

- 422 (widgets) A theory of widgets is presented in the form of some new syntax and some axioms. An implementation of widgets is written.
- (a) How do we know whether the theory of widgets is consistent or inconsistent?
 - (b) How do we know whether the theory of widgets is complete or incomplete?
 - (c) How do we know whether the implementation of widgets is correct or incorrect?

After trying the question, scroll down to the solution.

- (a) How do we know whether the theory of widgets is consistent or inconsistent?
§ If we implement it, and we prove the implementation is correct, and the theories used in the implementation are consistent, then we know that the theory of widgets is consistent also. If we can prove \perp from the theory of widgets, then it is inconsistent.
- (b) How do we know whether the theory of widgets is complete or incomplete?
§ To show completeness, show the result of all combinations of the new functions and operations. To show incompleteness, implement the theory twice so that some binary expression is a theorem according to one implementation and an antitheorem according to the other.
- (c) How do we know whether the implementation of widgets is correct or incorrect?
§ To prove it correct, prove that each of the axioms of widget theory becomes a theorem using the definitions and theories of the implementation. To say the same thing differently, prove that the implementation implies the theory. To prove incorrectness, find a behavior (values for initial and final state, time, ...) that satisfy the implementation but not the theory.