75 Express formally that L is a longest sorted sublist of M where

(a) the sublist must be consecutive items (a segment).

(b) the sublist must be consecutive (a segment) and nonempty.

(c) the sublist contains items in their order of appearance in M, but not necessarily consecutively (not necessarily a segment).

After trying the question, scroll down to the solution.

§ Let T be the type of item in the lists.

(a) the sublist must be consecutive items (a segment).

§ Define relation S so that $S\ L\ M$ says that list L is a sorted segment of list M as follows:

$$S = \langle L, M: [*T]\cdot\ \exists i, j: nat\cdot\ 0\le i\le j\le \#L\le \#M \wedge L=M[i;..j] \wedge \forall k, l\cdot\ i\le k\le l\le j \Rightarrow M\ k \le M\ l\rangle$$

The answer is $S\ L\ M\ \wedge\ \neg\ \exists K: [*T]\cdot\ S\ K\ M\ \wedge\ \#K>\#L$.

This question can be interpreted differently. It might mean that L is a sorted segment of M that cannot be extended on either end to be a longer sorted segment. In other words, that it is locally longest, rather than globally longest.

(b) the sublist must be consecutive (a segment) and nonempty.

§ Define relation S so that $S\ L\ M$ says that list L is a sorted nonempty segment of list M as follows (T is the type of item in the lists):

$$S = \langle L, M: [*T]\cdot\ \exists i, j: nat\cdot\ 0\le i<j\le \#L\le \#M \wedge L=M[i;..j] \wedge \forall k, l\cdot\ i\le k\le l\le j \Rightarrow M\ k \le M\ l\rangle$$

The answer is $S\ L\ M\ \wedge\ \neg\ \exists K: [*T]\cdot\ S\ K\ M\ \wedge\ \#K>\#L$.

(c) the sublist contains items in their order of appearance in M , but not necessarily consecutively (not necessarily a segment).

§ Define (domains are lists)

$$S = \langle L, M: [*T]\cdot\ \#L=0\ \vee\ \exists i: \square M\cdot\ L\ 0 = M\ i \wedge S\ (L[1;..\#L])\ (M[i+1;..\#M])\rangle$$

so $S\ L\ M$ means that L is a sublist of M with items in the same order but not necessarily consecutively. Then the desired expression is

$$S\ L\ M \wedge\ \neg\exists K: [*T]\cdot\ S\ K\ M \wedge \#K>\#L$$

Another solution might be

$$\exists N: [*T]\cdot\ \#N=\#L\ \wedge\ \Sigma N + \#N \le \#M\ \wedge\ \forall i: \square L\cdot\ L\ i = M((\Sigma N[0;..i+1])+i)$$

but I'm not sure.