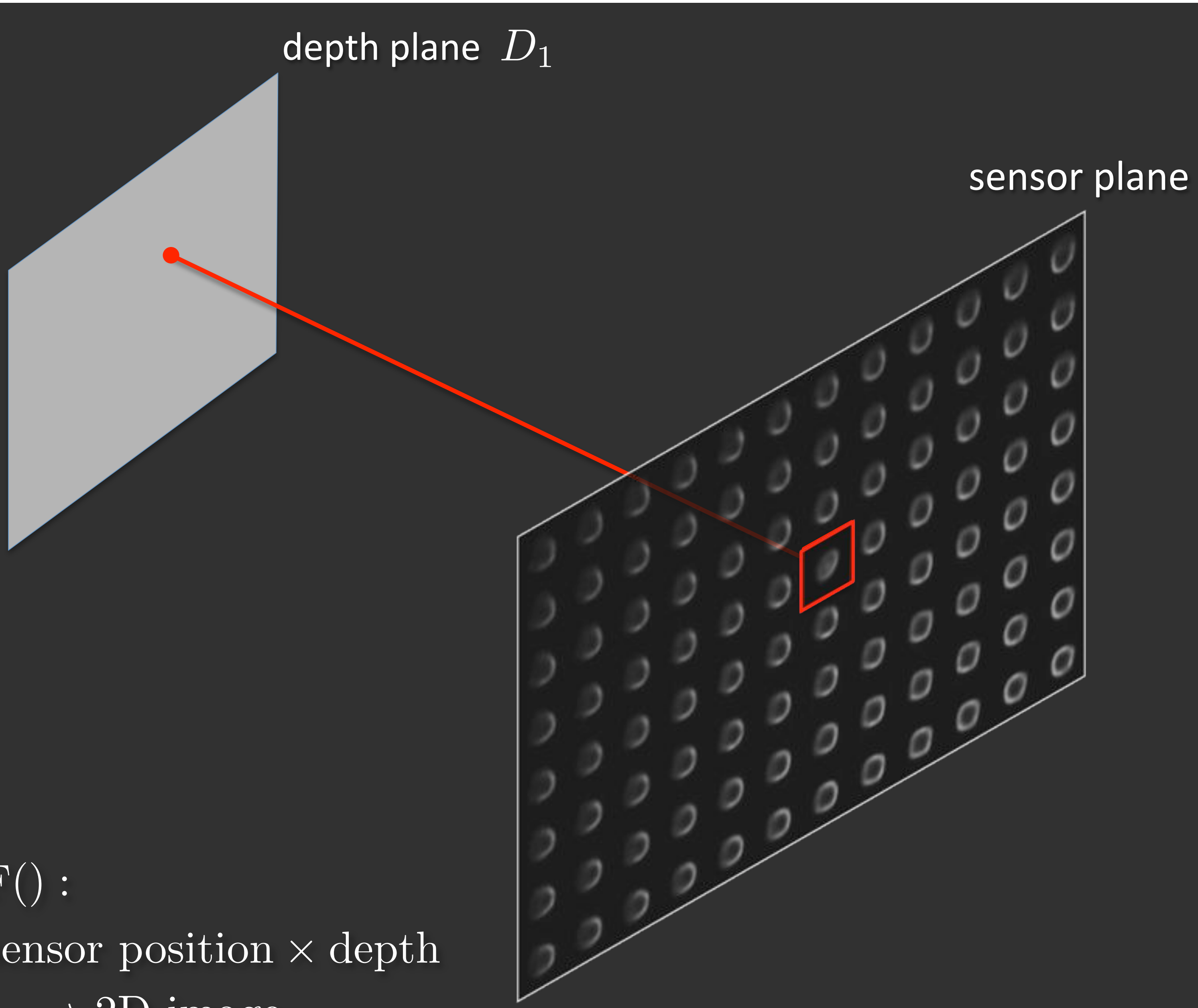

scene point
pupil plane
sensor plane
aberrated rays
2D image (defocus + aberrations)



PSF() :

sensor position × depth

→ 2D image



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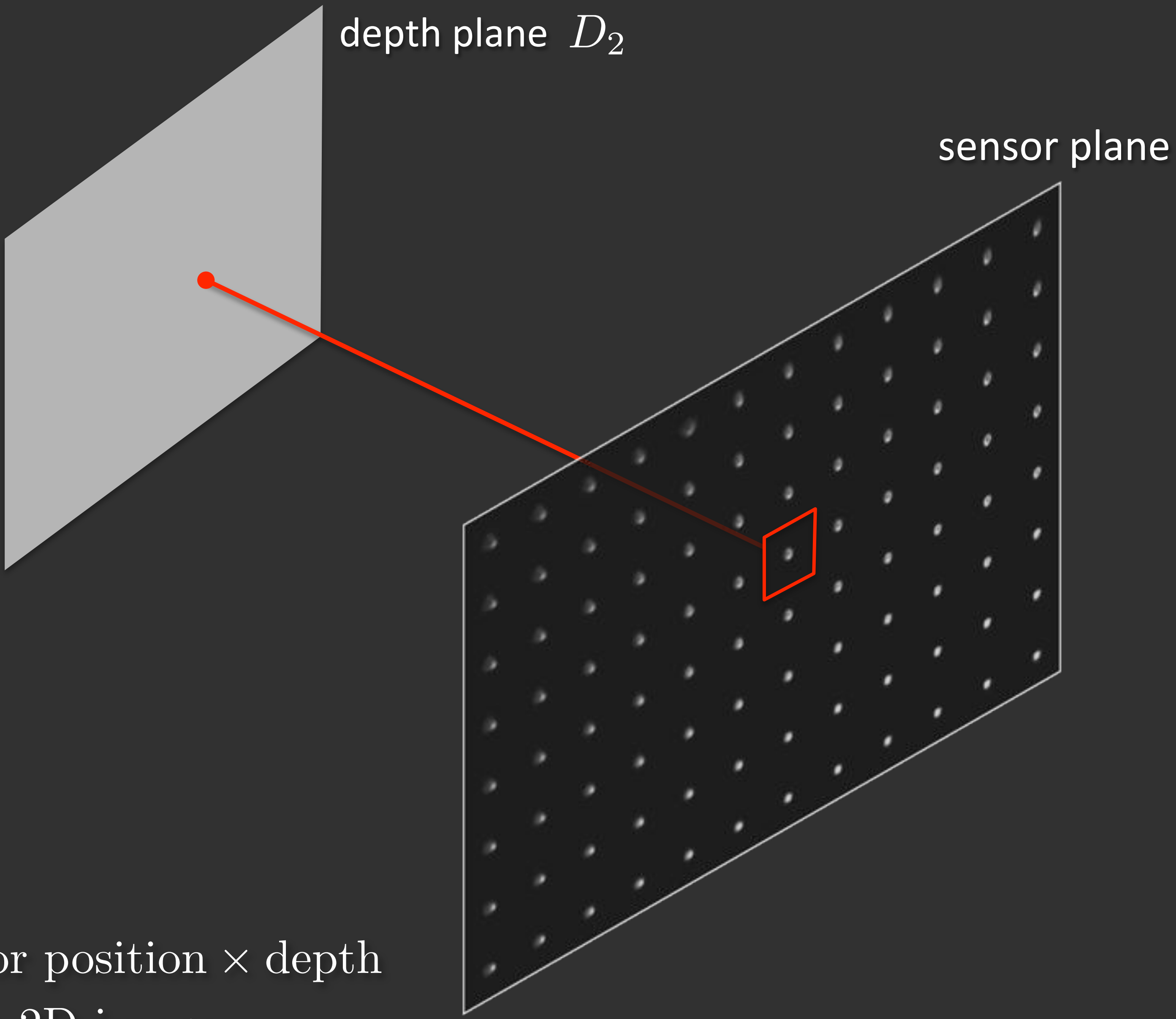

PSF() :

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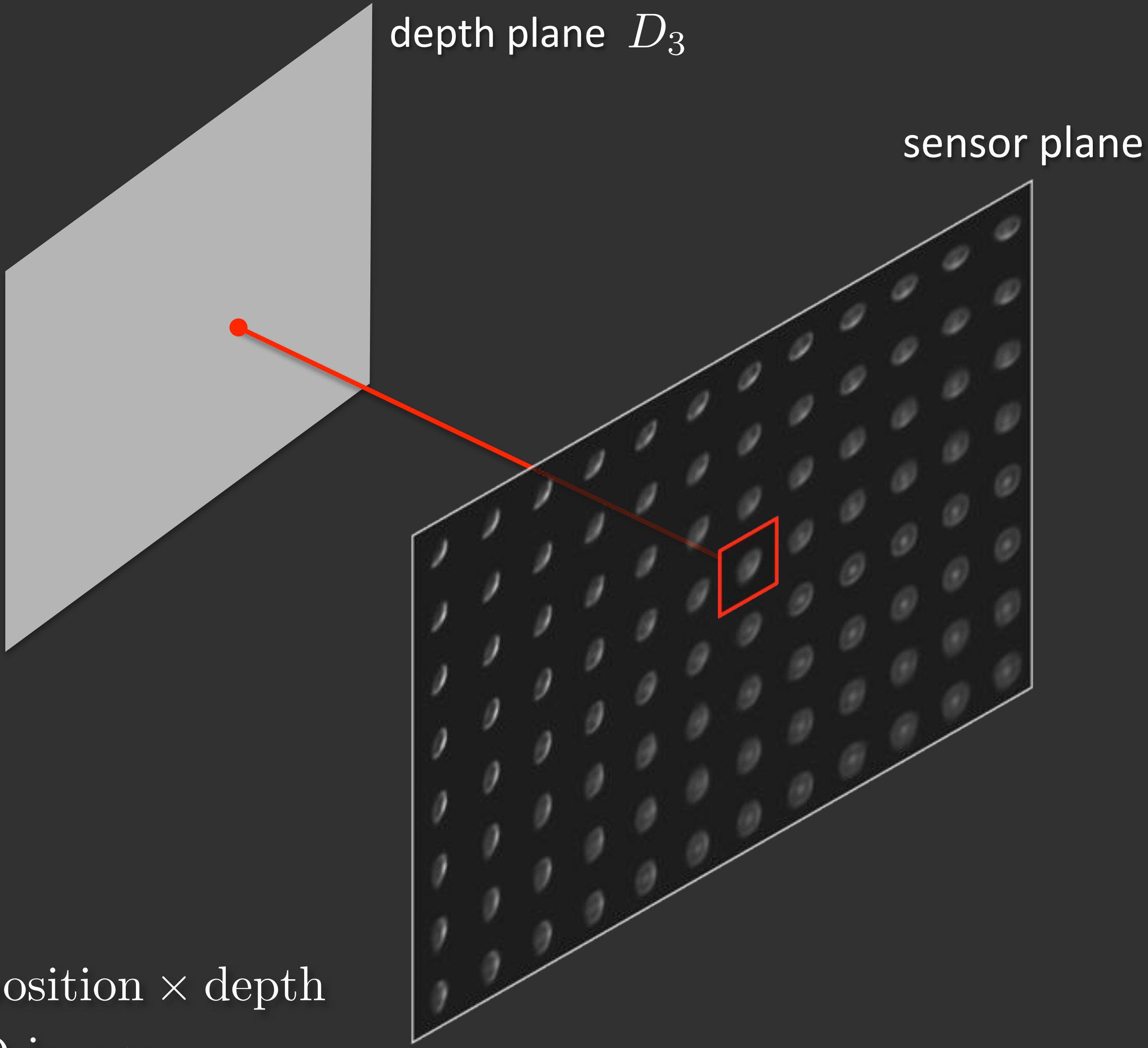
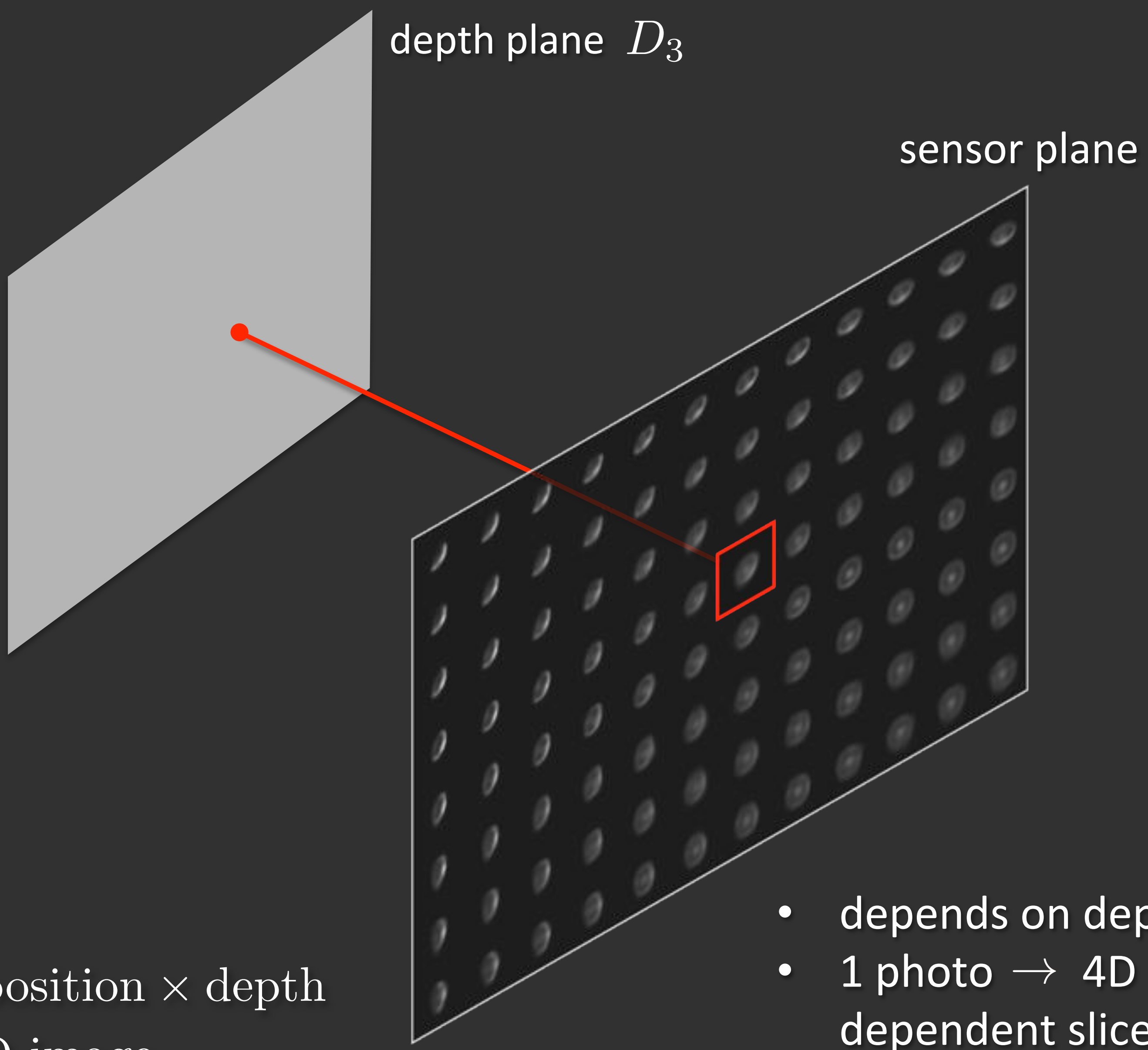


PSF() :

sensor position × depth

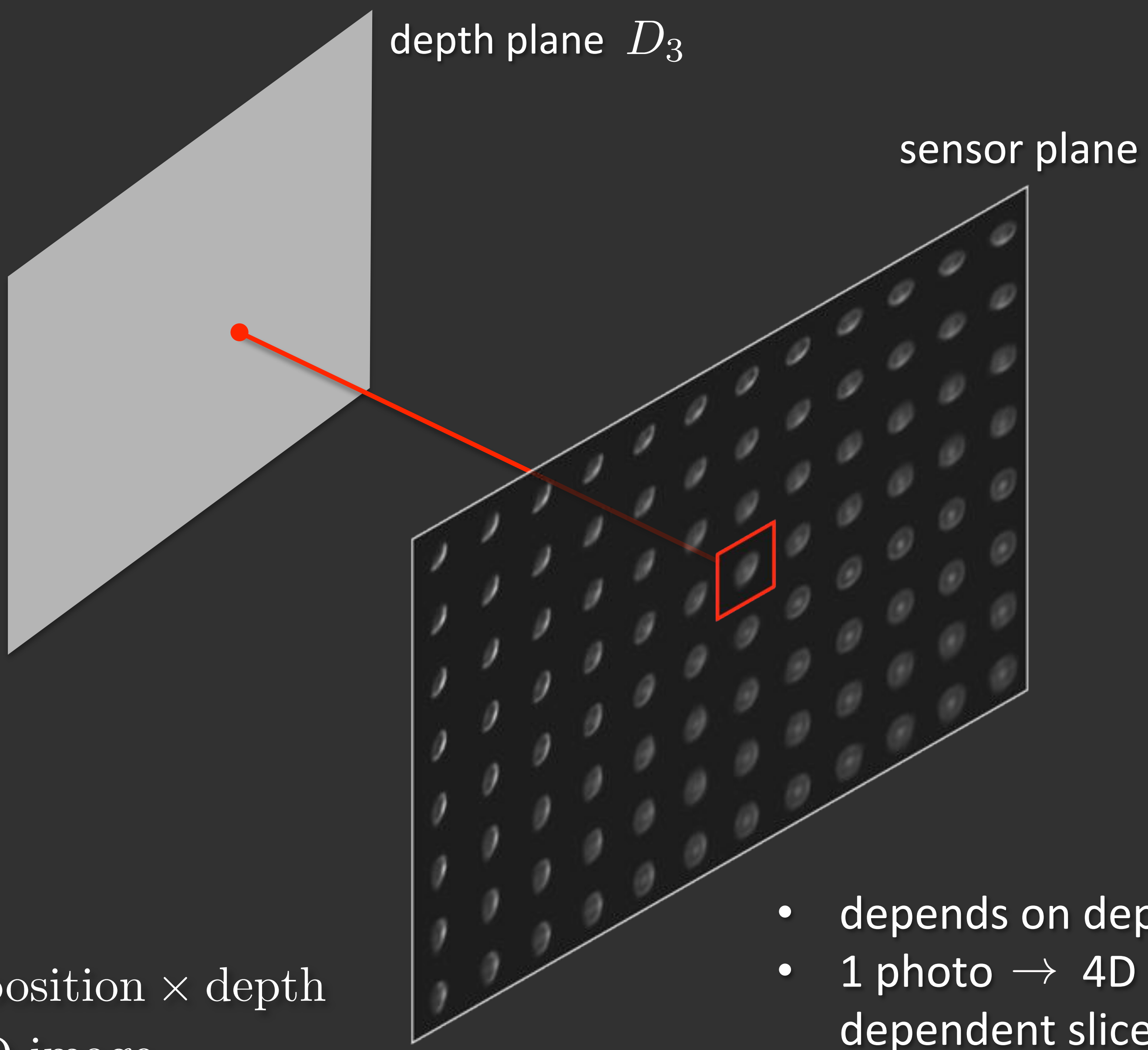
→ 2D image



PSF() :

sensor position × depth
→ 2D image

- depends on depth & lens
- 1 photo → 4D scene-dependent slice of PSF
- hard to capture
- hard to invert



