

402 (decimal-point numbers) Using recursive data definition, define the bunch of all decimal-point numbers. These are the rationals that can be expressed as a finite string of decimal digits containing a decimal point. Note: you are defining a bunch of numbers, not a bunch of texts.

After trying the question, scroll down to the solution.

§ If we're just allowed digits and a decimal point, they're non-negative. Let's call it dec . We can define it as

$$dec = nat / 10^{nat}$$

but that doesn't use recursive data definition that we were asked to use. The following axioms do.

$$nat, dec/10: dec \quad \text{construction}$$

$$nat, B/10: B \Rightarrow dec: B \quad \text{induction}$$

or

$$dec = nat, dec/10 \quad \text{fixed-point construction}$$

$$B = nat, B/10 \Rightarrow dec: B \quad \text{fixed-point induction}$$

But these axioms use nat , which already gives us construction and induction. So maybe we should try for a definition that doesn't use nat .

$$0, dec+1, dec/10: dec \quad \text{construction}$$

$$0, B+1, B/10: B \Rightarrow dec: B \quad \text{induction}$$

or

$$dec = 0, dec+1, dec/10 \quad \text{fixed-point construction}$$

$$B = 0, B+1, B/10 \Rightarrow dec: B \quad \text{fixed-point induction}$$