418 In real variable x, consider the equation

$$P = P.\ x:=x^2$$

(a) Find 7 distinct solutions for P.

(b) Which solution does recursive construction give starting from $\top$? Is it the weakest solution?

(c) If we add a time variable, which solution does recursive construction give starting from $t'\ge t$? Is it a strongest implementable solution?

(d) Now let x be an integer variable, and redo the question.

After trying the question, scroll down to the solution.

(a) Find 7 distinct solutions for P .

§ Here are 6 solutions: $x'=0$; $x'>0$; $0<x'<1$; $x'=1$; $x'>1$; $\bot$. The disjunction of any two solutions is also a solution. For any binary expression b and solutions A and B , **if** b **then** A **else** B **fi** is also a solution.

(b) Which solution does recursive construction give starting from $\top$? Is it the weakest solution?

§ $P_0 = \top$

$P_1 = P_0.\quad x:= x^2$

$= \top.\quad x' = x^2$

$= \exists x''\cdot\ \top \land\ x' = x''^2$

$= \exists x''\cdot\ x' = x''^2$ I don't have a law to quote here, but here's my reasoning. If x'' is any real value, its square is nonnegative.

$= x'\ge 0$

It gives $x'\ge 0$, which is the weakest solution.

(c) If we add a time variable, which solution does recursive construction give starting from $t'\ge t$? Is it a strongest implementable solution?

§ It gives $t'=\infty \land x'\ge 0$, which is not a strongest implementable solution because $t'=\infty \land x'=0$ is a stronger implementable solution.

(d) Now let x be an integer variable, and redo the question.

§ The solutions are: $x'=0$; $x'=1$; $\bot$; the disjunction of any two solutions is also a solution; for any binary expression b and solutions A and B , **if** b **then** A **else** B **fi** is also a solution. Starting from $\top$ we get $x'=0 \lor x'=1$ which is the weakest solution. Starting from $t'\ge t$ we get $t'=\infty \land (x'=0 \lor x'=1)$ which is not a strongest implementable solution because $t'=\infty \land x'=0$ is a stronger implementable solution.