461 Let p be a user's binary variable, and let m be an implementer's natural variable. The operations allow the user to assign a value n to the implementer's variable, and to test whether the implementer's variable is a prime number.

assign n = $m := n$

check = $p :=$ *prime m*

assuming *prime* is suitably defined. If *prime* is an expensive function, and the *check* operation is more frequent than the *assign* operation, we can improve the solution by making *check* less expensive even if that makes *assign* more expensive. Using data transformation, make this improvement.

After trying the question, scroll down to the solution.

§ I replace the implementer's natural variable m by a new implementer's binary variable q . The data transformer is

$q = prime\ m$

We have to check that this is a data transformer.

$\forall q\cdot \exists m\cdot\ q = prime\ m$

$=$ $(\exists m\cdot\ \top = prime\ m) \wedge (\exists m\cdot\ \bot = prime\ m)$ generalization twice

$\Leftarrow$ $(\top = prime\ 2) \wedge (\bot = prime\ 4)$

$=$ $\top$

Using this transformer, *assign n* is transformed to

$\forall m\cdot\ q = prime\ m \Rightarrow \exists m'\cdot\ q' = prime\ m' \wedge (m := n)$ expand assignment

$=$ $\forall m\cdot\ q = prime\ m \Rightarrow \exists m'\cdot\ q' = prime\ m' \wedge m'=n \wedge p'=p$ one-point m'

$=$ $\forall m\cdot\ q = prime\ m \Rightarrow q' = prime\ n \wedge p'=p$ change of variable from m to r

$=$ $\forall r: prime\ nat\cdot\ q=r \Rightarrow q' = prime\ n \wedge p'=p$ one-point r

$=$ $q' = prime\ n \wedge p'=p$

$=$ $q := prime\ n$

Using this transformer, *check* is transformed to

$\forall m\cdot\ q = prime\ m \Rightarrow \exists m'\cdot\ q' = prime\ m' \wedge (p := prime\ m)$ expand assignment

$=$ $\forall m\cdot\ q = prime\ m \Rightarrow \exists m'\cdot\ q' = prime\ m' \wedge m'=m \wedge p' = prime\ m$ one-point m'

$=$ $\forall m\cdot\ q = prime\ m \Rightarrow q' = prime\ m \wedge p'=prime\ m$ change of variable from m to r

$=$ $\forall r: prime\ nat\cdot\ q=r \Rightarrow q'=r \wedge p'=r$ one-point r

$=$ $q'=q \wedge p'=q$

$=$ $p := q$