

52 The compound axiom says

$$x:A, B \equiv x:A \vee x:B$$

There are 16 two-operand binary operators that could sit where \vee sits in this axiom if we just replace bunch union $(,)$ by a corresponding bunch operator. Which of the 16 two-operand binary operators correspond to useful bunch operators?

After trying the question, scroll down to the solution.

§ What is “useful”? It's not a well-defined question. I suppose any non-degenerate operator is useful (which means it uses both its operands; on the value table below, if the comment to the right mentions both A and B then the operator is not degenerate). One could argue that the degenerate operators are useful for throwing away information, or that they aren't useful because there is a perfectly good zero-operand or one-operand operator that could be used in their place.

Let $\backslash A$ be the complement of bunch A (those elements that are not in A , \backslash has precedence 2), defined formally by

$$x: \backslash A \equiv \neg x: A$$

	$\top\top$	$\top\perp$	$\perp\top$	$\perp\perp$	
	\top	\top	\top	\top	$\backslash null$ (universal bunch)
\vee	\top	\top	\top	\perp	A, B
\Leftarrow	\top	\top	\perp	\top	$A, \backslash B$
	\top	\top	\perp	\perp	A
\Rightarrow	\top	\perp	\top	\top	$\backslash A, B$
	\top	\perp	\top	\perp	B
$=$	\top	\perp	\perp	\top	$A \dot{\cup} B, \backslash A \dot{\cup} \backslash B$
\wedge	\top	\perp	\perp	\perp	$A \dot{\cap} B$
	\perp	\top	\top	\top	$\backslash A, \backslash B$
\neq	\perp	\top	\top	\perp	$A \dot{\cap} \backslash B, \backslash A \dot{\cap} B$
	\perp	\top	\perp	\top	$\backslash B$
	\perp	\top	\perp	\perp	$A \dot{\cap} \backslash B$
	\perp	\perp	\top	\top	$\backslash A$
	\perp	\perp	\top	\perp	$\backslash A \dot{\cap} B$
	\perp	\perp	\perp	\top	$\backslash A \dot{\cap} \backslash B$
	\perp	\perp	\perp	\perp	$null$