92 Simplify (all domains are *nat*)

(a) $\forall y\cdot\ y = x+2 \ \Rightarrow\ y>5$

(b) $\forall y\cdot\ y = x+2 \ \lor\ y = x+1 \ \ \Rightarrow\ \ y>5$

After trying the question, scroll down to the solution.

§(a) $\quad \forall y\cdot\ y = x+2 \ \Rightarrow\ y>5$ $\qquad$ one-point

$=\quad x+2 > 5$ $\qquad$ arithmetic

$=\quad x>3$

§(b) $\quad \forall y\cdot\ y = x+2\ \lor\ y = x+1\ \Rightarrow\ y>5$ $\qquad$ antidistribution

$=\quad \forall y\cdot\ (y = x+2\ \Rightarrow\ y>5) \land (y = x+1\ \Rightarrow\ y>5)$ $\qquad$ distribution

$=\quad (\forall y\cdot\ y = x+2\ \Rightarrow\ y>5) \land (\forall y\cdot\ y = x+1\ \Rightarrow\ y>5)$ $\qquad$ one-point twice

$=\quad x+2 > 5\ \land\ x+1 > 5$ $\qquad$ generic connection

$=\quad x+2 \downarrow x+1 > 5$ $\qquad$ distributivity

$=\quad x+(2\downarrow 1) > 5$ $\qquad$ arithmetic

$=\quad x+1 > 5$ $\qquad$ cancellation

$=\quad x>4$