PSF() :

sensor position × depth

→ 2D image

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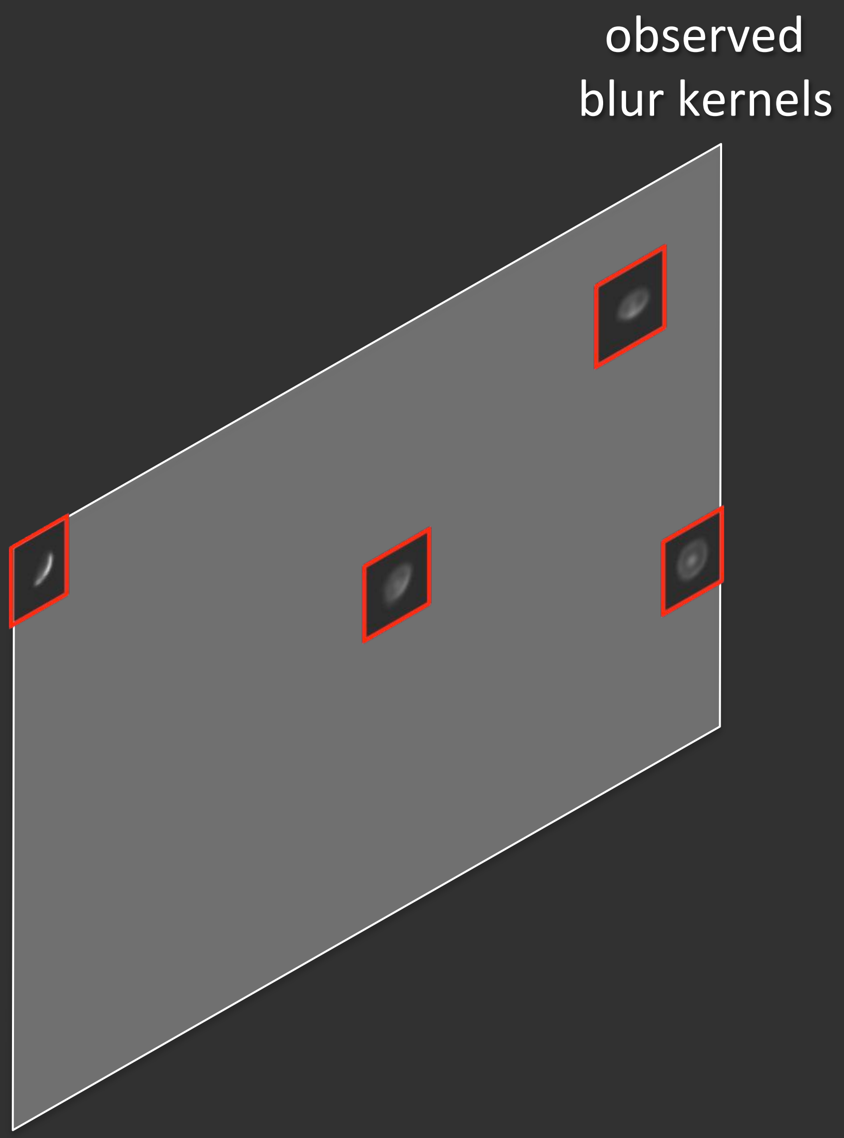
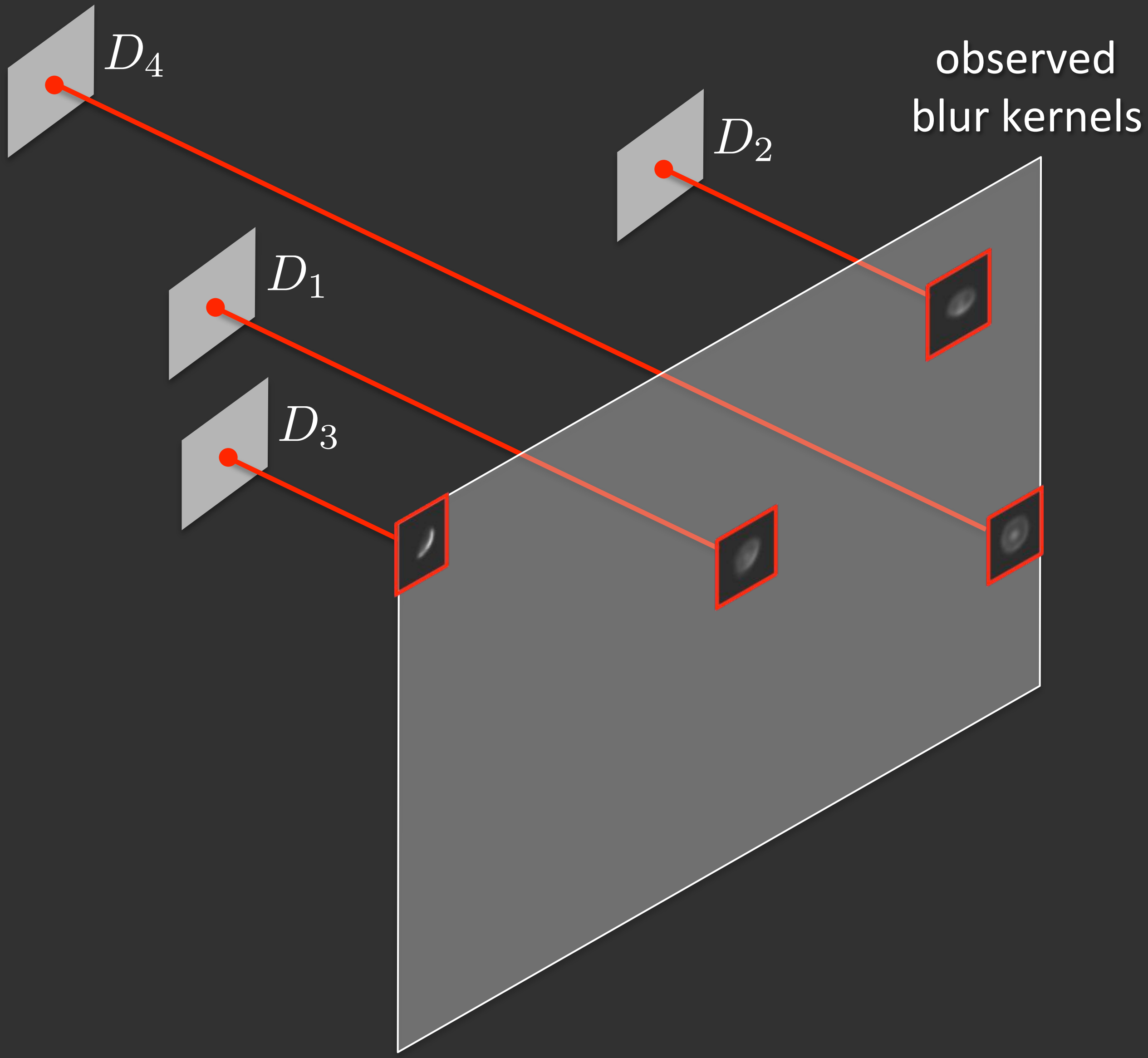
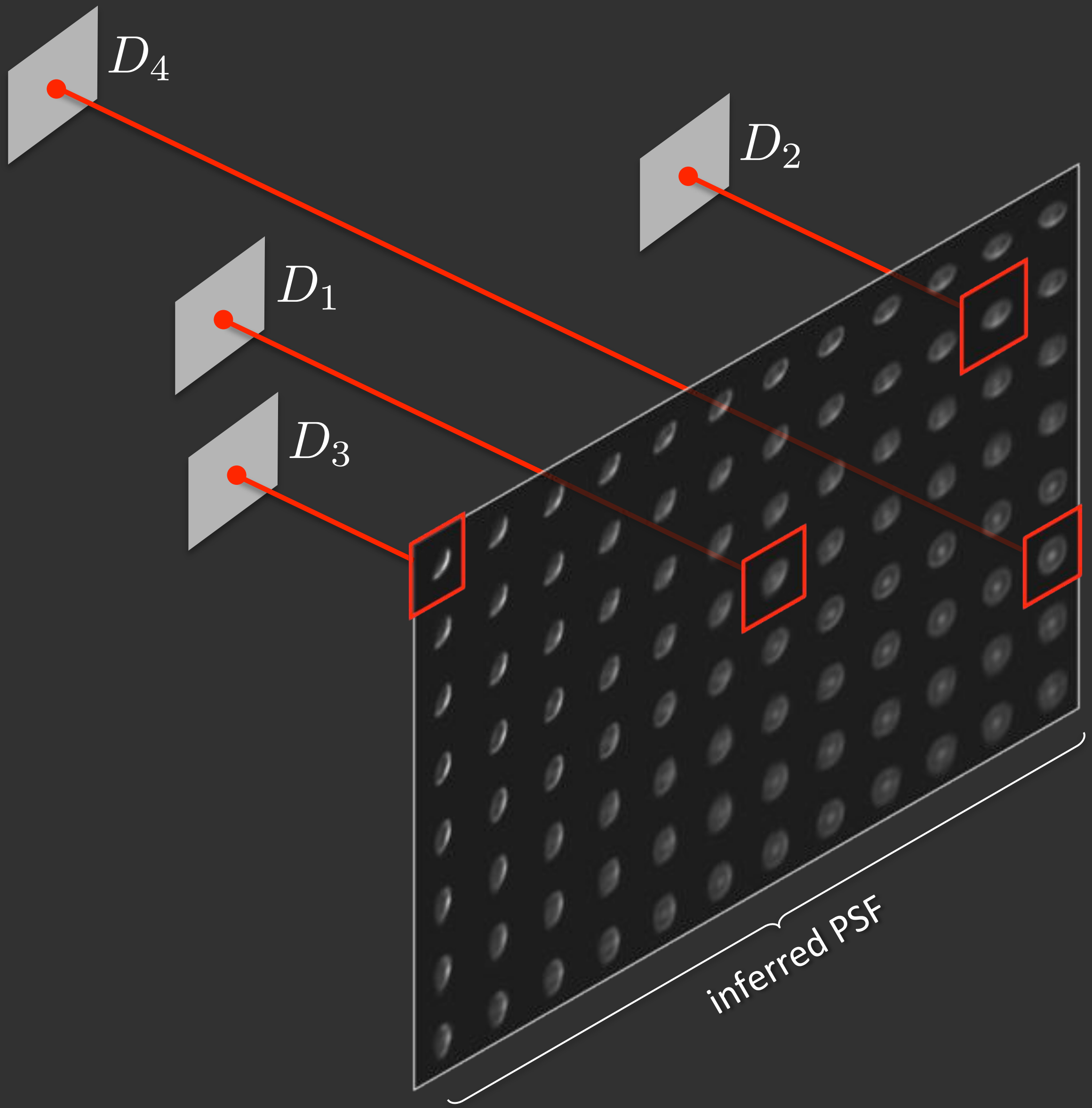
given K≥1 blur kernels from one photo ...



... can we infer depth?



… can we infer the full 5D PSF?



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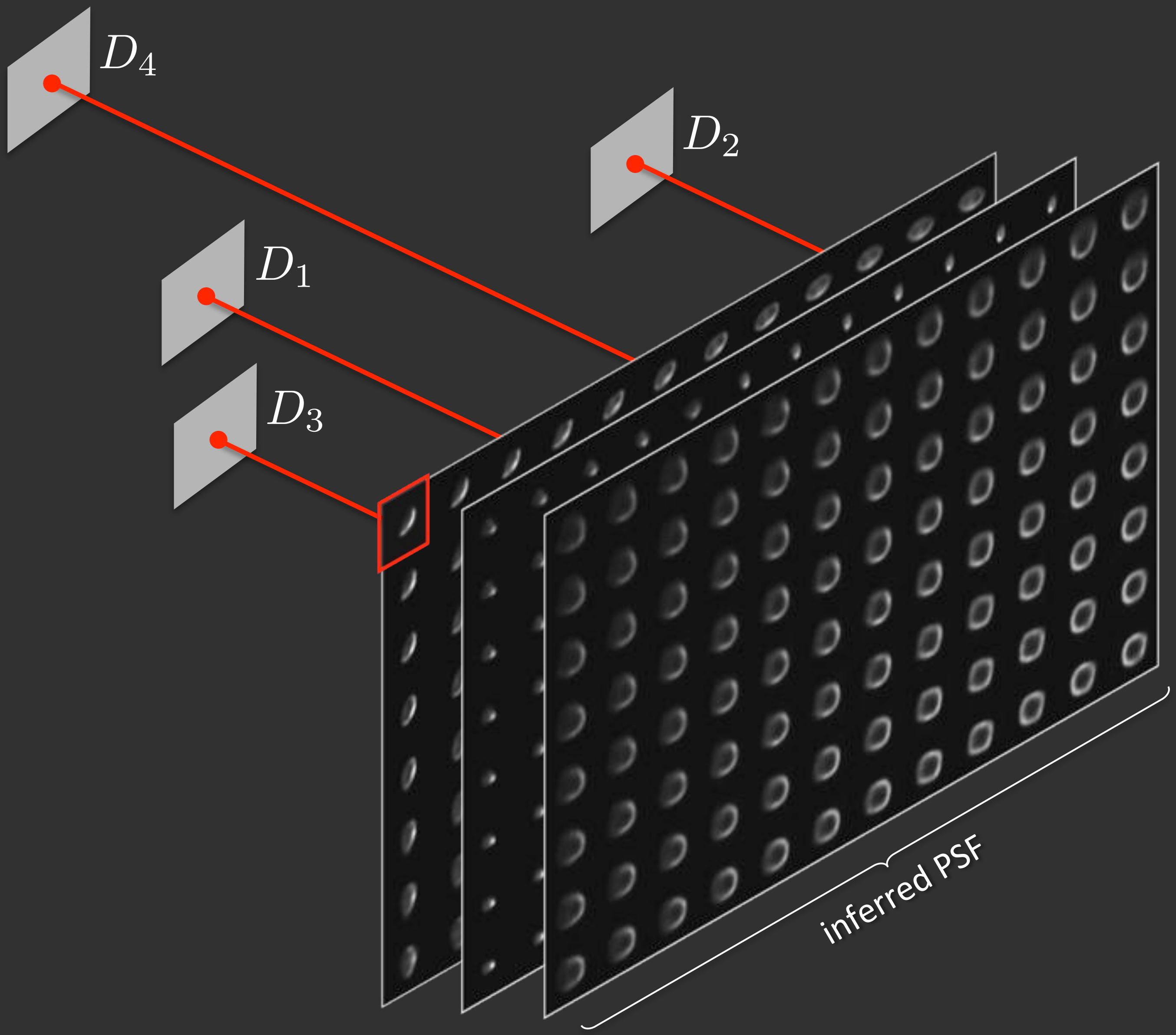
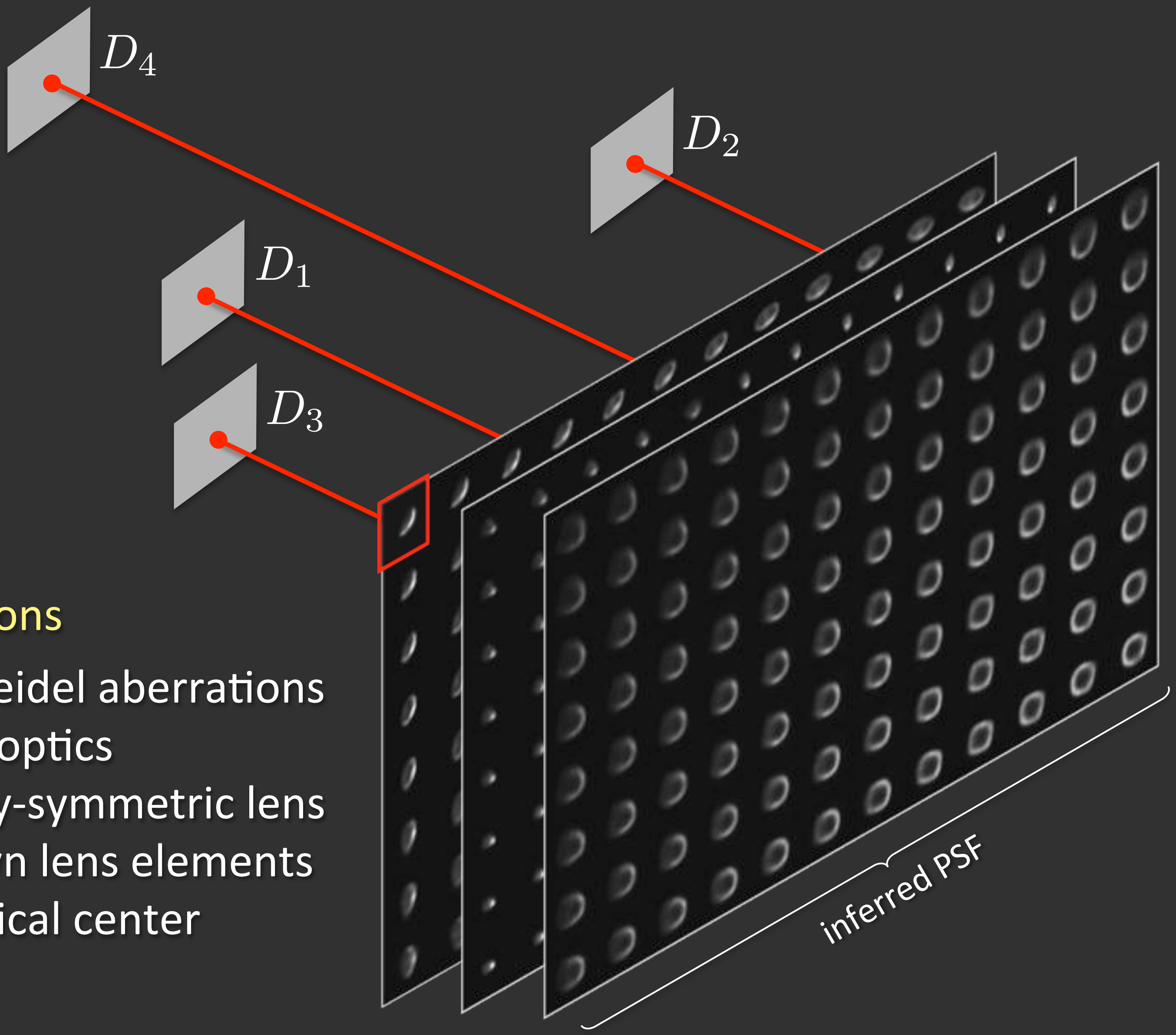


… and can we infer the lens parameters?

