389 Let n be a natural number. From the fixed-point equation

$$ply = n, ply+ply$$

we obtain a sequence of bunches ply_i by recursive construction.

(a) State ply_i in English, and formally (no proof needed).

(b) What is ply_∞ ?

(c) Is ply_∞ a solution? If so, is it the only solution?

After trying the question, scroll down to the solution.

(a) State ply_i in English, and formally (no proof needed).

§ For $i=0$ it is empty, and for $i>0$ it is multiples of n, the multipliers being from 1 up to (including) 2^{i-1}. Formally,

$$ply_0 = null$$
$$ply_{i+1} = (1+(0,..2^i))\times n$$

(b) What is ply_∞ ?

§ $$ply_\infty = (1+nat)\times n$$

(c) Is ply_∞ a solution? If so, is it the only solution?

§ It is a solution. Proof:

	$n, ply_\infty + ply_\infty$	replace ply_∞ twice
$=$	$n, (1+nat)\times n + (1+nat)\times n$	factor out $\times n$
$=$	$n, ((1+nat) + (1+nat))\times n$	insert $1\times$ and add $1+1=2$ and $nat+nat=nat$
$=$	$1\times n, (2+nat)\times n$	factor out $\times n$
$=$	$(1, (2+nat))\times n$	combine 1 and $2+nat$
$=$	$(1+nat)\times n$	
$=$	ply_∞	

Another solution is nat .