393(a) Considering E as the unknown, find three solutions of $E, E{+}1 = nat$.
(b) Now add the induction axiom $B, B{+}1 = nat \Rightarrow E{:}\, B$. What is E ?

After trying the question, scroll down to the solution.

(a) Considering E as the unknown, find three solutions of $E, E+1 = nat$.

§ Here are 4 solutions:

$2\times nat$

nat

$1, 2\times nat$

$0, 2\times nat + 1$

(b) Now add the induction axiom $B, B+1 = nat \Rightarrow E: B$. What is E ?

§ We now have inconsistency, so we can prove anything. From the first solution above and the induction axiom we have $E: 2\times nat$. From the last solution above and the induction axiom we have $E: 0, 2\times nat + 1$. From a distributive bunch axiom we have

$E: (2\times nat)‘(0, 2\times nat + 1)$

which says $E: 0$ and this contradicts the axiom of part (a).