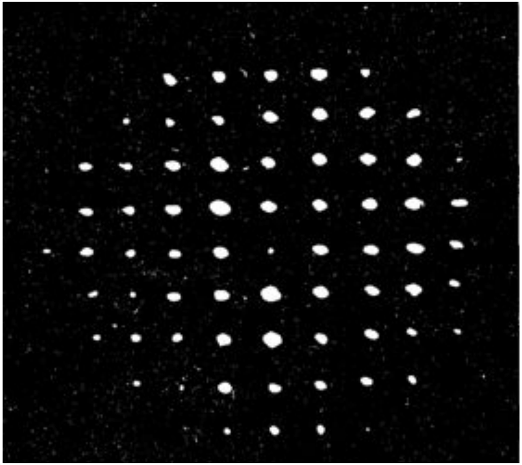


Key assumptions

- 3^{rd} order Seidel aberrations
- geometric optics
- rotationally-symmetric lens
- ≥1 unknown lens elements
- known optical center

related work

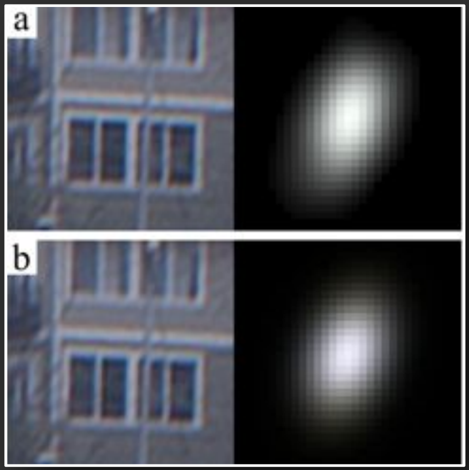
direct aberration measurement



[Liang et al 94][Ng & Hanrahan 06]

requires physical access to camera

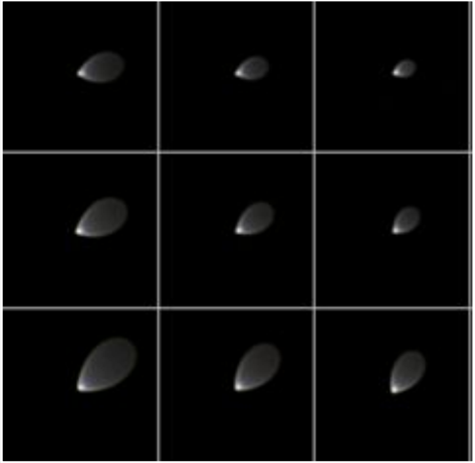
empirical PSF modeling



[Joshi et al 08][Kee et al 11]

significant data acquisition, limited inference power

blind/non-blind aberration correction

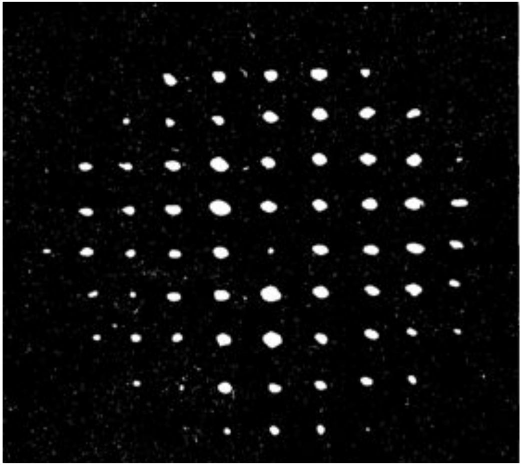


[Shih et al 12][Schuler et al 12]

interaction w/ defocus not well understood

related work

direct aberration measurement



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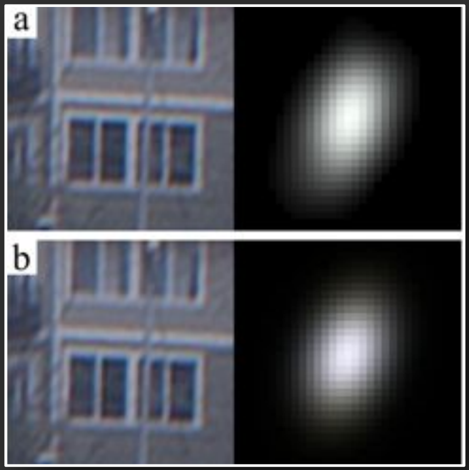

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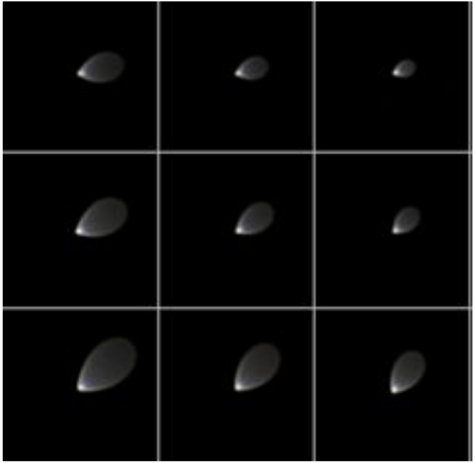

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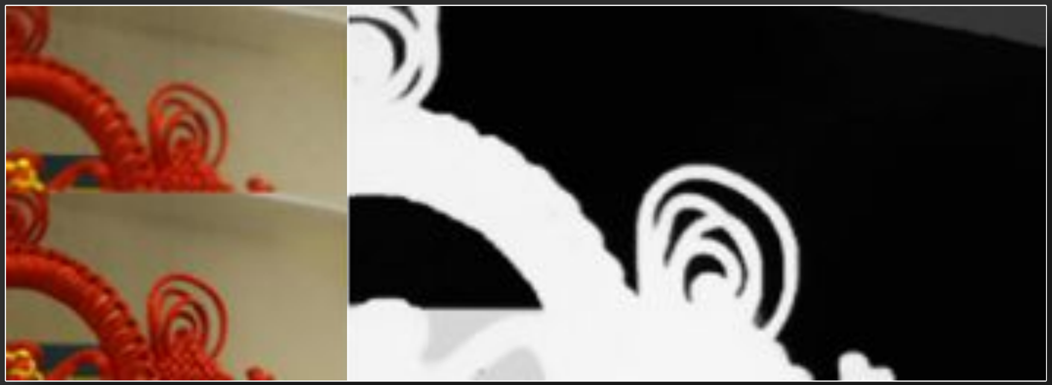


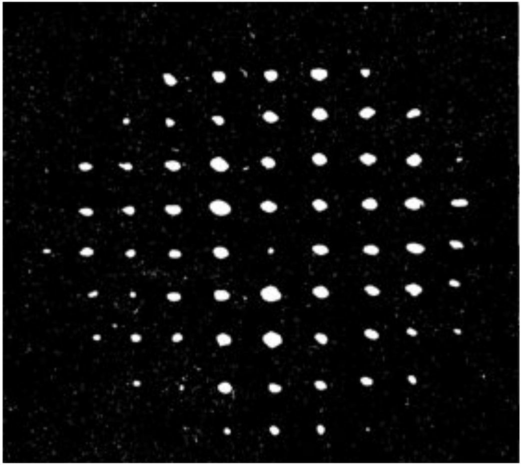
[Favaro 10]

depth from defocus

assumes aberration-free imaging

related work

direct aberration measurement

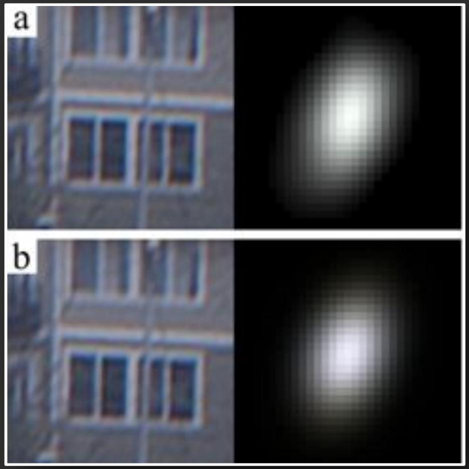


[Liang et al 94][Ng & Hanrahan 06]

requires physical access to camera

inference from 1 aberrated photo

empirical PSF modeling

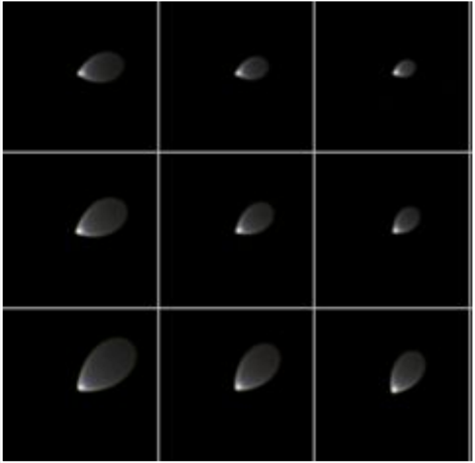


[Joshi et al 08][Kee et al 11]

significant data acquisition, limited inference power

physics-based model of 5D PSF formation

blind/non-blind aberration correction

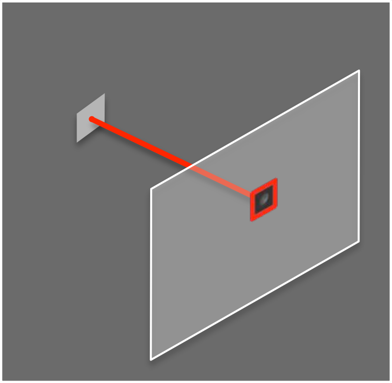


[Shih et al 12][Schuler et al 12]

interaction w/ defocus not well understood

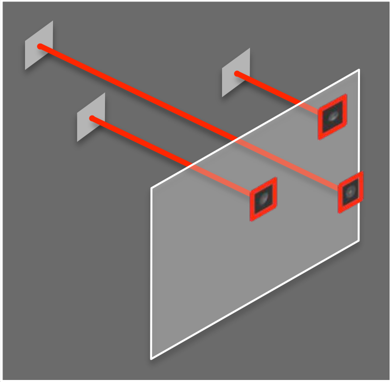
unified analysis of aberration & defocus

three main results



single-point inference

regardless of deblurring quality, cannot predict lens blur at other depths/image locations



multi-point inference

reconstruction of depth & 5D PSF may be possible from just 1 shot

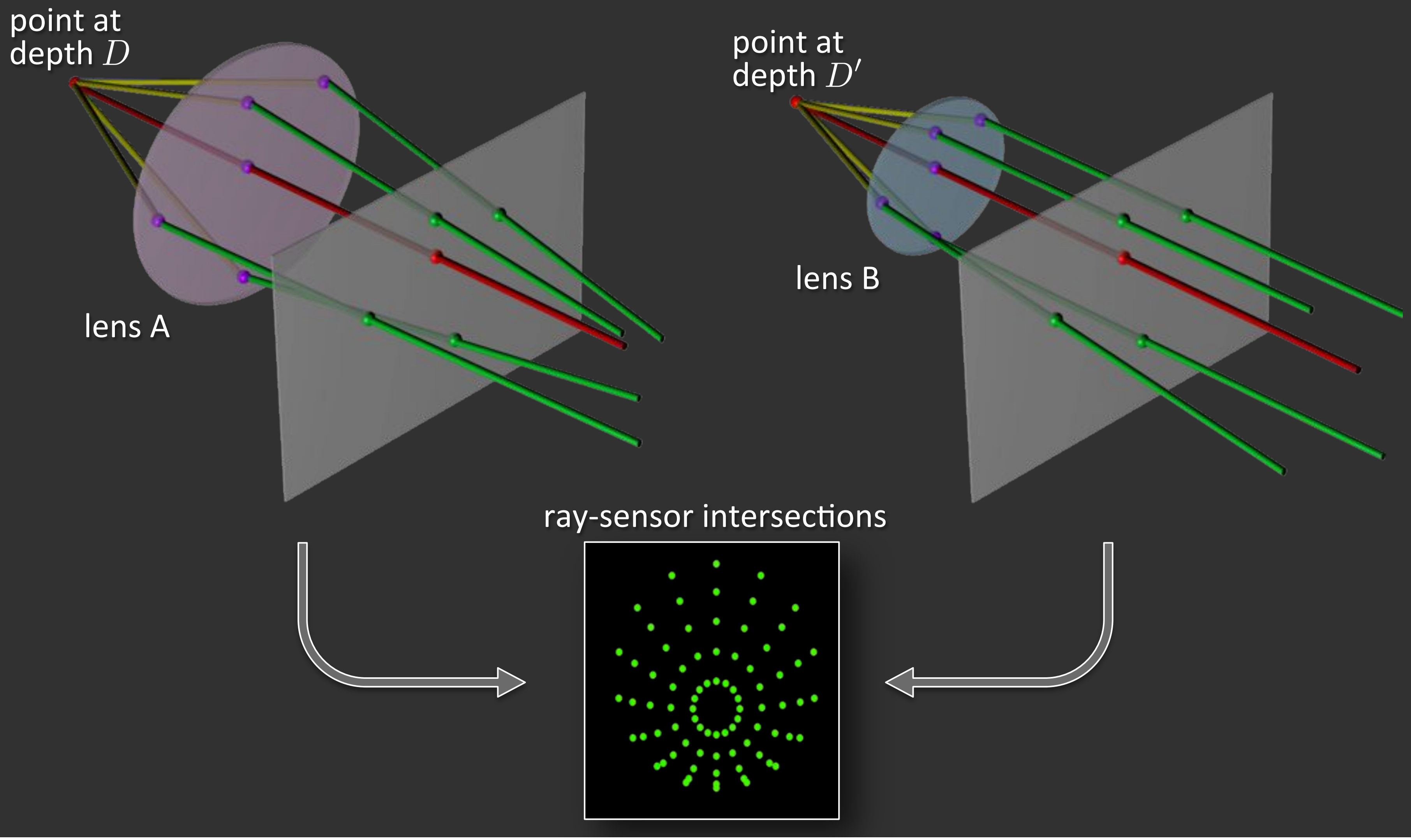
e ambiguity
in Seidel coefs & K defocus levels

practical modeling/inference of 5D PSFs

instead of modeling the PSF directly, model the aberrated rays

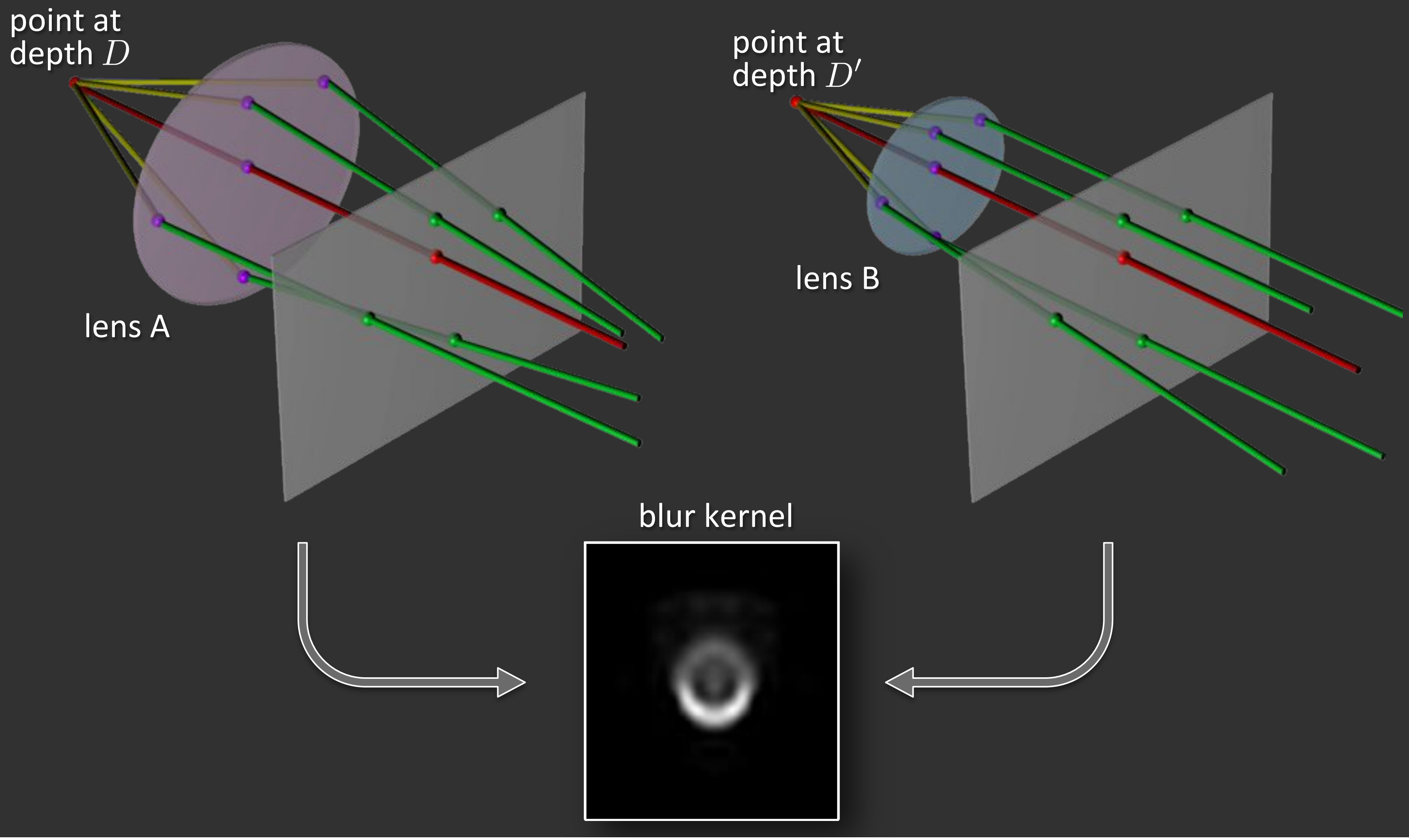
blur-equivalent arrangements

point-lens arrangements that send aberrated rays to identical pixels



Blur-consistent arrangements

point-lens arrangements that send aberrated rays to identical pixels



point at
depth D

lens A

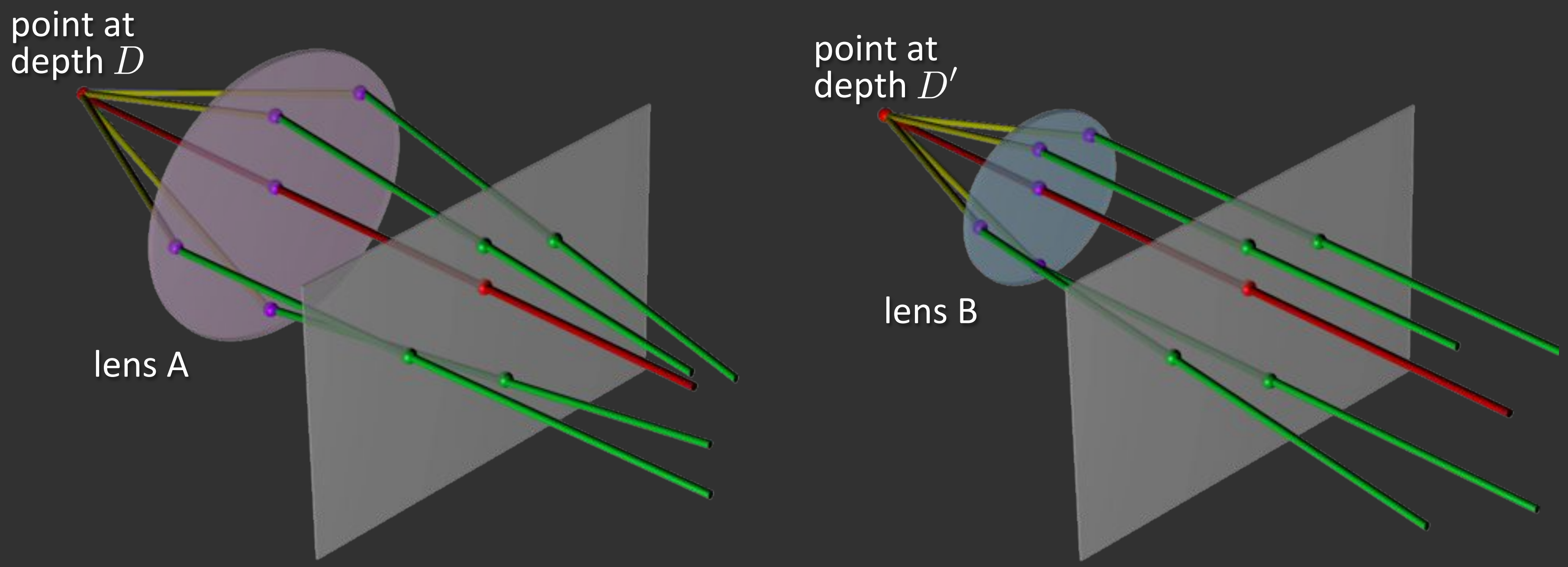
point at
depth D'

