

X4.1 Let s and c be integer variables. Prove

if $c=1$ then $s := -1$ else if $c=2$ then $s := 0$ else if $c=3$ then $s := 1$ else $\top$ fi fi fi
 $\Leftarrow$ $s := c-2$

After trying the question, scroll down to the solution.

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if $c=1$ **then** $s := -1$ **else if** $c=2$ **then** $s := 0$ **else if** $c=3$ **then** $s := 1$ **else** $\top$ **fi fi fi**

one-case

$=$ **if** $c=1$ **then** $s := -1$ **else if** $c=2$ **then** $s := 0$ **else** $c=3 \Rightarrow (s := 1)$ **fi fi** case analysis

$=$ **if** $c=1$ **then** $s := -1$ **else** $(c=2 \Rightarrow (s := 0)) \wedge (c \neq 2 \Rightarrow (c=3 \Rightarrow (s := 1)))$ **fi** portation

$=$ **if** $c=1$ **then** $s := -1$ **else** $(c=2 \Rightarrow (s := 0)) \wedge (c \neq 2 \wedge c=3 \Rightarrow (s := 1))$ **fi**

$=$ **if** $c=1$ **then** $s := -1$ **else** $(c=2 \Rightarrow (s := 0)) \wedge (c=3 \Rightarrow (s := 1))$ **fi** case analysis

$=$ $(c=1 \Rightarrow (s := -1)) \wedge (c \neq 1 \Rightarrow (c=2 \Rightarrow (s := 0)) \wedge (c=3 \Rightarrow (s := 1)))$ distribution

$=$ $(c=1 \Rightarrow (s := -1)) \wedge (c \neq 1 \Rightarrow (c=2 \Rightarrow (s := 0))) \wedge (c \neq 1 \Rightarrow (c=3 \Rightarrow (s := 1)))$

portation twice

$=$ $(c=1 \Rightarrow (s := -1)) \wedge (c \neq 1 \wedge c=2 \Rightarrow (s := 0)) \wedge (c \neq 1 \wedge c=3 \Rightarrow (s := 1))$

$=$ $(c=1 \Rightarrow (s := -1)) \wedge (c=2 \Rightarrow (s := 0)) \wedge (c=3 \Rightarrow (s := 1))$ context 3 times

$=$ $(c=1 \Rightarrow (s := c-2)) \wedge (c=2 \Rightarrow (s := c-2)) \wedge (c=3 \Rightarrow (s := c-2))$ monotonicity 3 times

$\Leftarrow$ $(s := c-2) \wedge (s := c-2) \wedge (s := c-2)$ idempotence

$=$ $s := c-2$