87 Define $\approx$ to give the deep contents of a string or set or list or function. Here are some examples.

$\approx(10; 11; 12; 13) = 10, 11, 12, 13$

$\approx\{10, \{11, 12\}, 13\} = 10, 11, 12, 13$

$\approx[10; [11; 12]; 13] = 10, 11, 12, 13$

$\approx\{10, [11; 12], 13\} = 10, 11, 12, 13$

$\approx[10; \{11, 12\}; 13] = 10, 11, 12, 13$

$\approx\langle x: nat \cdot \langle y: nat \cdot 2{\times}x{\times}y\rangle\rangle = 2{\times}nat$

$\approx\langle x: 0,..4 \cdot \langle y: 0,..x \cdot x+y\rangle\rangle = 1, 2, 3, 4, 5$

The contents operator $\sim$ removes one level of structure from a set or list. The deep contents operator $\approx$ removes all levels of structure.

After trying the question, scroll down to the solution.

§ It is convenient to start by defining $\approx$ for base cases. If x is an element, and x is not a set, string, list, or function, then

$$\approx x \;=\; x$$

Now for bunches,

$$\approx null \;=\; null$$

$$\approx(A, B) \;=\; \approx A, \approx B$$

Now for sets,

$$\approx\{A\} \;=\; \approx A$$

For strings,

$$\approx nil \;=\; null$$

$$\approx(A; B) \;=\; \approx A, \approx B$$

For lists,

$$\approx[S] \;=\; \approx S$$

For functions,

$$\approx\langle v\colon D\cdot\ e\rangle \;=\; \langle v\colon D\cdot \approx e\rangle\ D$$

Or if you prefer,

$$\approx f \;=\; \langle v\colon \Box f\cdot \approx f\ v\rangle\ (\Box f)$$

For functions we could instead define $\approx$ the way we usually define quantifiers.

$$\approx v\colon null\cdot\ e \;=\; null$$

$$\approx v\colon x\cdot\ e \;=\; \langle v\colon x\cdot \approx e\rangle\ x \qquad \text{for element } x$$

$$\approx v\colon A, B\cdot\ e \;=\; (\approx v\colon A\cdot\ e), (\approx v\colon B\cdot\ e)$$

$$\approx v\colon (\S v\colon D\cdot\ b)\cdot\ c \;=\; \langle v\colon D\cdot\ \textbf{if}\ b\ \textbf{then} \approx c\ \textbf{else}\ null\ \textbf{fi}\rangle\ D$$