lens B

blur kernel

Blur-consistent arrangements

point-lens arrangements that send aberrated rays to identical pixels



approach: we characterize the space of blur-equivalent arrangements

i. the PSF integral

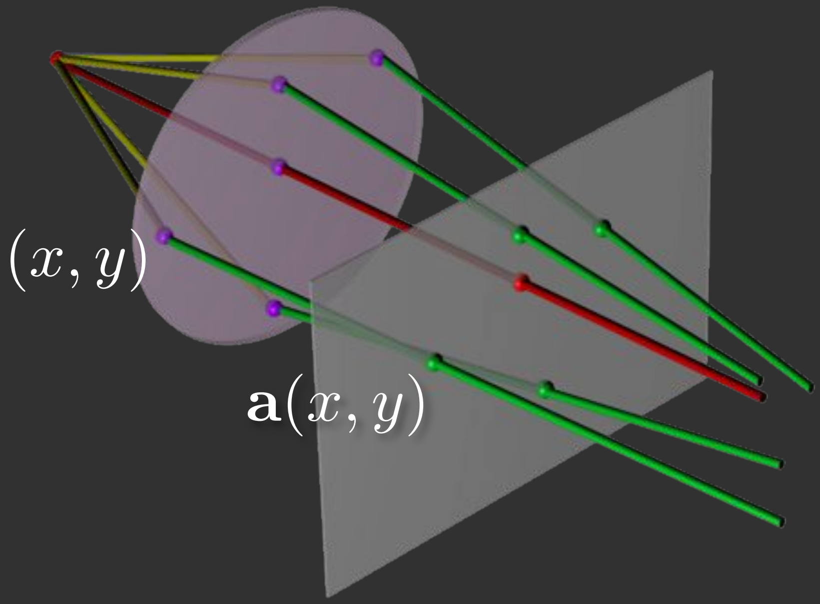
ii. the ray intersection function

iii. single-source inference

iv. multi-source inference

iv. preliminary experiments

blur formation model

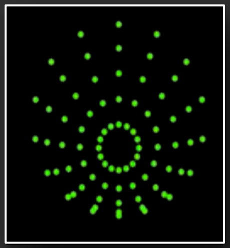


blur
kernel

complex function of
lens params & defocus

$$= \int_{x^2+y^2 \leq R}$$

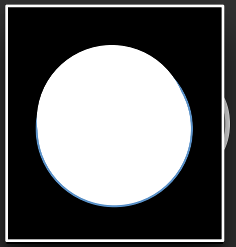
ray-sensor
intersections



simple 3rd order
polynomial

$$\cdot$$

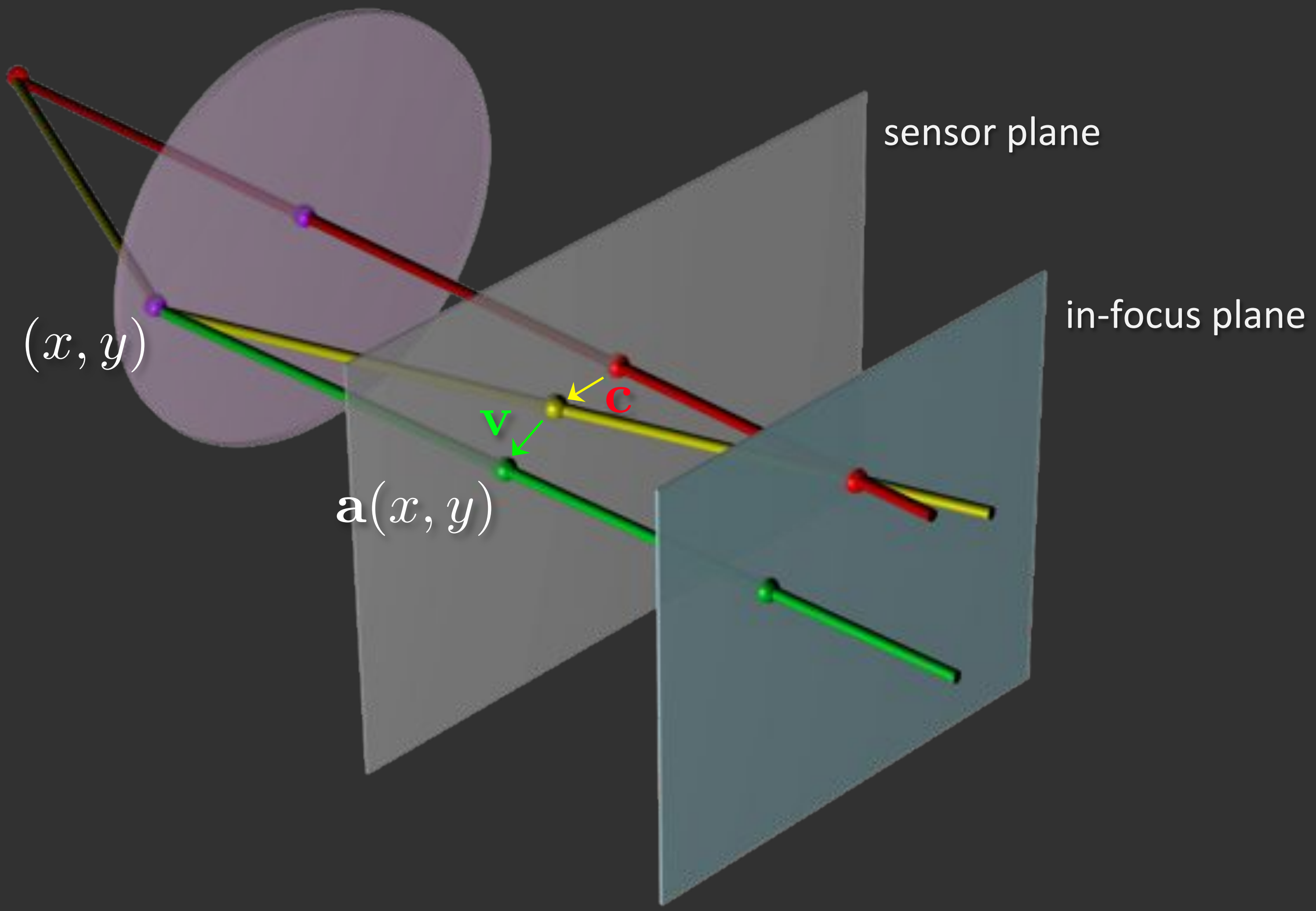
vignetting
function



intersection
of discs

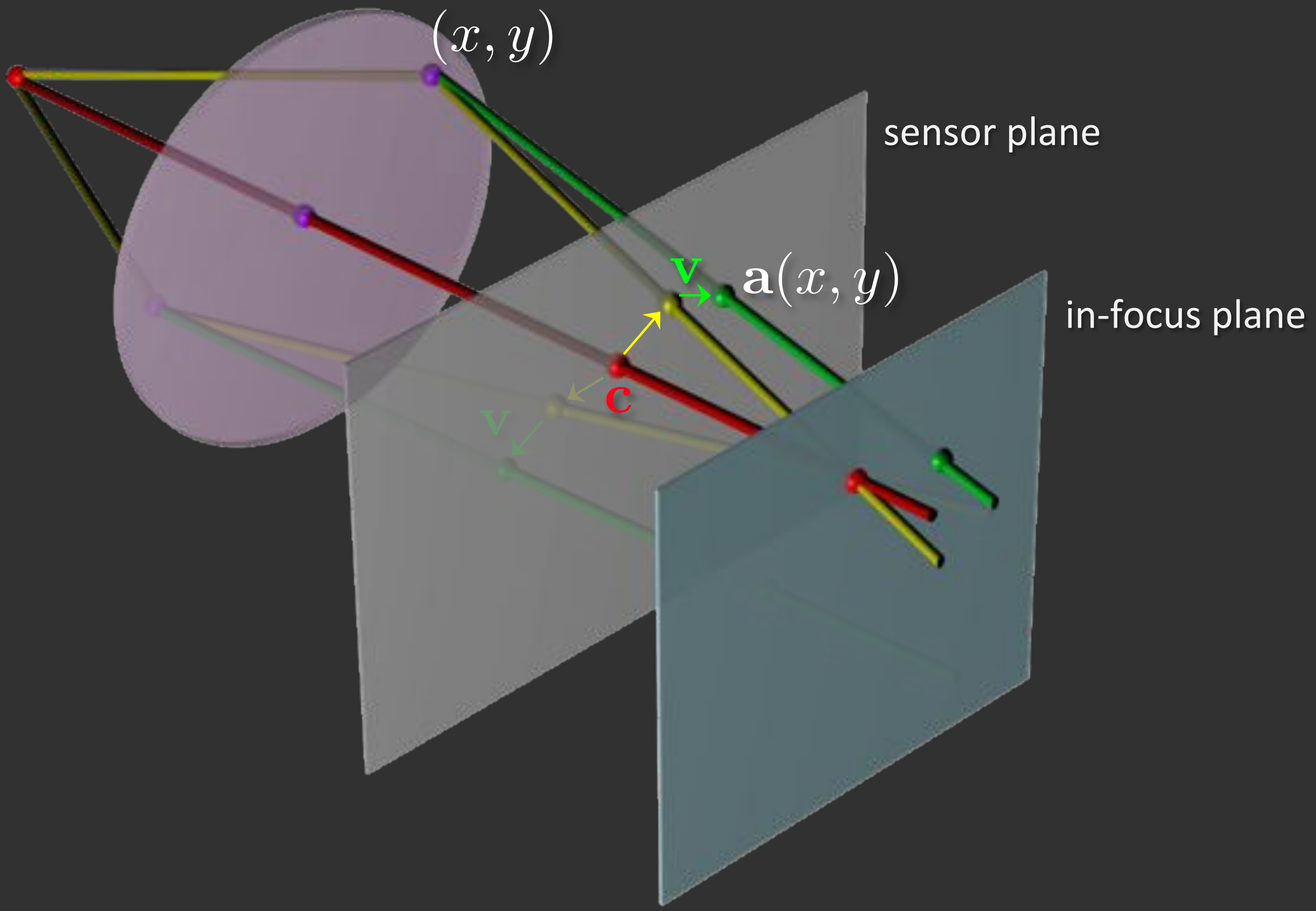
$$\mathrm{d}x\mathrm{d}y$$

ii. the ray intersection function (for 3rd order Seidel aberrations)



ray intersection = perspective + defocus + aberration

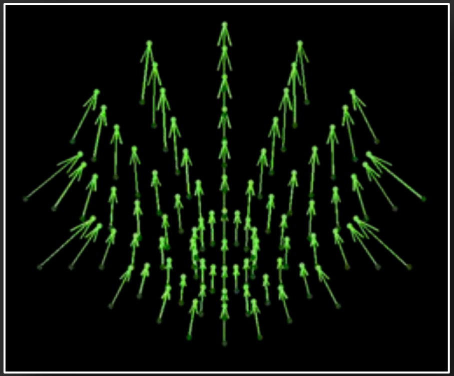
$\mathbf{a}(x, y)$ $\quad$ $\mathbf{c}$ $\quad$ $(x, y) \cdot D$ $\quad$ $\mathbf{v}$



ray intersection = perspective + defocus + aberration

$\mathbf{a}(x, y)$ $\qquad$ $\mathbf{c}$ $\qquad$ $(x, y) \cdot D$ $\qquad$ $\mathbf{v}(x, y, \mathbf{c}, D)$

the Seidel displacement field



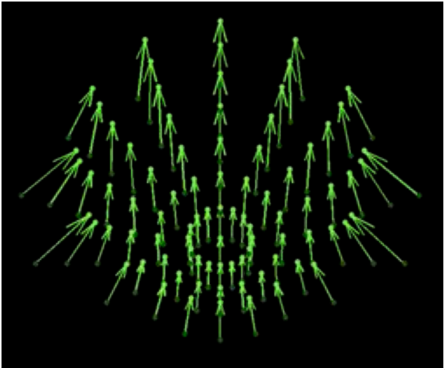
$\mathbf{v}(x, y, \mathbf{c}, D)$

- 3rd order polynomial of pupil coordinates, perspective projection & defocus
- fully determined by 5 lens-specific aberration parameters

ray intersection = perspective + defocus + aberration

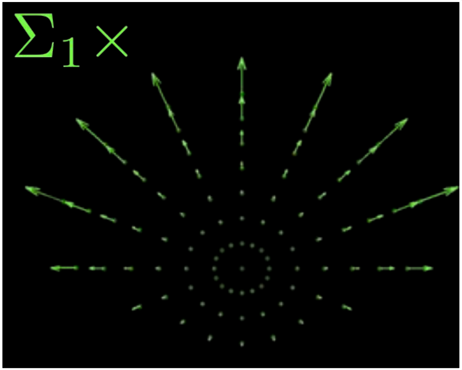
$\mathbf{a}(x, y) \quad \mathbf{c} \quad (x, y) \cdot D \quad \mathbf{v}(x, y, \mathbf{c}, D)$

the Seidel displacement field



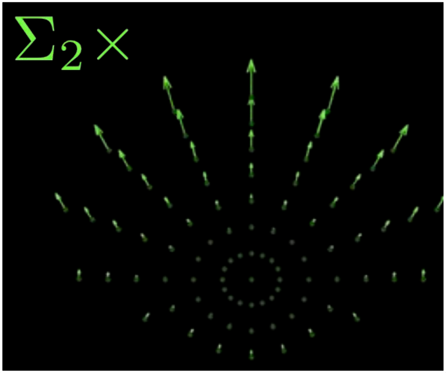
$\mathbf{v}(x, y, \mathbf{c}, D)$

$=$

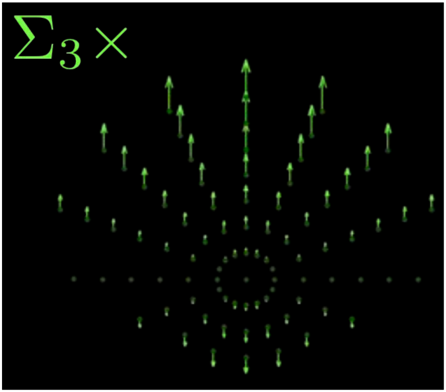


spherical

$+$

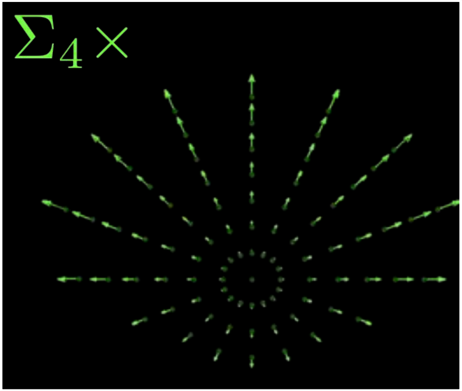


coma



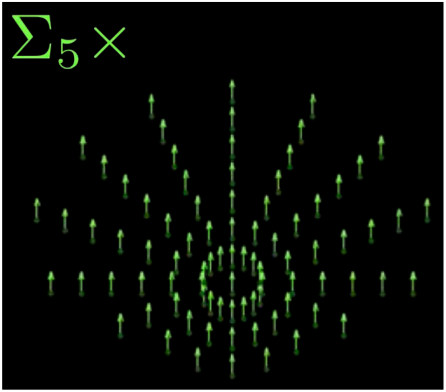
astigmatism

$+$



field curvature

$+$



field distortion

ray intersection	=	perspective	+	defocus	+	aberration
$\mathbf{a}(x, y)$		$\mathbf{c}$		$(x, y) \cdot D$		$\mathbf{v}(x, y, \mathbf{c}, D)$

the ray intersection polynomial

$$\left(\begin{bmatrix}0\\0\\0\\D\\|c|\end{bmatrix}+\begin{bmatrix}1 & D & D^2 & D^2 & D^3\\0 & |c| & D|c| & 2D|c| & 3D|c|^2\\0 & 0 & |c|^2 & 0 & 2D|c|^2\\0 & 0 & 0 & |c|^2 & D|c|^2\\0 & 0 & 0 & 0 & |c|^3\end{bmatrix}\begin{bmatrix}\Sigma_1\\\Sigma_2\\\Sigma_3\\\Sigma_4\\\Sigma_5\end{bmatrix}\right)^T\begin{bmatrix}[x^2+y^2](x,y)\\(3x^2+y^2,2xy)\\(x,0)\\(x,y)\\(1,0)\end{bmatrix}$$

perspective & defocus aberrations pupil coords

$$\langle \mathbf{c}, D, R, \Sigma_1, \Sigma_2, \Sigma_3, \Sigma_4, \Sigma_5 \rangle$$

specific pupil to point

radius

aberration parameters

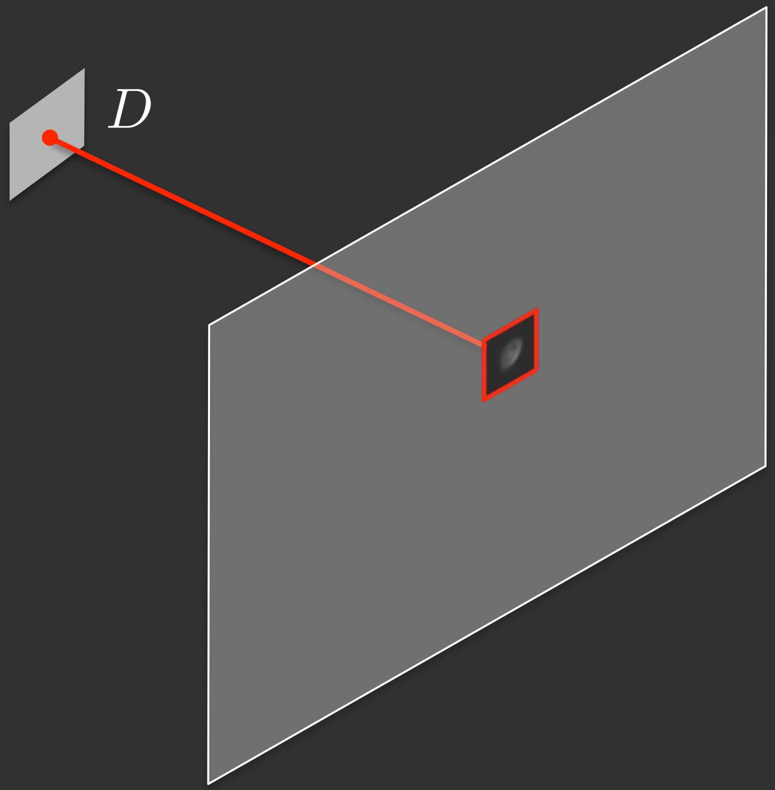
ray intersection = perspective + defocus + aberration

