70 Some mathematicians like to use a notation like $\exists! x \colon D \cdot P\, x$ to mean "there is a unique x in D such that $P\, x$ holds". Define $\exists!$ formally.

After trying the question, scroll down to the solution.

§ The word “unique” means “there is exactly one value of x such that $P\,x$ holds.

$(\exists x\colon D\cdot\ P\,x)\ \wedge\ \neg(\exists x, y\colon D\cdot\ x\ne y \wedge P\,x \wedge P\,y)$

or $\exists x\colon D\cdot\ P\,x\ \wedge\ \neg\exists y\colon D\cdot\ x\ne y \wedge P\,y$

or $(\exists x\colon D\cdot\ P\,x)\ \wedge\ (\forall x, y\colon D\cdot\ P\,x \wedge P\,y \Rightarrow x=y)$

or $\exists x\colon D\cdot\ P\,x\ \wedge\ \forall y\colon D\cdot\ P\,y \Rightarrow x=y$

or $\exists x\colon D\cdot\ \forall y\colon D\cdot\ P\,y = (x=y)$

or $¢(\S x\colon D\cdot\ P\,x) = 1$