85 We have defined several quantifiers by starting with an associative symmetric operator with an identity. Bunch union is also such an operator. Does it yield a quantifier?

After trying the question, scroll down to the solution.

§ Yes. I'll use $\mathcal{U}$ for the quantifier.

$\mathcal{U}v{:}\ null{\cdot}\ e \ = \ null$
$\mathcal{U}v{:}\ x{\cdot}\ e \ = \ \langle v{:}\ x{\cdot}\ e\rangle\, x$ for element x
$\mathcal{U}v{:}\ A, B{\cdot}\ e \ = \ (\mathcal{U}v{:}\ A{\cdot}\ e), (\mathcal{U}v{:}\ B{\cdot}\ e)$
$\mathcal{U}v{:}\ (\S v{:}\ D{\cdot}\ b){\cdot}\ c \ = \ \mathcal{U}v{:}\ D{\cdot}$ **if** b **then** c **else** $null$ **fi**

Application of function f distributes over bunch union, so the $\mathcal{U}$ quantifier gives the range of a function.

$\mathcal{U}f \ = \ f\,(\Box f)$