421 Suppose we define **while** b **do** P **od** by fixed-point construction and induction, ignoring time.

while b **do** P **od** $=$ **if** b **then** P. **while** b **do** P **od else** ok **fi**

$\forall \sigma, \sigma' \cdot W = \textbf{if } b \textbf{ then } P.\ W \textbf{ else } ok \textbf{ fi} \implies \forall \sigma, \sigma' \cdot \textbf{while } b \textbf{ do } P \textbf{ od} \Leftarrow W$

Prove that ordinary construction and induction

if b **then** P. **while** b **do** P **od else** ok **fi** $\Leftarrow$ **while** b **do** P **od**

$\forall \sigma, \sigma' \cdot \textbf{if } b \textbf{ then } P.\ W \textbf{ else } ok \textbf{ fi} \Leftarrow W \implies \forall \sigma, \sigma' \cdot \textbf{while } b \textbf{ do } P \textbf{ od} \Leftarrow W$

are theorems. Warning: this is hard, and requires the use of limits.

§ Ordinary construction is implied immediately by fixed-point construction. What remains is to prove ordinary induction from both fixed-point construction and fixed-point induction.