515 (input implementation) Let W be "wait for input on channel c and then read it".

(a)√ $W \;=\; t:= t\uparrow(\mathcal{T}_r + 1).\ c?$

Prove $W \;\Leftarrow\;$ **if** $\surd c$ **then** $c?$ **else** $t:= t+1.\ W$ **fi** where time is an extended natural.

(b) Now let time be a nonnegative extended real, redefine W, and reprove the refinement.

After trying the question, scroll down to the solution.

(a)√ $W = t:= t\uparrow(\mathcal{T}_{\boldsymbol{r}} + 1).\ c?$

Prove $W \Leftarrow$ **if** $\surd c$ **then** $c?$ **else** $t:= t+1.\ W$ **fi** where time is an extended natural.

§ see book Subsection 9.1.2

(b) Now let time be a nonnegative extended real, redefine W appropriately, and reprove the refinement.

§ With extended natural time, we took communication transit time to be 1 , and recursive call time to be 1 , and the computation reads the input the instant it becomes available. If time is a nonnegative extended real, we can't expect transit time to equal recursive call time, and we can't expect to read the input the instant it is available. The best we can do is to read it at the first opportunity after it is available. Let the communication transit time be a , and let the recursive call time be 1 (because that's what's in the refinement). I redefine

$$W = t:= t\uparrow(t + ceil\,(\mathcal{T}_{\boldsymbol{r}} + a - t)).\ c$$

The input check $\surd c$ is $\mathcal{T}_{\boldsymbol{r}} + a \le t$. Now I prove the refinement by cases. First

$$\begin{aligned}
& \surd c \wedge (\boldsymbol{r}:= \boldsymbol{r}+1)\\
= \quad & \mathcal{T}_{\boldsymbol{r}} + a \le t \ \wedge\ (t:= t.\ \boldsymbol{r}:= \boldsymbol{r}+1)\\
= \quad & \mathcal{T}_{\boldsymbol{r}} + a \le t \ \wedge\ (t:= t\uparrow(t + ceil\,(\mathcal{T}_{\boldsymbol{r}} + a - t)).\ \boldsymbol{r}:= \boldsymbol{r}+1)\\
\Rightarrow \quad & W
\end{aligned}$$

And the other case:

$$\begin{aligned}
& \neg\surd c \wedge (t:= t+1.\ W)\\
= \quad & \mathcal{T}_{\boldsymbol{r}} + a > t \ \wedge\ (t:= (t+1)\uparrow(t + 1 + ceil\,(\mathcal{T}_{\boldsymbol{r}} + a - t - 1)).\ \boldsymbol{r}:= \boldsymbol{r}+1)\\
= \quad & \mathcal{T}_{\boldsymbol{r}} + a > t \ \wedge\ (t:= (t+1)\uparrow(t + ceil\,(\mathcal{T}_{\boldsymbol{r}} + a - t)).\ \boldsymbol{r}:= \boldsymbol{r}+1)
\end{aligned}$$

Since $\mathcal{T}_{\boldsymbol{r}} + a > t$, then $ceil\,(\mathcal{T}_{\boldsymbol{r}} + a - t) \ge 1$, so $\uparrow$ is its second argument, and it does no harm to decrease the first argument

$$\begin{aligned}
= \quad & \mathcal{T}_{\boldsymbol{r}} + a > t \ \wedge\ (t:= t\uparrow(t + ceil\,(\mathcal{T}_{\boldsymbol{r}} + a - t)).\ \boldsymbol{r}:= \boldsymbol{r}+1)\\
\Rightarrow \quad & W
\end{aligned}$$