$\mathbf{a}(x,y)$ $\mathbf{c}$ $(x,y)\cdot D$ $\mathbf{v}(x,y,\mathbf{c},D)$

iii. single-source inference

limits of local inference

what does 1 blur kernel in 1 photo tell us about the lens & the scene?



limits of local inference

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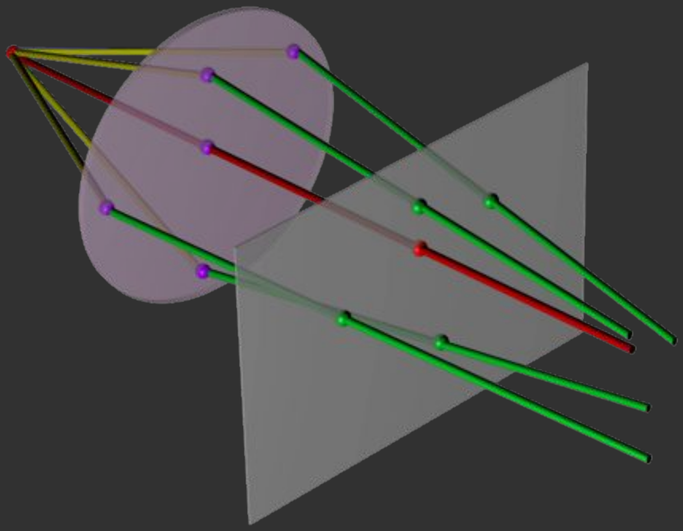
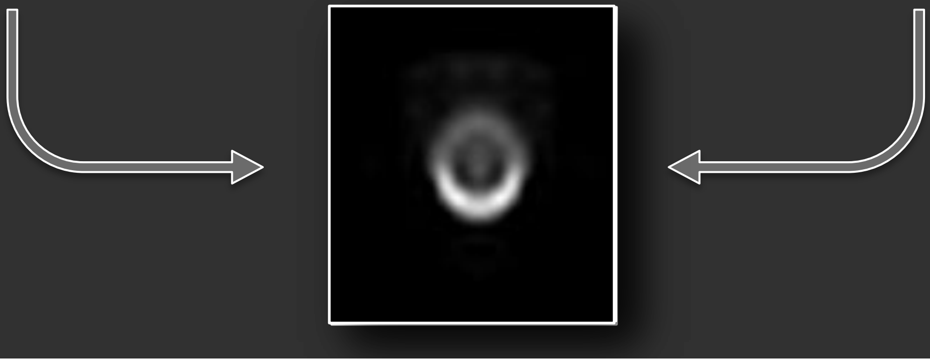
$\langle \mathbf{c}, D, R, \Sigma_1, \Sigma_2, \Sigma_3, \Sigma_4, \Sigma_5 \rangle$



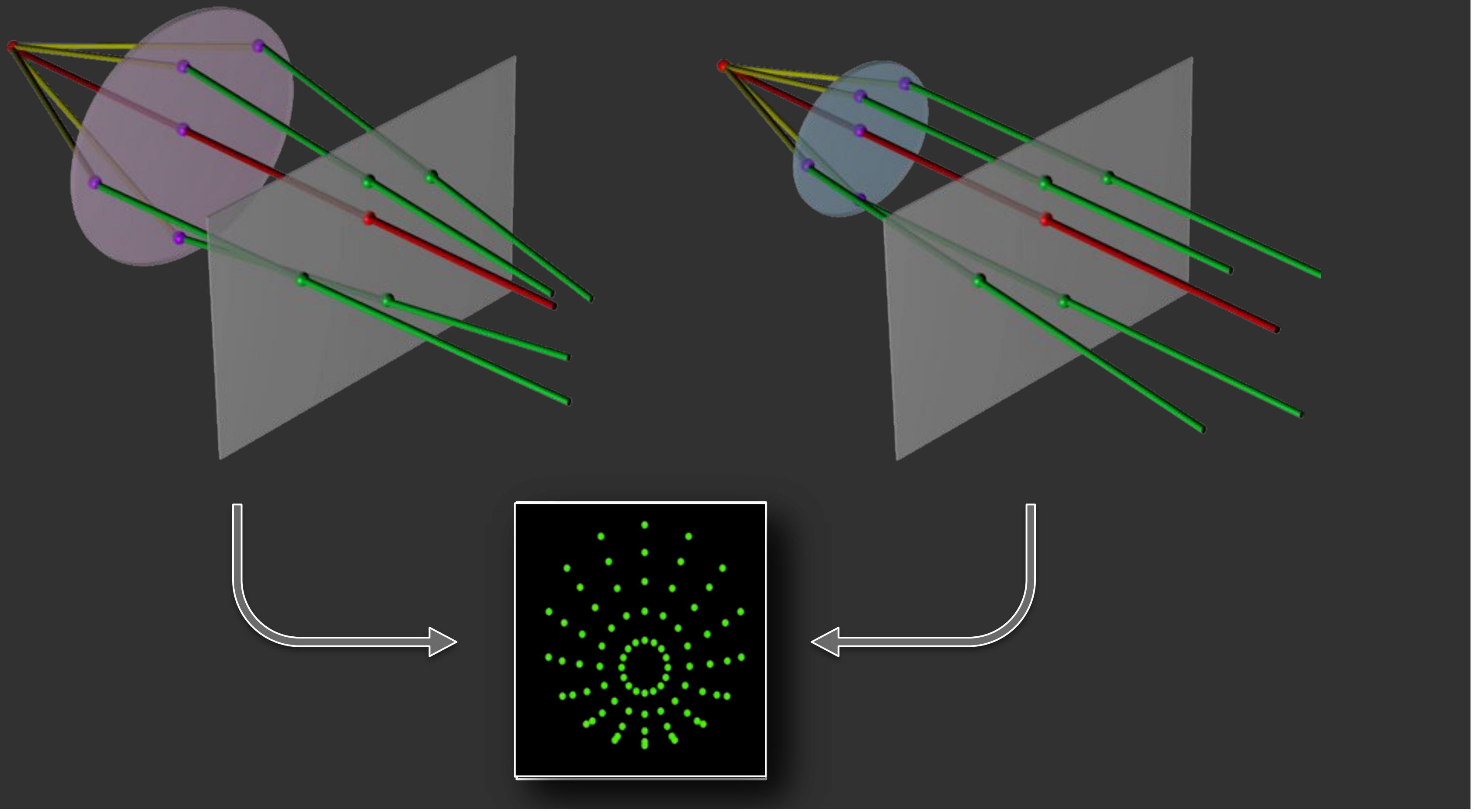
?



pupil radius ambiguity

Lemma: there is a blur-compatible solution for every $R' > 0$

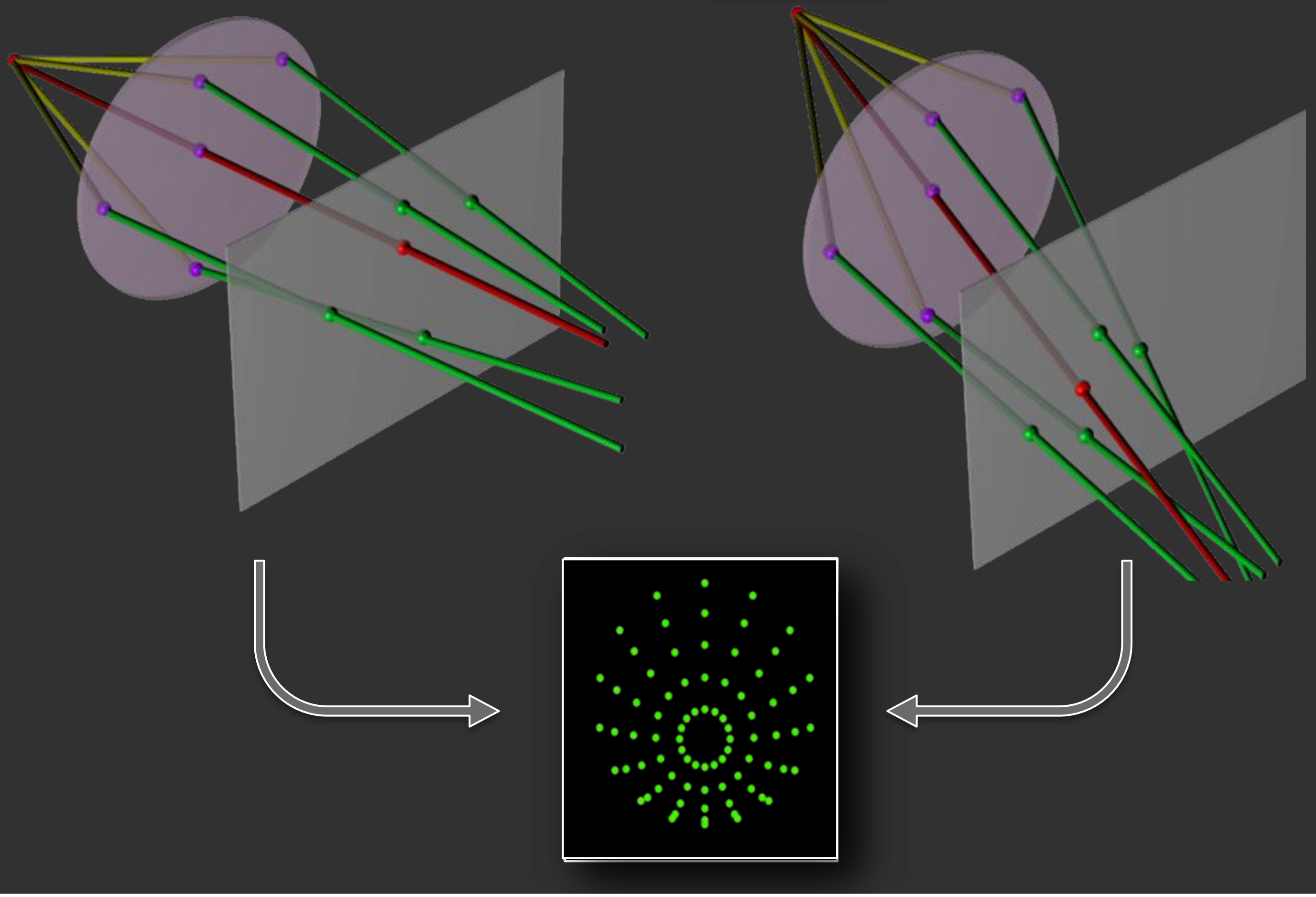
$$\langle \mathbf{c}, D, R, \Sigma_1, \Sigma_2, \Sigma_3, \Sigma_4, \Sigma_5 \rangle \quad \langle \mathbf{c}, D', \boxed{R'}, \Sigma_1', \Sigma_2', \Sigma_3', \Sigma_4', \Sigma_5 \rangle$$



depth & projection ambiguity

Lemma: there is a blur-compatible solution for every $D', \mathbf{c}' > 0$

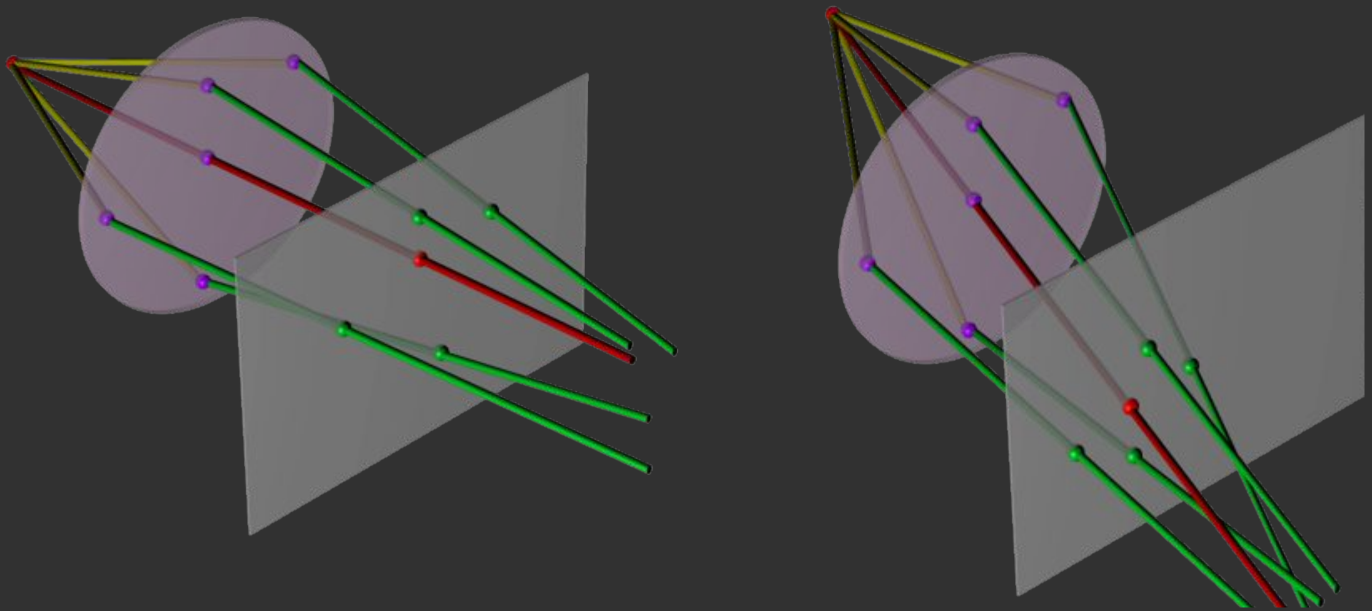
$$\langle \mathbf{c}, D, R, \Sigma_1, \Sigma_2, \Sigma_3, \Sigma_4, \Sigma_5 \rangle \quad \langle \boxed{\mathbf{c}', D'}, R, \Sigma_1', \Sigma_2', \Sigma_3', \Sigma_4', \Sigma_5' \rangle$$



depth & projection ambiguity

Lemma: there is a blur-compatible solution for every $D', \mathbf{c}' > 0$

$$\langle \mathbf{c}, D, R, \Sigma_1, \Sigma_2, \Sigma_3, \Sigma_4, \Sigma_5 \rangle \quad \langle \mathbf{c}', D', R, \Sigma_1', \Sigma_2', \Sigma_3', \Sigma_4', \Sigma_5' \rangle$$

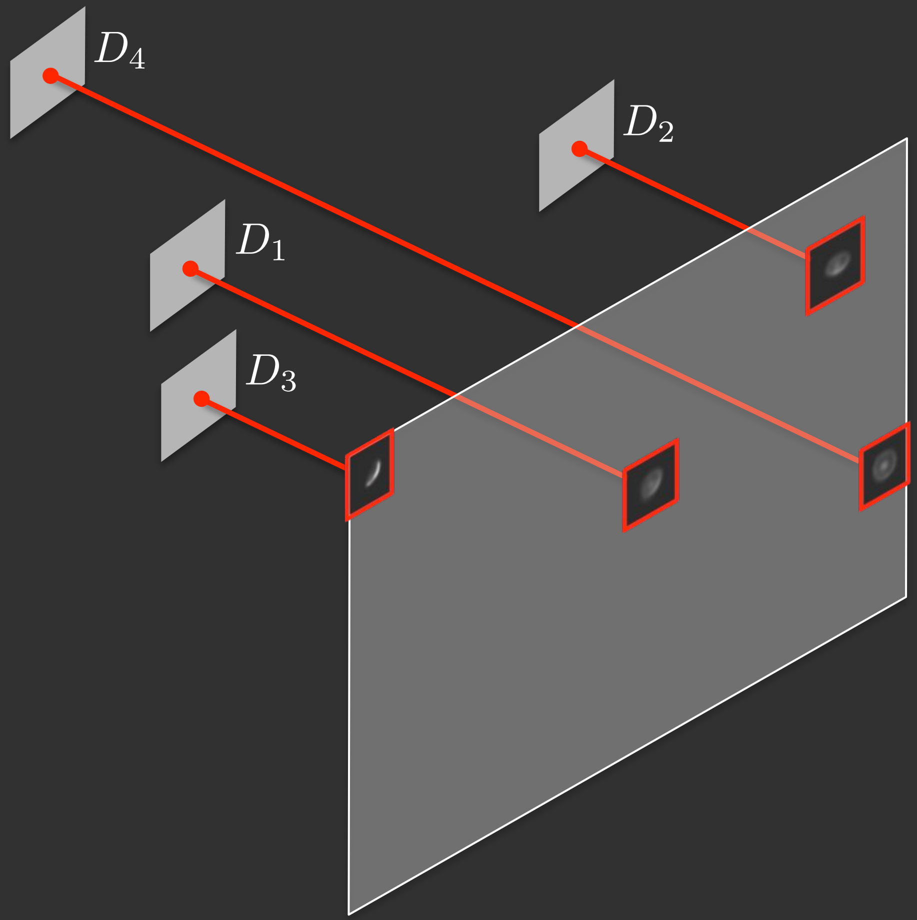


$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ D \\ |c| \end{bmatrix} + \begin{bmatrix} 1 & D & D^2 & D^2 & D^3 \\ 0 & |c| & D|c| & 2D|c| & 3D|c|^2 \\ 0 & 0 & |c|^2 & 0 & 2D|c|^2 \\ 0 & 0 & 0 & |c|^2 & D|c|^2 \\ 0 & 0 & 0 & 0 & |c|^3 \end{bmatrix} \begin{bmatrix} \Sigma_1 \\ \Sigma_2 \\ \Sigma_3 \\ \Sigma_4 \\ \Sigma_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ D' \\ |c'| \end{bmatrix} + \begin{bmatrix} 1 & D' & D'^2 & D'^2 & D'^3 \\ 0 & |c'| & D'|c'| & 2D'|c'| & 3D'|c'|^2 \\ 0 & 0 & |c'|^2 & 0 & 2D'|c'|^2 \\ 0 & 0 & 0 & |c'|^2 & D'|c'|^2 \\ 0 & 0 & 0 & 0 & |c'|^3 \end{bmatrix} \begin{bmatrix} \Sigma_1' \\ \Sigma_2' \\ \Sigma_3' \\ \Sigma_4' \\ \Sigma_5' \end{bmatrix}$$

iii. multi-point inference

multi-local inference

what do K blur kernels in 1 photo tell us about the lens & the scene?



