

- 475 An old implementer's variable $c: -1, 0, 1$ is being replaced by new implementer's variables $a, b: \text{bin}$ such that $c=-1$ is replaced by a and b both being \perp , $c=1$ is replaced by a and b both being \top , and $c=0$ is replaced by a and b being unequal.
- (a) What is the transformer?
 - (b) Use your transformer to transform $c:=0$.

After trying the question, scroll down to the solution.

(a) What is the transformer?

$$\begin{aligned} \S \quad & c := -1 \wedge \neg a \wedge \neg b \\ & \vee \quad c := 0 \wedge a \neq b \\ & \vee \quad c := 1 \wedge a \wedge b \end{aligned}$$

This is a transformer because

$$\begin{aligned} & \forall a, b. \exists c. c := -1 \wedge \neg a \wedge \neg b \vee c := 0 \wedge a \neq b \vee c := 1 \wedge a \wedge b && \text{splitting} \\ = & \forall a, b. (\exists c. c := -1 \wedge \neg a \wedge \neg b) \vee (\exists c. c := 0 \wedge a \neq b) \vee (\exists c. c := 1 \wedge a \wedge b) && \text{one-point} \\ = & \forall a, b. a \wedge b \vee \neg a \wedge \neg b \vee a \neq b && \text{laws of equality and inequality} \\ = & \forall a, b. a = b \vee \neg(a = b) && \text{excluded middle} \\ = & \forall a, b. \top && \text{identity} \\ = & \top \end{aligned}$$

(b) Use your transformer to transform $c := 0$.

$$\begin{aligned} \S \quad & \forall c. \quad c := -1 \wedge \neg a \wedge \neg b \vee c := 0 \wedge a \neq b \vee c := 1 \wedge a \wedge b \\ & \Rightarrow \exists c'. (c' := -1 \wedge \neg a' \wedge \neg b' \vee c' := 0 \wedge a' \neq b' \vee c' := 1 \wedge a' \wedge b') \wedge c' = 0 && \text{one-point} \\ = & \forall c. \quad c := -1 \wedge \neg a \wedge \neg b \vee c := 0 \wedge a \neq b \vee c := 1 \wedge a \wedge b \\ & \Rightarrow 0 := -1 \wedge \neg a' \wedge \neg b' \vee 0 := 0 \wedge a' \neq b' \vee 0 := 1 \wedge a' \wedge b' && \text{arithmetic, base, identity} \\ = & \forall c. \quad c := -1 \wedge \neg a \wedge \neg b \vee c := 0 \wedge a \neq b \vee c := 1 \wedge a \wedge b \\ & \Rightarrow a' \neq b' && \text{antidistributive law} \\ = & (\exists c. c := -1 \wedge \neg a \wedge \neg b \vee c := 0 \wedge a \neq b \vee c := 1 \wedge a \wedge b) \\ & \Rightarrow a' \neq b' && \text{above proof that we have a transformer} \\ = & \top \Rightarrow a' \neq b' && \text{identity} \\ = & a' \neq b' \\ \Leftarrow & a := \top. \quad b := \perp \end{aligned}$$