504 (consensus) Some parallel processes are connected in a ring. Each process has a local integer variable with an initial value. These initial values may differ, but otherwise the processes are identical. Execution of all processes must terminate in time linear in the number of processes, and in the end the values of these local variables must all be the same, and equal to one of the initial values. Write the processes.

After trying the question, scroll down to the solution.

§ Let us say there are n processes, but n is unknown to each process. Number the processes from 0 through $n{-}1$, and each process knows its process number. Each process $P\ i$ is connected to process $P(i{+}1)$ by channel $c\ i$ (all additions and subtractions are modulo n) which communicates integer values, and by channel $d\ i$ which communicates process numbers. Let the local integer variable be $x\ i$.

$$P\ i \ = \ c\ i!\ x\ i.\ \ d\ i!\ i.\ \ Q\ i$$
$$Q\ i \ = \ c(i{-}1)?.\ \ d(i{-}1)?.\ \ \textbf{if}\ d(i{-}1){=}i\ \textbf{then}\ ok$$
$$\textbf{else}\ x\ i{:=}\ (x\ i){\downarrow}(c(i{-}1)).\ \ c\ i!\ x\ i.\ \ d\ i!\ d(i{-}1).\ \ Q\ i\ \textbf{fi}$$

Now prove that for all processes, $x'i$ is the minimum of all $x\ i$, and that the execution time is n.