multi-local inference

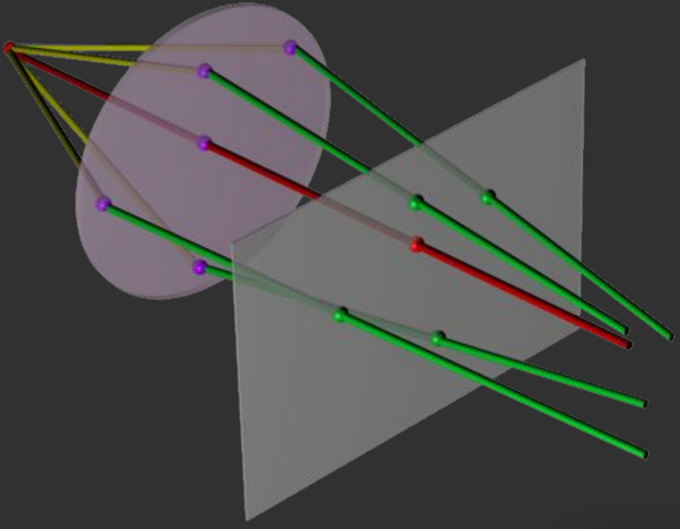
what do K blur kernels in 1 photo tell us about the lens & the scene?

$$\langle \boxed{\mathbf{c}_1, D_1,} R, \boxed{\Sigma_1, \Sigma_2, \Sigma_3, \Sigma_4, \Sigma_5} \rangle$$

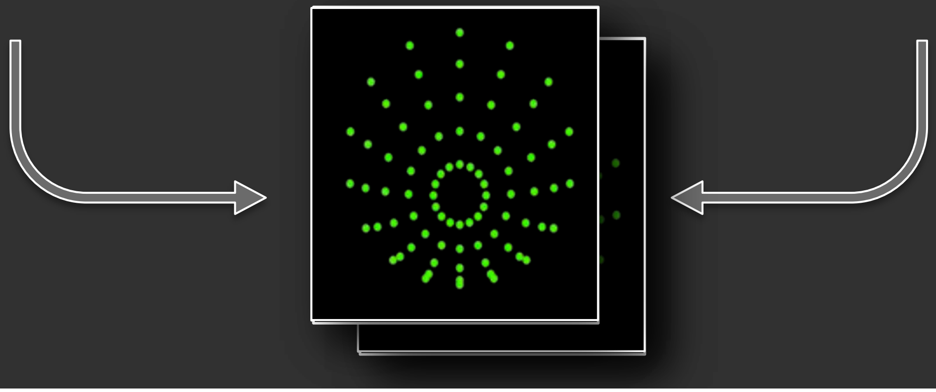
$$\langle \boxed{\mathbf{c}_2, D_2,} R, \boxed{\Sigma_1, \Sigma_2, \Sigma_3, \Sigma_4, \Sigma_5} \rangle$$

5 lens unknowns

2K scene unknowns



?

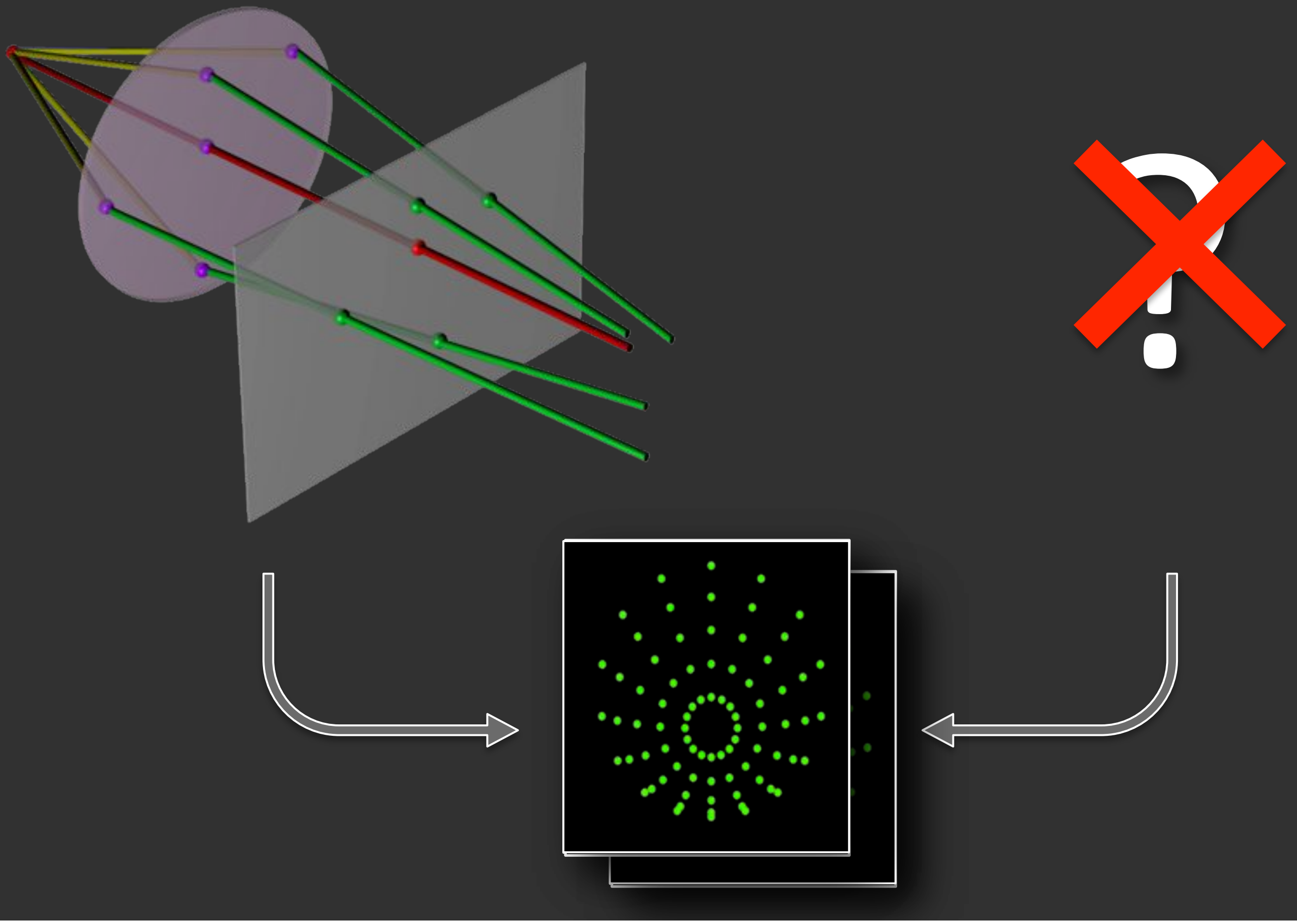


power of multi-local inference

Lemma: space of blur-consistent arrangements is discrete for K≥3

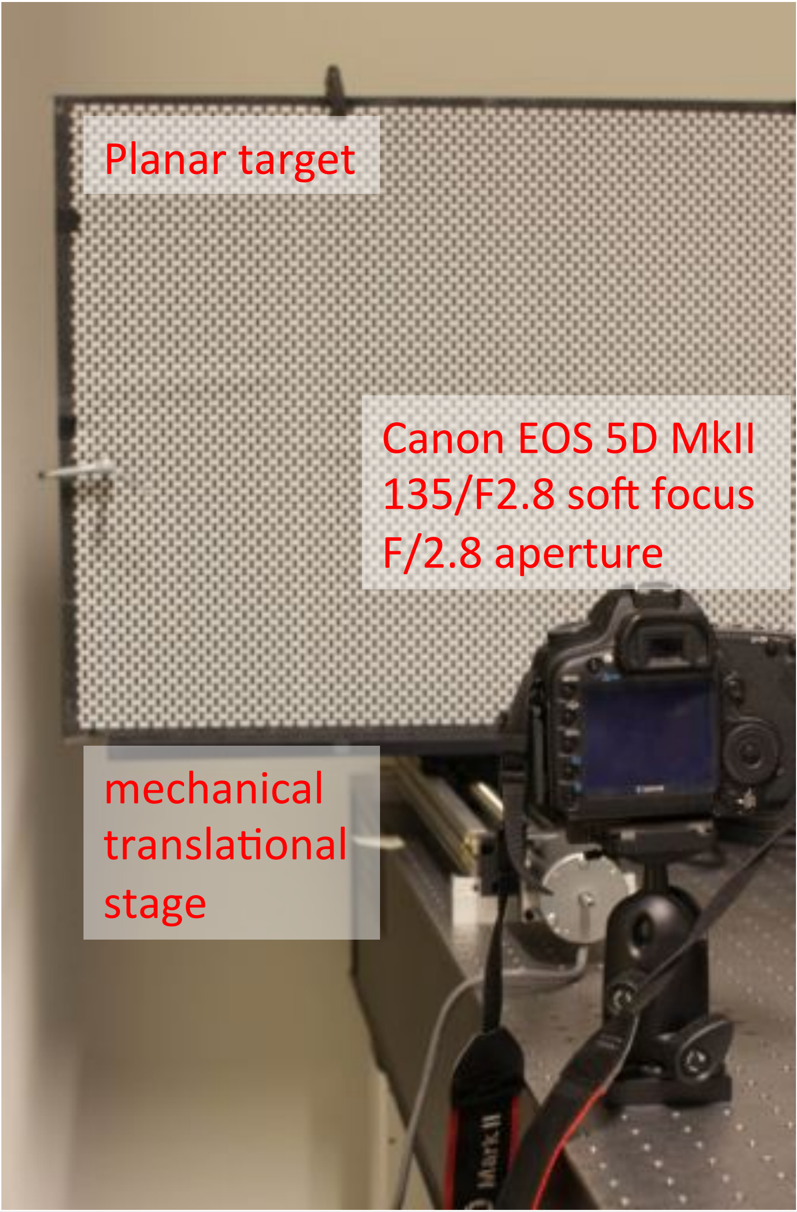
$$\langle \mathbf{c}_1, D_1, R, \Sigma_1, \Sigma_2, \Sigma_3, \Sigma_4, \Sigma_5 \rangle$$

$$\langle \mathbf{c}_2, D_2, R, \Sigma_1, \Sigma_2, \Sigma_3, \Sigma_4, \Sigma_5 \rangle$$



v. preliminary experiments

single-point & multi-point inference



ground-truth acquisition of 5D PSF

1. focus at depth D
2. for $D' \in [D \pm 20\text{dof}, D + 20\text{dof}]$
   take narrow & wide-aperture shots
   estimate local blur kernels [Joshi 08]
3. repeat

inference

1. choose the shot for a pair (D, D')
2. choose K blur kernels from that shot
3. estimate D' $\Sigma_1, \Sigma_2, \Sigma_3, \Sigma_4, \Sigma_5$
4. predict blur kernels for all depths

evaluation

compare predicted &
ground-truth blur kernels

single-point & multi-point inference

ground-truth acquisition of 5D PSF

1. focus at depth D
2. for $D' \in [D \pm 20\text{dof}, D + 20\text{dof}]$
    take narrow & wide-aperture shots
    estimate local blur kernels [Joshi 08]
3. repeat

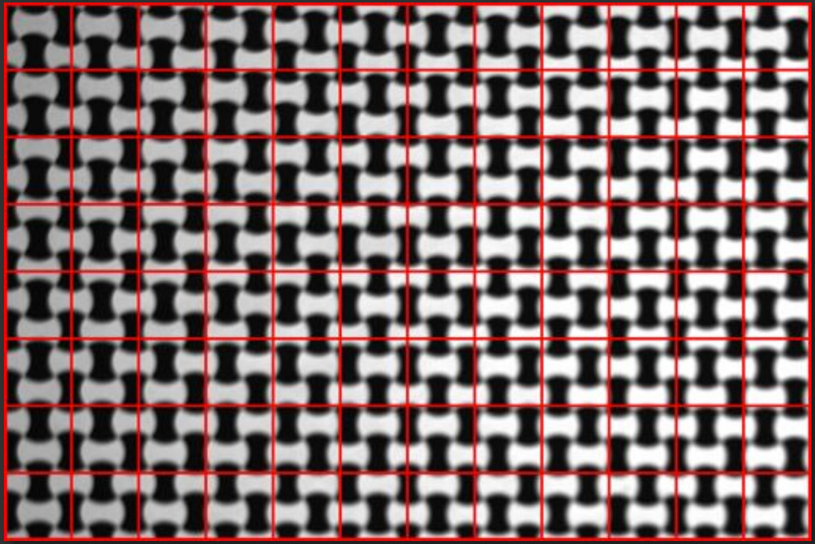
41 target positions

single-point & multi-point inference

ground-truth acquisition of 5D PSF

1. focus at depth D
2. for $D' \in [D \pm 20\mathrm{dof}, D + 20\mathrm{dof}]$
    - take narrow & wide-aperture shots
    - estimate local blur kernels [Joshi 08]
3. repeat

12x8 spatial position samples in ¼ field of view

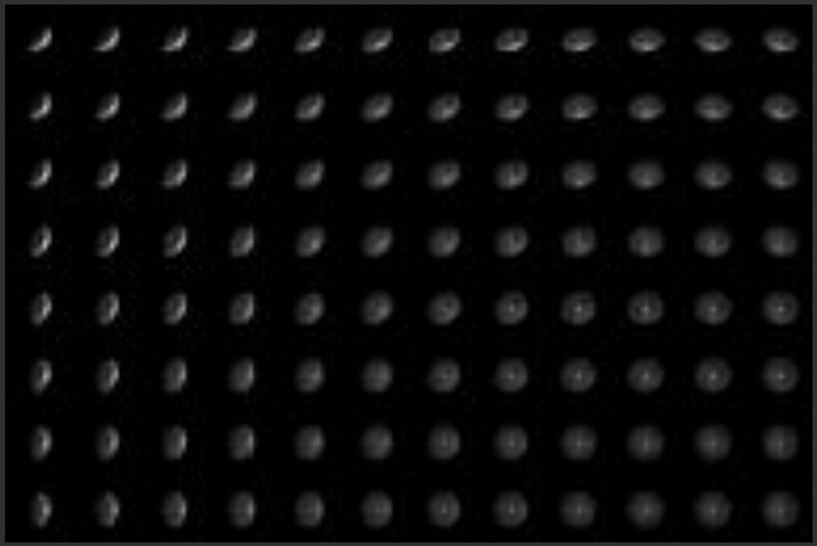


single-point & multi-point inference

ground-truth acquisition of 5D PSF

1. focus at depth D
2. for $D' \in [D \pm 20\mathrm{dof}, D + 20\mathrm{dof}]$
   - take narrow & wide-aperture shots
   - estimate local blur kernels [Joshi 08]
3. repeat

12x8x41 blur kernels



single-point & multi-point inference

ground-truth acquisition of 5D PSF

1. focus at depth D
2. for $D' \in [D \pm 20\text{dof}, D + 20\text{dof}]$
   take narrow & wide-aperture shots
   estimate local blur kernels [Joshi 08]
3. repeat

inference (K=1)

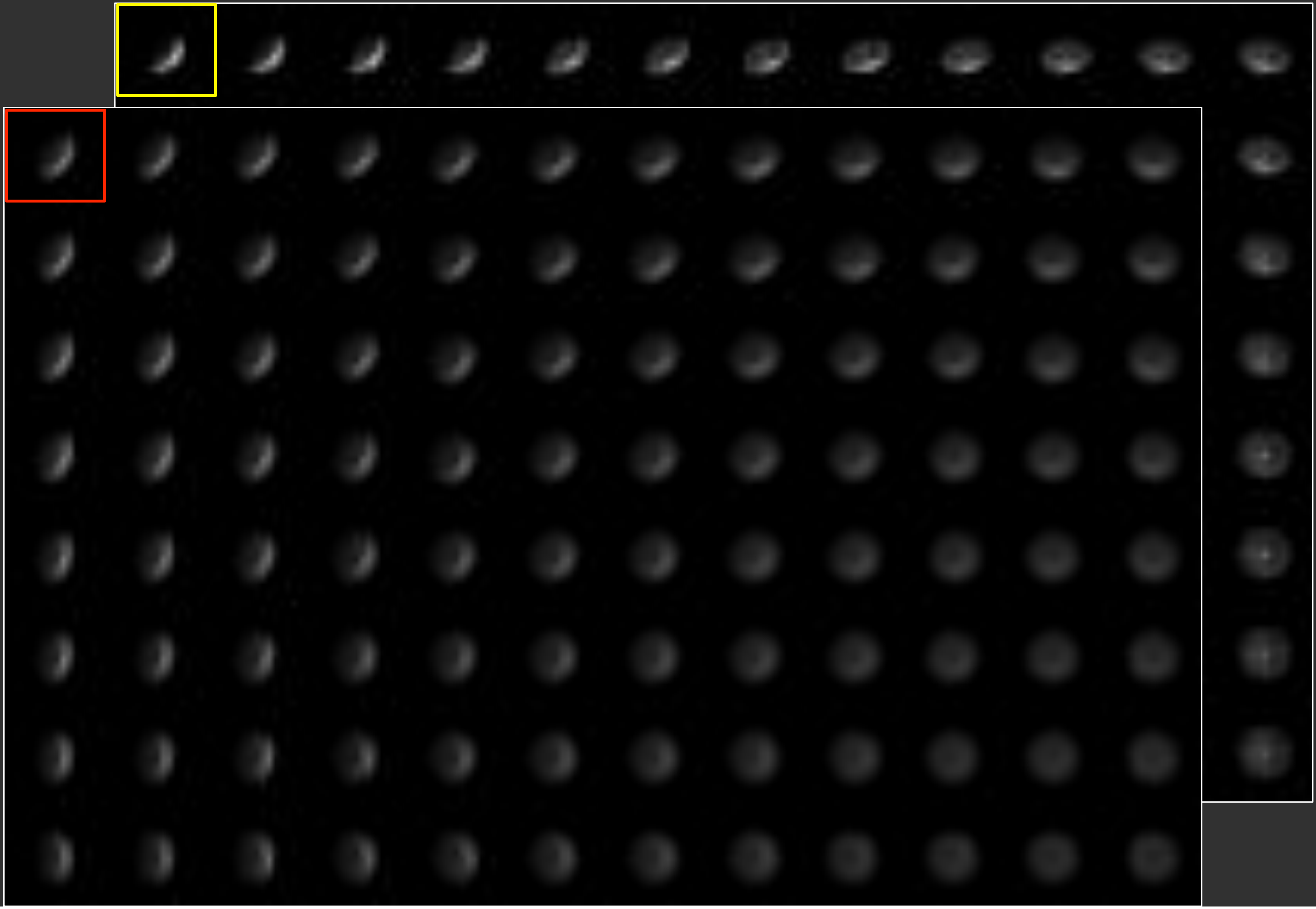
1. choose the shot for a pair (D, D')
2. choose K blur kernels from that shot
3. estimate $\Sigma_1, \Sigma_2, \Sigma_3, \Sigma_4$
4. predict blur kernels for all depths

optimization

$$\max_{\Sigma_1 \cdots \Sigma_4} \mathbf{corr}(\mathbf{PSF}_{c,D',\Sigma_1 \cdots \Sigma_5}, \mathbf{PSF}_{data})$$

