384 What elements can be proven in P from the axiom $P = 1, x, -P, P+P, P \times P$? Prove $2 \times x^2 - 1 : P$

After trying the question, scroll down to the solution.

§ All polynomials in one variable x (so really that's monomials) with integer coefficients are in P.

$1:P$ and $P+P:P$ therefore $1+1:P$ therefore $2:P$.

$x:P$ and $P\times P:P$ therefore $x\times x:P$ therefore $x^2:P$.

$2:P$ and $x^2:P$ and $P\times P:P$ therefore $2\times x^2:P$.

$2\times x^2:P$ and $-P:P$ therefore $-2\times x^2:P$.

$-2\times x^2:P$ and $1:P$ and $P+P:P$ therefore $-2\times x^2+1:P$.

$-2\times x^2+1:P$ and $-P:P$ therefore $-(-2\times x^2+1):P$ therefore $2\times x^2-1:P$.