67 Which step(s) in this proof is(are) wrong?

(a) $2 = 2^1 = 2^{2 \times 1/2} = (2^2)^{1/2} = 4^{1/2} = ((-2)^2)^{1/2} = (-2)^{2 \times 1/2} = (-2)^1 = -2$

(b) $4 = 4^1 = 4^{1/2+1/2} = 4^{1/2} \times 4^{1/2} = (2, -2) \times (2, -2) = 4, -4$

After trying the question, scroll down to the solution.

§(a) The step $2^{2\times 1/2} = (2^2)^{1/2}$ is wrong, and the step $((-2)^2)^{1/2} = (-2)^{2\times 1/2}$ is wrong.
Note that $4^{1/2} = 2, -2$. So $(4^{1/2})^2 = (2, -2)^2 = (-2)^2, 2^2 = 4, 4 = 4$.
But $(4^2)^{1/2} = 16^{1/2} = 4, -4 \neq 4 = 4^{2\times 1/2}$. Similarly $2^{2\times 1/2} \neq (2^2)^{1/2}$.
The law about Multiplying Exponents, in the Bunches section 11.3.3, says $x^{y\times z} : (x^y)^z$.
So the proof should say

$$2 = 2^1 = 2^{2\times 1/2} : (2^2)^{1/2} = 4^{1/2} = ((-2)^2)^{1/2} :: (-2)^{2\times 1/2} = (-2)^1 = -2$$

(b) The step $4^{1/2+1/2} = 4^{1/2} \times 4^{1/2}$ is wrong.
The law about Adding Exponents, in the Bunches section 11.3.3, says $x^{y+z} : x^y \times x^z$.
So the proof should say

$$4 = 4^1 = 4^{1/2+1/2} : 4^{1/2} \times 4^{1/2} = (2, -2) \times (2, -2) = 4, -4$$