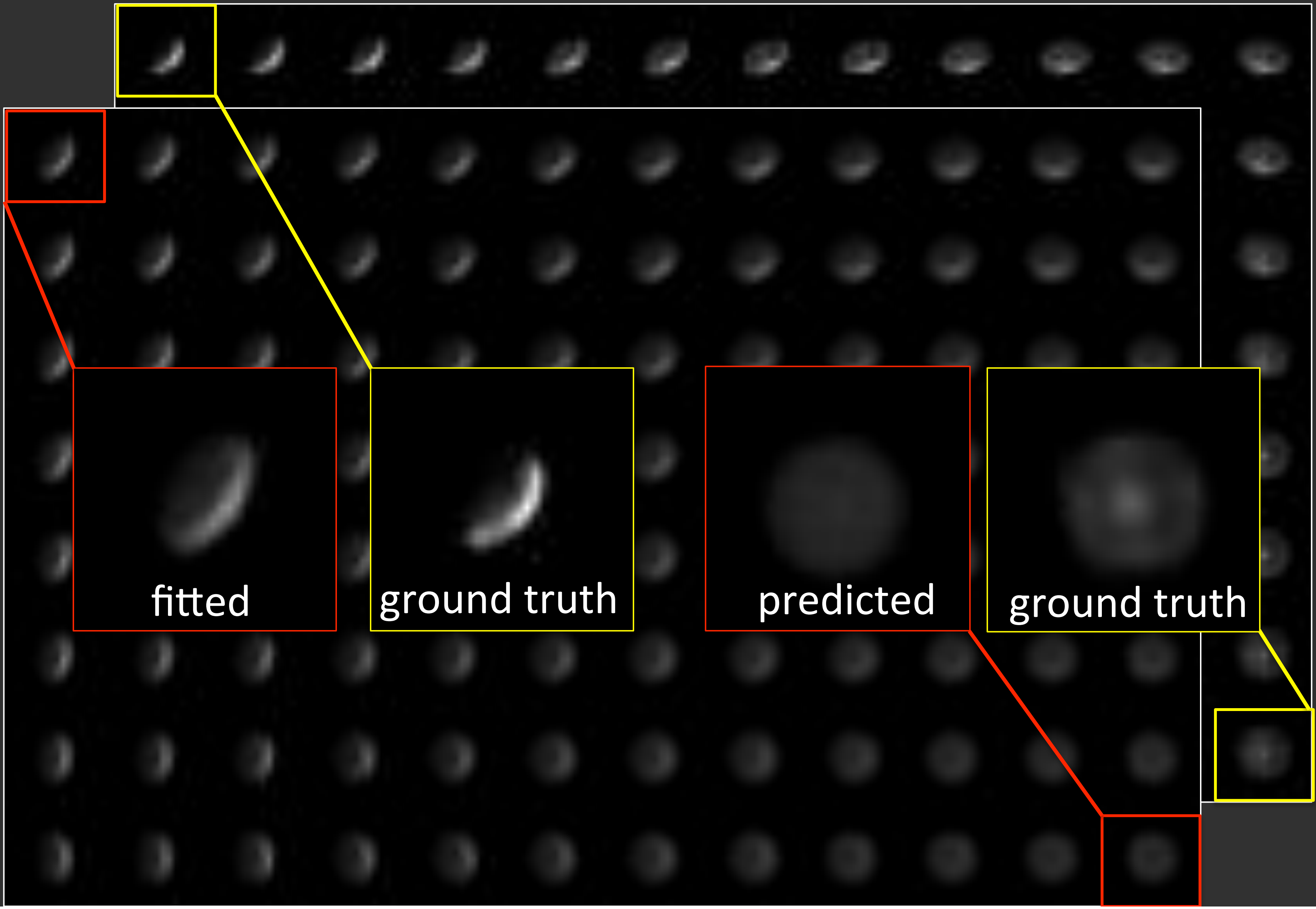
fitted depth = -20 DoF

predicted depth = -20 DoF

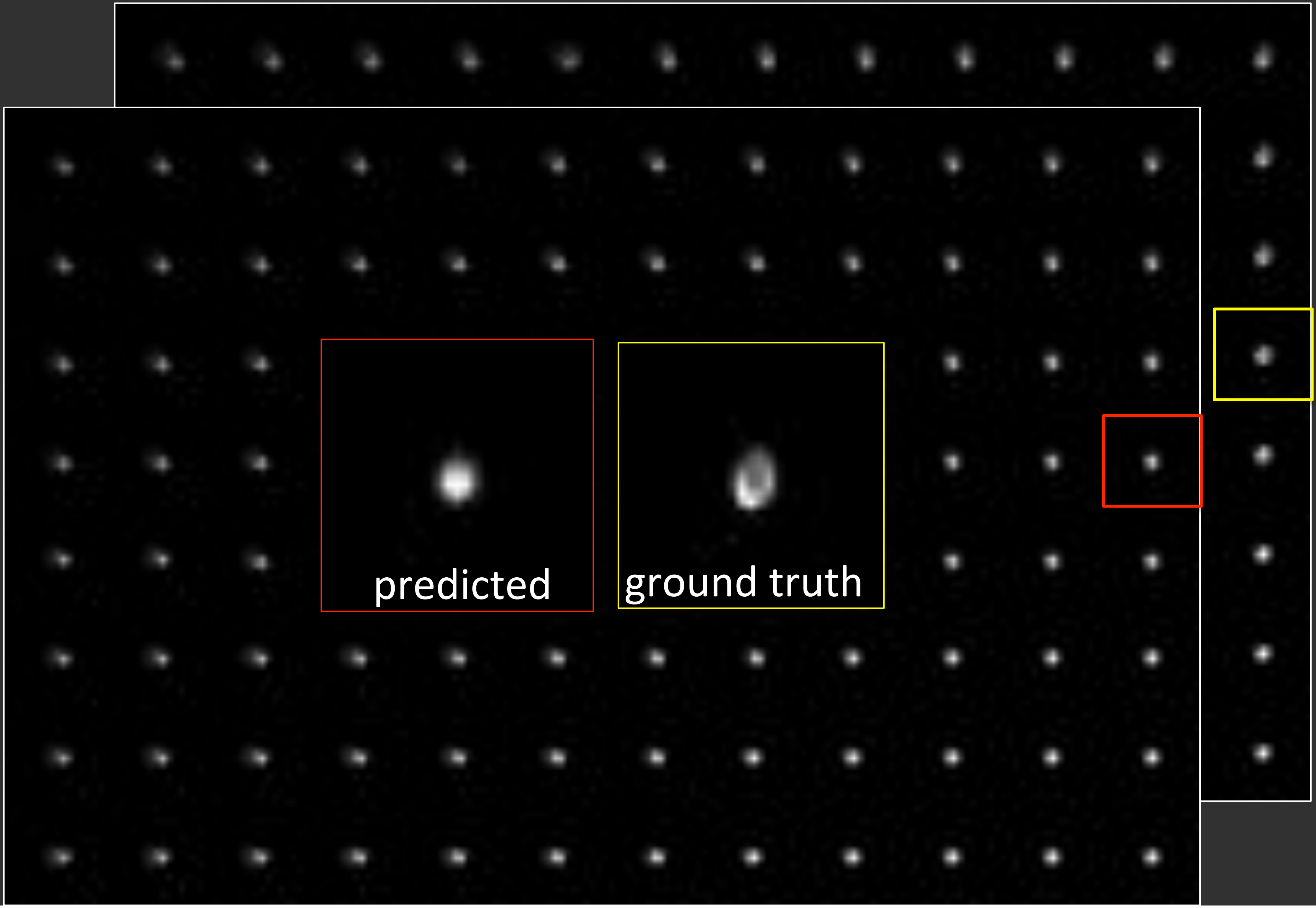


fitted depth = -20 DoF

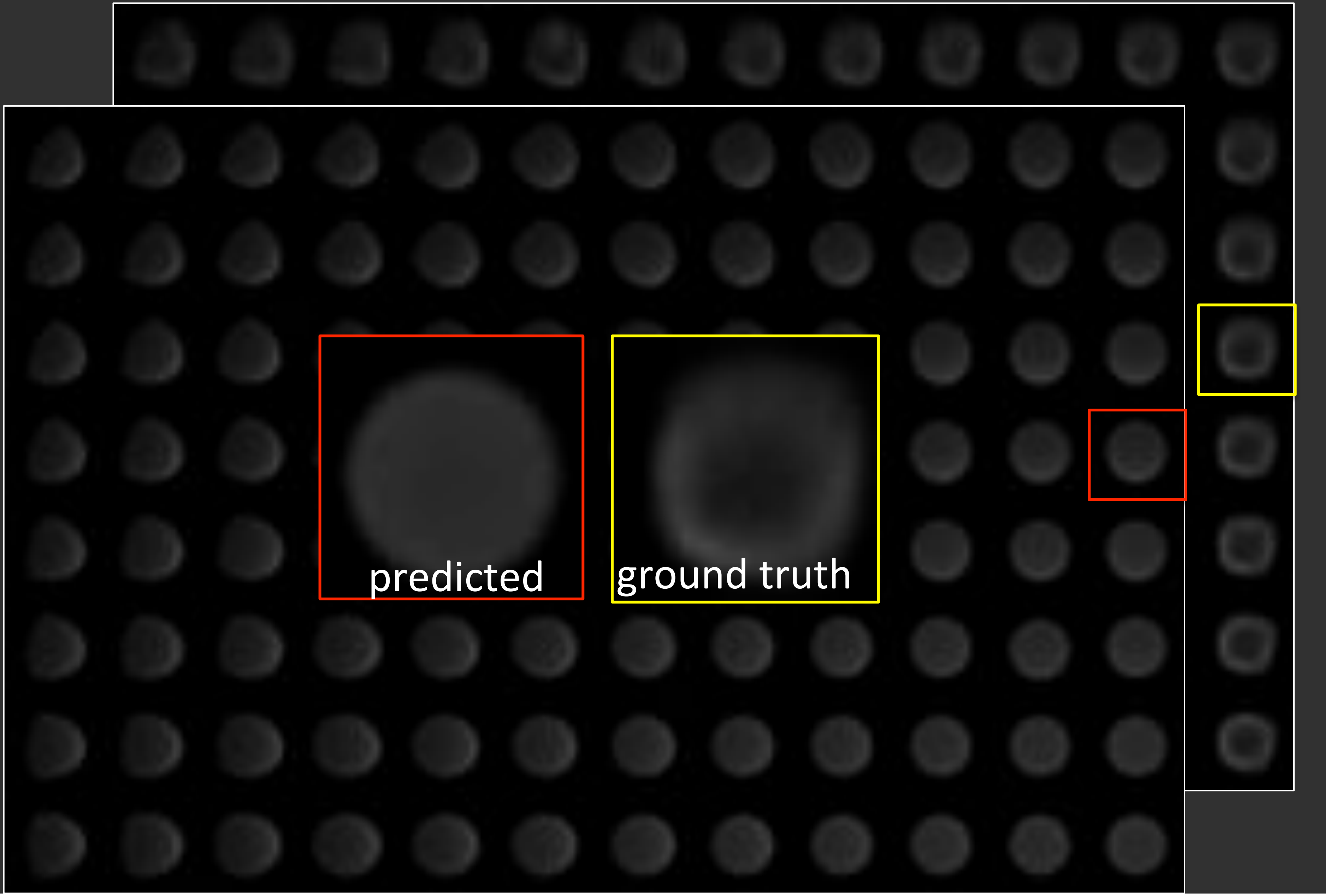
predicted depth = -20 DoF



fitted depth = -20 DoF predicted depth = 0 DoF



fitted depth = -20 DoF predicted depth = 20 DoF



multi-point inference

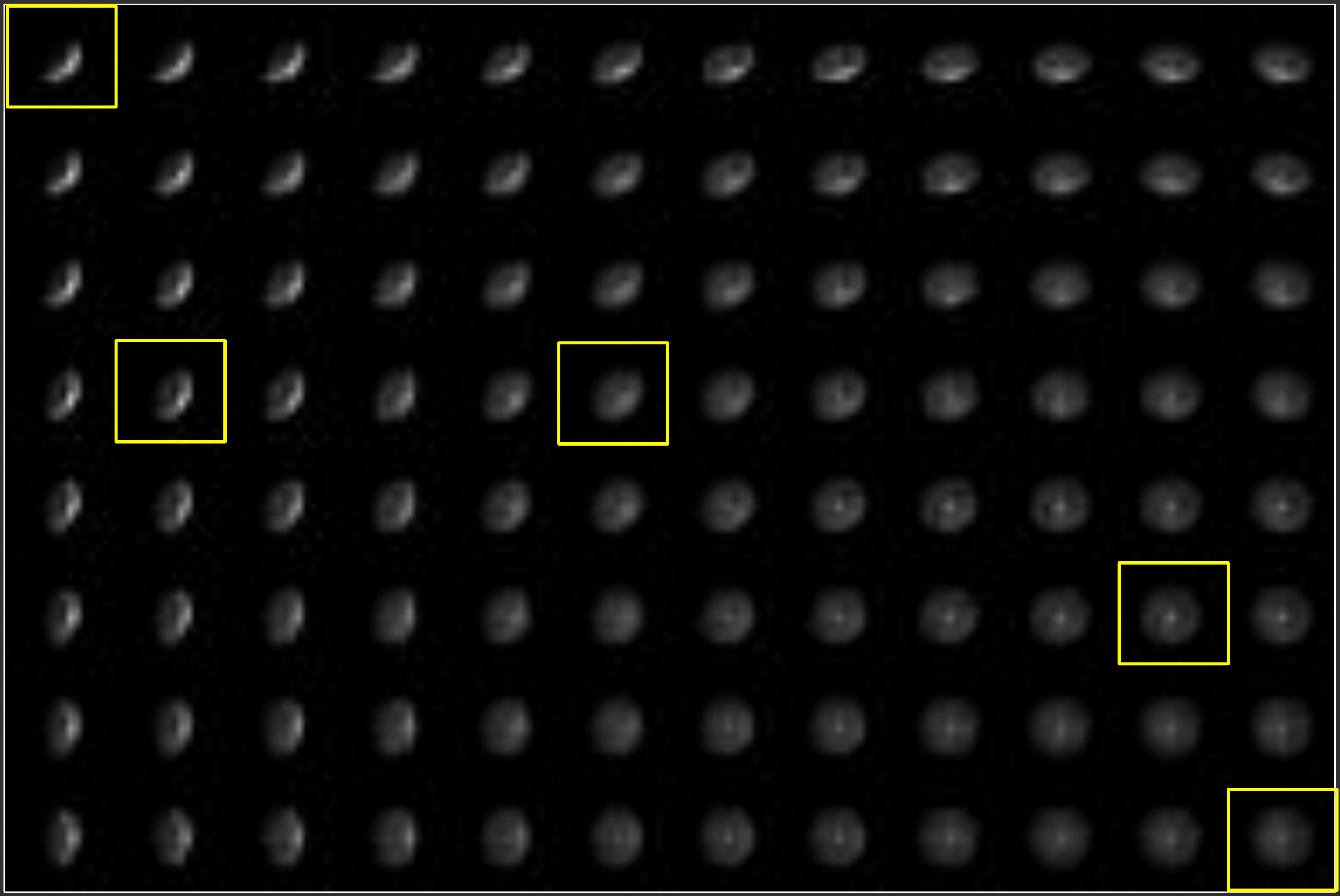
ground-truth acquisition of 5D PSF

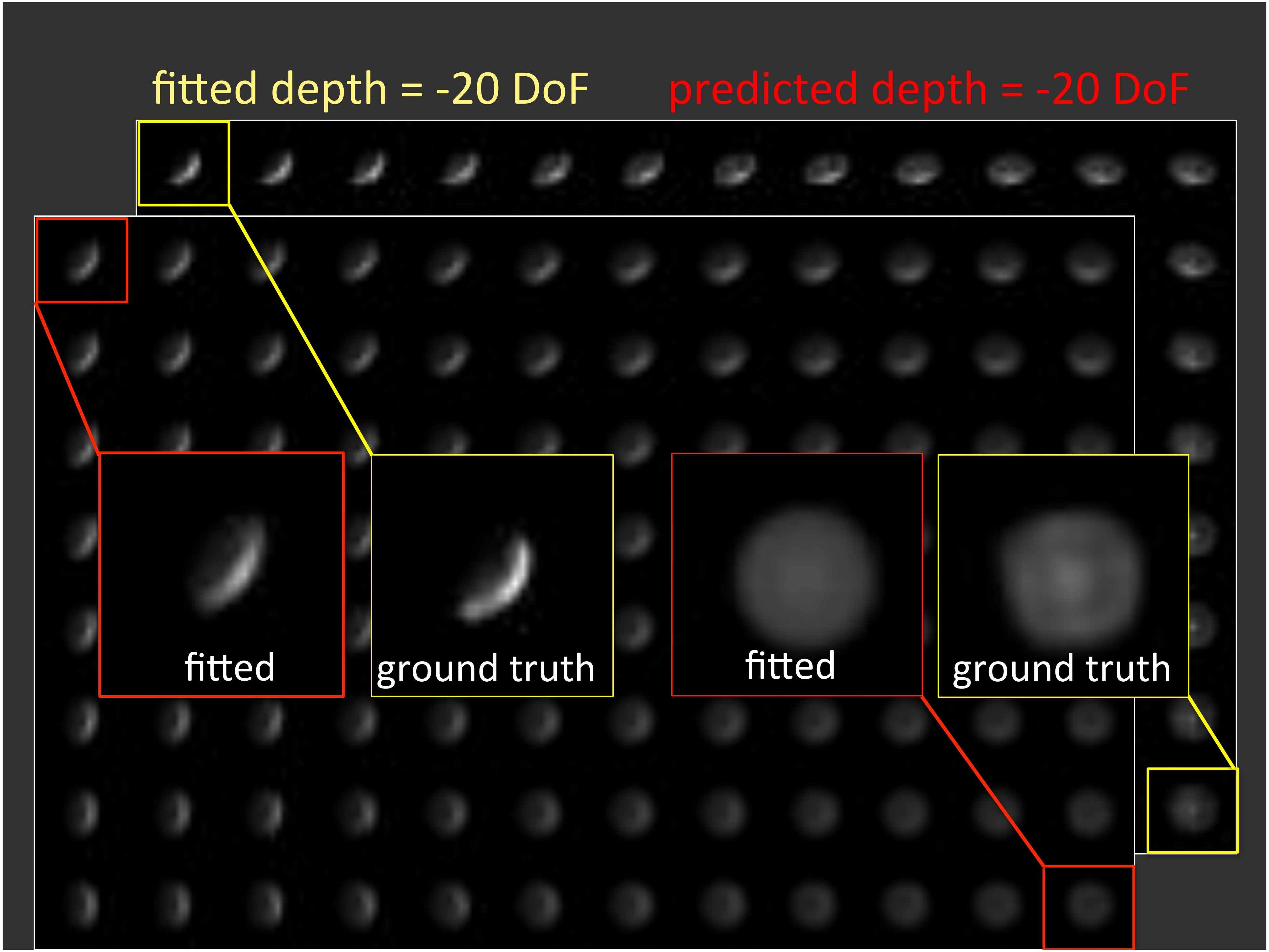
1. focus at depth D
2. for $D' \in [D \pm 20\mathrm{dof}, D + 20\mathrm{dof}]$
   - take narrow & wide-aperture shots
   - estimate local blur kernels [Joshi 08]
3. repeat

inference (K>1)

