470 Let u be a binary user's variable. Let a and b be old binary implementer's variables. We replace a and b by new integer implementer's variables x and y using the convention (from the C language) that 0 stands for $\bot$ and non-zero integers stand for $\top$.

(a) What is the transformer?

(b) Transform $a := \neg a$.

(c) Transform $u := a \wedge b$.

After trying the question, scroll down to the solution.

(a) What is the transformer?

§ $a{=}(x{\ne}0) \land b{=}(y{\ne}0)$

(b) Transform $a{:=}\ \neg a$.

§ $\forall a, b\cdot\ a{=}(x{\ne}0) \land b{=}(y{\ne}0) \Rightarrow \exists a', b'\cdot\ a'{=}(x'{\ne}0) \land b'{=}(y'{\ne}0) \land (a{:=}\ \neg a)$ replace asmt

$=$ $\forall a, b\cdot\ a{=}(x{\ne}0) \land b{=}(y{\ne}0)$

$\Rightarrow \exists a', b'\cdot\ a'{=}(x'{\ne}0) \land b'{=}(y'{\ne}0) \land a'{=}\neg a \land b'{=}b \land u'{=}u$ 1-pt a' and b'

$=$ $\forall a, b\cdot\ a{=}(x{\ne}0) \land b{=}(y{\ne}0) \Rightarrow \neg a{=}(x'{\ne}0) \land b{=}(y'{\ne}0) \land u'{=}u$ 1-pt a and b

$=$ $\neg(x{\ne}0){=}(x'{\ne}0) \land (y{\ne}0){=}(y'{\ne}0) \land u'{=}u$ case idempotent

$=$ $\mathbf{if}\ x{=}0\ \mathbf{then}\ \neg(x{\ne}0){=}(x'{\ne}0) \land (y{\ne}0){=}(y'{\ne}0) \land u'{=}u$

$\mathbf{else}\ \neg(x{\ne}0){=}(x'{\ne}0) \land (y{\ne}0){=}(y'{\ne}0) \land u'{=}u\ \mathbf{fi}$ context

$=$ $\mathbf{if}\ x{=}0\ \mathbf{then}\ \neg\bot{=}(x'{\ne}0) \land (y{\ne}0){=}(y'{\ne}0) \land u'{=}u$

$\mathbf{else}\ \neg\top{=}(x'{\ne}0) \land (y{\ne}0){=}(y'{\ne}0) \land u'{=}u\ \mathbf{fi}$

$\Leftarrow$ $\mathbf{if}\ x{=}0\ \mathbf{then}\ x'{\ne}0 \land y'{=}y \land u'{=}u\ \mathbf{else}\ x'{=}0 \land y'{=}y \land u'{=}u\ \mathbf{fi}$ assignment twice

$\Leftarrow$ $\mathbf{if}\ x{=}0\ \mathbf{then}\ x{:=}\ 1\ \mathbf{else}\ x{:=}\ 0\ \mathbf{fi}$

(c) Transform $u{:=}\ a{\land}b$.

§ $\forall a, b\cdot\ a{=}(x{\ne}0) \land b{=}(y{\ne}0) \Rightarrow \exists a', b'\cdot\ a'{=}(x'{\ne}0) \land b'{=}(y'{\ne}0) \land (u{:=}\ a{\land}b)$ replace asmt

$=$ $\forall a, b\cdot\ a{=}(x{\ne}0) \land b{=}(y{\ne}0)$

$\Rightarrow \exists a', b'\cdot\ a'{=}(x'{\ne}0) \land b'{=}(y'{\ne}0) \land a'{=}a \land b'{=}b \land u'{=}a{\land}b$ 1-pt a' and b'

$=$ $\forall a, b\cdot\ a{=}(x{\ne}0) \land b{=}(y{\ne}0) \Rightarrow a{=}(x'{\ne}0) \land b{=}(y'{\ne}0) \land u'{=}a{\land}b$ 1-pt a and b

$=$ $(x{\ne}0){=}(x'{\ne}0) \land (y{\ne}0){=}(y'{\ne}0) \land u'{=}(x{\ne}0){\land}(y{\ne}0)$

$\Leftarrow$ $u{:=}\ (x{\ne}0){\land}(y{\ne}0)$