

- 83 Prove that if variables i and j do not appear in predicates P and Q , then
- $$(\forall i. P i) \Rightarrow (\exists i. Q i) = (\exists i, j. P i \Rightarrow Q j)$$

After trying the question, scroll down to the solution.

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If the domain is *null*, then

$$\begin{aligned} & ((\forall i \cdot P i) \Rightarrow (\exists i \cdot Q i)) = (\exists i, j \cdot P i \Rightarrow Q j) \\ = & (\top \Rightarrow \perp = \perp) \\ = & \top \end{aligned}$$

\forall and \exists axioms
truth tables

If the domain is not *null*, then

$$\begin{aligned} & (\forall i \cdot P i) \Rightarrow (\exists i \cdot Q i) && \text{change second variable } i \text{ to } j \\ = & (\forall i \cdot P i) \Rightarrow (\exists j \cdot Q j) && \text{last antidistributive law; } a \text{ is } (\exists j \cdot Q j) \text{ and } b \text{ is } P i \\ = & \exists i \cdot P i \Rightarrow (\exists j \cdot Q j) && \text{last distributive law; } a \text{ is } P i \text{ and } b \text{ is } Q j \\ = & \exists i \cdot \exists j \cdot P i \Rightarrow Q j \end{aligned}$$