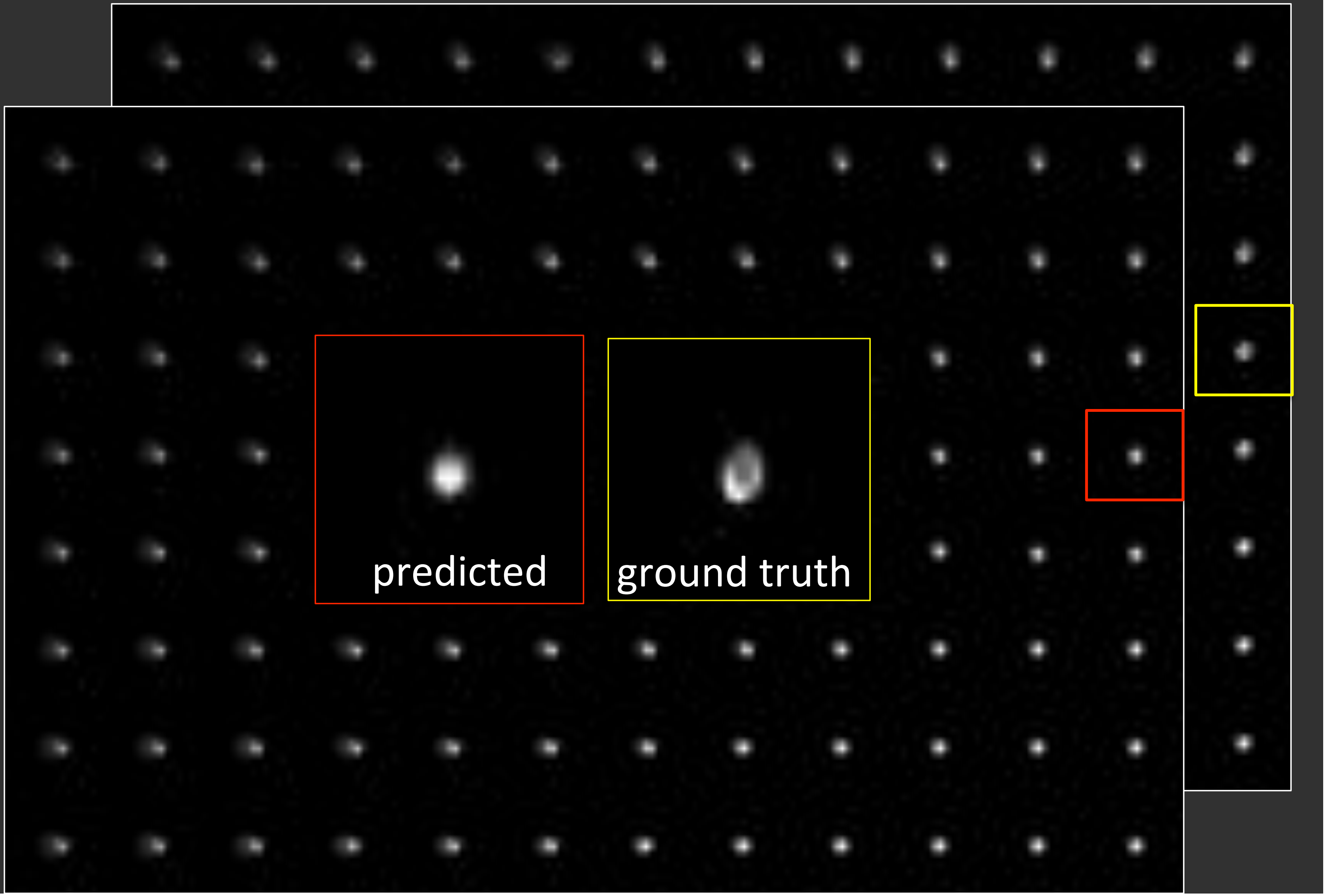
1. choose the shot for a pair (D, D')
2. choose K blur kernels from that shot
3. estimate D' $\Sigma_1, \Sigma_2, \Sigma_3, \Sigma_4, \Sigma_5$
4. predict blur kernels for all (D, D')

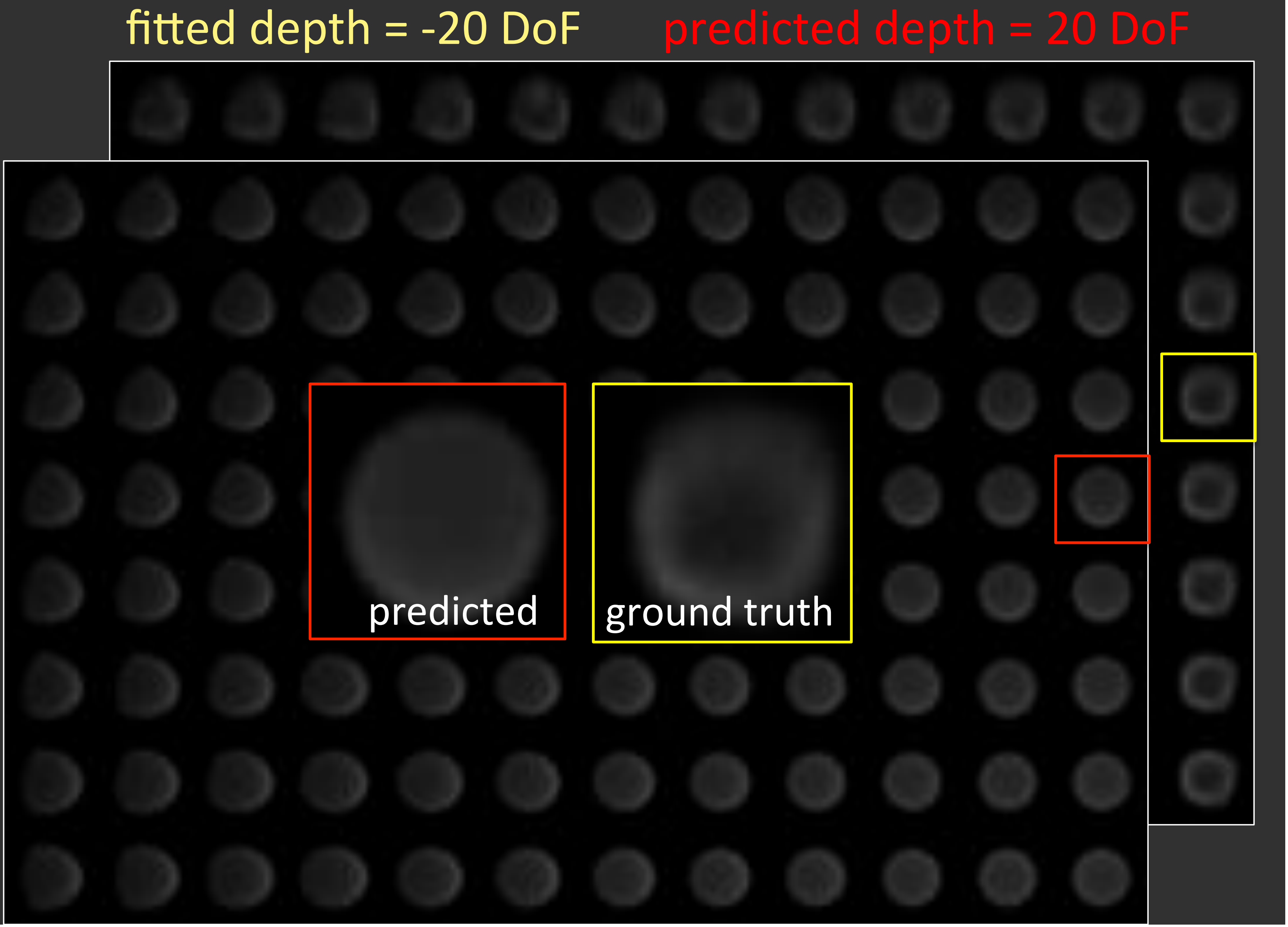
fitted depth = -20 DoF



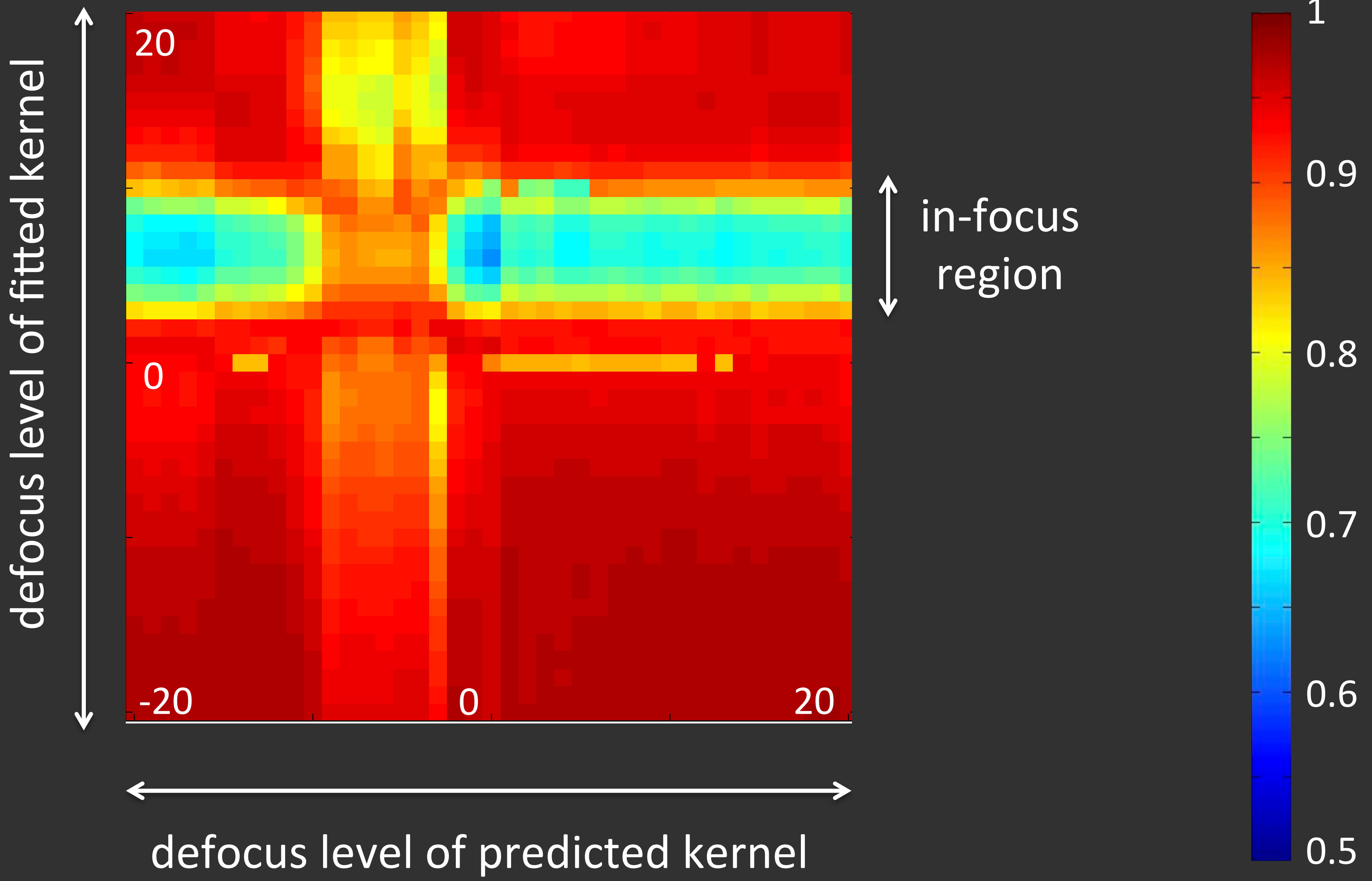

fitted depth = -20 DoF
predicted depth = -20 DoF
fitted
ground truth
fitted
ground truth

fitted depth = -20 DoF predicted depth = 0 DoF

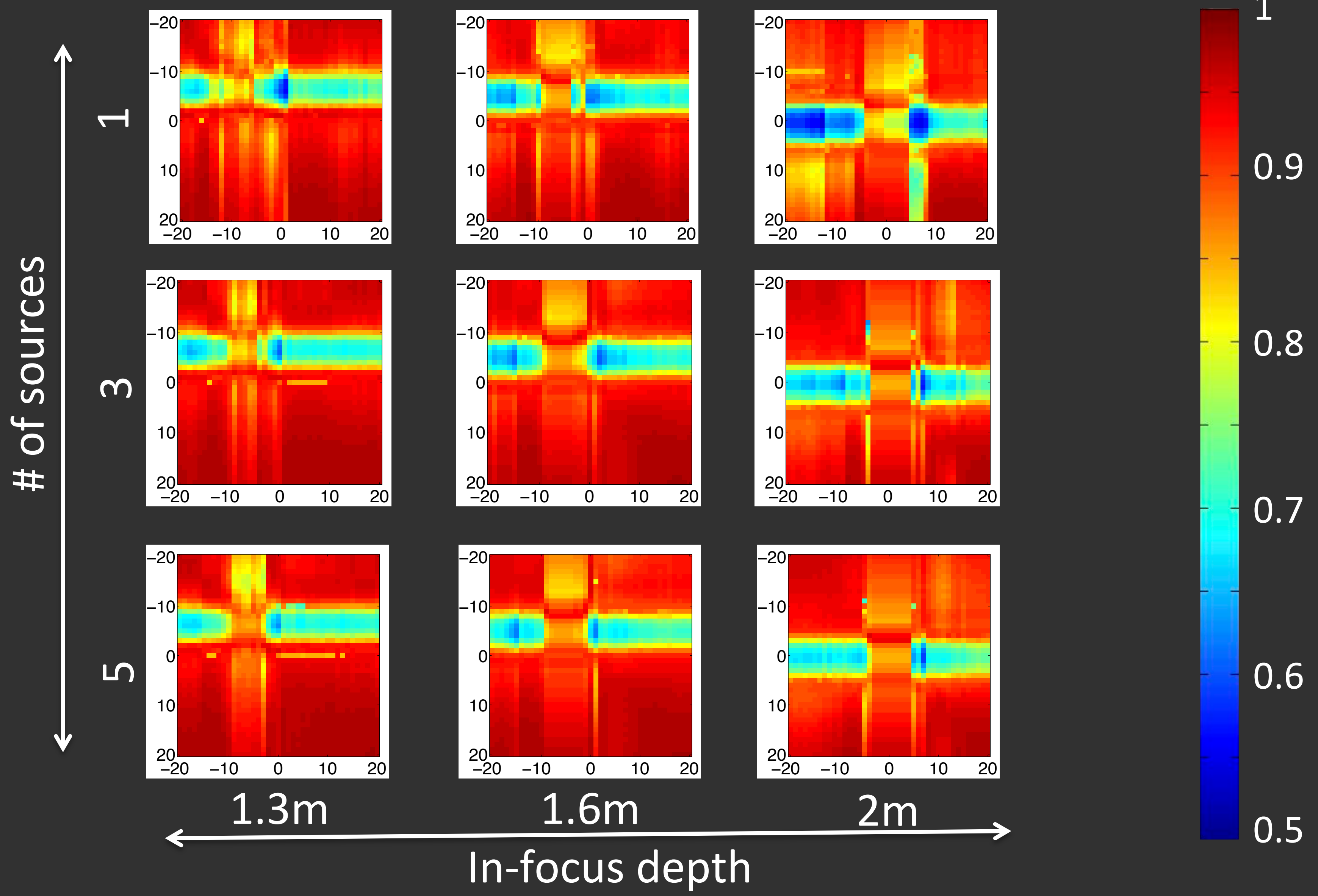



fitted depth = -20 DoF
predicted depth = 20 DoF
predicted
ground truth

quantitative evaluation (K=5)



Quantitative results for 3 different focus settings



concluding remarks

aberration & defocus blur give a great deal of info about the lens & scene

one-shot recovery of depth map & 5D lens PSF may be possible

modeling ray-sensor intersections easier & more general than modeling blur kernels directly

open problems

- blind estimation of blur kernels, depth & 5D PSF
- modeling mechanical vignetting
- accounting for changes in focus setting & zoom