81 (arity) The arity of a function is the number of variables (parameters) it introduces, and the number of arguments it can be applied to. Write axioms to define αf (arity of f).

After trying the question, scroll down to the solution.

§ As a first effort, we might try

$\alpha x = 0$ if x is not a function

$\alpha(A \to B) = 1 + \alpha B$

Unfortunately, $null \to null \to 3 = null \to 3$, so we would prove $2=1$. So we try again.

$\alpha x = 0$ if x is not a function

$\alpha f = 1 + \alpha(f(\Box f))$ if f is a function

Let's try an example.

$\alpha\langle n\colon nat\cdot\ n=0 \vDash 0 \dashv \langle m\colon nat\cdot\ m\rangle\rangle$

$= \quad 1 + \alpha(f\ nat)$

$= \quad 1 + \alpha(0, \langle m\colon nat\cdot\ m\rangle)$ Maybe we should let α distribute over bunch union.

$= \quad 1 + (\alpha 0, \alpha\langle m\colon nat\cdot\ m\rangle)$

$= \quad 1 + (0, 1 + \alpha(\langle m\colon nat\cdot\ m\rangle nat))$

$= \quad 1 + (0, 1 + \alpha\ nat)$

$= \quad 1 + (0, 1 + 0)$

$= \quad 1, 2$

and that seems reasonable to me.