459 (sparse array) An array $A$: [*[*$rat$]] is said to be sparse if many of its items are $0$. We can represent such an array compactly as a list of triples $[i; j; x]$ of all nonzero items $A\ i\ j = x \neq 0$. Using this idea, find a data transformer and transform the programs

After trying the question, scroll down to the solution.

§ We replace variable $A$ with variable $L$ with a domain such that $L$ satisfies

$L$: [*[*nat*; *nat*; §$r$: *rat*· $r \neq 0$]] $\wedge$ $\neg \exists n, m$: $\Box L$· $n \neq m \wedge L\,n\,0 = L\,m\,0 \wedge L\,n\,1 = L\,m\,1$

Now the data transformer $D$ is defined as

$\forall i, j$: *nat*· $\forall x$: *rat*· $(A\,i\,j = x \neq 0) = (\exists n$: $\Box L$· $L\,n = [i; j; x])$

Now to prove that this is a data transformer:

$\forall L$· $\exists A$· $\forall i, j$: *nat*· $\forall x$: *rat*· $(A\,i\,j = x \neq 0) = (\exists n$: $\Box L$· $L\,n = [i; j; x])$

$=$ UNFINISHED

(a) $A$:= [100*[100*0]]

§ SHOULD BE $L$:= [*nil*]

(b) $x$:= $A\,i\,j$

§ $\forall A$· $(\forall i, j$: *nat*· $\forall x$: *rat*· $(A\,i\,j = x \neq 0) = (\exists n$: $\Box L$· $L\,n = [i; j; x]))$

$\Rightarrow \exists A'$· $(\forall i, j$: *nat*· $\forall x$: *rat*· $(A'\,i\,j = x \neq 0) = (\exists n$: $\Box L'$· $L'n = [i; j; x]))$

$\wedge$ ($x$:= $A\,i\,j$) expand assignment

$=$ $\forall A$· $(\forall i, j$: *nat*· $\forall x$: *rat*· $(A\,i\,j = x \neq 0) = (\exists n$: $\Box L$· $L\,n = [i; j; x]))$

$\Rightarrow \exists A'$· $(\forall i, j$: *nat*· $\forall x$: *rat*· $(A'\,i\,j = x \neq 0) = (\exists n$: $\Box L'$· $L'n = [i; j; x]))$

$\wedge$ $x' = A\,i\,j$ $\wedge$ $A'$=$A$ one-point $A'$

$=$ $\forall A$· $(\forall i, j$: *nat*· $\forall x$: *rat*· $(A\,i\,j = x \neq 0) = (\exists n$: $\Box L$· $L\,n = [i; j; x]))$

$\Rightarrow$ $(\forall i, j$: *nat*· $\forall x$: *rat*· $(A\,i\,j = x \neq 0) = (\exists n$: $\Box L'$· $L'n = [i; j; x]))$

$\wedge$ $x' = A\,i\,j$ UNFINISHED

$\Leftarrow$ $L'$=$L$

$\wedge$ **if** $\exists x$: *rat*· $\exists n$: 0,..#$L$· $L\,n = [i; j; x]$ **then** $\exists n$: $\Box L$· $L\,n = [i; j; x']$ **else** $x'$=0 **fi**

$\Leftarrow$ $k$:= 0. $Q$

where

$Q$

$=$ $L'$=$L$

$\wedge$ **if** $\exists x$: *rat*· $\exists n$: $k$,..#$L$· $L\,n = [i; j; x]$ **then** $\exists n$: $k$,..#$L$· $L\,n = [i; j; x']$ **else** $x'$=0 **fi**

$\Leftarrow$ **if** $k$=#$L$ **then** $x$:= 0

**else if** $L\,k\,0 = i \wedge L\,k\,1 = j$ **then** $x$:= $L\,k\,2$

**else** $k$:= $k$+1. $Q$ **fi fi**

(c) $A$:= $(i;j) \rightarrow x \mid A$

SHOULD BE find $n$ such that $L\,n\,0 = i \wedge L\,n\,1 = j$

**if** $x$=0 **then if** *found* **then** remove $L\,n$

**else** *ok* **fi**

**else if** *found* **then** replace $L\,n$ with $[i; j; x]$

**else** add $[i; j; x]$ **fi fi**