

65 Simplify, assuming $i: \square L$

(a) $i \rightarrow L i \mid L$

(b) $(L i \rightarrow i \mid L) i$

(c) $L [0;..i] ;; [x] ;; L [i+1;..#L]$

After trying the question, scroll down to the solution.

$$(a) \quad \begin{array}{l} i \rightarrow L i \mid L \\ \S \quad L \end{array}$$

$$(b) \quad \begin{array}{l} (L i \rightarrow i \mid L) i \\ \S \quad L i \end{array}$$

Proof:

$$\begin{array}{l} (L i \rightarrow i \mid L) i \\ = \mathbf{if} \ L i = i \ \mathbf{then} \ i \ \mathbf{else} \ L i \ \mathbf{fi} \\ = \mathbf{if} \ L i = i \ \mathbf{then} \ L i \ \mathbf{else} \ L i \ \mathbf{fi} \\ = L i \end{array} \quad \begin{array}{l} \text{use law } (n \rightarrow i \mid L) m = \mathbf{if} \ n = m \ \mathbf{then} \ i \ \mathbf{else} \ L m \ \mathbf{fi} \\ \text{context} \\ \text{case idempotent} \end{array}$$

$$(c) \quad \begin{array}{l} L [0;..i] ;; [x] ;; L [i+1;..#L] \\ \S \quad i \rightarrow x \mid L \end{array}$$