411 Let x be an integer variable.

(a) Using the recursive time measure, add time and then find the strongest implementable specification S that you can find for which

$$\begin{aligned} S \;\Leftarrow\;& \textbf{if } x{=}0 \textbf{ then } ok \\ & \textbf{else if } x{>}0 \textbf{ then } x{:=}\, x{-}1.\; S \\ & \qquad \textbf{else } x' \ge 0.\; S \textbf{ fi fi} \end{aligned}$$

Assume that $x' \ge 0$ takes no time.

(b) What do we get from recursive construction starting with $t' \ge t$?

After trying the question, scroll down to the solution.

(a) Using the recursive time measure, add time and then find the strongest implementable specification S that you can find for which

$$S \Leftarrow \textbf{if } x=0 \textbf{ then } ok \textbf{ else if } x>0 \textbf{ then } x:=x-1.\ S \textbf{ else } x' \ge 0.\ S \textbf{ fi fi}$$

Assume that $x' \ge 0$ takes no time.

§ Adding time,

$$S \Leftarrow \textbf{if } x=0 \textbf{ then } ok \textbf{ else if } x>0 \textbf{ then } x:=x-1.\ t:=t+1.\ S \textbf{ else } x' \ge 0 \wedge t'=t.\ t:=t+1.\ S \textbf{ fi fi}$$

the strongest implementable solution for S is

$$x'=0 \wedge \textbf{if } x\ge 0 \textbf{ then } t' = t+x \textbf{ else } t' \ge t+1 \textbf{ fi}$$

If we replace $x' \ge 0 \wedge t'=t$ by $x:=c$ where c is an arbitrary natural number, then we can prove

$$x'=0 \wedge \textbf{if } x\ge 0 \textbf{ then } t' = t+x \textbf{ else } t'= t+1+c \textbf{ fi}$$

(b) What do we get from recursive construction starting with $t' \ge t$?

§

$$\begin{aligned} S_n \ =\ & 0\le x<n \wedge x'=0 \wedge t'=t+x \\ \vee\ & \neg\ 0\le x<n \wedge t'\ge t+n \\ \vee\ & x<0 \wedge x'=0 \wedge t+1\le t'<t+n \\ S_\infty \ =\ & 0\le x \wedge x'=0 \wedge t'=t+x \\ \vee\ & x<0 \wedge t'=\infty \\ \vee\ & x<0 \wedge x'=0 \wedge t+1\le t'<\infty \end{aligned}$$

S_∞ is a solution to the given implication, but not as strong as the solution shown in part (a). It is interesting to note that if the given implication were an equation, then S_∞ would not be a solution (fixed-point), but the solution of part (a) would still be a solution. $\Updownarrow n\cdot S_n$ is the same as S_∞ .